Testing for the martingale hypothesis in Asian stock prices: a wild bootstrap approach

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Abstract

This paper tests for the martingale (or random walk) hypothesis in the stock prices of a group of Asian countries. The selected countries include well-developed markets (Hong Kong and Japan) as well as emerging markets (Korea, Taiwan and Thailand). This paper adopts a joint variance ratio test based on the wild bootstrap technique. This test has been found to exhibit superior small sample properties to those of the conventional tests. In addition, the test for the martingale hypothesis is conducted with moving-subsample windows, to control the sensitivity of the results to the particular sample periods. The overall result indicates that the stock prices of Hong Kong, Japan and Korea follow the martingale, suggesting that these stock markets have been efficient.

Keywords: Martingale hypothesis, Stock Market Efficiency, Variance Ratio Test, Wild bootstrap

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1. Introduction

Testing for the martingale hypothesis in financial time series has been a subject of much attention in the empirical finance literature. While the random walk requires its increments to be identically and independently distributed, the martingale allows for uncorrelated increments with a general form of heteroskedasticity. Hence, the martingale is a relaxed version of the random walk, and is better suited for financial time series which shows a strong degree of (conditional) heteroskedasticity. If the time series of an asset price follows a martingale, then its return is purely non-predictable and investors are unable to make abnormal returns consistently over time. Hence, the question as to whether an assets price follows a martingale sequence has strong implications to the market efficiency. There have been numerous empirical studies which applied the VR tests to testing for the martingale property in stock prices. Notable examples include Lo and MacKinlay (1988), Kim et al. (1991), Frennberg and Hansson (1993) for western countries; Felix Ayadi and Pyun (1994), Huang (1995), Malliaropulos and Priestley (1999), Chang and Ting (2000) and Ryoo and Smith (2002) for Asian countries.

The most popular statistical tool to test for the martingale hypothesis is the variance ratio (VR) test. Since originally proposed by Lo and MacKinlay (1988), a number of alternative VR tests have been proposed. They are based on the property that, if the time series of an asset return is a purely random, the variance of k-period return is k times the variance of one-period return. Hence, the VR, defined as the ratio of $1/k$ times the variance of k-period return to that of one-period return, should be equal to one for all values of k. The VR tests can be classified into individual and joint versions, depending on the structure of the underlying null hypothesis. The former tests whether the VR is equal to one for a particular holding period (k), while the latter tests whether a set of VR's over a number of holding periods are jointly equal to one. The former includes the original VR tests proposed by Lo and MacKinlay (1988) and non-parametric tests of Wright (2000), while the latter includes the multiple VR test of Chow and Denning (1993), the Wald test of Richardson and Smith (1991), and the subsampling test of Whang and Kim (2003).

Although the past empirical studies provide rather mixed results, the general conclusion is to reject the martingale hypothesis of stock prices. This is contrary to the conventional belief that stock markets are efficient. It should be noted, however, there are a few problems associated with the past studies which may have contributed to these mixed results. First, most VR tests adopted in past studies are asymptotic tests which may have deficient small samples properties. For example, Lo and MacKinlay (1989) found that the sampling distribution of their VR statistic can be far different from the standard normal distribution in small sample, with severe bias and right skew. This can lead to size distortions or low power in small samples, resulting in misleading statistical inference. Second, many past empirical studies used the individual VR tests. However, the martingale hypothesis requires that the VR's for all of holding periods be equal to one, and the test should be conducted jointly over a number of holding periods. Conducting separate individual tests over a multiple of k values may provide misleading inferential outcomes as it tends to over-reject the joint null hypothesis (see Savin 1984). In addition, the size of the test can also be undermined by serial correlations among individual VR test statistics induced by overlapping observations, as pointed out by Richardson and Smith (1991). On this basis, recent empirical studies such as Fong et al. (1997) and Yilmaz (2003) preferred to use the joint tests.

The foregoing arguments suggest that it is desirable to employ a joint test with controlled size which does not rely on asymptotic approximations. In this paper, we employ a joint test based on the bootstrap method. The bootstrap (Efron, 1979) is a resampling method which approximates the sampling distribution of a test statistic. It has been applied widely in many areas of time series econometrics and statistics, and often found to perform better than asymptotic approximations in small samples (see, Li and Maddala, 1996; and Berkowitz and Kilian, 2000). To implement the bootstrap to time series data with unknown forms of (conditional) heteroskedasticty, the wild bootstrap of Wu (1986) is adopted in this paper. This is because the conventional bootstrap based on random resampling is misspecified when the data is heteroskedastic. In a recent study, Kim (2004) has found that the wild bootstrap test should be preferred to the existing joint VR tests such as the Chow-Denning, Wald, and Whang-Kim tests. This is based on the Monte Carlo simulation findings that the wild bootstrap test shows desirable size properties over a wide range of data generation processes. The other joint tests are found to suffer from severe size distortions, especially when the data contains strong conditional heteroskedasticity. The wild bootstrap test has excellent power especially when the sample size is reasonably large, for a wide range of alternative hypotheses including long memory time series.

In this paper, attention is paid to Asian stock prices, which are attracting increasing attention from both practitioners and academics. The region represents dynamic economies with high growth and increasing volume in the stock market. In addition, its major players consist of the well-developed and emerging markets. In this paper, we consider five Asian countries including Hong Kong, Japan, Korea, Thailand and Taiwan, using the weekly data form 1975 to 2002. Another distinct feature of this paper from the past studies is that it conducts the VR test using a moving sub-sample window. This is to accommodate the dynamic nature of the Asian stock markets, and to obtain inferential outcomes robust to possible structural changes to the market. This technique has been adopted by Yilmaz (2003) who investigated the martingale behavior of exchange rates.

The main finding of the paper is that the stock prices of Hong Kong, Japan and Korea follow the martingale, while those of Taiwan and Thailand show non-martingale behavior. This suggests that Hong Kong, Japanese and Korea stock markets are efficient in the sense that the stock returns are purely non-predictable. The structure of the paper is as follows. In the next section, the wild bootstrap test is presented. Section 3 presents the details about the data, and Section 4 presents the empirical results. The conclusion is drawn in Section 5.

2. The wild bootstrap test

Let x_t be an asset return at time t, where t =1,…, T. It is assumed that x_t is a realization of the underlying stochastic process X_t, which follows a martingale difference sequence.

This means that X_t's are serially uncorrelated, but are allowed to be conditionally or unconditionally heteroskedastic. A formal definition of the martingale difference time series can be found in Lo and MacKinlay (1988; Assumption H*). Following the formula given by Wright (2000), the VR statistic be written as

$$VR(x;k) = \left\{ \frac{1}{Tk} \sum_{t=k}^{T} (x_t + x_{t-1} + \ldots + x_{t-k+1} - k\hat{\mu})^2 \right\} \div \left\{ \frac{1}{T} \sum_{t=1}^{T} (x_t - \hat{\mu})^2 \right\}, \qquad (1)$$

where $\hat{\mu} = T^{-1} \sum_{t=1}^{T} x_t$. This is an estimator for the unknown population VR, denoted as $V(k)$, which is the ratio of $1/k$ times the variance of k-period return to the variance of one-period return.

The null hypothesis, $V(k_i) = 1$ for $i = 1, \ldots, l$ against the alternative hypothesis that $V(k_i) \neq 1$ for some i. Given the observed data $(x_1, .., x_T)$, the statistic of interest is

$$MV = \sqrt{T} \max_{1 \le i \le l} \left| (VR(x;k_i) - 1) \right|. \qquad (2)$$

This is the form of the statistic also considered by Whang and Kim (2003). The sampling distribution of (2) is unknown and the wild bootstrap can provide an approximation to the sampling distribution of the joint VR statistic given in (5). It can be conducted in three stages as below:

(i) Form a bootstrap sample of T observations $x_t^* = \eta_t x_t$ (t= 1,…, T) where η_t is a random sequence with zero mean and unit variance.

(ii) Calculate MV^*, which is the MV_3 statistic given in (4) obtained from the bootstrap sample generated in stage (i).

(iii) Repeat (i) and (ii) sufficiently many, say m, times to form a bootstrap distribution of the test statistic $\left\{MV^{*j}\right\}_{j=1}^{m}$

The bootstrap distribution $\left\{MV^{*j}\right\}_{j=1}^{m}$ is used to approximate the sampling distribution of the MV statistic given in (5). The 100α% critical value of the test can be obtained as the (1-α)th percentile of $\left\{MV^{*j}\right\}_{j=1}^{m}$, while the p-value of the test can be estimated as the proportion of $\left\{MV^{*j}\right\}_{j=1}^{m}$ greater than the MV statistic calculated from the original data.

In implementing the wild bootstrap, a specific form of η_t should be chosen. It is random weights which ensure that $E(x_t^* \mid x_t) = 0$ and $Var(x_t^* \mid x_t) = x_t^2$. Note that the conditions E(η_t) = 0 and E(η_t^2) = 1 are essential for the validity of the wild bootstrap. In this paper, we consider three alternative choices of η_t. The first is the standard normal distribution; the second is the two-point distribution proposed by Mammen (1993), which can be written as $\eta_t = -(5^{1/2}-1)/2$ with probability $p = (5^{1/2}+1)/(2\times 5^{1/2})$ and $\eta_t = (5^{1/2}-1)/2$ with probability 1-p; while the third is the empirical distribution of standardized returns adopted by Malliaropulos and Priestley (1999). Further details on the wild bootstrap can be found in Liu (1988) and Mammen (1993).

The use of the bootstrap method for the VR test is not new. A review of related studies appears in the survey article by Ruiz and Pascual (2002). In particular, Pan et al. (1997) used the bootstrap based on random resampling to test for the martingale hypothesis in daily currency future prices. As mentioned above, random resampling is not suitable for time series with conditional heteroskedasticity. According to simulation results reported by Whang and Kim (2003), the bootstrap test based on random resampling exhibits inferior small sample properties to those of their sub-sampling test. Malliaropulos and Priestley (1999) considered a wild bootstrap version of the Lo-MacKinlay test. However, their wild bootstrap test is an individual test and its small sample properties are completely unknown.

3. Data and computation details

We consider the stock prices of five Asian countries: Hong Kong, Japan, Korean, Taiwan and Thailand. The data is weekly from 7 May 1975 to 3 July 2002, comprising 1418 observations. Following most of the past studies, we use weekly data to circumvent the bias caused by nonsynchronous trading. The data points are associated with Wednesday data points, but Tuesday data points are used when Wednesday is a holiday. According to the descriptive statistics and ARCH test, all time series are highly non-normal with a strong degree of conditional heteroskedasticity. The details of descriptive statistics are not reported for simpliity.

For the wild bootstrap test, the number of bootstrap replications m is set to 1000 for all cases. The test conducted with moving sub-sample windows of 260, 520 and 780 observations, which represent moving windows of 5, 10 and 15 years respectively. The values of k_i's considered are (2, 4), (2, 4, 8) and (2, 4, 8, 16) corresponding to sub-sample windows of 260, 520 and 780. For each case, the p-values of the wild bootstrap test are plotted against the last time points of moving sub-sample windows. If the p-value of the test is less than 5% (10%), the martingale hypothesis is rejected at 5% (10%) level of significance for that particular sub-sample period.

4. Empirical results

Figure 1 plots the p-values of the wild bootstrap test for all countries considered against time, when the moving sub-sample window is 260. The levels of significance 0.05 and 0.10 are also plotted. Japan shows all p-values larger than 0.10 and 0.05, indicating that the martingale hypothesis cannot be rejected at 10% level of significance for all time periods. Korea shows a similar pattern, except that the p-values in the late eighties are less than 0.05 and 0.10. Overall, this indicates that the Korean stock price has been found to follow a martingale sequence. As for the other markets, the p-values can frequently be less than 0.10, indicating that these stock prices do not follow martingale sequences overall. Hence, when the 5 year horizon is considered, only Japanese and Korean stock prices found to follow the martingale.

Figure 2 plots the graphs with moving sub-sample window of 520. The Japanese and Korean stock prices show strong evidence of the martingale, as all p-values are greater than 0.10. For Hong Kong, the p-values are less than 0.10 in most cases until 1994. After 1994, however, all p-values are greater than 0.10 indicating that the martingale cannot be rejected at 10% level of significance after the mid 1990's. Thailand shows all p-values less than 0.10, while Taiwan stock prices show similar pattern except for the brief period around 2000. Hence, when the time horizon of 10 years is considered, the stock prices of Hong Kong, Japan and Korea are found to follow martingales, while those of Taiwan and Thailand are found to be otherwise.

Figure 3 present the graphs with moving sub-sample window of 780. The Japanese and Hong Kong stock prices show all p-values larger than 0.10, indicating that the martingale hypothesis cannot be rejected at 10% level of significance for all periods. The Korean stock price show similar pattern except for brief period in late 1990's. As for the Taiwanese and Thailand stock prices, the martingale is rejected for all time periods since all p-values are less than 10%. Hence, over the 15 year period, only the stock prices of Hong Kong, Japan and Korea are found to follow the martingale sequence.

The empirical results provide evidence that the stock markets of Hong Kong, Japan and Korea are efficient, in the sense that the stock prices are non-predictable as the stock prices follow martingale sequences. The evidences are particularly strong for the Korean and Japanese markets. There is no evidence to suggest that Taiwanese and Thailand markets are efficient. It would be interesting to compare the results obtained in this paper with those of past studies. The evidence for the efficiency of Korean stock market is in agreement with the findings of Felix Ayadi and Pyun (1994) and Ryoo and Smith (2002). On the other hand, Huang (1995) concluded that the stock prices of Korea and Hong Kong do not follow random walk, which is different from the conclusion of this study.

The evidence against the stock market efficiency of Taiwan is in agreement with Chang and Ting (2000). The current study is distinct from the past studies in that it used a joint VR test based on small sample approximations, which is conducted with moving sub-sample windows. It seems highly likely that the conclusion made in this paper is based on more convincing and reliable inferential outcomes.

5. Conclusion

This paper empirically evaluated whether selected Asian stock prices (Hong Kong, Japan, Korea, Taiwan and Thailand) follow the martingale sequences. This exercise is interesting because the martingale property has strong implications to the stock market efficiency. A new joint variance ratio test based on the wild bootstrap method is adopted in this paper. In a recent paper, Kim (2004) has found that this test shows superior small sample performances to those of the conventional joint variance ratio tests. Weekly time series data from 1975 to 2002 are considered in this paper, and the inference based on the wild bootstrap is conducted with moving sub-sample windows. This paper finds the strong evidence in support of stock market efficiency of Hong Kong, Japan and Korea, while the stock markets of Taiwan and Thailand are found to be inefficient.

Reference:

BERKOWITZ, J., and L. KILIAN (2000): "Recent Developments in Bootstrapping Time Series," *Econometric Reviews*, 19, 1-48.

CHANG, K.-P., and K. S. TING (2000): "A Variance Ratio Test of the Random Walk Hypotheses for Taiwan's Stock Market," *Applied Financial Economics*, 10, 525-532.

CHOW, K. V., and K. C. DENNING (1993): "A Simple Multiple Variance Ratio Test," *Journal of Econometrics*, 58, 385-401.

EFRON, B. (1979): "Bootstrap Methods: Another Look at the Jackknife," *Annals of Statistics*, 7, 1-26.

Felix AYADI, O., and C. S. PYUN (1994): "An Application of Variance Ratio Test to the Korean Securities Market," *Journal of Banking & Finance*, 18, 643-658.

FONG, W. M., S. K. KOH, and S. OULIARIS (1997): "Joint Variance-Ratio Tests of the Martingale Hypothesis for Exchange Rates," *Journal of Business and Economic Statistics*, 15, 51-59.

FRENNBERG, P., and B. HANSSON (1993): "Testing the Random Walk Hypothesis on Swedish Stock Prices: 1919-1990," *Journal of Banking & Finance*, 17, 175-191.

HUANG, B.-N. (1995): "Do Asian Stock Market Prices Follow Random Walks? Evidence from Variance Ratio Test," *Applied Financial Economics*, 5, 251-256.

KIM, J. H. (2004): "A Joint Variance Ratio Test Based on the Wild Bootstrap," *Working Paper, Monash University*.

KIM, M. J., C. R. NELSON, and R. STARTZ (1991): "Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence," *Review of Economic Studies*, 58, 515-528.

LI, H., and G. S. MADDALA (1996): "Bootstrapping Time Series Models," *Econometric Reviews*, 15, 115-158.

LIU, R. Y. (1988): "Bootstrap Procedures under Some Non-I.I.D. Models," *Annals of Statistics*, 167, 1696-1708.

LO, A. W., and A. C. MACKINLAY (1988): "Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test," *The Review of Financial Studies*, 1, 41-66.

LO, A. W., and A. C. MACKINLAY (1989): "The Size and Power of the Variance Ratio Test in Finite Samples," *Journal of Econometrics*, 40, 203-238.

MALLIAROPULOS, D., and R. PRIESTLEY (1999): "Mean Revision in Southeast Asian Stock Markets," *Journal of Empirical Finance*, 6, 355-384.

MAMMEN, E. (1993): "Bootstrap and Wild Bootstrap for High Dimensional Linear Models," *The Annals of Statistics*, 21, 255-285.

PAN, M.-S., K. C. CHAN, and R. C. W. FOK (1997): "Do Currency Futures Prices Follow Random Walks?," *Journal of Empirical Finance*, 4, 1-15.

RICHARDSON, M., and T. SMITH (1991): "Tests of Financial Models in the Presence of Overlapping Observations," *The Review Financial Studies*, 4, 227-254.

RUIZ, E., and L. PASCUAL (2002): "Bootstrapping Financial Time Series," *Journal of Economic Surveys*, 16, 271-300.

RYOO, H.-J., and G. SMITH (2002): "Korean Stock Prices under Price Limits: Variance Ratio Tests of Random Walks," *Applied Financial Economics*, 12, 475-484.

SAVIN, N. E. (1984): "Multiple Hypothesis Testing," in *Handbook of Econometrics*, ed. by Z. Griliches, and M. D. Intriligator. Oxford: North-Holland.

WHANG, Y.-J., and J. KIM (2003): "A Multiple Variance Ratio Test Using Subsampling," *Economics Letters*, 79, 225-230.

WRIGHT, J. H. (2000): "Alternative Variance-Ratio Tests Using Ranks and Signs," *Journal of Business & Economic Statistics*, 18, 1-9.

WU, C. F. J. (1986): "Jackknife, Bootstrap and Other Resampling Methods in Regression Analysis," *The Annals of Statistics*, 14, 1261-1295.

YILMAZ, K. (2003): "Martingale Property of Exchange Rates and Central Bank Intervention," *Journal of Business and Economic Statistics*, 21, 383-395.

Figure 1. P-values of the wild bootstrap test
(Moving window length = 260; ki's = 2 4)

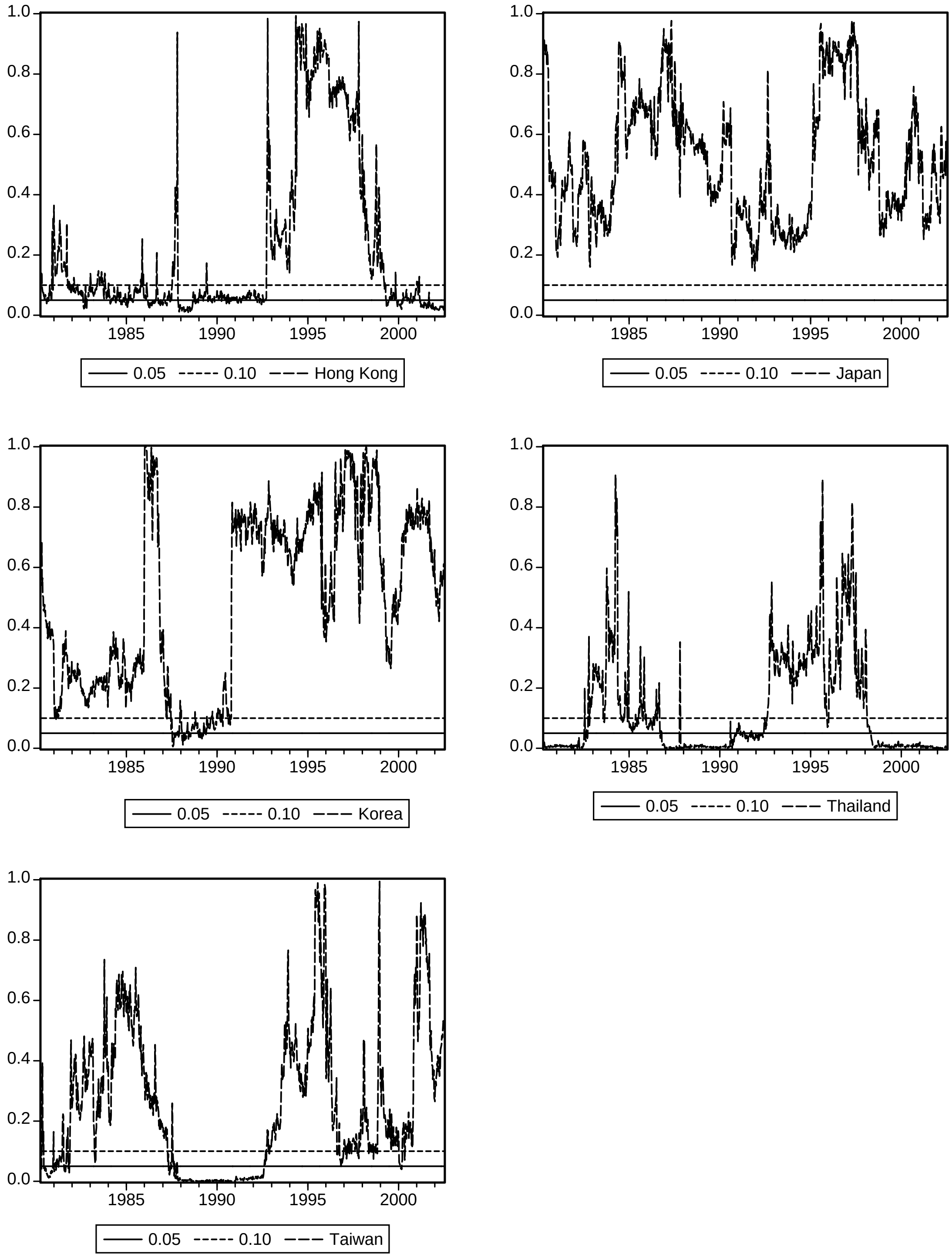

Figure 2. P-values of the wild bootstrap test
(Moving window length = 520; ki's = 2 4 8)

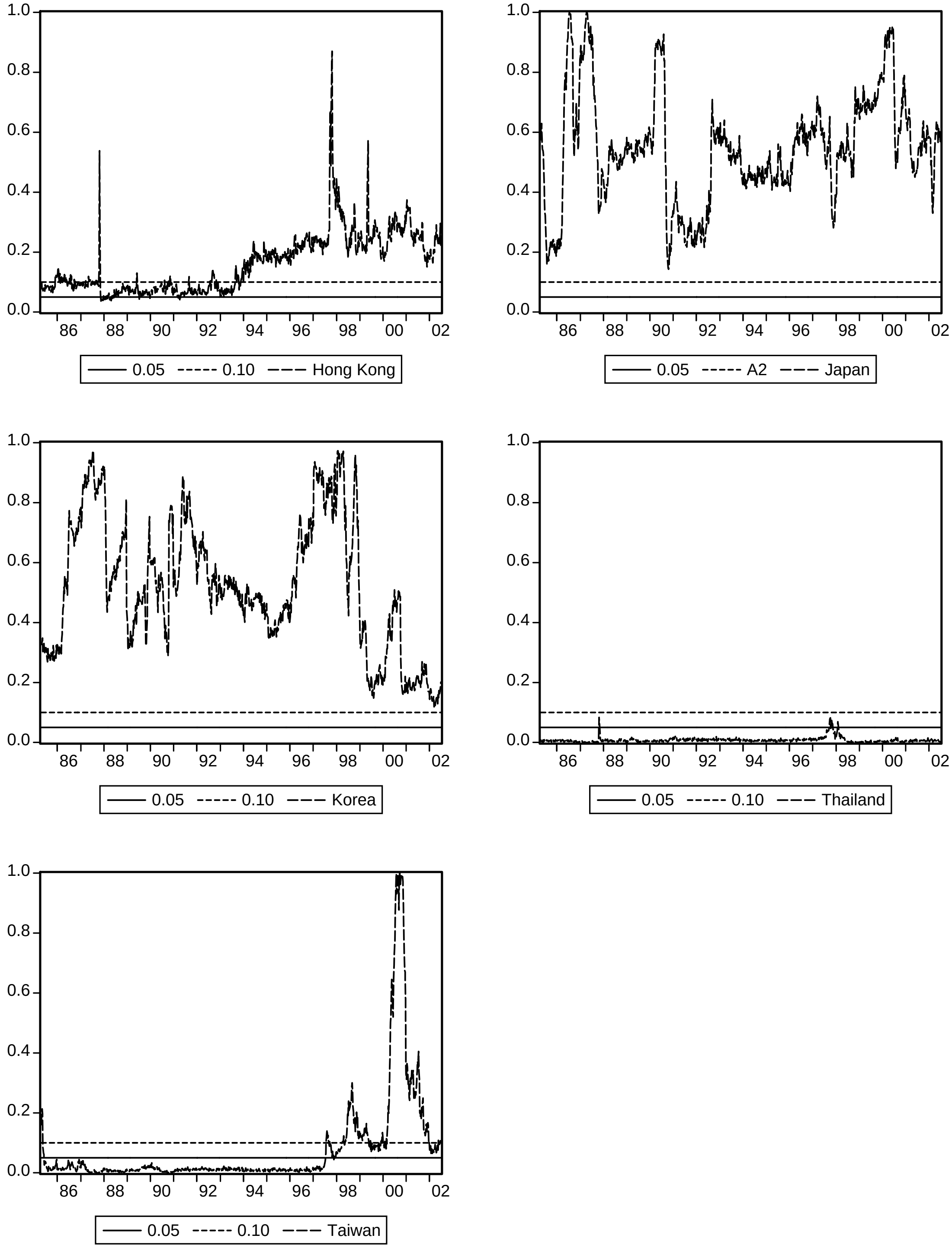

Figure 3. P-values of the wild bootstrap test
(Moving window length = 780; ki's = 2 4 8 16)