# Analysis of Deviation and Correction of Covered Interest Parity in Korean FX/Swap Market

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Abstract: Covered Interest Parity(CIP) often breaks or systemically breaks in between Won-Dollar spot rate and forward rate. This partly comes from foreign investor's arbitrage profit. We propose to modify CIP to take account this arbitrage effect and examine this effect alone explain systematic CIP violation. We also tried to find other factors such as market sentiment(measured by VIX) that might affect this violation. Basic concurrent regression using OLS are performed followed by vector autoregression to see causality relationship between variables. We find out modified CIP still has systemic violation. Furthermore, VIX and Won-dollar Spot rate has forecasting power on this violation with significance.

Keyword : Covered Interest Rate Parity, OLS, Market Sentiment, Cross-Currency Swap, Vector Autoregression

According to Baba(2008), Non-US financial Institution tend to fund US dollar through FX swap/Cross Currency Basis Swap(CCR or CRS) rather than direct borrowing. Due to Non-US financial Institution's credit status, especially for emerging market, CCR basis arise. Until the mid 2000, CCR basis was small compared to emerging market's domestic swap rate. Through the economic crisis such as Lehman, CCR basis considerably bigger , sometimes overwhelms domestic swap rate and ended up as even negative CRS rate in return to Libor.

INSERT FIGURE 1 ABOUT HERE

This is the point where Covered Interest Rate Parity(CIP) seriously violates too. Baba(2008) also suggest that credit/counter-party risk may cause this violation.

**This paper incorporate CIP and CRS together. We empirically study how seriously CIP violates and how well this violation is explained by CRS rate related times series.** CRS rate also implied arbitrage opportunity. Detailed mechanics of arbitrage relationship beyond CIP is somewhat described in Kang(2009) and Lee(2010).

In this paper, we propose CIP needed to be modified to take account this arbitrage and reconcile credit discrepancy between US/non-US, Developed/Emerging so as to explain FX basis/CIP violation.

We performed OLS to check proposed model's validity and factors in modified CIP will be further analyzed through Vector Autoregressive(VAR) model with actual market data.

We believe this study will explain how emerging market's FX and Interest rate is inter-correlated and will help to forecast one by the other. Especially who needs to predict CIP violation can be beneficial to know when the violation goes up and down.

Using similar variables to this paper, Kim(2010) tried to identify causality between several factors including FX & CRS in forecasting Korean domestic bond return. Our study focuses on CIP deviation itself, which we believe, is central factor to forecast another variable in CIP.

This paper takes following steps:

1. Propose modified CIP
2. Select variables
3. Check Stability of each variable in time-series sense(unit root test).
4. Set hypothesis for OLS

5. Perform OLS to check validity of proposed model
6. Perform VAR to find out causality and lead-leg relationship between variables
7. Interpret the result

Our study differs from Kim and Jang(2010) since we use CDS spread as our key endogenous variable to explain others and also differs from Kim(2010), which considers CDS ,in the sense that we try to explain lead-leg relationship among variables.

**Covered Interest Rate Parity(CIP)**

$$\mathbf{F = S\, e^{(r-r_f)T}}$$

$r_f$ : foreign risk-free rate

r : Domestic risk-free rate

F : Currency Forward price

S : Spot Currency

T : maturity of currency forward

Traditional CIP, which represents cost of carry for a currency, shows no-arbitrage relationship between two different currency. If someone try to exploit risk-free rate difference by locking exchange rate risk using currency forward/futures, forward/futures price will be determined not to allow arbitrage gain in equilibrium.

Baba(2008) indicates that CIP frequently get violated as credit/counter-party risk increases. For a specific emerging market's condition such as Korea, there's some arbitrage opportunity concerning cross currency basis swap described in[3] and [4].

Since currency forward market and cross-currency market are related, arbitrage opportunity in one end should be considered in other end. Otherwise, there will be arbitrage profit between currency forward and CRS.

To incorporate this violation and keep arbitrage condition considered**, this paper propose to modify CIP as** :

from $\mathbf{F} < S\, \mathbf{e^{(r-r_f)T}}$ (without convenience yield modification)

to $\mathbf{F = S\, e^{(r-r_f)T} e^{-xT}}$

**X can be interpreted as additional benefits/gain from holding local currency like convenience yield in commodity**. Potential arbitrageur will aim not only risk-free rate difference but also Cross currency basis swap arbitrage profit. According to [2] and [3], an investor who can borrow at US Libor will have arbitrage profit of (KTB-CRS) or (IRS-CRS).

X in reality can be actual cross currency swap arbitrage profit which you don't explicitly expect by entering forward market. But the amount should be considered in forward market, otherwise CRS and forward market arbitrage can be executed.

Theoretically, X can be interpreted credit risk investor should consider for risk premium.

$$\mathbf{F = S\, e^{(r^* - r^*_f)T}}$$

$$\mathbf{= S\, e^{(r - r_f)T} e^{-xT}}$$

**(r* : credit adjusted domestic rate, $r^*_f$: credit adjusted foreign rate)**

In this way one can argue that **x** may be sovereign credit risk(difference)
With all of these intuition in mind,
**Potential Candidate of X can be, anything represents arbitrage profit or anything represents sovereign credit spreads, such as,**
KTB-CRS, IRS-CRS,CDS premium, Max(KTB-CRS, IRS-CRS)
Other types of Mixture or Composite of above three
(KTB: Korean Treasury Bond, IRS: Domestic Interest Rate Swap)
So, X should amount to one of these. Then the new parity will holds or get closer than traditional parity.
Even after this arbitrage considered, there can be some sentiment factor to deviate from parity. We believe as market turbulence deepens, market participants sentiment/fear get unstable, deviation from parity also increase.
We will perform ordinary linear regression to check the validity of the proposed model in reality using historical data from Feb. 2002 and Oct. 2011.
Then we moved to vector autoregression to see causality relationship between factors in CIP.

In summary, we have 3 propositions.

1. The more arbitrage return(KTB-CRS), The more deviation either in concurrent and casual relation
2. The more credit risk(Korean CDS), The more deviation either in concurrent and casual relation
3. The more market sentiment get unstable, The more deviation either in concurrent and casual

relation

# Hypothesis

To select regression variable, we define CIP violation, modified CIP violation and express modified CIP violation in terms of CIP violation. There could be many ways of representing violation. One can represent violation as spread in forward price or spot price. Here, we represent the violation in terms of spread on domestic interest rate.

Our choice of **Y** and **X** comes from manipulation of both CIP.

$F = S * (1 + r_d * T)/(1 + r_f * T)$…traditional CIP (Simple Compounding Version)

$F/S * (1 + r_f * T) = (1 + r_d * T)$

$S/F * (1 + r_d * T)/(1 + r_f * T) = 1$

$S/F * (1 + r_d * T)/(1 + r_f * T) - 1 = 0$

**Let LHS $=$ Y**

$Y = S/F * (1 + r_d * T)/ (1 + r_f * T) - 1$ ……..(1)

**and this represents amount of CIP violation**.(Since if CIP holds, Y = 0)

For modified CIP,

$F = S * (1 + r_d * T)/[(1 + x * T) * (1 + r_f * T)]$…modified CIP

$F/S * (1 + r_f * T) = (1 + r_d * T) / (1 + x * T)$

$(1 + x * T) = S/F * (1 + r_d * T)/ (1 + r_f * T)$

$x * T = S/F * (1 + r_d * T)/ (1 + r_f * T) - 1$

Let$\mathbf{X} = \mathbf{x} * \mathbf{T}$

$\mathbf{X} = S/F * (1 + r_d * T)/ (1 + r_f * T) - 1 \ldots\ldots.(2)$

$\mathbf{X} = \mathbf{Y}$

For **X**, we used KTB-CRS which corresponds to foreign investor's arbitrage return. (Sometimes, depends on tenor, arbitrage return corresponds to IRS-CRS).

Then,

Basis 1(or CIP violation) : $Y = S/F * (1 + r_d * T)/ (1 + r_f * T) - 1$

Basis 2(Adjustment in Modified CIP) : $X = (r_d * T - CRS * T)$

Y-X represents amount of modified CIP violation (If modified CIP holds, $Y - X = 0$)

Null Hypothesis for regression is

$Y - X = a * dY + B * Z + e\ (dY = Y_t - Y_{t-1})$
$a = e = 0\ \ B = [b1, b2 \ldots], Z = [z1, z2, \ldots]'$

Null hypothesis(h0) : $a = e = 0, B = 0$ (i.e. Modified CIP violation is zero, Modified CIP holds)

In this hypothesis, we test whether Y-X is statistically zero or not. If all regression coefficient and intercept is zero, Y-X is technically white noise, which is zero in time series sense. The variance in this white noise can be explained by bid-offer bouncing or random noise. Either of them is not interest of this paper. We focus on systematic violation of CIP and modified CIP.

# Variables and Data Description

Considering liquidity of the currency forward, our choice of maturity is 1 year. For domestic and foreign interest rate, Korean Treasury Bond and US Treasury Bond is selected respectively. Forward and spot currency price is chosen as Korea Foreign Exchange close price.

rd : Korean Treasury Bond(KTB) 1y rate
rf : US Treasury(UST) 1y rate **interpolated from UST 6m and 2y**.
T : 1y
F : currency forward 1 y (denoted by KRW) i.e. 1300\ (KRW/USD)
S: spot currency(denoted by KRW)

INSERT TABLE 1 ABOUT HERE

Additional Variables Used as Z in regression.

INSERT TABLE 2 ABOUT HERE

Since our analysis are based on regression, stability of time-series variable is great concern.
X , Y, VIX have unit root at 5% significance level with augumented dicky-fuller test ,so first order differential is taken for regression variable. CDS marginally rejects unit root null, we decided to differentiate it.(See Appendix.A)

# Visual Inspection of X and Y

INSERT FIGURE 2 ABOUT HERE

Y, which represents CIP violation, shows calm movement until mid 2000. Around 2008, it shows serious downward peak(serious deviation from CIP). This is the time when Lehman Bros. bankruptcy and credit/counter party risk soars up worldwide. CIP violation is unstable AR(1) process, which mean violation is systematic.

X, represent foreign investor's arbitrage gain(KTB-CRS), which shows similar fluctuation as Y and it better be noted that despite of its spread nature, it still has unit root.

Obviously, After deducting X, corrected Basis, modified CIP violation, Y-X become neutralized in its temporal fluctuation ,reducing downward peak by about 1/3. Y-X turn out to be stable time series without unit root(See Appendix. A)

Next chart shows KRWUSD spot exchange rate, which shows similar volatility profile and peaks.

INSERT FIGURE 3 ABOUT HERE

# OLS Result

Based on hypothesis described above, we set regression equation as following,

Following is result from OLS

$$Y - X = a * dY + b1 * FX\_return + b2 * CDS\_diff + b3 * VIX\_diff + b4 * KOSPI\_return + e$$

Where $X = KTB - CRS$

INSERT TABLE 3 ABOUT HERE

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.003821 on 2490 degrees of freedom

Multiple R-squared: 0.01878, Adjusted R-squared: 0.01681

F-statistic: 9.531 on 5 and 2490 DF, p-value: $5.091e-09$

Two coefficients, both the intercept e and slope a of dY are statistically significant.
This result rejects null hypothesis, which means modified CIP violation is still systematic. It needs to be noted that all other variables do not have any significance.
With these variables, we move to vector autoregression(VAR) to see any causality between these.

# VAR Result

For VAR, we stack variable to form vector $\mathbf{V} = [\ \mathrm{Y-X\ \ dY\ \ FX\_return\ \ VIX\ \ dCDS}\ ]'$

$\mathbf{V\ =\ A1 * V_{t-1}\ +\ A2 * V_{t-2}\ +\ E}$

Following is result from VAR.

Estimation results for equation Y_X_:

$\mathrm{Y-X = (Y-X).l1 +\ dY.l1\ +\ FX_r.l1\ +\ dVIX.l1\ +\ dCDS.l1\ + (Y-X).l2\ +\ dY.l2\ +\ FX_r.l2}$
$\mathrm{+\ dVIX.l2\ +\ dCDS.l2\ +\ const}$

INSERT TABLE 4 ABOUT HERE

Lag1 coefficient of VIX has negative sign.
Positive change of VIX give negative change to change of X.

Residual standard error: 0.001132 on 2483 degrees of freedom
Multiple R-Squared: 0.9141, Adjusted R-squared: 0.9137
F-statistic: 2641 on 10 and 2483 DF, p-value: $< 2.2e-16$

Estimation results for equation dY:

$\mathrm{dY\ = (Y-X).l1 +\ dY_{.l1}\ +\ FX_r.l1\ +\ dVIX.l1\ +\ dCDS.l1\ + (Y-X).l2\ +\ dY.l2\ +\ FX_r.l2}$
$\mathrm{+\ dVIX\_.l2\ +\ dCDS.l2\ +\ const}$

INSERT TABLE 5 ABOUT HERE

Lag1 coefficient of VIX has positive sign.
Positive change of VIX give rise to change of Y.

Signif. codes: 0 ‘ $***$ ’ 0.001 ‘ $**$ ’ 0.01 ‘ $*$ ’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.001461 on 2483 degrees of freedom

Multiple R-Squared: 0.06129, Adjusted R-squared: 0.05751

F-statistic: 16.21 on 10 and 2483 DF, p-value: $< 2.2e-16$

To see effect of dX, form vector slightly different way,

$\mathbf{V} = [\ \mathrm{Y-X}\ \ \mathrm{dX}\ \ \mathrm{FX\_return}\ \ \mathrm{VIX}\ \ \mathrm{dCDS}\ ]'$

Estimation results for equation dX:

==========

$\mathrm{dX} = (\mathrm{Y-X}).\mathrm{l1} + \mathrm{dX.l1} + \mathrm{FX_r.l1} + \mathrm{dVIX.l1} + \mathrm{dCDS.l1} + (\mathrm{Y-X}).\mathrm{l2} + \mathrm{dX.l2} + \mathrm{FX_r.l2}$
$+ \mathrm{dVIX.l2} + \mathrm{dCDS.l2} + \mathrm{const}$

INSERT TABLE 6 ABOUT HERE

Lag1 coefficient of VIX has positive sign.

Positive change of VIX give rise to change of X.

Signif. Codes : 0 ‘ $***$ ’ 0.001 ‘ $**$ ’ 0.01 ‘ $*$ ’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.001107 on 2483 degrees of freedom

Multiple R-Squared: 0.07281, Adjusted R-squared: 0.06908

F-statistic: 19.5 on 10 and 2483 DF, p-value: $< 2.2e-16$

VAR result shows interesting relation. Counter-intuitively, VIX lead Y-X negatively. However, it leads Y and X positively.

# Conclusion

We modified CIP to take account known arbitrage relation to hope this remedy can holds parity back. However, modified CIP Violation, **Y-X**, still shows systematic violation, whose variance explained by dY exclusively. Even though Y and X shows high correlation they **do not cancel each other**. This may imply their sensitivities to some common variable is different.

Suppose VIX represents **market sentiment** or turbulence that may cause to violate parity. Positive change or VIX give rise to both Y and X according to VAR results. However, Y is more sensitive to VIX so that give Y-X is negatively proportional to change of VIX.

In other words, traditional violation is positively proportional to change of VIX **and modified violation is negatively proportional to VIX.**

One possible reason for this unequal sensitivities between Y & X is due to **safety asset preference** amid turbulence.

We can suppose following mechanics,

**VIX goes up(market turbulence increase) -> safety asset long tendency ->Korean Treasury Bond long-> KTB rate down -> Reduce X(=KTB-CRS) who has KTB component.**

For domestic investor, even under market turbulence, KTB is still their safety asset.

Moreover, if X increase, it will attract arbitrageur to make them take KTB long and CRS pay position. On that they because of this demand KTB rate can be reduced temporarily

Noting significance of intercept, it needs further study to find out the source of this intercept. It's also notable that 5Y Korean CDS premium doesn't have any role in regression. One reason can be maturity mismatching between 1 year currency forward and 5 year CDS spread. This is another counter-intuitive result needs to be resolved. Another possible reason, more importantly, by considering **X** as cross-currency basis, further counter-party risk index is redundant to explain deviation.

**Reference**

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[3] Kang Byung-Jin, Understanding and Modeling the KRW IRS Swap Spreads and Their Term Structure, 2009

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market and its effect on the won-dollar foreign exchange market. 2010

**Appendix A.**

# Unit Root Test of variables

**We use 5% level test for lag-3 Augmented Dickey-Fuller Test.**

Critical values for test statistics:

| | 1pct | 5pct | 10pct |
|---|---|---|---|
| tau3 | -3.96 | -3.41 | -3.12 |
| phi2 | 6.09 | 4.68 | 4.03 |
| phi3 | 8.27 | 6.25 | 5.34 |

$> \mathrm{Y.df} <- ur.df(y = Y, lags = 3, type = 'trend')$

$> \mathrm{summary(Y.df)}$

###############################################

# Augmented Dickey-Fuller Test Unit Root Test #

###############################################

Test regression trend

Call:

$\mathrm{lm(formula = z.diff \sim z.lag.1 + 1 + tt + z.diff.lag)}$

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---|---|---|---|---|
| -0.0142365 | -0.0004473 | -0.0000623 | 0.0003637 | 0.0158621 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| (Intercept) | 3.17E-05 | 5.97E-05 | 0.531 | 0.59535 | |
| z.lag.1 | -7.40E-03 | 2.83E-03 | -2.61 | 0.0091 | ** |
| tt | 7.34E-08 | 4.88E-08 | 1.503 | 0.13302 | |
| z.diff.lag1 | -1.94E-01 | 2.01E-02 | -9.662 | < 2e-16 | *** |
| z.diff.lag2 | -8.53E-02 | 2.04E-02 | -4.191 | 2.88E-05 | *** |
| z.diff.lag3 | -5.88E-02 | 2.00E-02 | -2.934 | 0.00337 | ** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.001473 on 2487 degrees of freedom

Multiple R-squared: 0.04431, Adjusted R-squared: 0.04239

F-statistic: 23.06 on 5 and 2487 DF, p-value: $< 2.2e-16$

Value of test-statistic is: -2.6105 2.3529 3.4125

**Y has unit root.**

$> \mathrm{Y.df} <- ur.df(y = X, lags = 3, type = 'trend')$

$> \mathrm{summary(Y.df)}$

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################
```

Test regression trend

Call:

lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---|---|---|---|---|
| -0.0068848 | -0.0002993 | -0.0000554 | 0.0002189 | 0.0129257 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| (Intercept) | 1.52E-05 | 4.59E-05 | 0.331 | 0.74043 | |
| z.lag.1 | -8.84E-03 | 2.96E-03 | -2.983 | 0.002884 | ** |
| tt | 7.07E-08 | 3.85E-08 | 1.837 | 0.066301 | . |
| z.diff.lag1 | -6.44E-02 | 2.00E-02 | -3.217 | 0.001311 | ** |
| z.diff.lag2 | -7.34E-02 | 2.00E-02 | -3.668 | 0.00025 | *** |
| z.diff.lag3 | -7.60E-02 | 2.00E-02 | -3.8 | 0.000148 | *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.001138 on 2487 degrees of freedom

Multiple R-squared: 0.01861, Adjusted R-squared: 0.01664

F-statistic: 9.434 on 5 and 2487 DF, p-value: $6.372e-09$

Value of test-statistic is: -2.9828 3.0567 4.4644

**X has unit root too.**

$> \ Y.df <- ur.df(y = CDS, lags = 3, type = 'trend')$
$> \ summary(Y.df)$

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend
Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
    Min      1Q  Median      3Q     Max
-126.198  -1.116  -0.287   0.702 185.489

Coefficients:
```

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| (Intercept) | 0.219341 | 0.32197 | 0.681 | 0.49578 | |
| z.lag.1 | -0.00723 | 0.002079 | -3.477 | 0.000515 | *** |
| tt | 0.000348 | 0.000234 | 1.487 | 0.137239 | |
| z.diff.lag1 | 0.214304 | 0.02 | 10.715 | < 2e-16 | *** |
| z.diff.lag2 | 0.090357 | 0.020431 | 4.423 | 1.02E-05 | *** |
| z.diff.lag3 | -0.03608 | 0.020084 | -1.796 | 0.072551 | . |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.825 on 2487 degrees of freedom

Multiple R-squared: 0.06333, Adjusted R-squared: 0.06145

F-statistic: 33.63 on 5 and 2487 DF, p-value: $< 2.2e-16$

Value of test-statistic is: -3.4772 4.0684 6.0736

$-3.477 < -3.41$

**CDS reject unit root null but marginally at 5% level.**

$> \ Y.df <- ur.df(y = VIX, lags = 3, type = 'trend')$

$> \ summary(Y.df)$

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################
Test regression trend
Call:
```

$\mathrm{lm(formula\ =\ z.diff \sim z.lag.1\ +\ 1\ +\ tt\ +\ z.diff.lag)}$

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---|---|---|---|---|
| -15.8370 | -0.6703 | -0.1537 | 0.5101 | 16.7892 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| (Intercept) | 2.06E-01 | 9.88E-02 | 2.08 | 0.03764 | * |
| z.lag.1 | -1.23E-02 | 3.68E-03 | -3.337 | 0.00086 | *** |
| tt | 5.58E-05 | 5.15E-05 | 1.084 | 0.27861 | |
| z.diff.lag1 | -1.46E-01 | 2.01E-02 | -7.278 | 4.52E-13 | *** |
| z.diff.lag2 | -9.19E-02 | 2.02E-02 | -4.544 | 5.80E-06 | *** |
| z.diff.lag3 | -5.06E-02 | 2.01E-02 | -2.522 | 0.01172 | * |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.814 on 2487 degrees of freedom

Multiple R-squared: 0.03393, Adjusted R-squared: 0.03199

F-statistic: 17.47 on 5 and 2487 DF, p-value: < 2.2e-16

Value of test-statistic is: -3.3368 3.806 5.6663

**VIX has unit root.**

$> \mathrm{Y.df} < - ur.df(y = Y\_X, lags = 3, type = 'trend')$

$> \mathrm{summary(Y.df)}$

###############################################

# Augmented Dickey-Fuller Test Unit Root Test #

###############################################

Test regression trend

Call:

$\mathrm{lm(formula\ =\ z.diff \sim z.lag.1\ +\ 1\ +\ tt\ +\ z.diff.lag)}$

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---|---|---|---|---|
| -0.0133001 | -0.0003693 | -0.0000171 | 0.0003446 | 0.0133688 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| (Intercept) | 5.00E-05 | 4.51E-05 | 1.111 | 0.266842 | |
| z.lag.1 | -2.26E-02 | 6.25E-03 | -3.616 | 0.000305 | *** |
| tt | 4.05E-08 | 3.27E-08 | 1.24 | 0.215154 | |
| z.diff.lag1 | -5.70E-01 | 2.02E-02 | -28.171 | < 2e-16 | *** |
| z.diff.lag2 | -3.85E-01 | 2.17E-02 | -17.682 | < 2e-16 | *** |
| z.diff.lag3 | -1.63E-01 | 1.98E-02 | -8.263 | 2.27E-16 | *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.001086 on 2487 degrees of freedom
Multiple R-squared: 0.2708, Adjusted R-squared: 0.2694
F-statistic: 184.7 on 5 and 2487 DF, p-value: $< 2.2e-16$

Value of test-statistic is: $-3.616\ 4.3742\ 6.5492$

**Y-X rejects unit root null**

**Figure1.**

Historical fluctuation of Currency Rate Swap(CRS) rate between USD and KRW. Rate is represented by KRW in exchange of USD Libor. This rate includes cross-currency basis, which sometimes bigger than domestic interest swap rate(IRS), so ended up as negative CRS rate. Vertical axis has scale of basis point. During Lehman related crisis during 2008 and 2009, there was negative CRS period peaked about 200 basis point. This is the time when Korean currency also exhibit serious weakness over USD.

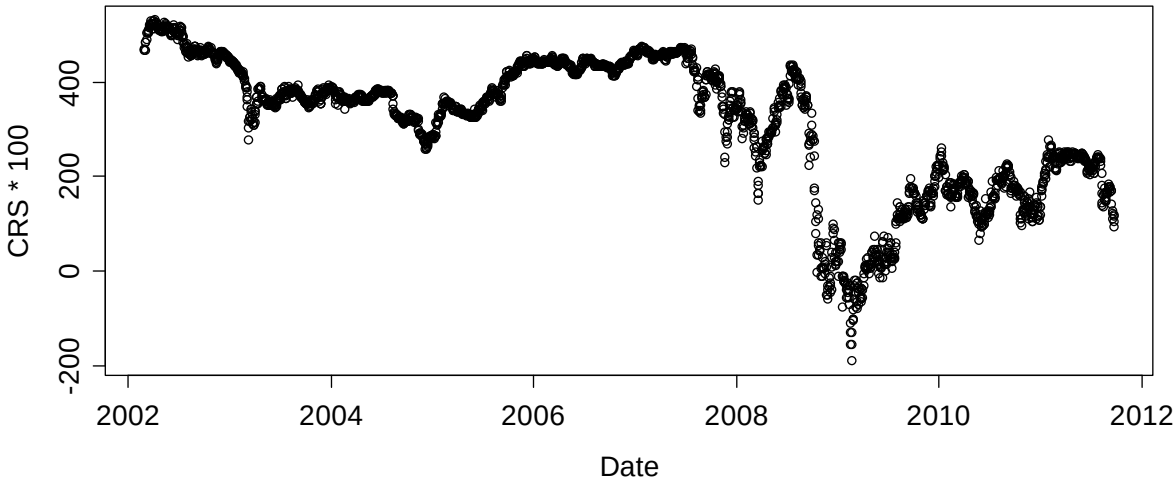


**Table 1.**

**Variables and Correspoding data used to express covered interest rate parity(CIP).** Theoretically, domestic rate(**rd**) and foreign rate(**rf**) should be risk-free rate. For Korea and US, treasury bond rate is chosen. For 1 year currency forward price and currency spot price, **F** and **S**, daily close price of Korean foreign currency exchange is selected each. For convience yield term, **X**, we chose Korean treasury bond(**KTB**) 1 year rate minus currency swap(**CRS**) 1 year rate, **KTB-CRS**. For bonds and currency forward, maturity(**T**) is chosen for 1 year.

| Expression in CIP | Data Used | Notation | Period |
|---|---|---|---|
| rd | Korean Treasury Bond(KTB) 1y rate | KTB | Feb 2002~ Oct 2011 |
| rf | US Treasury 1y rate<br>Interpolated between 6 month and 1 year | UST | |
| F | Currency forward 1y(denoted by KRW) | F | |
| S | Currency Spot | S | |
| X | Korean Treasury Bond 1 y rate minus Cross Currency Swap 1 y rate | KTB-CRS | |
| T | 1 | Time to maturity | |

**Table 2.**

Additional Variables Used as Z in regression.

| | Description | Related Notation | Comment |
|---|---|---|---|
| VIX | CBOE volatility Index | dVIX | Used in first order differential ( $\mathrm{dVIX} = \mathrm{VIX}_t - \mathrm{VIX}_{t-1}$ ) |

| | | | |
|---|---|---|---|
| KOSPI | KRX index | KOSPI_return | Used in log return ( $\mathrm{KOSPI\_return} = \log(\mathrm{KOSPI}_t) - \log(\mathrm{KOSPI}_{t-1})$ ) |
| CDS | 5 year Korean Credit default swap | dCDS | Used in first order differential ( $\mathrm{dCDS} = \mathrm{CDS}_t - \mathrm{CDS}_{t-1}$ ) |
| FX | Won-Dollar exchange rate | FX_return | Used in log return ( $\mathrm{FX\_return} = \log(\mathrm{FX}_t) - \log(\mathrm{FX}_{t-1})$ ) |

**FIGURE 2.**

Visual Inspection of CIP deviation and chosen X.

Notably, CIP deviation, Y , is unstable time series, even though it has subtraction and several division operation. X, the difference between KTB and CRS has also exhibited unstable behavior in visual estimate. Y-X shows stable behavior supported by unit root test. But it is still systematic violation according to either X or Y.

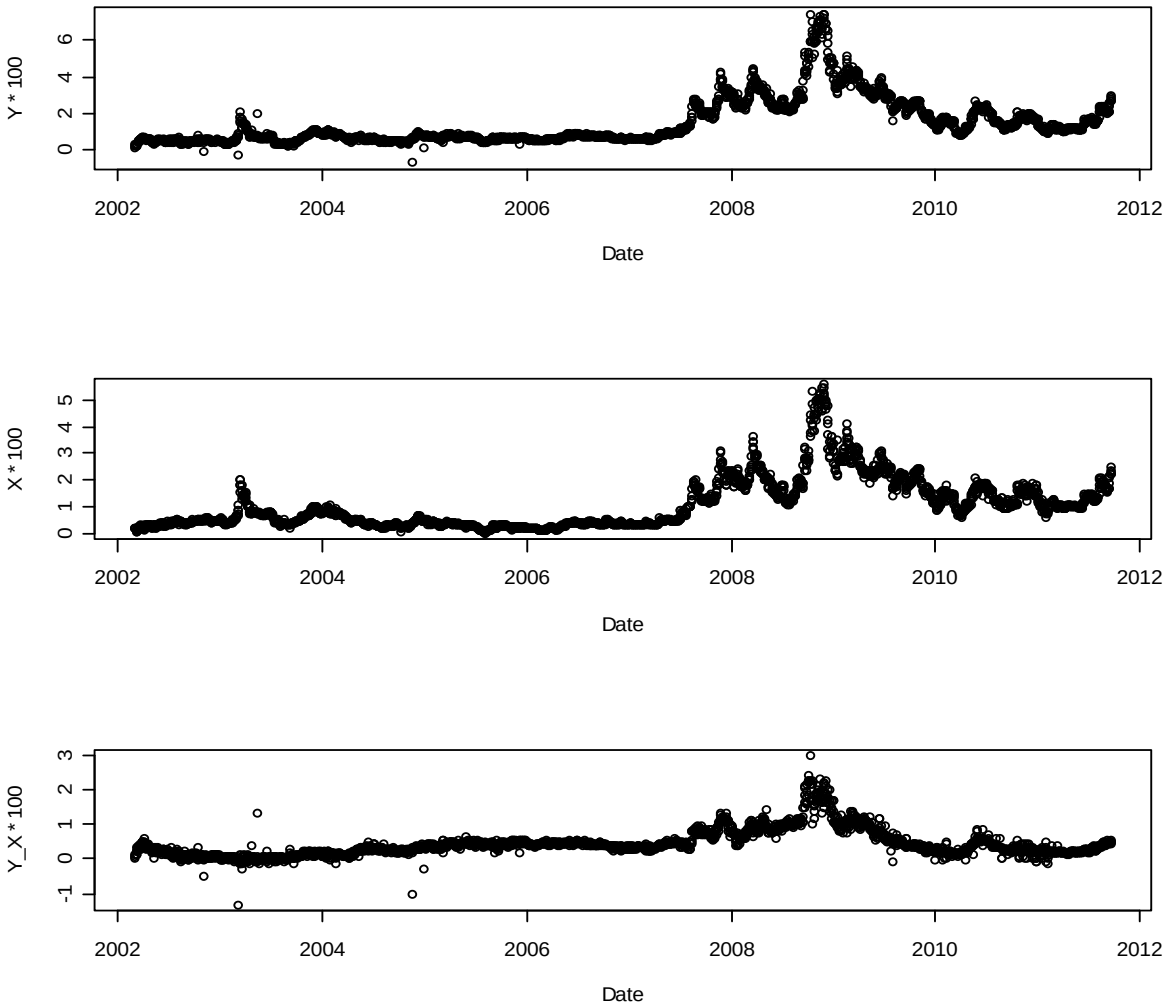


**FIGURE 3.**

Historical Fluctuation of KRW-USD FX rate(Won-Dollar exchange rate) in Seoul Foreign Currency Exchange.

It shows similar behavior with CIP deviation.

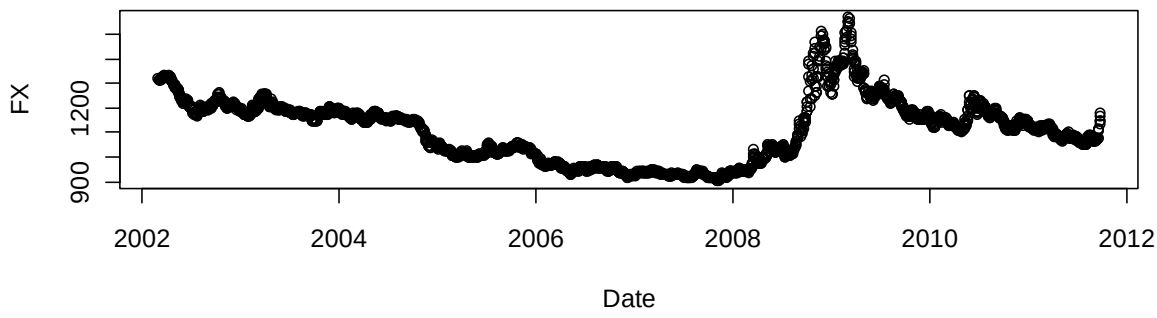


**Table 3.**

Result of Ordinary Regression of the proposed hypothesis :

$Y - X = a * dY + \mathbf{B} * \mathbf{Z} + e\ (dY = Yt - Yt - 1)$

$\mathbf{B} = [b1, b2 \ldots], \mathbf{Z} = [z1, z2, \ldots]' = [FX\_return, dCDS, dVIX, KOSPI\_return]'$

Note that, for independent variable, first order differential or log differential is taken while independent variable Y-X stay as it is.(Based on unit root test)

**Null hypothesis(h0)** : **a = e = 0, B=0** (i.e. Modified **CIP violation is zero, Modified CIP holds)**

| | Estimate Coefficients | Std. Error | **t value** | Pr(>\|t\|) | Pr(>\|t\|) |
|---|---|---|---|---|---|
| **(Intercept)** | **4.305e-03** | **7.650e-05** | **56.271** | **< 2e-16** | ******* |
| **dY** | **3.174e-01** | **5.399e-02** | **5.879** | **4.67e-09** | ******* |
| FX_return | 1.281e-02 | 1.185e-02 | 1.081 | 0.280 | |
| dCDS | 4.443e-03 | 1.112e-01 | 0.040 | 0.968 | |
| dVIX | 4.220e-05 | 4.366e-05 | 0.967 | 0.334 | |
| KOSPI_return | 3.012e-03 | 5.647e-03 | 0.533 | 0.594 | |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.003821 on 2490 degrees of freedom

Multiple R-squared: 0.01878, Adjusted R-squared: 0.01681

F-statistic: 9.531 on 5 and 2490 DF, p-value: 5.091e-09

Crucial variable to check is dY and it turned out to reject null hypothesis. Therefore, Y-X is regressible, so Modified CIP still has systematic violation.

LEAD-LAG relation analysis using Vector Autoregression(VAR) with lag 2. Variables in Z is not significant for concurrent regression, but may have forecasting power on lead-lag analysis.
For VAR, we stack variable to form vector, $\mathbf{V} = [\,\mathrm{Y-X}\ \ \mathrm{dY}\ \ \mathrm{FX\_return}\ \ \mathrm{VIX}\ \ \mathrm{dCDS}\,]'$

$$\mathbf{V} = \mathbf{A1} * \mathbf{V_{t-1}} + \mathbf{A2} * \mathbf{V_{t-2}} + \mathbf{E}$$

**Table 4.**

First row of vector equation is,

$$\mathrm{Y-X} = (\mathrm{Y-X}).\mathrm{l1} + \mathrm{dY}.\mathrm{l1} + \mathrm{FX_r}.\mathrm{l1} + \mathrm{dVIX}.\mathrm{l1} + \mathrm{dCDS}.\mathrm{l1} + (\mathrm{Y-X}).\mathrm{l2} + \mathrm{dY}.\mathrm{l2} + \mathrm{FX_r}.\mathrm{l2} + \mathrm{dVIX}.\mathrm{l2} + \mathrm{dCDS}.\mathrm{l2} + \mathrm{const}$$

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| Y_X.l1 | 5.87E-01 | 2.61E-02 | 22.531 | < 2e-16 | *** |
| dY.l1 | -7.43E-02 | 2.20E-02 | -3.383 | 0.000728 | *** |
| FX_r.l1 | 8.00E-03 | 3.48E-03 | 2.301 | 0.021496 | * |
| dVIX.l1 | -4.60E-05 | 1.32E-05 | -3.482 | 0.000506 | *** |
| dCDS.l1 | 1.25E-02 | 3.40E-02 | 0.369 | 0.71218 | |
| Y_X.l2 | 3.84E-01 | 2.61E-02 | 14.684 | < 2e-16 | *** |
| dY.l2 | -1.27E-01 | 1.70E-02 | -7.447 | 1.31E-13 | *** |
| FX_r.l2 | 6.54E-03 | 3.41E-03 | 1.915 | 0.055587 | . |
| dVIX.l2 | -4.60E-05 | 1.38E-05 | -3.325 | 0.000899 | *** |
| dCDS.l2 | 1.43E-02 | 3.26E-02 | 0.44 | 0.659946 | |
| const | 1.33E-04 | 3.44E-05 | 3.854 | 0.000119 | *** |

**Table 5.**

Second row of vector equation is,

$$\mathrm{dY} = (\mathrm{Y-X}).\mathrm{l1} + \mathrm{dY}.\mathrm{l1} + \mathrm{FX_r}.\mathrm{l1} + \mathrm{dVIX}.\mathrm{l1} + \mathrm{dCDS}.\mathrm{l1} + (\mathrm{Y-X}).\mathrm{l2} + \mathrm{dY}.\mathrm{l2} + \mathrm{FX_r}.\mathrm{l2} + \mathrm{dVIX\_}.\mathrm{l2} + \mathrm{dCDS}.\mathrm{l2} + \mathrm{const}$$

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| Y_X_.l1 | -2.04E-01 | 3.36E-02 | -6.052 | 1.65E-09 | *** |
| dY.l1 | -1.10E-01 | 2.83E-02 | -3.863 | 0.000115 | *** |
| FX_r.l1 | 8.09E-03 | 4.49E-03 | 1.801 | 0.071873 | . |
| dVIX_.l1 | 5.57E-05 | 1.71E-05 | 3.265 | 0.001109 | ** |
| dCDS.l1 | -4.88E-02 | 4.39E-02 | -1.114 | 0.265574 | |
| Y_X_.l2 | 1.84E-01 | 3.37E-02 | 5.443 | 5.76E-08 | *** |
| dY.l2 | -1.07E-01 | 2.20E-02 | -4.878 | 1.14E-06 | *** |
| FX_r.l2 | 5.96E-03 | 4.40E-03 | 1.353 | 0.176265 | |
| dVIX.l2 | -2.39E-05 | 1.78E-05 | -1.337 | 0.181222 | |
| dCDS.l2 | 4.22E-03 | 4.21E-02 | 0.1 | 0.920076 | |
| const | 1.01E-04 | 4.44E-05 | 2.272 | 0.023186 | * |

**Table 6.**

To see equation of dX, we change vector as **V**=[ Y-X  dX  FX_return  VIX  dCDS ]'
(substitute dY for dX)

$$dX = (Y-X).l1 + dX.l1 + FX_r.l1 + dVIX.l1 + dCDS.l1 + (Y-X).l2 + dX.l2 + FX_r.l2 + dVIX.l2 + dCDS.l2 + const$$

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| Y_X.l1 | 1.70E-01 | 1.87E-02 | 9.081 | < 2e-16 | *** |
| dX.l1 | -3.69E-02 | 2.15E-02 | -1.718 | 0.086001 | . |
| FX_r.l1 | -5.01E-04 | 3.40E-03 | -0.147 | 0.883045 | |
| dVIX.l1 | 9.75E-05 | 1.30E-05 | 7.521 | 7.53E-14 | *** |
| dCDS.l1 | -6.13E-02 | 3.32E-02 | -1.844 | 0.065292 | . |
| Y_X.l2 | -1.61E-01 | 1.87E-02 | -8.578 | < 2e-16 | *** |
| dX.l2 | -7.29E-02 | 2.07E-02 | -3.521 | 0.000438 | *** |
| FX_r.l2 | 3.78E-03 | 3.39E-03 | 1.117 | 0.263919 | |
| dVIX.l2 | 2.69E-05 | 1.34E-05 | 2.005 | 0.045122 | * |
| dCDS.l2 | -6.26E-03 | 3.19E-02 | -0.196 | 0.844284 | |
| const | -3.08E-05 | 3.36E-05 | -0.917 | 0.358978 | |

From table 4~6, dVIX consistently shows statistical significance on forecasting Y-X, dY, dX, which

represents Modified CIP deviation, Traditional CIP deviation, CIP correction, respectively. But its forecasting direction is not consistent.

For tradition CIP deviation and correction, dVIX lead them positively, which is intuitively sensible. Increasing market turmoil or sentiment measured by VIX may logically increase CIP deviation and its counterpart. It may increase any kind of deviation.

For modified CIP deviation, however, dVIX lead it negatively, which is counter-intuitive. One possible explanation is, modified CIP deviation has two components that has different sensitivities to market sentiment.