HEDGE PERFORMANCES OF OIL FUTURES CONTRACTS USING MIXED NORMAL DYNAMIC CONDITIONAL CORRELATION MODELS

By

Sang-Kuck Chung
Department of Economics and
Center for Research on Northeast Asian Economy
INJE University
607 obang-dong
Kimhae City, Kyungnam 621-749, Korea.
E-mail: tradcsk@inje.ac.kr. Office: +82-55-320-3124. PhoneFax: +82-55-337-2902.

Jong-Ha Weon
Department of International Trade
Inje University, Kimhae City, Kyungnam 621-749, Korea.

February 2012

Abstract

In this study, bivariate mixed normal time varying GARCH models are proposed for capturing the skewness and kurtosis detected in both conditional and unconditional return distributions. Overall, this study investigates that separating the effects of positive and negative basis on market volatility and the correlation between two markets as well as jointly incorporating the long memory effect of the basis on market returns not only provides better descriptions of the dynamic behavior of the oil prices, but also plays an important role in discovering the dynamic hedging strategies. The proposed models are compared with different standard bivariate GARCH models in terms of both the percentage variance reduction of the out-of-sample hedged portfolio and the statistical significance test of the performance improvements. All models are applied to the crude and heating oil markets and the out-of-sample evaluation is carried out by comparing the hedged portfolio variances over one to 50 days hedge horizons.

JEL Classification: C13, C32, G13
Keywords: Dynamic Hedge Performances, Regime-dependent Conditional Correlations, Bivariate Mixed Normal DCC GARCH, SPA tests

I. INTRODUCTION

Since 1970s, the oil market has been continually expanded to have now become the world's biggest commodity market. Especially, the crude oil and heating oil are indispensable for industrial and residential uses. It is therefore that issues regarding crude oil risk management are of great importance for a wide range of participants, such as crude oil producers, crude oil physical traders, and refining and oil companies.

The previous studies on oil markets centers mainly on the issues such as price discovery and market interrelationships. For example, many studies investigate the issue of price discovery, efficiency and causal relationship between oil spot and futures prices; Crowder and Hamed (1993), Moosa and Al-Loughani (1994), Peroni and McNown (1998) and Silvapulle and Moosa (1999). A number of studies also investigate linkages both in conditional return and variances between spot and futures of crude oil markets in different geographical locations; Ewing and Harter (2000), Lin and Tamvakis (2001), Lanza et al. (2006), Manera et al. (2006), Chang et al. (2009a, 2009b).

It has not been widely acknowledged in the literature that risk in the oil market can be minimized through futures hedging. Knill et al. (2006) suggest that if an oil and gas company uses futures contracts to hedge risk, they hedge only the downside risk. On the while, Daniel (2001) shows that hedging strategies can substantially reduce oil price volatility without significantly reducing returns, and with the added benefit of greater predictability and certainty. Haigh and Holt (2002) specify the time-varying hedge ratio of BEKK model of Engle and Kroner (1995) for crude oil (WTI), heating oil and unleaded gasoline futures contracts to examine volatility spillovers. Using the VECM and BEKK models, Alizadeh et al. (2004) examine the effectiveness of hedging marine bunker price fluctuations in Rotterdam, Singapore and Houston using different crude oil and petroleum futures contracts traded on the New York Mercantile Exchange (NYMEX) and the International Petroleum Exchange (IPE) in London. Jalali-Naini and Kazemi-Manesh (2006) find that the OHRs are time varying for all contracts, and higher duration contracts had higher perceived risk, a higher OHR mean, and standard deviations using weekly spot prices of WTI and futures prices of crude oil contracts one month to four months on NYMEX.

In order to estimate time varying optimal hedge ratios, two distinct approaches have been developing. One approach is basically to follow a Markov regime-switching model, which is firstly used for estimating optimal hedge ratios by Alizadeh and Nomikos (2004). Lee *et al*. (2006) and Lee and Yoder (2007a,b) propose various forms of Markov regime-switching models with allowing the hedge ratio to be both time varying and state-dependent, and find that all of these models outperform state-independent GARCH models.

The other approach is to estimate time varying optimal hedge ratios by using mixed normal GARCH models. In fact, finite mixing two or more conditionally normal and heteroskedastic components exhibit quite complex dynamics, as often observed in financial markets. For example, there may be components provided by nonstationary dynamics, another is not, but the overall mixing process might be a covariance stationary. This implies that markets are stable most of the time, but, occasionally, subject to severe and temporal fluctuations. In this regards, Alexander and Lazar (2004, 2005, 2006), and Haas *et al*. (2002, 2004) recently proposed family of univariate mixed normal GARCH processes, which has been shown to be particularly well suited for analyzing and forecasting financial volatility. However, estimating time varying optimal hedge ratios is inherently multivariate and Haas *et al*. (2006) and Bauween *et* al. (2006) thus generalize the univariate mixed normal GARCH model to the multivariate specification. As a consequence, the hedge ratios estimated from mixed normal GARCH models are both time varying and asymmetric.

This study differs from previous works in at least three ways. First, the fractionally integrated error correction term is considered in the conditional mean, developed by Granger

(1986) when fractional cointegration prevails. It is useful to know that these market prices are of the same fractional order, and a linear combination of them has a smaller fractional order. Second, as often observed in financial markets, conventional GARCH models with only one possible state for volatility cannot capture the full extent of skewness and excess kurtosis in data. Nor are they well suited for analyzing time-variation in the conditional higher moments unless it is added exogenously as, for instance, in Hansen (1994) and Harvey and Siddique (1999). Nor are they capable of differencing, via regime specific leverage effects, the mean reversion experienced during different market circumstances. To overcome these hurdles, we use bivariate mixed normal GARCH models. Third, the lagged basis is decomposed into positive and negative terms, which are used as separate explanatory variables in modeling both the conditional variances and the correlation of spot and futures returns. Following Kogan et al. (2005), Lien and Yang (2007) find that a model with asymmetric effect leads to lower risk than do the conventional models, implying that neglecting the asymmetric effect of the basis in the model could affect hedging performance. Fourth, hedge performance comparisons are compared with the use of out-of-sample point estimates and with respect to different hedge horizons, respectively. To test the statistical significance of these differences in hedging performance, Hansen and Lunde's (2005) SPA (superior predictive ability) test is applied.

The remainder of the paper is structured as follows. The next section provides the model specifications, and the hedge performance criterion and Hansen's (2001) SPA tests are introduced in the third section. In the forth section, the preliminary data analysis and the empirical results are discussed and the fifth section contains our concluding remarks.

II. THE MODELS

Let $R_{S,t} = S_t - S_{t-1}$ and $R_{F,t} = F_t - F_{t-1}$ denote spot and futures returns (in logarithms differences) at time *t*, respectively. We assume that the basis follows fractionally integrated processes, which exhibit long memory and generate very slow decay in the impulse-response weights. This implies that there exists a fractionally cointegrated relationship between spot and futures prices. Let *L* denote the lag operator such that $LZ_t = Z_{t-1}$ and Z_t be integrated of order *d*, the bivariate mean model are specified as

$$R_{S,t} = \phi_{S0} + \sum_{i=1}^{m} \phi_{Si} R_{S,t-i} + \sum_{j=1}^{m} \theta_{Fj} R_{F,t-j} + \delta_S \left[1-(1-L)^{1-d}\right](1-L)^d Z_t + \varepsilon_{St}\,, \tag{1}$$

$$R_{F,t} = \phi_{F0} + \sum_{i=1}^{m} \phi_{Fi} R_{F,t-i} + \sum_{j=1}^{m} \theta_{Sj} R_{S,t-j} + \delta_F \left[1-(1-L)^{1-d}\right](1-L)^d Z_t + \varepsilon_{Ft}\,. \tag{2}$$

where $Z_t = S_t - F_t$ is the basis, and ε_{St} and ε_{Ft} are random residual terms[1]. The traditional error correction model is thus encompassed in the fractionally integrated error correction model[2].

Allowing the conditional variance and correlation of spot and futures returns to change over time, the conditional variance-covariance matrix of residual series, $(\varepsilon_{St}\,\varepsilon_{Ft})$ is denoted by

[1] If Z_{t-1}>0, the Capesize price tends to be decreasing whereas the Panamax price tends to be increasing at time *t* in order to maintain the long-term relationship between Capesize and Panamax prices. Similarly, when Z_{t-1}<0, the Capesize price tends to be increasing and the Panamax price tends to be decreasing in the next period. This would lead one to predict that $\delta_S \le 0$ and $\delta_f \ge 0$.

[2] By expanding the operator $(1-L)^d$ in Z_t term, whereas the error correction model indicates that only the most recent price difference is relevant information variable, the fractionally integrated error correction model considers the whole history of the difference.

(3) $$Var\left(\varepsilon_t \equiv \begin{bmatrix}\varepsilon_{s,t}\\ \varepsilon_{f,t}\end{bmatrix} \mid \Omega_{t-1}\right) \equiv H_t = \begin{bmatrix} h_{St}^2 & h_{sf,t} \\ h_{fs,t} & h_{Ft}^2 \end{bmatrix},$$

where Ω_{t-1} is the information set at time *t-1*, and $h_{s,t}^2$ and $h_{f,t}^2$ are conditional variances of spot and futures returns, respectively.

Following Ball and Torous (1983), Kon (1984), Haas et al (2006) and Bauwen et al. (2006), a 2-dimensional random vectors $\varepsilon_t = (\varepsilon_{St}\, \varepsilon_{Ft})$ is said to have a 2-component multivariate finite normal mixture distribution, $\varepsilon_t \mid \Omega_{t-1} \sim MNM(p_1, p_2; \lambda_1, \lambda_2; H_{1t}, H_{2t})$, if its density is given by

(4) $$f(\varepsilon_t) = p_1\eta_1(\varepsilon_t; \lambda_1, H_{1t}) + p_2\eta_2(\varepsilon_t; \lambda_2, H_{2t})$$

where $[p_1, p_2]$ is the positive mixing law with $p_1 + p_2 = 1$, and the component densities are

(5) $$\eta_j(\varepsilon_t; \lambda_j, H_{jt}) = (2\pi)^{-M/2}\left|H_{jt}\right|^{-1/2} \exp\left\{-\frac{1}{2}(\varepsilon_t - \lambda_j)' H_{jt}^{-1}(\varepsilon_t - \lambda_j)\right\},\ M=2,\ j=1,\ 2,$$

where $\lambda_j = E(\varepsilon \mid \Omega_{t-1})$ and following McLachlan and Peel (2000) and Haas et al. (2006), the expected value and covariance matrix are thus given by

(6) $$E(\varepsilon) = p_1\lambda_1 + p_2\lambda_2,$$

(7) $$Cov(\varepsilon) = p_1H_1 + p_2H_2 + p_1(\lambda_1 - E(\varepsilon))(\lambda_1 - E(\varepsilon)) + p_2(\lambda_2 - E(\varepsilon))(\lambda_2 - E(\varepsilon)).$$

From the risk analysis perspective, the individual distributions in the mixed component represent different market states and the positive mixing law correspond the probabilities of these states. With two mixed normal densities they differentiate between 'normal' and 'abnormal' market conditions (Ball and Torous, 1983). Here, two approaches are developed to be capable of capturing the skewness and kurtosis detected in both conditional and unconditional return distributions. The first is the bivariate mixed normal CCC GARCH (henceforth BMN-CCC), which is defined as;

(8) $$H_{j,t} = \begin{bmatrix} h_{j,St}^2 & h_{j,sf,t} \\ h_{j,fs,t} & h_{j,Ft}^2 \end{bmatrix} = \begin{bmatrix} h_{j,St} & 0 \\ 0 & h_{j,Ft} \end{bmatrix} \begin{bmatrix} 1 & \rho_j \\ \rho_j & 1 \end{bmatrix} \begin{bmatrix} h_{j,St} & 0 \\ 0 & h_{j,Ft} \end{bmatrix},\ j=1,\ 2$$

where the variances $h_{j,St}^2$ and $h_{j,Ft}^2$ are assumed to follow univariate GARCH processes. Another bivariate mixed normal version of DCC GARCH model (henceforth, BMN-DCC) model is as follows;

(9) $$H_{j,t} = D_{jt}\Gamma_{jt}D_{jt} = \begin{bmatrix} h_{j,St}^2 & h_{j,sf,t} \\ h_{j,fs,t} & h_{j,Ft}^2 \end{bmatrix} = \begin{bmatrix} h_{j,St} & 0 \\ 0 & h_{j,Ft} \end{bmatrix} \Gamma_{jt} \begin{bmatrix} h_{j,St} & 0 \\ 0 & h_{j,Ft} \end{bmatrix},\ j=1,\ 2,$$

where $\Gamma_{jt} = \begin{bmatrix} \sqrt{q_{j,SSt}} & 0 \\ 0 & \sqrt{q_{j,FFt}} \end{bmatrix}^{-1} Q_{jt} \begin{bmatrix} \sqrt{q_{j,SSt}} & 0 \\ 0 & \sqrt{q_{j,FFt}} \end{bmatrix}^{-1}$, where the 2×2 symmetric positive definite matrix $Q_{jt} = [q_{j,uv,t}]$ for *u,v=S,F* is given by

(10) $$Q_{jt} = (1-\delta_{j,1}-\delta_{j,2})\overline{Q}_j + \delta_{j1}\xi_{j,t-1}\xi_{j,t-1}^{'} + \delta_{j2}Q_{j,t-1},$$

with $\xi_{jt} = \varepsilon_{j,ut} / h_{j,ut}$ for *u*=*S*, *F* and $\overline{Q}_j$ is 2×2 unconditional variance matrices of $\xi_{j,t}$

As in conditional correlation GARCH models, two approaches are developed to specify conditional correlation mixed normal GARCH models. The first specification takes the assumption that spot and futures prices are more divergent from each other as the size of basis increases. Incorporating these effects of basis, we have the following symmetric equations for conditional variances;

(11) $$h_{j,St}^2 = \omega_{j,S} + \alpha_{j,S}\varepsilon_{S,t-1}^2 + \beta_{j,S}h_{j,St-1}^2 + \pi_S Z_{t-1}^2,$$

(12) $$h_{j,Ft}^2 = \omega_{j,F} + \alpha_{j,F}\varepsilon_{F,t-1}^2 + \beta_{j,F}h_{j,Ft-1}^2 + \pi_F Z_{t-1}^2.$$

However, this model prescribes that positive and negative basis have the same effects on the return behavior. As a result, it is called the symmetric model[3].

The second type of conditional process comes directly from the work of Kogan *et al*. (2005). They suggest that the relationship between the volatility of spot or futures returns and the basis is asymmetric and presents a V-shape. To investigate this V-shape effect, Lien and Yang (2007), and Lien and Yang (2008) decompose the basis into positive and negative terms and use them as separate explanatory variables in modeling time-varying variances of spot and futures returns. Letting the conditional variance-covariance matrix of residual series be equation (3), the asymmetric conditional variances is specified by

(13) $$h_{j,St}^2 = \omega_{j,S} + \alpha_{j,S}\varepsilon_{S,t-1}^2 + \beta_{j,S}h_{j,St-1}^2 + \pi_{S,p}I(Z_{t-1}\geq 0)Z_{t-1} + \pi_{S,n}I(Z_{t-1}<0)Z_{t-1},$$

(14) $$h_{j,Ft}^2 = \omega_{j,F} + \alpha_{j,F}\varepsilon_{F,t-1}^2 + \beta_{j,F}h_{j,Ft-1}^2 + \pi_{F,p}I(Z_{t-1}\geq 0)Z_{t-1} + \pi_{F,n}I(Z_{t-1}<0)Z_{t-1},$$

where I(.) is the indicator function. The positive and negative basis at time *t-1* in equations (14)-(16) capture the asymmetric effect of the basis on the conditional variances of spot and futures returns, and on the conditional time-varying correlation between spot and futures returns[4].

Suppose that $\{y_t\}$, *t* = 1, …, *T*, are generated by the model (4)-(14). Denoting $\gamma = (\phi_1^{'}, \phi_2^{'})^{'}$, where $\phi_j^{'}$ is the row vector and $\theta = (p;\lambda;\kappa_1,\kappa_2)^{'}$, where $p = (p_1, p_2)^{'}$, $\lambda = (\lambda_1, \lambda_2)^{'}$, and $\kappa_j^{'}$ is the row vector containing all the parameters of $\kappa_j^{'} = (\omega_{j,u}, \alpha_{j,u}, \beta_{j,u}, \rho_j, \delta_{j1}, \delta_{j2})^{'}$, *u*=*S*, *F*, *j*=1, 2. Assuming that $\psi_0 = (\gamma_0^{'}, \theta_0^{'})^{'}$ is the true value of ψ, the approximate maximum likelihood estimator (MLE) maximizes the conditional log-likelihood, $L(\psi_0) = \sum_{t=1}^{T} l_t(\psi_0)$, which is given by

---

[3] We can also replace Z_{t-1}^2 with $|Z_{t-1}|$ and GARCH formulation with EGARCH in the conditional variance-covariance structure to investigate the hedging effectiveness: see the works of Zhong et al. (2004), and Gao and Wang (2005).

[4] Kogan, Livdan, and Yaron (2005) suggest that the relationship between the volatility of either spot or futures returns and the basis is positive and stronger when the basis is positive and larger. On the other hand, the relationship between the volatility and the basis is negative and stronger when the basis is negative and smaller. This implies that the coefficients, the coefficients of π_{Sp} and π_{Fp} (π_{Sn} and π_{Fn}) would be expected to be positive (negative).

(15) $$l_t(\psi_0) = \ln\left(\sum_{j=1}^{k} \rho_j \eta_j\right),$$

where k=2, η_j and ρ_j, j = 1,2 are the component densities and the mixing parameter, respectively. The estimated time-varying optimal hedge ratio $\hat{\chi}_t^*$ is calculated by using (7) with (8) and (9) for BMN-CCC and BMN-DCC, respectively.

The objective of futures hedging is to reduce the risk of adverse price changes in the spot market. Consider, at time t, an individual who has a long spot position and attempts to hedge his spot position in a corresponding futures market. Letting χ_t denote the time-varying hedge ratio at time t, then the hedged portfolio return can be expressed as $R_{p,t} = R_{s,t} - \chi_t R_{f,t}$, where for any hedge horizon η, $R_{S,t} = S_t - S_{t-\eta}$ and $R_{F,t} = F_t - F_{t-\eta}$. Based on the portfolio approach, the optimal futures position is chosen to minimize the variance of the return on this hedged portfolio of spot and futures positions, conditional on information available at time t. The variance of the hedged portfolio returns for a given optimal estimated hedge ratio $\hat{\chi}_t$ is calculated as $Var(R_{p,t}) = Var(R_{s,t} - \hat{\chi}_t R_{f,t})$, which is used to evaluate the hedging performance of each model.

Forecasts from alternative models are usually compared either with a pairwise or a joint test. In fact, it is far more sensible to compare the hedge performances of alternative models altogether, because with a pairwise comparison as in Diebold and Mariano (1995) we can only test two different models and decide which one is better. In this regard, Superior Predictive Ability (SPA) test of Hansen (2001) is applied to the statistical significance of the variance reduction[5].

We are interested to know whether any of the models, k = 1, . . . ,m, are better than the benchmark model. So we seek a test of the null hypothesis that the benchmark is not inferior to any of the alternatives. The Hansen's (2001) SPA tests are based on the following relative performance variables defined by

(16) $$d_{k,t} = \left(R_{s,t} - \hat{\chi}_0 R_{f,t+1}\right)^2 - \left(R_{s,t} - \hat{\chi}_k R_{f,t+1}\right)^2 \quad k = 1, \ldots, m,\ t = 1, \ldots, n,$$

where n is the prediction periods, $\hat{\chi}_0$ is the estimate of hedge ratio from the benchmark, and $\hat{\chi}_k$ is from the other competing models over the one, 5, 10, 20, 30, 40 and 50 days hedge horizons, respectively.

III. Preliminary Data Analysis

Daily spot and futures prices for the crude and heating oil traded on NYMEX are used. The sample period extends from January 02, 1990 through March 7, 2011 for both contracts (5311 observations for the crude oil and 5304 for the heating oil). The data for the period of January 02, 1990 to March 7, 2006 for the crude oil are used for in-sample estimation (January 02, 1990 to March 6, 2006 for the heating oil), which leaves four years for the out-of-sample forecasts. The spot and futures returns are calculated as the first difference in the logarithm of price multiplied by 100.

Table 1 gives the summary statistics for levels and returns of the spot and futures,

[5] A closely related test is the Reality Check (RC) of Sullivan *et* al. (1999) and White (2000) that applies the non-standardized test statistic, which causes the RC test to be sensitive to the inclusion of poor and irrelevant models, and to be less powerful, see Hansen (2001) and Hansen et al. (2005) for details.

respectively and for the basis. From the skewness and kurtosis of the series, it is evident that the returns are symmetric, while the dispersion of a large number of observed values is very small, which implies a leptokurtic frequency curve. This means that returns do not follow a normal distribution, instead presenting a sharp peak and fat tail distribution. This is confirmed by the Jaque-Bera test for normality. In addition, the Ljung and Box (1976) Q statistics, which evaluates the independence between the series in conditional mean, denotes that all return series investigated suffer from long-run dependencies. From the Ljung and Box (1976) Q^2 statistics, it is clear that there is a significant, nonlinear temporal dependence in the squared adjusted returns series, suggesting that the volatility of adjusted returns follows an ARCH-type model.

There are many previous studies to combine non-linearity and non-stationarity with the concept of long memory in mean or distribution within a time series framework; see Escribano (1985, 1986 and 1987), Hallman (1990), Granger and Hallman (1991) and Granger and Terasvirta (1993). These concepts are further developed by Granger (1995) who introduces the definition of extended memory, which generalizes the linear concept of integration to a nonlinear framework. However, since there exist the lack of Laws of Large Numbers and Functional Central Limit Theorems, applying these definitions is complicated both in estimation and testing. Thus, a testable definition of integration suggested by Davidson (1999) is considered for our inference analysis[6].

In particular, this study considers the modified rescaled range (R/S) statistic by Lo (1991), the KPSS statistic by Kwiatkowski, Phillips, Schmidt and Shin (1992), the S_1, S_2, S_3 and S_4 statistics by Bierens and Guo (1993) and Breitung's (2002) variance ratio statistic[7]. In table 2(A), the results of testing for short-memory are reported. All tests are conducted at a 5% level. Breitung's (2002) test is for the null hypothesis of non-stationarity and the other statistics test for the null of level stationarity. The KPSS and S_2 tests are one-sided, the other tests are two-sided. All tests were conducted by doing a distinction according to the type of null hypothesis, level or trend stationarity, and by evaluating the statistical significance of different forms of the deterministic trend in the testing regression. Nonlinear stationarity tests lead us to a clear conclusion that all variables are nonlinear integrated of order '1', NI(1), especially because of the results of Breitung's nonparametric test, even though some tests (KPSS with trend and S_4) led to conclusive results concerning its order of nonlinear integration.

In general, the single-equation analysis seems to display no significant loss of efficiency compared with a multivariate method. In order to allow for a wide range of nonlinear processes, the variance ratio statistic by Breitung (2002) is generalized to cointegrated system by using a Johansen-type multivariate framework. Briefly, the number of stochastic trend components is detected by this nonparametric cointegration test without specifying a linear vector error correction representation. As table 2(B) illustrates, the test results from Breitung's methodology provide clear evidence of cointegration at the 5% level for both crude oil and heating oil.

In order to examine whether spot and futures prices are fractionally cointegrated, Geweke and Porter-Hudak (1983)'s test is applied and its results are reported in table 2(C). The results vary little across the different values of μ under consideration. The statistics shows that all of the estimates of *d* for both samples lie between '0' and '1', suggesting possible fractional integration relationship between spot and futures prices[8].

---

[6] This definition does not require making any assumptions about the specific form of the data generation process and so it can be considered as a general enough definition of nonlinear integration.

[7] Unlike the first three testing devices, Breitung's (2002) testing procedure is a non-parametric methodology since it does not require the practitioner to specify the short-run dynamics of the process and to estimate the long-run variance.

[8] The series is covariance stationary if $d < 0.5$ and invertible if $d > -0.5$. The autocorrelation function of the series does not decline at an exponential rate, which is typical for covariance stationary ARMA processes, but rather at a

IV. Empirical Results and Out-of-Sample Hedging Performances

4.1 Dynamics in the Conditional Mean

Table 3 shows the estimation results of the mean equations with the fractionally cointegrated error correction term for the symmetric and asymmetric conditional variance specifications, respectively. It consists of eight models, in which the basic four models are considering the fractionally cointegrated error correction term in the conditional mean equation, but have the different symmetric conditional variance and correlation structures with constant correlation coefficients (SCCC), dynamic conditional correlation (SDCC), mixed normal CCC (BMN-SCCC) and mixed normal DCC (BMN-SDCC). The other four models are extended the basic model to considering the asymmetric effects in the conditional variance equation; that is, ACCC, ADCC, BMN-ACCC and BMN-ADCC. The parameters are estimated by maximizing the log-likelihood functions in (15) using CML and MAXLIK procedures in GAUSS. For both symmetric and asymmetric results, the long memory parameters, *d*, vary from 0.2800 to 0.3933 for crude oil and are all statistically significant. For the heating oil, the long memory parameters, *d*, vary from 0.0122 for the BMN-SDCC model to 0.1630 for the SDCC model and are all statistically significant.

The other symmetric results of various models for crude oil and heating oil are also summarized in the table 3. The lag orders are identified as one. The mean equations results indicate that the spot prices for both crude oil and heating oil exhibit mean reversion by responding positively to lagged futures prices and the futures market is also statistically significant affected by the lagged values of the spot prices. In particular, the coefficients of the futures lags in the spot equations are broadly larger in magnitude than the coefficients of the spot lags in the futures equations, implying that futures prices play a leading role in incorporating new information. On the while, the asymmetric estimation results are shown the similar results as in symmetric results.

From the results of table 3, the coefficients for the error correction term are all statistically significant for the spot and not for the futures both in crude and heating oil markets. Of particular interest are the signs of the estimates for the error correction term, which are all negative for the spot. The negative coefficient for the error-correction term in the spot equation indicates that when the spot price is too high relative to the futures price the current spot price will be adjusted downward to meet the long-run equilibrium relationship[9]. Additionally, the coefficient of the error correction term in the spot equation is larger in magnitude than in the futures equation, implying that the spot prices react quicker to information and reach equilibrium faster.

4.2 Time Varying Dynamics in the Conditional Variances and Correlations

Tables 4 and 5 present the results of fitting both symmetric and asymmetric models to the crude and heating oil markets data, respectively. Note that the two versions of bivariate mixed normal GARCH models have two components in the normal mixture GARCH formulation; that is, $k = 2$. Further, the off-diagonal elements of β_i's are set to zeros. Alexander and Lazar (2006) and others find that substantial estimation biases appear when $k > 2$

(much) slower hyperbolic rate. In particular, for $0 < d < 0.5$, the series possesses long memory, in the sense that the autocorrelations are not absolutely summable. For $0.5 < d < 1$, the series is nonstationary, but the limiting value of the impulse response function is equal to 0, such that shocks do not have permanent effects.

[9] On the contrary, the positive coefficient for the error-correction term in the futures equation indicates that when the futures price is too low relative to the spot price the current futures price will be adjusted upward to meet the long-run equilibrium.

and allowing for non-zero off-diagonal elements in β_i 's does not improve the empirical performance of mixed normal GARCH models they considered.

When discussing the parameter estimates from mixed normal models, the components of the mixture distributions are identified with distinctly different volatility dynamics. For example, the first higher long-term volatility component with the lower mixing weight, is stationary in the sense that $\alpha_i + \beta_i < 1$, implying that it has more weight on the reaction parameters in α_i and less weight on the persistence parameters in β_i, relative to the second lower volatility component. In particular, the second component in all mixed normal GARCH models is nonstationary, implying that the high volatility component reacts more strongly to shocks, but has a shorter memory. In all cases the lower (higher) long-term volatility component has the higher (lower) value for the mixing parameter. Also if nonzero component means are allowed for, the high long-term volatility component is associated with positive means, which are all statistically significant. The mixed normal models embed two different regimes in the spot and futures prices volatility; one is a usual volatility which occurs most of the time, and the other is an unusual volatility which occurs rarely. Thus the estimated mixing weights imply the frequencies with which these two states occurred during the sample period.

Modifying the conditional variance specification to a mixed normal GARCH process yields an obvious improvement in performance. The log likelihood values of the mixed normal GARCH models are quite large comparable with those of the usual GARCH models for both crude and heating oil markets. An astute observer will point out that a simple comparison of log likelihood values is not a vigorous way to select a specification among these different models. Because the models under examination are not all properly nested, there is no simple testing procedure to compare their degrees of goodness-of-fit. The estimates of the mixing parameters are consistent with the presence of two GARCH processes driving the conditional volatility of the crude and heating oil markets data. The component GARCH process with a larger mixing parameter estimate has a level of persistence, measured by the sum of ARCH and GARCH parameter estimates, similar to level of persistence estimated from the simple GARCH processes.

On the while, the estimation results of the variance and the correlation equations are also provided in tables 4 and 5. The estimation results for the traditional CCC and DCC models support the existence of conditional heteroskedasticity for both crude and heating oil markets prices. There is strong persistence in the variance movement. As expected, the two series are highly correlated from the coefficient ρ_i in the Tables, implying that we need the dynamic conditional correlation in the conditional variance specifications. For both symmetric and asymmetric estimation results, the correlation coefficients δ_{1i} and δ_{2i}, for i=1, 2, are all positive and statistically significant for both in crude and heating oil markets.

Tables 4 and 5 present the estimation results of conditional variance and correlation equations for the asymmetric effect and the symmetric effect. For the asymmetric effect model, the results indicate that volatility increases as the absolute value of the basis increases or decrease with one exception of the asymmetric DCC model for the heating oil and all parameters are statistically different from zero. We also observe asymmetric effects of the basis on volatility. Using conventional significance level of 5% or above, the results suggest that in all models, the positive and negative basis has effects on both spot and futures volatilities. This implies that spot and futures prices are more volatile as the basis increases irrespective of positive and negative basis. Of special interests is that the negative spread has a stronger effect than a positive spread on both spot and futures volatility in all models for the crude oil and BMN-CCC and BMN-DCC models for the heating oil, but the reverse is true in usual ACCC and ADCC models for the heating oil. It suggests that both spot and futures market volatilities are more responsive to the negative basis than to the positive basis. For instance, in the BMN-ADCC model from Table 4, a one-unit increment in the positive basis of fuel oil increases the

variance of the spot return by 0.0985 while a one-unit increment in the negative basis causes the variance of the spot return to increase by 0.1335. This implies that the effects of basis on the variances of spot and futures returns are non-monotonic. Tables 4 and 5 also summarize the estimation results for the symmetric effect model. All qualitative conclusions observed from the estimated asymmetric effect model remain valid. In particular, the absolute value of the spread promotes volatility in all models for both crude and heating oil.

4.3 Out-of-Sample Hedging Performances

Using the estimation results for all models specified in section II, the out-of-sample hedge ratios are calculated and the hedge performance of each strategy is then compared. The performance of the models may vary according to the hedge horizons, which are 1, 5, 10, 20, 30, 40 and 50 days. The percentage variance reduction from each model is calculated as the difference between the sample variance of the unhedged/OLS position and the estimated variance of each model divided by the sample variance of the unhedged/OLS position.

Table 6 provides the point estimates of out-of-sample hedging performances of competing models for the crude and heating oil markets. Based on the out-of-sample hedging performances, some regularities seem to emerge. For the crude oil, the variance reductions of asymmetric models are slightly less than the ones of the symmetric models except BMN-DCC model. For the heating oil, the BMN-ADCC provides the lowest out-of-sample variance with 62.3% variance reduction compared to the other models, but the difference is trivial. In the longer horizons, say after 5 days, the BMN-ADCC also provides the lowest variance for the heating oil. In sum, allowing the conditional correlation to be both regime dependent and time varying clearly improves the out-of-sample hedging effectiveness.

To test the statistical significance of the performance improvements of these dynamic hedging models, the SPA test is performed. Table 7 reports *p*-values for the SPA test of each model against all the others at one to 50 days hedge horizons. A few observations are as follows. The *p*-values distinctly show how all these tests could not reject the null hypothesis of SPA when the benchmark is OLS at all horizons for the crude oil. However the tests could not reject the null hypothesis when the benchmark is ADCC at shorter horizons, say, 1 to 10 days for the heating oil. As the hedge horizon is extended to longer than 10 days, the null hypothesis that the benchmark is the BMN-ADCC model could not be rejected. These results imply that there is no competing model which is significantly better than the benchmark.

Figures 1 and 2 compare the hedge ratios of the selected models for the crude and heating oil markets, respectively and show that hedge ratios from the mixed normal models are generally less volatile than those from the other competing models. Figure 3 show time-varying correlations estimated from the selected models, respectively. The correlation coefficients for the crude oil are constant in figure 3(A) (0.9099 for the SCCC and 0.9080 for the ACCC) and the mixed normal CCC models have constant correlation estimates for regime 1 (0.9555 for the BMN-SCCC and 0.9558 for the BMN-ACCC) and regime 2 (0.0.7850 for the BMN-SCCC and 0.7805 for the BMN-ACCC) in figure 3(B). The correlation results for the heating oil are similar to the ones of the crude oil. The time varying correlation estimates from the mixed normal models for regime 1 and regime 2 suggest that the correlation is slightly less volatility in regime 1 than regime 2

V. Conclusion

In this study, bivariate mixed normal models with symmetric and asymmetric time varying correlation GARCH process are proposed to estimate the time-varying hedge ratio. The standard bivariate GARCH models are also employed. The main goal is to evaluate the performance of different models in terms of their ability to reduce variances of out-of-sample

hedged portfolio and to test the statistical significance of the performance improvements using SPA tests. The out-of-sample evaluation is carried out by comparing the hedged portfolio variances from all competing models over the one to 50 days hedge horizons.

Overall, three main empirical results are in order. First, upon fitting a bivariate mixed normal GARCH models to spot and futures returns of the crude and heating oil markets, a parsimonious version of the model captures the salient features of the data rather well. Second, the OLS model significantly outperforms the other competing models at all horizons for the crude oil, but the ADCC model performs better than the other competing models at shorter horizons (from one to 10 days) for the heating oil. Third, as the hedge horizon is extended to longer than 10 days, it is clearly evident that the BMN-ADCC model is the best at the usual significance level of 5% for the heating oil.

References

Alizadeh, A., & Nomikos, N. (2004). A Markov regime switching approach for hedging stock indices. The Journal of Futures Markets, 24, 649-674.

Alexander, C. , & Lazar E. (2004). Normal mixture GARCH(1,1) applications to exchange rate modelling. ISMA Centre Discussion Papers in Finance, ISMA Centre, University of Reading, UK.

Alexander, C., & Lazar E. (2005). Asymmetries and volatility regimes in the European equity market. ICMA Centre Discussion Papers in Finance 2005-14, The Business School for Financial Markets at the University of Reading.

Alexander, C., & Lazar E. (2006). Normal mixture GARCH(1,1) applications to exchange rate modelling. Journal of Applied Econometrics, 21, 307-336.

Ball, C.A., & Torous, W.N. (1983). A simplified jump process for common stock returns. Journal of Financial and Quantitative Analysis, 18, 53-65.

Bauwens, L., Hafner, C.M., & Rombouts, J.V.K. (2006). Multivariate mixed normal conditional heteroskedasticity. CORE DISCUSSION PAPER 2006/12, Universit´e catholique de Louvain.

Bierens, H.J. and Guo, S. (1993) Testing stationarity and trend stationarity against the unit root hypothesis. *Econometric Reviews*, 12, 1-32.

Breitung, J. (2002). Nonparametric tests for unit roots and cointegration. *Journal of Econometrics*, 108, 343-363.

Chang, C.-L., M. McAleer and R. Tansuchat (2009a), Modeling conditional correlations for risk diversification in crude oil markets, to appear in *Journal of Energy Markets*.

Chang, C.-L., M. McAleer and R. Tansuchat (2009b), Forecasting volatility and spillovers in crude oil spot, forward and futures market.

Crowder, W. and Hamed, A. (1993): " A Cointegration Test for Oil Futures Market Efficiency," *The Journal of Futures Markets*, 13: 933 – 941

Daniel, J. (2001), Hedging government oil price risk, *IMF Working Paper* 01/185.

Davidson, J. (Unpublished results) When is a time-series I(0)? Evaluating the memory properties of nonlinear dynamics models. Discussion paper, Cardiff Business School, U.K. 1999.

Diebold, F.X., & Mariano, R.S. (1995). Comparing predictive accuracy. Journal of Business and Economic Statistics, 13, 253-263.

Engle, R.F., & Kroner, K.F. (1995). Multivariate simultaneous generalized ARCH. Econometric Theory, 11, 122-150.

Escribano, A. (Unpublished results) Nonlinear error-correction: the case of money demand in the U.K. (1878-1970). Working paper, Department of Economics, University of California at San Diego, USA.

Escribano, A. (1986) Identification & modelling of economic relationships in a growing economy. Ph.D. Thesis, University of California at San Diego, USA.

Escribano, A. (Unpublished results) Error-correction systems: nonlinear adjustments to linear long-run relationships. Discussion paper 8730, Department of Economics, University of Louvain, CORE, Louvain-la-Neuve, Belgium.

Ewing, B. and Harter, C. (2000): "Co-movements of Alaska North Slope and UK Brent Crude Oil Prices," *Applied Economic Letters*, 7: 55. - 558

Gao, A.H. and G.H.K. Wang, 2005, Asymmetric volatility of basis and the theory of storage. *Journal of Futures Markets*, 25, pp. 399-418.

Geweke J. and Porter-Hudak, S. (1983) The estimation and application of long memory time series models. *Journal of Time Series Analysis*, 4, 221-238.

Granger, C.W.J., 1986, Developments in the study of cointegrated economic variables. *Oxford Bulletin of Economics and Statistics*, 48, pp. 213-228.

Granger, C. W. J. and Hallman, J. (1991) Long memory processes with attractors. *Oxford Bulletin of Economics and Statistics*, 53, 11-26.
Granger, C.W.J. and Terasvirta, T. (1993) *Modelling Nonlinear Economic Relationships*. Oxford University Press, Oxford.
Haas, M., Mittnik, S., & Paolella, M.S. (2002). Mixed normal conditional heteroskedasticity. CFS Working Paper No. 2002/10.
Haas, M., Mittnik, S., & Paolella, M.S. (2004). Mixed normal conditional heteroskedasticity. Journal of Financial Econometrics, 2, 211-250.
Haas, M., Mittnik, S., & Paolella, M.S. (2006). Multivariate normal mixture GARCH. CFS Working Paper No. 2006/09.
Haigh, M.S. and M. Holt (2002), Crack spread hedging: Accounting for time-varying spillovers in the energy futures markets, *Journal of Applied Econometrics*, 17, 269-289.
Hallman, J. (1990) Nonlinear integrated series, cointegration and an application. Ph.D. Thesis, University of California at San Diego, USA.
Hansen BE. 1994. Autoregressive conditional density estimation. *International Economic Review* **35**: 705–730.
Hansen, P.R. (2001). A test for superior predictive ability. Department of EconomicsWorking Paper 2001–06, Brown University.
Hansen, P.R., & Lunde, A. (2005). A forecast comparison of volatility models: Does anything beat a GARCH(1,1)? Journal of Applied Econometrics, 20, 873-889.
Harvey CR, Siddique A. 1999. Autoregressive conditional skewness. *Journal of Financial and Quantitative Studies* **34**(4): 465–487.
Jalali-Naini, A. and M. Kazemi-Manesh (2006), Price volatility, hedging, and variable risk premium in the crude oil market, *OPEC Review*, 30(2), 55-70.
Knill, A., M. Kristina and A. Nejadmalayeri (2006), Selective hedging, information, asymmetry, and futures prices, *Journal of Business*, 79(3), 1475-1501.
Kogan, L. D. Livdan. and A. Yaron, 2005, Futures prices in a production economy with investment constraints. NBER WP 11509.
Kon, S.J. (1984). Models of stock returns: A comparison. Journal of Finance, 39, 147-165.
Kroner, K.F., & Sultan, J. (1993). Time-varying distribution and dynamic hedging with foreign currency futures. Journal of Financial and Quantitative Analysis, 28, 535-551.
Kwiatkowski, D, Phillips, P.C.B., Schmidt, P. and Shin, Y. (1992) Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54, 159-178.
Lanza, A., M. Manera and M. McAleer (2006), Modeling dynamic conditional correlations in WTI oil forward and future returns, *Finance Research Letters*, 3, 114-132.
Lee, H.T., Yoder, J.K., Mittelhammer, R.C., & McCluskey, J.J. (2006). A random coefficient autoregressive Markov regime switching model for dynamic futures hedging. The Journal of Futures Markets, 26, 103-129.
Lee, H.T., & Yoder, J.K. (2007a). A bivariate Markov regime switching GARCH approach to estimate the time varying minimum variance hedge ratio. Applied Economics, 39, 1253-1265.
Lee, H.T., & Yoder, J.K. (2007b). Optimal hedging with a regime-switching time-varying correlation GARCH model. The Journal of Futures Markets, 27, 495-516.
Lien, D. and L. Yang, 2007, Asymmetric effects of basis on dynamic futures hedging: Empirical evidence from commodity futures. *Journal of Banking and Finance*, 32, pp. 187-198.
Lien, D. and L. Yang, 2008, Hedging with Chinese Metal Futures. *Global Finance Journal*, 19, pp. 123-138.
Lin, X. S. and Tamvakis, M. (2001): "Spillover Effects in Energy Futures Markets," *Energy Economics*, 23: 43 - 56.

Lo, A.W. (1991) Long memory in stock market prices. *Econometrica*, 59, 1279-1313.
Manera, M., M. McAleer and M. Grasso (2006), Modelling time-varying conditional correlations in the volatility of Tapis oil spot and forward returns, *Applied Financial Economics*, 16, 525-533.
McLachlan, G. J. and D. Peel, Finite Mixture Models, John Wiley & Sons, New York, 2000.
Moosa, I.A. and Loughani, N.E., 1994, Unbiased and time varying risk premia in the crude oil futures market, *Energy Economics* 16, pp. 99–105.
Peroni, E. and McNown, R. 1998, Noninformative and Informative Tests of Efficiency in Three Energy Futures Markets, *Journal of Futures Markets*, 18, 939-964.
Silvapulle, P. and Moosa, I. (1999): "The Relationship between Spot and Futures Prices: Evidence from the Crude Oil Market," *The Journal of Futures Markets*, 19: 175 - 193.
Sullivan, R., Timmermann, A., & White, H. (1999). Data-snooping, technical trading rule performance, and the bootstrap. Journal of Finance, 54, 1647–1692.
White, H. (2000). A reality check for data snooping. Econometrica, 68, 1097-1126.
Zhong, M., A.F. Darrat and R. Otero, 2004, Price Discovery and Volatility Spillovers in Index Futures Markets: Some Evidence from Mexico. *Journal of Banking and Finance*, 28, pp. 3037-3054.

Table 1: Preliminary data analysis

	Mean	*Skewness*	*Kurtosis*	*Q(10)*	*Q^2(10)*	*JB*
			In Sample			
crude oil: Spot, Futures, and Basis						
S_t	3.00	1.12[0.00]	5.47[0.00]	11430.6[0.00]	11403.4[0.00]	587.9[0.00]
$R_{s,t}$	-0.02	-2.71[000]	49.19[0.00]	48.66[0.00]	72.29[0.00]	113243[0.00]
F_t	3.00	1.12[0.00]	5.54[0.00]	11444[0.00]	11414[0.00]	606.1[0.00]
$R_{F,t}$	-0.02	-2.96[0.00]	50.16[0.00]	51.41[0.00]	86.15[0.00]	118262[0.00]
B_t	-0.003	1.79[0.00]	61.73[0.00]	514.18[0.00]	492.01[0.00]	181220[0.00]
Heating oil: Spot, Futures, and Basis						
S_t	0.58	1.90[0.00]	7.01[0.00]	10769.1[0.00]	10472.6[0.00]	1598.2[0.00]
$R_{s,t}$	-0.06	-2.95[0.00]	48.21[0.00]	37.40[0.00]	60.12[0.00]	108825[0.00]
F_t	0.57	2.01[0.00]	7.68[0.00]	11218.3[0.00]	10973.7[0.00]	2001.1[0.00]
$R_{F,t}$	-0.04	-3.24[0.00]	49.15[0.00]	49.60[0.00]	43.89[0.00]	113668[0.00]
B_t	0.21	3.32[0.00]	20.35[0.00]	6473.9[0.00]	1476.9[0.00]	18075.6[0.00]
			Out of Sample			
Crude oil: Spot, Futures, and Basis						
S_t	3.58	0.21[0.00]	1.90[0.00]	39986.6[0.00]	39904.1[0.00]	234.7[0.00]
$R_{s,t}$	0.04	-0.18[0.00]	7.62[0.00]	33.71[0.00]	804.52[0.00]	3638.1[0.00]
F_t	3.58	0.22[0.00]	1.89[0.00]	40012.8[0.00]	40002.3[0.00]	236.6[0.00]
$R_{F,t}$	0.04	-0.10[0.00]	7.08[0.00]	28.08[0.002]	1023.29[0.00]	2823.9[0.00]
B_t	0.02	-3.10[0.00]	124.9[0.00]	1178.2[0.00]	979.2[0.00]	257820[0.00]
Heating oil: Spot, Futures, and Basis						
S_t	1.20	1.08[0.00]	3.70[0.00]	39724.4[0.00]	39369.3[0.00]	877.5[0.00]
$R_{s,t}$	0.04	-1.46[0.00]	38.96[0.00]	55.19[0.00]	1636.5[0.00]	219501[0.00]
F_t	1.20	1.06[0.00]	3.61[0.00]	39803.0[0.00]	39411.3[0.00]	829.5[0.00]
$R_{F,t}$	0.05	-0.54[0.00]	7.72[0.00]	12.57[0.25]	183.47[0.00]	3963.9[0.00]
B_t	-0.43	9.68[0.00]	197.62[0.00]	14859.2[0.00]	3007.7[0.00]	6449145[0.00]

Note: The sample is daily observations from January 2, 1990 to March 7, 2011 for both crude and heating oil contracts. R and B denote return and basis of each of the spot and futures prices, respectively. The Q(10) and Q^2(10) are the Ljung-Box statistics for tenth-order serial correlation in the residuals and squared residuals, respectively. *JB* means the Jaque-Bera test for normality. The critical values at the 0.05 significance level is 18.31 for 10 degrees of freedom. The standard errors for skewness and kurtosis are $(6/T)^{0.5}$ =0.033 and $(24/T)^{0.5}$ =0.067, respectively for both crude and heating oil, where T is the number of observations. The critical values of the rejection of null hypothesis of normal distribution for skewness, kurtosis, and Jarque-Bera statistics at 5% level are $\pm$ 0.091, $\pm$ 0.182, and 5.99, respectively.

Table 2: Unit Root Tests

Variables		Crude Oil			Heating Oil		
Sub-samples		Spot	Future	Basis	Spot	Future	Basis
Panel A: Linear and Nonlinear Unit root Tests							
KPSS	level	205.06(2)	84.09(4)	13.07(3)	99.51 (3)	80.61(4)	3.18(4)
	trend	18.14(3)	11.66(6)	2.71(4)	18.43(3)	12.73(5)	0.31(5)
R/S		13.74(4)	15.61(3)	6.95(4)	13.71(4)	15.42(3)	2.83(5)
s_1		2149.02	2251.98	1.08	2049.46	2080.62	0.37
s_2		5085.18	5303.16	1.08	5043.26	5244.52	0.37
s_3		131.81	227.55	1.51	115.66	143.67	0.41
s_4		136.82(5)	12.75(5)	1.60(3)	113.53(4)	146.50(2)	0.42(2)
Breitung	const	0.0772	0.0791	0.0093	0.0749	0.0759	0.0026
	trend	0.0136	0.0153	0.0024	0.0138	0.0143	0.0003
Results		NI(1)		NI(1)	NI(1)		NI(1)
Panel B:Breitung's Cointegration Test of Spot and Futures							
Rank r_0		Λ_q^a	Λ_q^b		Λ_q^a	Λ_q^b	
0		903.72	1294.78		4643.38	4825.12	
1		12.44	57.16		13.11	68.12	
Panel C: GPH Fractional Cointegration Test of Spot and Futures							
μ		$\mu=.55$		$\mu=.575$		$\mu=.60$	
Hypothesis		d H_0:d=1 H_0:d=0		d H_0:d=1 H_0:d=0		d H_0:d=1 H_0:d=0	
Crude Oil		0.72 -4.29 11.60		0.77 -3.78 13.37		0.79 -3.83 14.92	
Heating Oil		0.41 -7.30 5.15		0.42 -8.78 6.48		0.46 -9.55 8.27	

Note: All tests were conducted at a 5% level. In Panel (a), the critical values are referred to Lo (1991) for the modified rescaled range (R/S) statistic, Kwiatkowski, Phillips, Schmidt and Shin (1992) for the KPSS statistic, Bierens and Guo (1993) for the S_1, S_2, S_3 and S_4 statistics, and Breitung's (2002) for the variance ratio statistic. In Panel (B), the multivariate variables under study are capesize and Panamax prices (n=2). The null hypothesis is that the process can be decomposed into a q-dimensional vector of stochastic trend components and a ($n-q$)-dimensional vector of transitory components. The number q of stochastic trend components is related to the cointegration rank r by $q = n-r$. The hypothesis $r = r_0$ is rejected if the test statistic Λ_q exceeds the respective critical value. The 5% critical values correspond to both the demeaned (Λ_q^a) and detrended cases (Λ_q^b). From Panel (c), the sample size for the GPH test is given by $n=T^{\mu}$. There exist two alternatives; One is that the hypothesis H_0: d=1 is tested against the one-sided alternative. The other one is that the hypothesis H_0: d=0 is tested against the two-sided alternative. Critical values at the 5% level are -1.84 for μ =.55, -1.82 for μ =.575, and -1.75 for μ =.60, respectively.

Table 3: Estimation Results of Mean Equations

Models	SCCC	SDCC	BMN-CCC	BMN-DCC	ACCC	ADCC	BMN-ACCC	BMN-ADCC
				Crude Oil				
d	0.3856 (0.0520)	0.3472 (0.0406)	0.2850 (0.0305)	0.3882 (0.0474)	0.3792 (0.0515)	0.3456 (0.0414)	0.2800 (0.0305)	0.3933 (0.0485)
ϕ_{S0}	0.0630 (0.0255)	0.0201 (0.0261)	0.0211 (0.0296)	0.0584 (0.0240)	0.0476 (0.0261)	0.0055 (0.0263)	0.0190 (0.0296)	0.0570 (0.0240)
ϕ_{S1}	-0.2264 (0.0266)	-0.1716 (0.0268)	-0.1095 (0.0312)	-0.1291 (0.0301)	-0.2222 (0.0265)	-0.1777 (0.0268)	-0.1093 (0.0318)	-0.1258 (0.0302)
θ_{F1}	0.2622 (0.0317)	0.2021 (0.0303)	0.1075 (0.0358)	0.1287 (0.0327)	0.2593 (0.0318)	0.2103 (0.0306)	0.1063 (0.0364)	0.1245 (0.0327)
δ_S	-0.2030 (0.0359)	-0.1702 (0.0336)	-0.1254 (0.0279)	-0.1353 (0.0330)	-0.1999 (0.0343)	-0.1686 (0.0334)	-0.1232 (0.0275)	-0.1292 (0.0330)
ϕ_{F0}	0.0579 (0.0211)	0.0256 (0.0227)	0.0194 (0.0253)	0.0667 (0.0216)	0.0453 (0.0222)	0.0131 (0.0227)	0.0173 (0.0253)	0.0655 (0.0216)
ϕ_{F1}	-0.0562 (0.0252)	-0.1436 (0.0246)	-0.0881 (0.0306)	-0.1843 (0.0283)	-0.0551 (0.0253)	-0.1416 (0.0248)	-0.0860 (0.0314)	-0.1872 (0.0284)
θ_{S1}	0.0964 (0.0184)	0.1763 (0.0174)	0.1027 (0.0253)	0.1930 (0.0229)	0.0949 (0.0185)	0.1743 (0.0174)	0.0998 (0.026)	0.1953 (0.0230)
δ_F	-0.0055 (0.0253)	0.0302 (0.0276)	0.0112 (0.0233)	0.0464 (0.0281)	-0.0055 (0.0238)	0.0318 (0.0273)	0.0104 (0.0232)	0.0499 (0.0283)
				Heating Oil				
d	0.1428 (0.0316)	0.1630 (0.0307)	0.0122 (0.0197)	0.0364 (0.0197)	0.1444 (0.0319)	0.1627 (0.0308)	0.0150 (0.0195)	0.0377 (0.0196)
ϕ_{S0}	-0.0901 (0.0262)	-0.0424 (0.0267)	0.0052 (0.0316)	0.0031 (0.0261)	-0.0803 (0.0267)	-0.0418 (0.0267)	0.0039 (0.0315)	-0.0007 (0.0258)
ϕ_{S1}	-0.2908 (0.0227)	-0.1982 (0.0253)	-0.1101 (0.0317)	-0.1751 (0.0272)	-0.2918 (0.0230)	-0.1978 (0.0255)	-0.1109 (0.0317)	-0.1726 (0.0271)
θ_{F1}	0.2938 (0.0239)	0.2118 (0.0275)	0.1113 (0.0319)	0.1825 (0.0268)	0.2953 (0.0241)	0.2115 (0.0276)	0.1126 (0.0319)	0.1828 (0.0267)
δ_S	-0.2342 (0.0262)	-0.1619 (0.0233)	-0.0807 (0.0209)	-0.0899 (0.0150)	-0.2315 (0.0266)	-0.1613 (0.0234)	-0.0826 (0.0206)	-0.0903 (0.0148)
ϕ_{F0}	-0.0454 (0.0246)	0.0262 (0.0257)	0.0421 (0.0310)	0.0636 (0.0250)	-0.0351 (0.0252)	0.0268 (0.0259)	0.0409 (0.0311)	0.0610 (0.0247)
ϕ_{F1}	-0.0874 (0.0236)	-0.2144 (0.0256)	-0.1462 (0.0312)	-0.1690 (0.0271)	-0.0884 (0.0240)	-0.2148 (0.0257)	-0.1456 (0.0312)	-0.1695 (0.0270)
θ_{S1}	0.0923 (0.0192)	0.2313 (0.0198)	0.1568 (0.0294)	0.1836 (0.0253)	0.0938 (0.0197)	0.2317 (0.0200)	0.1566 (0.0294)	0.1832 (0.0252)
δ_F	-0.1073 (0.0199)	-0.0068 (0.0161)	0.0047 (0.0177)	0.0237 (0.0125)	-0.1017 (0.0203)	-0.0064 (0.0162)	0.0047 (0.0176)	0.0241 (0.0123)

Note: The numbers within the parentheses are the corresponding standard errors.

Table 4: Estimation Results of Variance Equations for Crude Oil

Models	SCCC	SDCC	BMN-CCC	BMN-DCC	ACCC	ADCC	BMN-ACCC	BMN-ADCC
ω_{S1}	0.4684 (0.0203)	0.4409 (0.0193)	0.0480 (0.0069)	0.4902 (0.0491)	0.3448 (0.0191)	0.3925 (0.0214)	0.0424 (0.0061)	0.5086 (0.0504)
ω_{S2}	-	-	0.2932 (0.1108)	0.4933 (0.1124)	-	-	0.2566 (0.1051)	0.4898 (0.1142)
α_{S1}	0.2017 (0.0076)	0.2261 (0.0073)	0.0182 (0.0020)	0.1043 (0.0112)	0.1821 (0.0073)	0.2160 (0.0071)	0.0164 (0.0018)	0.1033 (0.0113)
α_{S2}	-	-	0.5449 (0.0785)	0.5848 (0.0627)	-	-	0.5370 (0.0766)	0.5777 (0.0615)
β_{S1}	0.6309 (0.0086)	0.6276 (0.0099)	0.9500 (0.0044)	0.4864 (0.0319)	0.6873 (0.0087)	0.6555 (0.0098)	0.9550 (0.0039)	0.4695 (0.0325)
β_{S2}	-	-	0.7910 (0.0291)	0.7502 (0.0252)	-	-	0.7950 (0.0281)	0.7497 (0.0251)
π_{SP}	0.1520 (0.0073)	0.1130 (0.0067)	0.0056 (0.0009)	0.1027 (0.0107)	0.1030 (0.0060)	0.0870 (0.0062)	0.0042 (0.0008)	0.0985 (0.0109)
π_{Sn}	-	-	-	-	0.1702 (0.0084)	0.1420 (0.0100)	0.0070 (0.0015)	0.1335 (0.0173)
δ_{11}	-	0.2288 (0.0061)	-	0.2502 (0.0230)	-	0.2254 (0.0061)	-	0.2480 (0.0233)
δ_{12}	-	0.3130 (0.0276)	-	0.2924 (0.0548)	-	0.3321 (0.0274)	-	0.2831 (0.0558)
ρ_1	0.9099 (0.0018)	-	0.9555 (0.0017)	-	0.9080 (0.0018)	-	0.9558 (0.0017)	-
P_1	-	-	0.8481 (0.0096)	0.8062 (0.0146)	-	-	0.8486 (0.0096)	0.8049 (0.0146)
ω_{F1}	0.2927 (0.0224)	0.3148 (0.0224)	0.0330 (0.0052)	0.3450 (0.0390)	0.2154 (0.0181)	0.2722 (0.0209)	0.0281 (0.0045)	0.3616 (0.0404)
ω_{F2}	-	-	0.3288 (0.1319)	0.5009 (0.1621)	-	-	0.3104 (0.1262)	0.5017 (0.1624)
α_{F1}	0.1573 (0.0083)	0.2086 (0.0081)	0.0207 (0.0023)	0.1024 (0.0108)	0.1395 (0.0074)	0.1976 (0.0077)	0.0183 (0.0020)	0.1019 (0.0110)
α_{F2}	-	-	0.3683 (0.0703)	0.5362 (0.0699)	-	-	0.3648 (0.0694)	0.5378 (0.0696)
β_{F1}	0.7192 (0.0125)	0.6724 (0.0118)	0.9505 (0.0043)	0.5496 (0.0320)	0.7654 (0.0109)	0.7016 (0.0112)	0.9565 (0.0037)	0.5356 (0.0329)
β_{F2}	-	-	0.7913 (0.0430)	0.7517 (0.0325)	-	-	0.7917 (0.0427)	0.7497 (0.0325)
π_{Fp}	0.0580 (0.0042)	0.0554 (0.0041)	0.0055 (0.0007)	0.0572 (0.0067)	0.0425 (0.0037)	0.0447 (0.0038)	0.0041 (0.0006)	0.0575 (0.0071)
π_{Fn}	-	-	-	-	0.0521 (0.0048)	0.0555 (0.0061)	0.0069 (0.0010)	0.0652 (0.0101)
δ_{11}	-	-	-	0.1951 (0.0185)	-	-	-	0.1984 (0.0184)
δ_{12}	-	-	-	0.4177 (0.0978)	-	-	-	0.4260 (0.0932)
ρ_2	-	-	0.7850 (0.0145)	-	-	-	0.7805 (0.0150)	-
P_2	-	-			-	-		
λ_{11}	-	-	0.0812 (0.0253)	0.0238 (0.0107)	-	-	0.0836 (0.0253)	0.0242 (0.0108)
λ_{12}	-	-	0.0901 (0.0209)	0.0336 (0.0110)	-	-	0.0930 (0.0209)	0.0338 (0.0110)
LLK	-13951.7	-13690.8	-13036.4		-13947.3	-13684.9	-13033.2	

Note: The numbers within the parentheses are the corresponding standard errors and LLK denotes the value of log likelihood.

Table 5: Estimation Results of Variance Equations for Heating Oil

Models	SCCC	SDCC	BMN-CCC	BMN-DCC	ACCC	ADCC	BMN-ACCC	BMN-ADCC
ω_{S1}	0.1174 (0.0135)	0.1146 (0.0126)	0.4259 (0.0397)	0.1543 (0.0177)	0.1427 (0.0154)	0.1175 (0.0137)	0.4144 (0.0402)	0.1391 (0.0173)
ω_{S2}	-	-	0.0496 (0.0678)	0.0289 (0.1182)	-	-	0.0487 (0.0733)	0.0261 (0.1230)
α_{S1}	0.1239 (0.0038)	0.1366 (0.0036)	0.0710 (0.0066)	0.0767 (0.0058)	0.1255 (0.0040)	0.1370 (0.0036)	0.0698 (0.0065)	0.0736 (0.0056)
α_{S2}	-	-	0.2682 (0.0258)	0.4984 (0.0465)	-	-	0.2763 (0.0274)	0.5364 (0.0520)
β_{S1}	0.8399 (0.0047)	0.8463 (0.0045)	0.7559 (0.0161)	0.8321 (0.0107)	0.8357 (0.0049)	0.8455 (0.0046)	0.7603 (0.0159)	0.8357 (0.0105)
β_{S2}	-	-	0.8656 (0.0130)	0.8705 (0.0113)	-	-	0.8636 (0.0136)	0.8600 (0.0126)
π_{SP}	0.0372 (0.0021)	0.0093 (0.0012)	0.0387 (0.0041)	0.0045 (0.0011)	0.0444 (0.0027)	-0.0094 (0.0013)	0.0366 (0.0041)	0.0041 (0.0010)
π_{Sn}	-	-	-	-	0.0231 (0.0045)	-0.0085 (0.0039)	0.0437 (0.0083)	0.0166 (0.0043)
δ_{11}	-	0.0605 (0.0018)	-	0.1757 (0.0095)	-	0.0606 (0.0018)	-	0.1732 (0.0094)
δ_{12}	-	0.9090 (0.0019)	-	0.7982 (0.0115)	-	0.9087 (0.0019)	-	0.8002 (0.0115)
ρ_1	0.9249 (0.0015)	-	0.9686 (0.0013)	-	0.9256 (0.0015)	-	0.9683 (0.0013)	-
P_1	-	-	0.8259 (0.0109)	0.8755 (0.0096)	-	-	0.8301 (0.0107)	0.8773 (0.0096)
ω_{F1}	0.0792 (0.0062)	0.0822 (0.0068)	0.3480 (0.0327)	0.1250 (0.0161)	0.0959 (0.0078)	0.0857 (0.0078)	0.3400 (0.0339)	0.1112 (0.0156)
ω_{F2}	-	-	0.0053 (0.0315)	0.0300 (0.0709)	-	-	0.0077 (0.0350)	0.0323 (0.0756)
α_{F1}	0.0976 (0.0026)	0.1211 (0.0029)	0.0580 (0.0055)	0.0675 (0.0055)	0.0989 (0.0027)	0.1217 (0.0030)	0.0569 (0.0054)	0.0647 (0.0053)
α_{F2}	-	-	0.1965 (0.0157)	0.4052 (0.0412)	-	-	0.2012 (0.0166)	0.4150 (0.0419)
β_{F1}	0.8755 (0.0022)	0.8683 (0.0028)	0.7962 (0.0143)	0.8519 (0.0109)	0.8727 (0.0022)	0.8675 (0.0029)	0.8002 (0.0143)	0.8563 (0.0106)
β_{F2}	-	-	0.9070 (0.0065)	0.9014 (0.0095)	-	-	0.9065 (0.0068)	0.8983 (0.0096)
π_{Fp}	0.0241 (0.0012)	0.0068 (0.0007)	0.0221 (0.0027)	0.0011 (0.0006)	0.0275 (0.0015)	0.0071 (0.0007)	0.0211 (0.0028)	0.0009 (0.0006)
π_{Fn}	-	-	-	-	0.0155 (0.0029)	0.0052 (0.0024)	0.0237 (0.0058)	0.0087 (0.0032)
δ_{11}	-	-	-	0.1390 (0.0233)	-	-	-	0.1480 (0.0222)
δ_{12}	-	-	-	0.0221 (0.0941)	-	-	-	0.0326 (0.0981)
ρ_2	-	-	0.8385 (0.0089)	-	-	-	0.8345 (0.0094)	-
P_2	-	-			-	-		
λ_{11}	-	-	0.0121 (0.0270)	0.0027 (0.0082)	-	-	0.0097 (0.0268)	0.0029 (0.0081)
λ_{12}	-	-	0.0269 (0.0276)	0.0083 (0.0084)	-	-	0.0253 (0.0276)	0.0081 (0.0083)
LLK	-14248.1	-14012.0	-13319.4		-14242.5	-14011.9	-13319.6	

Note: The numbers within the parentheses are the corresponding standard errors and LLK denotes the value of log likelihood.

Table 6 Out-of-Sample Hedging Performance

Models	SCCC	SDCC	BMN-CCC	BMN-DCC	ACCC	ADCC	BMN-ACCC	BMN-ADCC
Crude Oil								
1 day	1.2000 (-56.1%) [-6.8%]	1.0927 (60.0%) [-15.2%]	1.1670 (-57.3%) [-9.4%]	1.2084 (-55.8%) [-6.2%]	1.2525 (-54.2%) [-2.8%]	1.1277 (-58.8%) [-12.5%]	1.1660 (-57.4%) [-9.5%]	1.2121 (-55.7%) [-5.9%]
5 days	2.1635 (-65.5%) [-18.8%]	2.3143 (-63.1%) [-13.1%]	2.2448 (-64.2%) [-15.7%]	2.7130 (-56.7%) [1.8%]	2.1688 (-65.4%) [-18.6%]	2.2820 (-63.6%) [-14.3%]	2.2558 (-63.4%) [-15.3%]	2.6989 (-56.9%) [1.3%]
10 days	2.3867 (-71.1%) [-17.7%]	2.4248 (-70.6%) [-16.4%]	2.3983 (-71.0%) [-17.3%]	2.8487 (-65.5%) [-1.8%]	2.3846 (-71.1%) [-17.8%]	2.3938 (-71.0%) [-17.5%]	2.4097 (-70.8%) [-16.9%]	2.8289 (-65.7%) [-2.5%]
20 days	2.7860 (-76.3%) [-17.6%]	2.9620 (-74.8%) [-12.4%]	2.8045 (-76.2%) [-17.1%]	3.4885 (-70.4%) [3.1%]	2.7817 (-76.4%) [-17.8%]	2.8967 (-75.4%) [-14.4%]	2.8188 (-76.1%) [-16.7%]	3.4457 (-70.7%) [1.9%]
30 days	3.0305 (-79.7%) [-21.4%]	3.3140 (-77.8%) [-14.1%]	3.0845 (-79.3%) [-20.0%]	4.1301 (-72.3%) [7.1%]	3.0752 (-79.4%) -20.3%]	2.2131 (-85.2%) [-42.6%]	3.1046 (-79.2%) [-19.5%]	4.0533 (-72.8%) [5.1%]
40 days	3.4121 (-80.8%) [-19.6%]	3.6461 (-79.5%) [-14.1%]	3.4419 (-80.6%) [-18.9%]	4.7034 (-73.5%) [10.8%]	3.4831 (-80.4%) [-17.9%]	3.5340 (-80.1%) [-16.7%]	3.4717 (-80.4%) [-18.2%]	4.6151 (-74.0%) [8.8%]
50 days	3.7815 (-82.0%) [-15.6%]	4.0224 (-80.9%) [-10.2%]	3.7386 (-82.2%) [-16.5%]	5.5128 (-73.8%) [23.1%]	3.9268 (-81.3%) [-12.3%]	3.8337 (-81.8%) [-14.4%]	3.7876 (-82.0%) [-15.4%]	5.3822 (-74.4%) [20.2%]
Heating Oil								
1 day	0.9094 (-60.5%) [-3.1%]	0.8677 (-62.3%) [-7.5%]	0.9152 (-60.3%) [-2.5%]	0.8679 (-62.3%) [-7.5%]	0.9100 (-60.5%) [-3.0%]	0.8674 (-62.3%) [-7.6%]	0.9156 (-60.3%) [-2.4%]	0.8687 (-62.3%) [-7.4%]
5 days	1.1465 (-77.5%) [-8.4%]	1.1152 (-78.1%) [-10.9%]	1.1774 (-76.9%) [-6.0%]	1.1474 (-77.5%) [-8.4%]	1.1482 (-77.4%) [-8.3%]	1.1144 (-78.1%) [-11.0%]	1.1807 (-76.8%) [-5.7%]	1.1485 (-77.4%) [-8.3%]
10 days	1.3251 (-81.5%) [-11.2%]	1.3329 (-81.4%) [-10.7%]	1.3802 (-80.8%) [-7.5%]	1.3286 (-81.5%) [-10.9%]	1.3313 (-81.4%) [-10.8%]	1.3327 (-81.4%) [-10.7%]	1.3861 (-80.7%) [-7.1%]	1.3266 (-81.5%) [-11.1%]
20 days	1.6213 (-84.5%) [-12.5%]	1.5955 (-84.7%) [-13.9%]	1.6966 (-83.7%) [-8.4%]	1.4774 (-85.8%) [-20.3%]	1.6308 (-84.4%) [-12.0%]	1.5968 (-84.7%) [-13.8%]	1.7096 (-83.6%) [-7.7%]	1.4696 (-85.9%) [-20.7%]
30 days	1.8913 (-85.3%) [-13.1%]	1.8992 (-85.2%) [-12.7%]	1.9824 (-84.6%) [-8.9%]	1.7502 (-86.4%) [-19.6%]	1.9027 (-85.2%) [-12.6%]	1.9012 (-85.2%) [-12.7%]	1.9979 (-84.5%) [-8.2%]	1.7384 (-86.5%) [-20.1%]
40 days	2.0360 (-86.5%) [-13.7%]	2.0300 (-86.5%) [-14.0%]	2.1372 (-85.8%) [-9.4%]	1.7876 (-88.1%) [-24.2%]	2.0499 (-86.4%) [-13.1%]	2.0331 (-86.5%) [-13.8%]	2.1585 (-85.7%) [-8.5%]	1.7756 (-88.2%) [-24.7%]
50 days	2.3008 (-87.0%) [-13.3%]	2.3107 (-86.9%) [-12.9%]	2.4263 (-86.3%) [-8.6%]	1.9787 (-88.8%) [-25.4%]	2.3120 (-86.9%) [-12.9%]	2.3142 (-86.9%) [-12.8%]	2.4541 (-86.1%) [-7.5%]	1.9762 (-88.8%) [-25.5%]

Note: The table reports the variance of the hedged portfolio return. The numbers in the parentheses and brackets are the percentage reduction in the variance of alternative models against unhedged and OLS strategies, respectively.

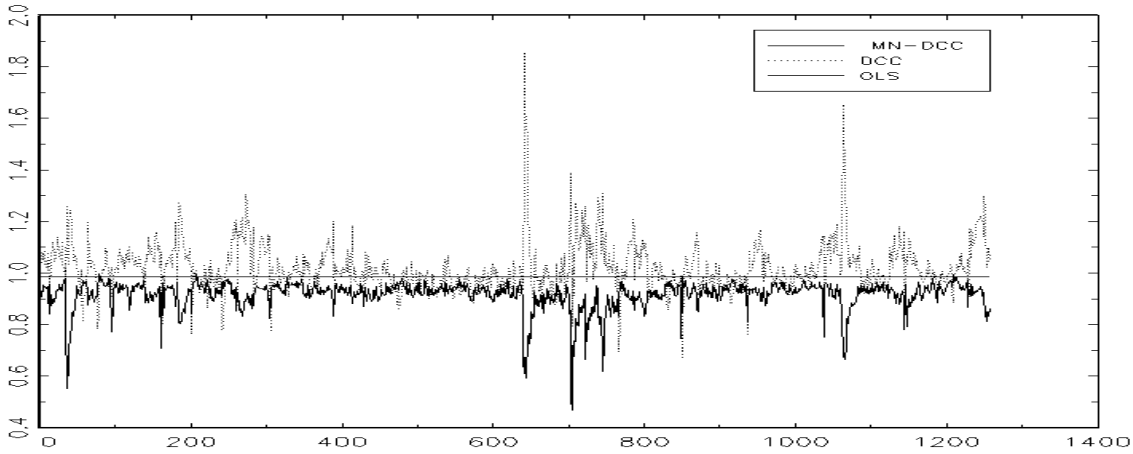
Table 7: SPA Tests for all models

Models		OLS	SCCC	SDCC	BMN-CCC	BMN-DCC	ACCC	ADCC	BMN-ACCC	BMN-ADCC
Crude Oil										
1 day	SPA_l	0.9452	0.6212	0.8790	0.6122	0.5094	0.5048	0.8730	0.6172	0.4822
	SPA_c	0.9428	0.6206	0.8772	0.6274	0.5088	0.5156	0.8754	0.6036	0.4910
	SPA_u	0.9992	0.6278	0.9798	0.6510	0.5476	0.5112	0.9590	0.6490	0.5236
5 days	SPA_l	0.9268	0.0406	0.1084	0.0188	0.0	0.0518	0.1112	0.0130	0.0
	SPA_c	0.9282	0.0350	0.0962	0.0176	0.0	0.0500	0.1074	0.0164	0.0014
	SPA_u	1	0.2064	0.1650	0.0340	0.0	0.2486	0.1934	0.0336	0.0
10 days	SPA_l	0.9780	0.0040	0.0370	0.0038	0.0	0.0050	0.0458	0.0042	0.0
	SPA_c	0.9756	0.0052	0.0372	0.0046	0.0	0.0052	0.0440	0.0038	0.0
	SPA_u	1	0.0058	0.0570	0.0048	0.0	0.0058	0.0698	0.0040	0.0
20 days	SPA_l	0.9412	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SPA_c	0.9436	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SPA_u	1	0.0	0.0	0.0	0.0	0.0012	0.0	0.0	0..0
30 days	SPA_l	0.8796	0.0	0.0	0.0	0.0012	0.0	0.0	0.0	0.0
	SPA_c	0.8886	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0016
	SPA_u	1	0.0826	0.0	0.0088	0.0	0.0632	0.0026	0.0040	0.0
40 days	SPA_l	0.9142	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SPA_c	0.9194	0.0	0.0	0.0	0.0	00	0.0	0.0	0.0
	SPA_u	1	0.0124	0.0	0.0	0.0	0.0032	0.0	0.0	0.0
50 days	SPA_l	0.9284	0.0	0.0	0.0	0.0012	0.0	0.0	0.0	0.0
	SPA_c	0.9316	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SPA_u	1	0.0780	0.0	0.0672	0.0	0.0276	0.0306	0.0416	0.0
Heating Oil										
1 day	SPA_l	0.3650	0.7230	0.9608	0.7048	0.9308	0.7304	0.9764	0.6988	0.9288
	SPA_c	0.3606	0.7170	0.9628	0.6906	0.9218	0.7140	0.9792	0.6916	0.9224
	SPA_u	0.3680	0.7524	0.9944	0.7172	0.9960	0.7462	0.9980	0.7116	0.9940
5 days	SPA_l	00964	0.7196	0.9068	0.4892	0.7180	0.7076	0.9524	0.4722	0.7160
	SPA_c	0.0894	0.7012	0.9076	0.4842	0.7252	0.7196	0.9564	0.4702	0.7186
	SPA_u	0.0858	0.7386	0.9940	0.4874	0.8154	0.7276	0.9996	0.4718	0.8118
10 days	SPA_l	0.0266	0.9250	0.9486	0.4962	0.9142	0.9080	0.9518	0.4502	0.9218
	SPA_c	0.0260	0.9268	0.9556	0.4882	0.9174	0.9032	0.9536	0.4462	0.9242
	SPA_u	0.0252	0.9998	0.9874	0.5146	0.9884	0.9692	0.9874	0.4600	0.9900
20 days	SPA_l	0.0	0.0174	0.0668	0.0	0.0468	0.0128	0.0618	0.0	0.5420
	SPA_c	0.0	0.0158	0.0618	0.0	0.0436	0.0130	0.0554	0.0	0.5330
	SPA_u	0.0	0.0182	0.1640	0.0	0.9812	0.0116	0.1582	0.0	1
30 days	SPA_l	0.0	0.0162	0.0538	0.0	0.0376	0.0156	0.0434	0.0	0.5242
	SPA_c	0.0	0.0144	0.0476	0.0	0.0374	0.0148	0.0414	0.0	0.5416
	SPA_u	0.0	0.0242	0.0828	0.0	0.9814	0.0154	0.0760	0.0	1
40 days	SPA_l	0.0	0.0	0.0034	0.0	0.0014	0.0	0.0062	0.0	0.5308
	SPA_c	0.0	0.0	0.0052	0.0	0.0020	0.0	0.0036	0.0	0.5040
	SPA_u	0.0	0.0	0.0116	0.0	0.9832	0.0	0.0094	0.0	1
50 days	SPA_l	0.0	0.0	0.0	0.0	0.2432	0.0	0.0	0.0	0.4946
	SPA_c	0.0	0.0	0.0012	0.0	0.2446	0.0	0.0	0.0	0.5000
	SPA_u	0.0	0.0	0.0026	0.0	0.9944	0.0	0.0	0.0	0.9986

Note: This table presents the *p*-values of the consistent (*SPAc*), lower bound (SPA_l) and upper bound (SPA_u) of Hansen's (2005) Superior Predictive Ability (SPA) test for the one to 50 days hedge horizon. Each model in the row is the benchmark versus all the other competitors. The null hypothesis is that none of the models is better than the benchmark.

Figure 1: Hedge ratios for the Crude Oil

(A) MN-DCC-21, dcc21, OLS



(B) MN-CCC-21, Ccc21, OLS

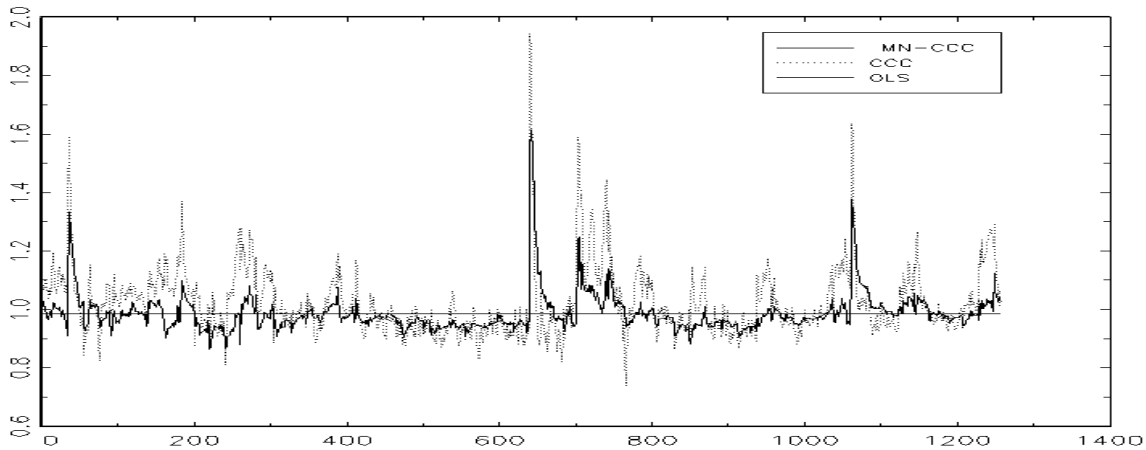
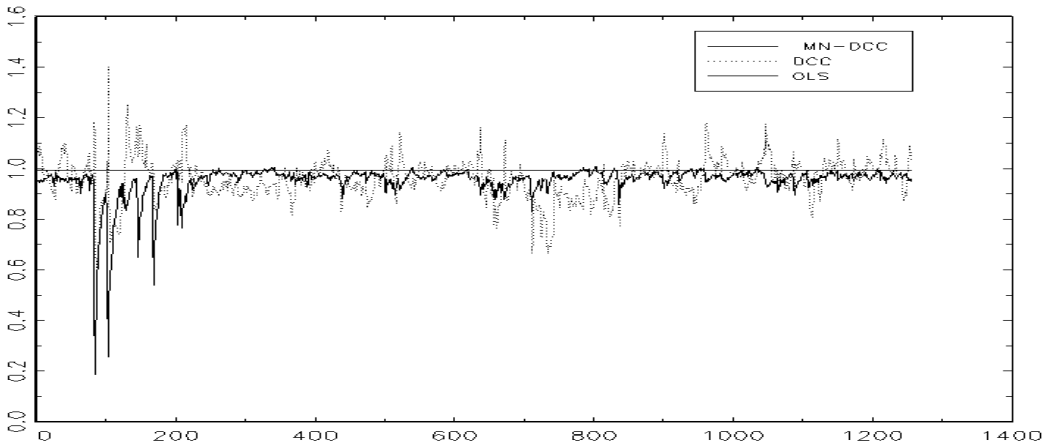


Figure 2 Hedge ratios for the Heating Oil

(A) MN-DCC-22, dcc22, OLS



(B) MN-CCC-22, Ccc22, OLS

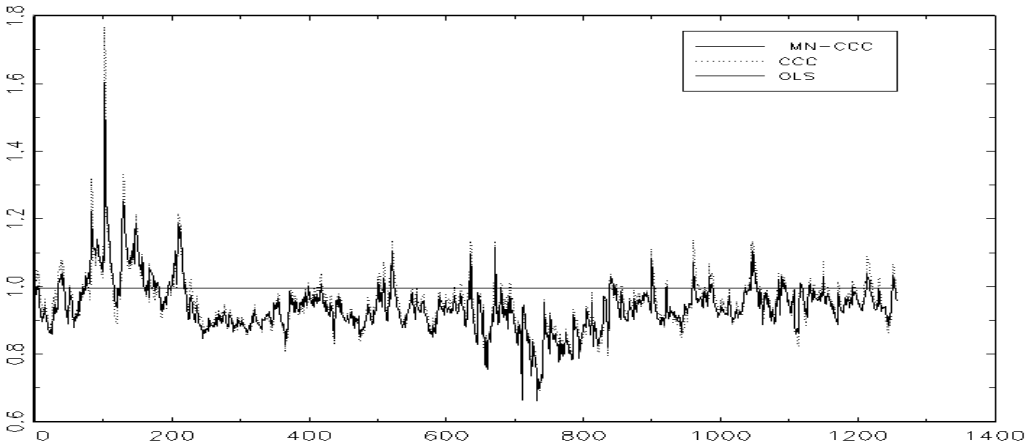
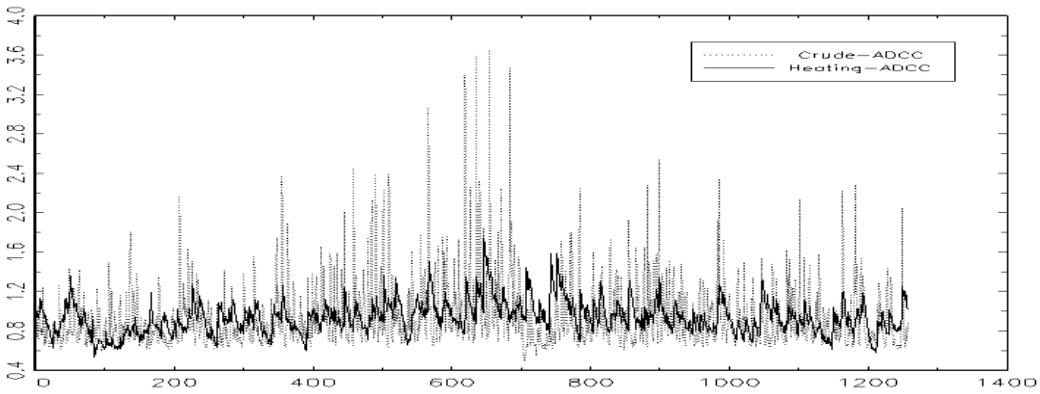
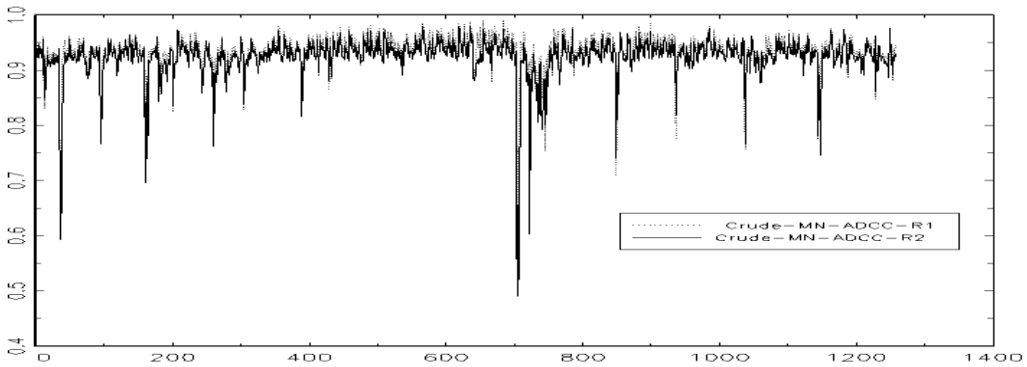


Figure 3 Time-varying correlations

(A) ADCC models of the crude oil and heating oil



(B) BMN-ADCC model for the crude oil



(C) BMN-ADCC model for the heating oil