Asset Pricing Implications of Equilibrium Business Cycle Models: Operating Leverage Redux*

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Abstract

Operating Leverage, the observation that the share of capital is riskier than the share of labour due to the priority status of wage claims over the business cycles, has been believed to relevant to a resolution of the equity premium puzzle. This paper asks whether asset pricing fluctuations induced by a DSGE model with operating leverage are empirically plausible.

Keywords: Operating Leverage; Staggered Nash Bargaining; business cycles; equity premium puzzle

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1 Introduction

As a possible solution to the equity premium puzzle, Donaldson and Mehra (1984) entertain the hypothesis that the share of capital is riskier than the share of labour, which is largely negotiated prior to the realization of output, which creates operating leverage. However, Rouwenhorst (1995) demonstrates that equilibrium business cycle models with operating leverage are irrelevant for the price of aggregate (non-diversifiable) risk. This paper asks whether operating leverage specified by GHH preferences and real wage rigidity in the form of staggered Nash bargaining wage in a DSGE model has an effect on the premium for aggregate risk.

First, with GHH preferences, there is no wealth effect on labour supply choice, so labour hours are strongly procyclical due to the substitution effect of wages over the business cycle, making it a less effective tool for smoothing fluctuations in marginal utility. Hence GHH preferences guarantee the procyclicality of hours worked over the business cycle even if wages are weakly procyclical. Second, as substitutes for wage rigidity, we choose a Taylor-type staggered Nash bargainin wage based on a model of the labour market with labour adjustment costs. The model, however, differs from the Mortensen-Pissarides labour search model because we abstract from explicit matching and search frictions of the labour market. Rather, our model can be viewed as a real version of the Cho-Cooley model with labour adjustment costs (Janko (2008)), although our wage is derived from the celebrated Nash bargaining problem. The model also can be viewed a real version of the monetary model with staggered Nash bargaining wage in Christiano et al (2007) but without matching and search frictions.

The rest of the paper is structured as follows. Section 2 presents the basic ingredients of the model. In Section 3, we present the basis for its calibration, examine the quantitative performance of our model and show that the model can account roughly for the basic features of the US data, including macroeconomic dynamics and financial statistics. Section 4 concludes the paper.

2 The Model

2.1 Households

There is a continuum of infinitely lived households uniformly distributed on a set of Lebesgue measure 1. Following Merz (1995) and Christiano et al. (2007), each household is viewed as a large extended family which contains a continuum of family members uniformly distributed on a set of Lebesgue measure 1. Individual workers in each household, each endowed with one unit of time, supply their labour hours. In the labour market employment varies at the extensive margin and labour hours worked varies at the intensive margin. Although their employment status varies at the extensive margin, individual workers in each household perfectly insure each other against labour income variation. Each household has the total fraction of workers employed, \( n_t \), within which the different cohorts \( l^i_t \), \( i = 0, ..., N - 1 \) are allocated
according to their staggered wage bargaining status. For instance, the first cohort \( l_1 \) will engage in wage bargaining with firm in period \( t \), receive the same wage from period \( t \) to period \( t + 1 + N \) and renegotiate in period \( t + N \), while the second cohort \( l_2 \) already bargained over wage in period \( t - 1 \) and receives the same wage from period \( t - 1 \) to period \( t - 2 + N \). Hence, the wage paid to an individual worker is determined by Nash bargaining (specified later), which occurs once every \( N \) periods. Therefore, our staggered wage setting is characterized by a Taylor-type friction. Households also have access to the financial markets, where they trade equity claims to the firm’s future dividend stream and one-period risk-free bonds (in zero net supply).

Given its information set \( \Omega^h_t \), the representative household maximizes its lifetime expected utility as given by:

\[
V^h(\Omega^h_0) = \max_{\{h_t, c_t, e_{t+1}, b_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t - n_t H(h_t))\right] \tag{1}
\]

s.t.

\[
c_t + p^e_t e_{t+1} + p^b_t b_{t+1} \leq (p^e_t + d_t)e_t + b_t + \sum_{i=0}^{N-1} w^i_t l^i_t h_t
\]

\[
l^i_{t+1} = (1 - \rho)l^i_t + x^i_t l^i_t \text{ for } i = 0, 1, ..., N - 1 \tag{2}
\]

In problem (1), \( u(\cdot) \) denotes the period utility function, \( H(\cdot) \) the household’s disutility of labour hours supplied, \( c_t \) its period \( t \) consumption, and \( h_t \) its period \( t \) labour hours supplied. \( d_t \) denotes the period \( t \) dividend payment by the firm to its capital owners; \( e_t \) and \( b_t \), respectively, the household’s period \( t \) stock and bond holdings; and the corresponding period \( t \) prices of these securities are \( p^e_t \) and \( p^b_t \). \( w^i_t \) is the bargained wage rate of each cohort. Equation (2) represents the law of motion of each cohort: \( \rho \) is the probability of separation and \( x^i_t \) denotes the firm’s hiring rate with respect to the \( i \)th cohort. \( n_t \) is the total fraction of workers employed and thus we have

\[
n_t = \sum_{i=0}^{N-1} l^i_t.
\]

The total number of unemployed workers, \( u_t \), is given by the difference between unity (the total population of workers) and the number of employed workers at the end of period \( t - 1 \), \( n_t \).

\[
u_t = 1 - n_t.
\]

Accordingly, \( E_t \equiv E(\cdot | \Omega^h_t) \) denotes the expectation operator conditional on his information set \( \Omega^h_t \), and \( \beta \) is the economy-wide subjective discount factor.

Here, we adopt GHH preferences for the representative household. It is known that the class of GHH preferences has an extremely weak short-run wealth effect on the labor supply. In fact, the Hicksian wealth effect of the real wage increase on hours worked is zero for this class of preferences.\(^1\) The class of

\(^1\)For more detail, see Jaimovich and Rebelo (2006).
GHH preferences implies that the marginal rate of substitution between consumption and labour supply depends only on the labour supply. Thus intertemporal substitution of consumption has no effect on the labour supply. Indeed, the marginal rate of substitution between consumption and labour supply in this model economy reads as:

$$- \frac{u_h(c_t - F_t - n_t H(h_t))}{u_c(c_t - F_t - n_t H(h_t))} = H'(h_t).$$

(3)

Conditional upon his information set $\Omega_t^h$, the recursive formulation of the representative’s problem is:

$$V^h(\Omega_t^h) = \max_{\{c_t, h_t, e_{t+1}, b_{t+1}\}} \left[ u(c_t - F_t - n_t H(h_t)) ight. \left. + \lambda_t \left[ p_t^e d_t e_t + b_t + \sum_{i=0}^{N-1} w^i h_t - c_t - p_t^e e_{t+1} - p_t^f b_{t+1} \right] + \beta E[V^h(\Omega_{t+1}^h) | \Omega_t^h] \right]$$

(4)

where $\lambda_t$ is the marginal utility of the representative household, i.e. $\lambda_t = u_c(c_t - n_t H(h_t))$.

The solution to the recursive problem (4) is characterized by

$$h_t : \sum_{i=0}^{N-1} w^i l_t^i = H'(h_t)$$

(5)

$$e_t : p_t^e = \beta E_t[\Lambda_{t,t+1}(p_{t+1} + d_{t+1}) | \Omega_t^h]$$

(6)

$$b_t : p_t^f = \beta E_t[\Lambda_{t,t+1} | \Omega_t^h]$$

(7)

where $\Lambda_{t,t+1}$ is the household’s marginal rate of substitution, $\frac{\lambda_{t+1}}{\lambda_t}$.

2.2 Firm

Each period, the firm can produce $y_t$ according to the following aggregate production function:

$$y_t = f(k_t, h_t n_t) z_t$$

where $z_t$, $k_t$, and $h_t n_t$ denote aggregate productivity shock, capital stock in period $t$, and total labour hours supplied by employed workers, respectively. Each cohort $l_t^i$ evolves according to the following law of motion:

$$l_{t+1}^i = (1 - \rho) l_t^i + x_t^i l_t^i$$

for $i = 0, 1, ..., N - 1$.

Each period, $\rho l_t^i$ separates exogenously from each cohort of employed workers $l_t^i$, and the firm organizes the new cohort $l_{t+1}^i$, augmented by new hires $x_t^i l_t^i$ from the pool of the unemployed. $x_t^i$ is the hiring rate of the representative firm within the existing cohort $i$: in other words,

$$x_t^i \equiv \frac{\text{new hires from pool of the unemployed}}{l_t^i}$$
The firm owns the (physical) capital stock, $k_t$. Each period the capital stock depreciates at the rate of $\delta$ and is augmented by new investment $i_t$.

We introduce two costs of adjusting the firm’s capital stock and labour force. Merz and Yashiv (2007) report that the simultaneous introduction of these two adjustment costs empirically affects the market value of the firm; ignoring either cost does not match with the empirical evidence.

Capital adjustment costs have a long tradition in the investment theory literature. Such costs form a wedge between the shadow price of capital installed within the firm and the price of an additional unit of capital. We replace the standard capital-accumulation technology with the specification employed in Jermann (1998):

$$k_{t+1} = (1 - \delta)k_t + G(\frac{i_t}{k_t})k_t.$$ 

The adjustment cost function $G(\cdot)$ is given by

$$G(\frac{i_t}{k_t}) = \frac{a_1}{1 - \xi} (\frac{i_t}{k_t})^{1-\xi} + a_2$$

where $a_1$ and $a_2$ are chosen so that $G(\delta) = \delta$, and $G'(\delta) = 1$. The elasticity parameter $\xi \equiv -\frac{1}{\partial G_G(\delta)} > 0$ is independent of the determination of the model’s steady-state equilibrium. A cost of adjusting labour force is a device for capturing the allocational feature of wages: wages affect employment at the extensive margin; they influence the rate at which firms add new workers to their existing labour forces. As in Gertler and Trigari (2007), the adjustment costs of the employment size of each cohort $l_i^j$ are given by

$$\frac{\kappa}{2}(x_i^j)^2l_i^j,$$

where $\kappa$ is a fixed cost.

The staggered wage setting specializes to the case of $N = 2$ for simplicity’s sake. Then the laws of motion of each cohort can be rewritten as:

$$l_{i+1}^1 = (1 - \rho)l_i^0 + x_i^0l_i^0$$

$$l_{i+1}^0 = (1 - \rho)l_i^1 + x_i^1l_i^1$$

$$n_t = l_t^0 + l_t^1.$$

The firm seeks to maximize its pre-dividend stock market value $d_t + p_t$ on a period-by-period basis given its information set $\Omega_I^t$:

$$\max_{\{i_t, k_t^t, x_t\}} d_t + p_t^e \equiv d_t + E(\beta \Lambda_{t+1}(p_{t+1} + d_{t+1}) \mid \Omega_I^t)$$

(9)
In problem (9), $w^i_t, i = 0, 1$ is the Nash bargaining wage for each cohort $i$ (specified later).

Letting $V^f(\Omega^f_t) \equiv d_t + p_t^f$, the recursive representation of the firm’s problem is written as:

$$V^f(\Omega^f_t) = d_t + \beta E(\Lambda_{t,t+1}V^f(\Omega^f_{t+1}) | \Omega^f_t).$$

The necessary and sufficient first-order condition for the firm’s optimal investment decision is given by:

$$i_t : (-1) + \beta E(\Lambda_{t,t+1}V^f_{k_t+1} | \Omega^f_t) \frac{\partial k_{t+1}}{\partial i_t} = 0.$$

By the envelope theorem,

$$k_t : \frac{\partial V^f(\Omega^f_t)}{\partial k_t} = f_t(k_t, h_t, n_t)z_t + \beta E(\Lambda_{t,t+1}V^f_{k_{t+1}} | \Omega^f_t) \frac{\partial k_{t+1}}{\partial k_t} = 0.$$

The Euler equation is represented as:

$$1 = \beta E(\Lambda_{t,t+1}^s G^s(\frac{i_t}{k_t})f_t(k_{t+1}, h_{t+1}, n_{t+1})z_{t+1} + \frac{(1 - \delta) + G^s(\frac{i_{t+1}}{k_{t+1}})}{G^s(\frac{i_{t+1}}{k_{t+1}})} - \frac{i_{t+1}}{k_{t+1}} | \Omega^f_t). \quad (10)$$

The first-order conditions for the firm’s optimal hiring decision of workers are given by

$$x^0_t : \kappa x^0_t = \beta E_1 A_{t,t+1}J^1_{t+1}$$
$$x^1_t : \kappa x^1_t = \beta E_1 A_{t,t+1}J^0_{t+1} \quad (11)$$

where $J^0_t \equiv \frac{\partial V^f(\Omega^f_t)}{\partial i^0_t}$ and $J^1_t \equiv \frac{\partial V^f(\Omega^f_t)}{\partial i^1_t}$, respectively, are the firm’s shadow value of one added worker from the cohort 0 and the shadow value of one added worker from the cohort 1.

### 2.3 Nash Bargaining

In this section, we introduce a (Nash) bargaining wage contract between the firm and workers with a Taylor-type friction. Unlike the standard Mortensen-Pissarides framework, we abstract from the explicit formulation of the labour market with search and matching frictions. Rather, the firm’s matching surplus and the household’s matching surplus are defined respectively as the firm’s shadow value of one added worker and as the household’s shadow value of one employed worker. We then apply the standard first-order condition of Nash bargaining, the constant surplus sharing rule. Again, for simplicity, the staggered wage setting specializes to the case of $N = 2$ for simplicity’s sake.
Firm’s matching surplus Presuming that the firm’s decision variables are chosen optimally, the firm value (its pre-dividend stock market value), \( V^f(t) \equiv V^f_t \equiv d_t + p^f_t \) can be represented in the recursive form as follows:

\[
V^f_t = d_t + \beta E(\Lambda_{t,t+1}(p^f_{t+1} + d_{t+1}) | \Omega^f_t)
\]

Let \( J^0_t \equiv \frac{\partial V^f_t}{\partial q^0_t} \) and \( J^1_t \equiv \frac{\partial V^f_t}{\partial q^1_t} \) be the firm’s shadow value of one added worker from the cohort 0 and the shadow value of one added worker from the cohort 1. \( J^0_t \) and \( J^1_t \), respectively, may be described recursively as:

\[
J^0_t = f_{nt} - w^0_n h_t + \frac{\kappa}{2}(x^0_t)^2 + (1 - \rho + x^0_t)\beta E_1 \Lambda_{t,t+1} J^1_{t+1}
\]

\[
J^1_t = f_{nt} - w^1_n h_t + \frac{\kappa}{2}(x^1_t)^2 + (1 - \rho + x^1_t)\beta E_1 \Lambda_{t,t+1} J^0_{t+1}
\]

where \( f_{nt} \equiv \frac{\partial y_t}{\partial n_t} \frac{\partial n_t}{\partial f_t} = \frac{\partial y_t}{\partial m_t} \frac{\partial m_t}{\partial f_t} = h_t f_2(k_t, h_t n_t) z_t \) is defined as "extensively marginal product labour of workers."²

The first-order conditions for hiring also can be written as:

\[
x^0_t : \quad \kappa x^0_t = \beta E_1 \Lambda_{t,t+1} J^1_{t+1}
\]

\[
x^1_t : \quad \kappa x^1_t = \beta E_1 \Lambda_{t,t+1} J^0_{t+1}
\]

Note that the wage bargaining occurs between the firm and the workers from cohort 0. Therefore, the shadow value \( J^0_t \) only plays the role of the firm’s matching surplus as one part of the standard Nash product.

Worker’s surplus We now derive the worker’s surplus from employment. This is defined as the shadow value of one worker employed in terms of current consumption of final goods. Because each household has two cohorts of workers employed, two surpluses should be taken into account.

²In the matching labour market for outsiders, we distinguish between "extensively marginal product of outsiders’ labour" and "intensively marginal product of insiders’ labour". Similarly, intensively marginal product labour, \( MPL_{h,n} \), is defined as \( \frac{\partial m}{\partial f_t} = n_t z_t f_3(k_t, h^*_t \cdot 1, h^*_t \cdot n_t) \).
When \( N = 2 \), the recursive representation of the household’s problem can be rewritten as:

\[
V^h(\Omega_t^h) = \max_{\{c_t,h_t,e_t,0,1\}} \left[ \lambda_t[(p^h + d_t)e_t + b_t + w_{t}^0h_t + w_{t}^1h_t - c_t - p^h_t e_{t+1} - p^h_t b_{t+1}] + \beta E(V^h(\Omega_{t+1}^h) | \Omega_t^h) \right]
\]

s.t.
\[
\begin{align*}
l_{t+1}^1 &= (1 - \rho)l_{t+1}^0 + x_{t+1}^0 l_{t}^0 \\
l_{t+1}^0 &= (1 - \rho)l_{t}^1 + x_{t}^1 l_{t}^1 \\
n_t &= l_{t}^0 + l_{t}^1.
\end{align*}
\]

From the above recursive representation (14), it is possible to derive the marginal benefits of one worker employed from each cohort:

\[
l_t^0 : \frac{\partial V^h_t}{\partial l_t^0} = \lambda_t(-H(h_t)) + \lambda_t w_t^0 h_t + \beta E_t(\frac{\partial V^h_{t+1}}{\partial l_{t+1}^1} \frac{\partial l_{t+1}^1}{\partial l_t^0})
\]

\[
l_t^1 : \frac{\partial V^h_t}{\partial l_t^1} = \lambda_t(-H(h_t)) + \lambda_t w_t^1 h_t + \beta E_t(\frac{\partial V^h_{t+1}}{\partial l_{t+1}^0} \frac{\partial l_{t+1}^0}{\partial l_t^1})
\]

where \( V_t^h = V^h(\Omega_t^h) \) and \( E_t \equiv E(\cdot | \Omega_t^h) \).

Then, the shadow values of one worker employed from each cohort in terms of current consumption, \( S_t^0 \) and \( S_t^1 \), are defined as

\[
\begin{align*}
S_t^0 &= \frac{1}{\lambda_t} \frac{\partial V_t^h}{\partial l_t^0} \\
S_t^1 &= \frac{1}{\lambda_t} \frac{\partial V_t^h}{\partial l_t^1}.
\end{align*}
\]

Therefore \( S_t^0 \) and \( S_t^1 \) read as:

\[
\begin{align*}
S_t^0 &= w_t^0 h_t - H(h_t) + (1 - \rho + x_t^0) \beta E_t(\frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^1) \\
S_t^1 &= w_t^1 h_t - H(h_t) + (1 - \rho + x_t^1) \beta E_t(\frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^0)
\end{align*}
\]

**Staggered Nash bargaining wage** At the beginning of period \( t \), the cohort 0 bargains over wage with the firm and receives \( w_t^0 \). Then the \( l_t^0 \) workers of cohort 0 supply their labour hours to the firm and production takes place. After production, the \( \rho l_{t+1}^0 \) workers separate from cohort 0 for exogenous reasons and cohort 0 is augmented by hiring the \( x_t^0 l_{t}^0 \) workers from the pool of unemployed workers. Next period, the cohort 0 is renamed as the cohort 1— in fact, \( l_{t+1}^1 = (1 - \rho)l_{t+1}^0 + x_t^0 l_{t}^0 \)— and the workers of cohort 1 supply their labour hours and receive the wage previously determined: in other words, \( w_{t+1}^1 = w_t^0 \). In the same manner, the \( \rho l_{t+1}^1 \) separates from cohort 1 and cohort 1 is augmented by hiring
the $x_{t+1}^1 l_{t+1}^1$ workers from the pool of unemployed workers. In period $t + 2$, cohort 1 is renamed as cohort 0, which will bargain over their wage. Thus, the wage bargaining (or renegotiation) occurs once every 2 periods.

Since the wage bargaining always involves cohort 0, the shadow value $J_t^0$ only plays the role of the firm’s matching surplus as one part of the standard Nash product, while the shadow value $S_t^0$ on the worker side constitutes the other part of the Nash product. The Nash bargaining problem is formulated as:

$$\max_{w_t^0 h_t} (J_t^0)^{1-\eta} \cdot (S_t^0)^{\eta}.$$ 

Under this Nash bargaining problem, the firm and the worker choose the wage $w_t^0$ and the hours of work $h_t$ to maximize the Nash product. The wage $w_t^0$ chosen by the bargaining must satisfy the optimality condition:\[1\]:

$$\eta J_t^0 = (1 - \eta) S_t^0. \quad (17)$$

Using the equations (12) and (13), $J_t^0$ can be represented as:

$$J_t^0 = \left\{ \begin{array}{c} f_{nt} - w_t^0 h_t + \frac{\xi}{2} (x_t^0)^2 \\ + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} [f_{nt+1} - w_{t+1}^1 h_{t+1} + \frac{\xi}{2} (x_t^1)^2 + (1 - \rho + x_{t+1}^1) \beta E_{t+1} \Lambda_{t+1,t+2} J_{t+2}^0] \end{array} \right\}$$

Similarly, using the equations (15) and (16), $S_t^0$ is written as:

$$S_t^0 = \left\{ \begin{array}{c} w_t^0 h_t - H(h_t) \\ + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} [w_{t+1}^1 h_{t+1} - H(h_{t+1}) + (1 - \rho + x_{t+1}^1) \beta E_{t+1} \Lambda_{t+1,t+2} S_{t+2}^0] \end{array} \right\}$$

Note that due to the Taylor-type staggered wage setting the wage $w_t^1$, paid to the workers of cohort 1, is equal to $w_{t-1}^0$, i.e.

$$w_t^1 = w_{t-1}^0. \quad (18)$$

Using relation (18) and the optimality condition of Nash bargaining (17), $J_t^0$ and $S_t^0$ can be reduced further to the tractable forms:

$$J_t^0 = \left\{ \begin{array}{c} f_{nt} + \frac{\xi}{2} (x_t^0)^2 + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} [f_{nt+1} + \frac{\xi}{2} (x_t^1)^2] \\ - w_t^0 h_t + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} h_{t+1} \end{array} \right\}$$

$$S_t^0 = \left\{ \begin{array}{c} w_t^0 h_t + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} h_{t+1} \\ - [H(h_t) + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} H(h_{t+1})] \end{array} \right\}$$

The reapplication of the optimality condition of Nash bargaining (17) generates the desired staggered Nash bargaining wage:

$$w_t^0 = \eta \frac{f_{nt} + \frac{\xi}{2} (x_t^0)^2 + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} [f_{nt+1} + \frac{\xi}{2} (x_t^1)^2]}{h_t + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} h_{t+1}} + (1 - \eta) \frac{H(h_t) + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} H(h_{t+1})}{h_t + (1 - \rho + x_t^0) \beta E_t \Lambda_{t,t+1} h_{t+1}}.$$ 

\[3\]This condition is the constant surplus sharing rule.
2.4 Equilibrium

In this economy, market clearing requires that for all $t$,

$$e_t = \int e_t d\varphi = 1$$
$$0 = \int h_t d\varphi$$
$$y_t = c_t + x_0 t + \frac{\kappa}{2} (x_0^t)^2 l_0^t + \frac{\kappa}{2} (x_1^t)^2 l_1^t$$

where $\varphi$ stands for the measure of households.

**Definition 1** Under the above market-clearing conditions, a decentralized stationary recursive equilibrium is defined as: a set of decision rules $\{c_t(\cdot); h_t(\cdot); e_{t+1}(\cdot); b_{t+1}(\cdot); i_t(\cdot); x_0^t(\cdot), x_1^t(\cdot)\}$ and a set of wage and price functions $\{w_0^t(\cdot), w_1^t(\cdot); p_e^t(\cdot), p_f^t(\cdot), d_t(\cdot)\}$ given the information set of aggregate states $\Omega = \{k_t, l_0^t, l_1^t; z_t\}$ such that (i) $\{c_t(\cdot), h_t(\cdot); e_{t+1}(\cdot), b_{t+1}(\cdot)\}$ satisfies a set of the optimality conditions (5), (6) and (7) (iii)$\{w_0^t(\cdot)\}$ satisfies the optimality condition for Nash bargaining (17) (iv) $\{i_t(\cdot), x_t(\cdot)\}$ satisfies the optimality conditions for investment and hiring (10) and (11) (vi) $w_1^t(\cdot)$ satisfies the condition (18) (vii)$\{p_e^t(\cdot), d_t(\cdot)\}$ satisfies the Lucas’ asset pricing equation (6), while $\{p_f^t(\cdot)\}$ satisfies the equation (7) (ix) The economy follows four laws of motion: $k_{t+1} = (1 - \delta)k_t + G\left(\frac{h}{k_t}\right)k_t$, and $l_{t+1} = (1 - \rho)l_0^t + x_0^t l_0^t$, $l_{t+1} = (1 - \rho)l_1^t + x_1^t l_1^t$.

2.5 Asset Pricing

Under the decentralized stationary recursive equilibrium defined in Section 2.4, it is possible to define and compute financial variables. Using the dividend series, the conditional price $p_e^t(\Omega_t)$ of an equity security is recursively computed according to the Lucas asset pricing equation:

$$p_e^t(\Omega_t) = \beta E\left(\frac{\lambda_{t+1}}{\lambda_t} [p^e(\Omega_{t+1}) + d(\Omega_{t+1})] \mid \Omega_t\right),$$

where $\Omega_t = \{k_t, l_0^t, l_1^t; z_t\}$ is the aggregate state of economy and $\lambda_t = u_e(c_t - n_t H(h_t))$ is the household’s marginal utility.

Using these prices, the time series of equity returns is computed in the conventional way:

$$R_{t,t+1}^e = \frac{p_e^t(\Omega_{t+1}) + d(\Omega_{t+1})}{p_e^t(\Omega_t)} - 1.$$  

In a similar fashion, the price of a one-period risk-free real bond is given by

$$p_f^t(\Omega_t) = \beta E\left(\frac{\lambda_{t+1}}{\lambda_t} \mid \Omega_t\right).$$

The risk-free rate of return, $R_{t}^f$, is computed using

$$R_{t}^f = \frac{1}{p_f^t(\Omega_t)} - 1.$$
3 Calibration

The time unit of the models is three months. We calibrate the process for aggregate productivity shocks to match the quarterly AR(1) process found by Cooley and Prescott (1995) to match the US Solow residual. The productivity shock $z_t$ evolves according to the law of motion:

$$\log z_{t+1} = 0.95 \log z_t + \epsilon_{t+1}$$

where $\epsilon$ is distributed normally, with mean zero and standard deviation $\sigma_\epsilon$.

The quarterly volatility of productivity shock, $\sigma_\epsilon$, is set to be 0.712% which is the standard parameterization in the business cycle literature.

For all simulation runs, the production function employed is the customary Cobb-Douglas function $z_t f(k_t, h_t, n_t) = z_t k_t^\alpha (h_t \cdot n_t)^{1-\alpha}$.

The parameter $\alpha$ is typically calibrated to reproduce the observed share of capital in total value added. We adopt the most commonly used value, 0.36. Thus, the share of income going to wages, 0.64, falls in the range $[0.60, 0.72]$ and is broadly consistent with the OECD cross-country data. The subjective discount factor $\beta$ is fixed at $\beta = 0.99$, corresponding to a steady state return on capital of 4%. Following Kydland and Prescott (1982), the quarterly capital depreciation rate $\delta$ is 0.025.

As in section 3.3, there exists observational equivalence between our labour market and the labour market with matching and search frictions. Therefore, we calibrate the labour market using the standard parameters for labour market search and matching.

The suggestions of the empirical literature vary with several measures of the US worker separation rate. We follow the report of Davis, Haltiwanger and Schuh (1996) so that the quarterly separation rate $\rho$ is set to be 8 percent. The steady state value of employment of outsider-nonstockholders $\bar{n}$ is set to be 0.89 as in den Haan, Ramey, and Watson (2000). We assume that the steady state value of the size of every cohort is equal, i.e. $\bar{l}^0 = \bar{l}^1 = \frac{n}{2}$; thus two laws of motion of labour cohorts (8) imply that the steady state values of the hiring rates, $\bar{x}^0$ and $\bar{x}^1$ are also equal. The vacancy cost $\kappa$ and the bargaining power parameter $\eta$ are derived from the steady state relationships so that the steady state ratio of adjustment costs to output $\bar{x}^2 / \bar{y}$ is 0.04; this ratio is broadly consistent with the empirical findings of labour adjustment costs in the US (Janko (2008)). The steady state value of the vacancy-filling probability $\bar{q}$ is taken to be 0.7 in the literature (for instance, Cooley and Quadrini (1999)) and this value enables us to derive the observationally equivalent number of job vacancies $\nu_t$. The period utility function of the representative household is postulated as

$$u(c_t - H(h_t)) = \frac{(c_t^{1-\psi} - B(h_t)^\psi)^{1-\gamma}}{1-\gamma} - 1$$

with $\psi = 1.3$. $\psi = 1.3$ implies that the Frisch elasticity of labour supply is $1 \frac{1}{1.3-1} = 3.33$. We choose the adjustment cost parameter $\xi = 0.23$, used by Jermann (1998).
4 Model Results

Equity premium Table 1 displays the statistics from the simulated models along with their empirical counterparts from the historical US data.

Table 1: Financial statistics for the US economy and model economies

<table>
<thead>
<tr>
<th></th>
<th>$E[R_f^t]$</th>
<th>$\sigma[R_f^t]$</th>
<th>$E[R_{t,t+1}^e-R_f^t]$</th>
<th>$\sigma[R_{t,t+1}^e-R_f^t]$</th>
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</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.80</td>
<td>5.67</td>
<td>6.18</td>
<td>16.67</td>
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<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma=10$</td>
<td>2.76</td>
<td>9.14</td>
<td>2.31</td>
<td>14.47</td>
</tr>
<tr>
<td>$\gamma=15$</td>
<td>2.31</td>
<td>11.50</td>
<td>3.35</td>
<td>18.27</td>
</tr>
<tr>
<td>$\gamma=20$</td>
<td>1.92</td>
<td>13.35</td>
<td>4.06</td>
<td>20.55</td>
</tr>
<tr>
<td>$\gamma=30$</td>
<td>1.40</td>
<td>16.17</td>
<td>5.08</td>
<td>23.84</td>
</tr>
</tbody>
</table>

4.1 Business Cycle Implications

In Table 2 the business-cycle implications of our baseline models are reported alongside the US data.

Table 2: Business cycle statistics for model economies

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_y$</th>
<th>$\sigma_c/\sigma_y$</th>
<th>$\sigma_i/\sigma_y$</th>
<th>$\sigma_w/\sigma_y$</th>
<th>$\sigma_u/\sigma_y$</th>
<th>$\sigma_v/\sigma_y$</th>
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</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.59</td>
<td>0.77</td>
<td>3.09</td>
<td>0.43</td>
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<td>8.27</td>
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</tr>
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<td>0.88</td>
<td>2.07</td>
<td>0.33</td>
<td>7.74</td>
<td>20.50</td>
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<tr>
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<td>1.17</td>
<td>0.63</td>
<td>2.79</td>
<td>0.32</td>
<td>7.80</td>
<td>20.64</td>
</tr>
<tr>
<td>$\gamma=30$</td>
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</tbody>
</table>

5 Concluding Remarks

Operating Leverage, the observation that the share of capital is riskier than the share of labour due to the priority status of wage claims over the business cycles, has been believed to relevant to a resolution of the equity premium puzzle. This paper asks whether asset pricing fluctuations induced by a DSGE model with operating leverage are empirically plausible.
References


