Risky assets allocation between stock and mutual fund: Dynamic Portfolio Choice under time costs

We examine the optimal dynamic portfolio decisions for investors who can invest in the stock market directly (through stocks) and indirectly (through mutual funds). We posit the time required for managing stock investment including conducting research and monitoring its performance. And we also posit the equilibrium management fees for managing mutual fund investment. An economic agent, who should allocate a limited amount of time to labor, leisure, stock and mutual fund investment, is subject to the opportunity time cost, which is forgone labor or leisure. We characterize the optimal stock proportions, mutual fund proportions, consumption, labor and leisure choices in the presence of such time costs. Then we will calibrate the results obtained in our model to the historical data from Survey of Consumer Finance, the Survey of Income and Program Participation, the Consumer Expenditure Survey and the American Time Use Survey."

Since the optimal portfolio theory was introduced in 1952 by Harry Markowiz with his paper "Portfolio Selection," scholars and practitioners have looked at the issue of how much money should an investor optimally allocate to different assets or asset classes and then extended to the problem of the individual's consumption and portfolio choices over time.

The model presented in this paper is based on a simplified assumption that an economic agent allocates a limited amount of time into four activities: labor, leisure, stock investment and mutual fund investment. We can also classify the time cost of stock investment as fixed time cost and variable time cost. For example, some direct cost such as brokerage time costand research cost (such as reading newspapers, web surfing, and consulting with brokerage agents) could be counted as fixed time cost. In contrast, monitoring time cost and indirect costs are close to variable time cost, which increases with an amount of investment in the stock. As an investor's investment in stock increases, his monitoring time also increases and his labor efficiency is more negatively affected. In this way the time cost ofstock investment resembles the "dollar" cost structure of stock investment.

In addition, we introduce the reciprocal relation between stock return volatility and time cost associated with stock management which is a novel feature of our model. T hus an investor can reduce volatility proportionately to the time he used to research, monitor and analyze his stock. Furthermore, using the same time, a young investor ca n acquire more information than an older one, thus he can control his stock return vol atility better than an older one. The curvature of the stock return volatility function is assumed to decrease as the investor ages.

Lastly, we introduce the forth alternative usage of time, time spent on mutual fund investment. Since with mutual funds, experienced professionals manage a portfolio of securities for investors full-time, and decide which securities to buy and sell based on extensive research, we only consider fixed time cost to mutual fund investment. This fixed time cost is the amount of entry time such as research and fund manager connection. Once an investor enters mutual fund market, no matter how much he invests, his time cost will not be changed. Thus as an investor's investment in mutual fund increases, his time cost does not increase and is not forgone labor or leisure. In such a case, investors should pay managing fees for delegating his mutual fund investment. And this management fee is monotonically increasing with the wealth that the investor invested in mutual fund investment.

The rest of the paper is organized as follows. Section 1 describes how to represent t he time opportunity cost of risky investment. Section 2 describes our economic model . Section 3 numerically solves the optimal consumption, stock proportions, mutual fu nd proportions, labor, and leisure time allocation. Section 4 calibrates our model to th e historical data based on the results of Section 3. Finally, Section 5 concludes the dis cussion.

1. The Time Cost Structure and the Time Constraint

In this paper, we impose a constraint that an investor be endowed with a limited am ount of time at each period t, which is normalized to one unitwithout loss of generalit y. This unit time will be allocated to labor activity ($j(t)$), leisure activity ($s(t)$), stock investment activity ($m(t)$) or mutual fund investment activity (E) at an investor's d isposal, and therefore

$$s(t) + j(t) + m(t) + E \cdot 1_{\beta>0} = 1$$

This representation is standard in the real business cycle model, as laid out by Prescot t(1986).

As mentioned above, we posit two alternative costs associated with stock investme nt: fixed time cost and variable time cost. As illustrated in Figure 1, the fixed time cos t, F , is the amount of entry time such as time spend on research and brokerage conne ction. The variable time cost is represented as a function of two determinants, $\alpha(t)$ and $W(t)$, where $\alpha(t)$ is the stock proportions at period t and $W(t)$ is the investor's net wealth at period $W(t)$. Obviously, the investor's monitoring time cost must be m onotonically increasing with $\alpha(t)$ since it requires a larger amount of time to discove r and process available information before and after the investment. In contrast, inclu ding $W(t)$ as the second determinant of the variable cost could be controversial. For th e same stock proportions, $\alpha(t)$, the amount of time associated with stock investment a t $W(t)=\$1{,}000$ could be quite different from when $W(t)=\$100{,}000$. The wealthier invest or might require more time in research and brokerage implementation, but the associa tion of monitoring time and psychological time with the wealth level is a priori not cl ear.

Thus we represent the variable time cost by $\varepsilon \cdot K(\alpha(t),W(t))$, where $K(\cdot,\cdot)$ is an inde x representing the strength of activity associated with stock investment. We consider t he following simple representation of $K(\cdot,\cdot)$:

$$K(\alpha(t),W(t)) = |\alpha(t)|^{\zeta} W(t)^{\xi}$$

And we assume the symmetry in time cost required for stock investment between a long position $(\alpha(t) > 0)$ and a short position $(\alpha(t) < 0)$.

Thus the participation time cost associated with stock investment , $m(t)$, can be des cribed as:

$$m(t) = [F + \varepsilon \cdot K(\alpha(t),W(t))] \cdot 1_{\alpha(t)>0} \tag{1}$$

where $1_{\alpha(t)>0}$ is an indicator variable with an appropriate argument.

However, we posit only fixed time cost associated with mutual fund investment, be cause experienced professionals manage the mutual fund for the investor full-time. This fixed cost, E, is the amount of entry time such as research and fund mana ger connection. Then the investor only needs to pay management fees to the fund man ager for mutual fund management. Therefore, the participation time cost associated w ith mutual fund investing can be written as $E \cdot 1_{\beta(t)>0}$, where $1_{\beta(t)>0}$ is an indicator var iable with an appropriate argument.

2. The Finite Horizon Economic Model

The economy consists of investors living for at most T periods, where T is a positive integer. This allows us to directly consider the impact of the investor's age (and increasing mortality) upon his optimal consumption, leisure, investment, and realization behavior. Let λ_j be the probability that the investor is alive at time j for $j = 0,\cdots,T$, given that he was alive at time $j-1$. We assume that $\lambda_j > 0$ for all j and that $\lambda_T = 0$. The probability that an individual investor lives up to period $t (t \le T)$ is giving by the following survival function:

$$\Phi(t) = \prod_{j=0}^{t} \lambda_j , \qquad (2)$$

Where $0 < \Phi(t) < 1$ for all $0 \le t < T$, and $\Phi(T) = 0$.

The investor in the economy derives utility from consuming a numeraire good, $c(t)$, and taking leisure, $s(t)$. In each time period, the investor also has age dependent full labor income, $D(t)$, which is the amount of income that the investor can achieve by allocating all available time (the unit time in our model) to labor, forgoing other activities such as leisure and stock investment. The process of the time varying behavior of the investor's full labor income before retirement at age J is as follows:

$$\Delta \log D(t) = \mu_D(t), \qquad \text{for } t = 0, \cdots J-1, \qquad (3)$$

Where the growth rate of full labor income, $\mu_D(t)$, is a deterministic function of age. After retirement at age J, the full labor income of an investor is assumed as a constan t fraction, θ, of preretirement full labor income at age $J-1$. Equation (3) assumes th at the full labor income is not a state variable. The entire uncertainty in our economy i s generated by the stock price and the mutual fund price. The reason for introducing s uch an assumption is that we want to focus exclusively on the effect of opportunity ti me cost.

We assume that the investor can invest in three financial assets: a riskless bond (B_t), a risky stock (S_t) and a risky mutual fund (MF_t). No transaction costs are incurred for trading these assets. The real gross return on the riskless bond is denoted as $1 + r_f$ and is assumed to be constant over time. The real gross return on the risky stoc k is denoted to have a lognormal distribution as

$$\log \tilde{R}_1(t) \sim N(\mu_1 - \frac{1}{2}\sigma_1^2, \sigma_1(t)) \qquad (4)$$

Then the expected gross return on the stock is $E(R_1(t)) = \exp(\mu_1) \cong 1 + \mu_1$, in which μ_1 r epresents the expected growth rate of the stock price at period t, and is assumed to be constant over time for simplicity. The volatility of the stock return is $\sigma_1(t) = \dfrac{1}{a \cdot t \cdot m(t) + b}$, and is assumed to be the reciprocal function of the participat ion time cost associated with stock investment $m(t)$, and the investor's age t. Thus an investor can use time cost to reduce his stock return volatility. And the curvature of σ_1 decreases as t increases, which means that an investor needs more time cost to re duce the same proportion of stock return volatility as he ages.

The real gross return on the risky mutual fund is denoted to be log-normally distributed as

$$\log \tilde{R}_2(t) \sim N(\mu_2 \frac{1}{2}\sigma_2^2, \sigma_2), \tag{5}$$

Then the expected gross return on mutual fund is $E(R_2(t)) = \exp(\mu_2) \cong 1 + \mu_2$, in which μ_2 represents the expected growth rate of the mutual fund price at period t, and σ_2 represents the volatility of the mutual fund. These two parameters are both assumed to be constant over time. The financial market does not allow borrowing at the riskles s interest rate to finance equity investments. This constraint is particularly to bind on an individual whose unconstrained optimal equity position is particularly large. We al so assume that in each period, the investor can costlessly adjust his portfolio.

Lastly, we make another assumption that the investor has a bequest motive. Since a n investor has the potential of dying at any period t, specifying his bequest function is not a trivial issue. Following Dammon, Spatt, and Zhang (2001), we assume that up on an investor's death the investor's asset holdings are liquidated, and are used to pur chase a $L-$period annuity for the benefit of the investor's beneficiary. We assume that the $L-$period annuity provides the investor's beneficiary with nominal consumptio n of

$$\frac{r_f(1+r_f)^L}{(1+r_f)^L - 1} W_t \equiv A_L W_t$$

at date k, $t+1 \le k \le t+L$, where W_t is the investor's wealth at the time of death, a nd $A_L = [r_f(1+r_f)^L]/[(1+r_f)^L - 1]$ is the $L-$period annuity factor.

The investor's problem here is to maximize his discounted expected utility of lifetime numeraire good and leisure consumption and bequest, subject to the intertemporal budget constraint and time allocation constraint, given his initial endowment and asset holding. To illustrate it, we define $M^o(t)$ and $N^o(t)$ as the following binary choice variables:

$$M^o(t) = \begin{cases} 0 & if \alpha(t) = 0 \\ 1 & if \alpha(t) > 0 \end{cases} \qquad for \quad t = 0, \cdots, T$$

$$N^o(t)=\begin{cases}0 & if\beta(t)=0\\ 1 & if\beta(t)>0\end{cases} \qquad for \quad t=0,\cdots,T$$

The investor's problem at time $t=0$ can now be represented as follows:

$$\max_{A(t)} E[\sum_{t=0}^{T}\beta^t\{\Phi(t)U(c(t),s(t))+[\Phi(t-1)-\Phi(t)]B(Q_t)\}] \tag{6}$$

$$A(t)=\{c(t),s(t),\alpha(t),\beta(t)\} \quad for \quad t=0,\cdots,J-1$$

s.t.

$$W(t+1)=(W(t)-c(t)+w(j(t)))[(1-\alpha(t)-\beta(t))(1+r_f)+\alpha(t)\tilde{R}_1(t)+\beta(t)(\tilde{R}_2(t)-\Psi) \tag{7}$$

$$w(j(t))=D(t)(1-s(t)-[F+\varepsilon\cdot K(\alpha(t),W(t))]\cdot M^o(t)-E\cdot N^o(t)) \tag{8}$$

$$D(t+1)=D(t)\mu_D(t+1) \quad for \quad t=0,\cdots,J-1 \tag{9}$$

$$D(J+k)=\theta\cdot D(J-1) \quad for \quad k=0,\cdots,T-J-1 \tag{10}$$

$$0\le s(t)\le 1 \tag{11}$$

$$0\le [F+\varepsilon\cdot K(\alpha(t),W(t))]\cdot M^o(t)\le 1 \tag{12}$$

$$0\le E\cdot N^o(t)\le 1 \tag{13}$$

$$0\le 1-s(t)-[F+\varepsilon\cdot K(\alpha(t),W(t))]\cdot M^o(t)-E\cdot N^o(t)\le 1 \tag{14}$$

$$c(t)>0, \qquad D(t)\ge 0 \tag{15}$$

Given the initial net worth, W_0, initial full labor income, D_0, initial stock holding ratio, α_0, and initial mutual fund holding ratio, β_0. The expression inside the square brackets in Equation (6) is the investor's probability-weighted utility at period t. The first term measures the investor's utility of numeraire good consumption and leisure in period t weighted by the probability of living through period t, while the second term is the investor's utility of bequest weighted by the probability of dying at period t. $U(\cdot)$ and $B(\cdot)$ denote the investor's utility function and bequest function, respectively. β is the subjective time discount factor. $\Phi(-1)$ is set to 1 to indicate that the investor has survived up to period 0.

Equation (7) defines the evolution of W_t, the investor's net worth at the beginning of the period. Equation (8) shows that the labor income is the product of "full labor income" and labor time consumed. An additional source of income stems from an investor's portfolio, the performance of which is also uncertain. If an economic agent allocates all available time only to labor activity, forgoing leisure and financial investment activities, he could achieve the full labor income at period t. Equation (9) and (10) define the evolution of the investor's full labor income. Equations (11), (12), (13), and (14) define the time constraint of the investor's four activities at period t, and Equation (15) states the positive condition of consumption and non-negative condition of full labor income.

We assume that the investor's preferences for numeraire good consumption and lei sure consumption are represented by the Cobb-Douglas utility function:

$$U(c(t),s(t)) = \frac{1}{1-\delta}(c(t)^{\gamma} h(s(t))^{1-\gamma})^{1-\delta} \quad (16)$$
$$\text{s.t.} \quad h(s(t)) = B(t)\cdot s(t)$$

where γ measures the relative importance of numeraire good consumption versus le isure consumption and δ is the curvature parameter. $c(t)$ represents the consumptio n at period t, $s(t)$ represents the amount of leisure time consumed at period t, and t hese two parameters are independent with each other. $h(s(t))$ is the leisure consumpti on measured at period t, and $B(t)$ is the appropriate parameter for transforming the amount of leisure time to the leisure consumption. Since people can obtain more leisu re utility from the same leisure time after retirement, the value of $B(t)$ is set to jump up when the investor is retired. An investor with the Cobb-Douglas utility will spend on $c(t)$ and $s(t)$ in a fixed proportion in a one-period model, and this property still holds in a multiperiod setup.

We assume that the annuity income from a bequest is used to pay for the beneficiar y's numeraire good consumption and leisure consumption costs. Furthermore, the ben eficiary's numeraire good and leisure consumption is set at the fixed proportion of $\gamma/(1-\gamma)$, the optimal level for the CobbDouglas utility function. Hence, the bequest f unction can be defined as

$$B(Q_t) \equiv \sum_{k=t+1}^{t+L} \beta^{k-t} \frac{B''}{1-\delta}\left(\frac{r_f(1+r_f)^L}{(1+r_f)^L - 1} W_t\right)^{1-\delta} = \frac{\beta(1-\beta^L)}{1-\beta}\frac{B''}{1-\delta}(A_L W_t)^{1-\delta} \quad (17)$$
$$\text{s.t.} \quad B'' = \left(\frac{1-\gamma}{\gamma D(t)}\right)^{(1-\gamma)(1-\delta)} B(t)^{(1-\gamma)(1-\delta)}$$

The value function of the investor's intertemporal consumption and investment pro blem can be written as

$$V_t(X_t) = \max_{A(t)}\{\lambda_t[B'\frac{1}{1-\delta}(c(t)^{\gamma}s(t)^{1-\gamma})^{1-\delta} + \beta E_t[V_{t+1}(X_{t+1})]] + (1-\lambda_t)\frac{\beta(1-\beta^L)}{1-\beta}\frac{B''}{1-\delta}(A_L W_t)^{1-\delta}\}$$

$$A(t) = \{c(t), \alpha(t), \beta(t), s(t)\}, \qquad for \quad t = 0, \cdots, T-1 \tag{18}$$

s.t.

$$B'' = \left(\frac{1-\gamma}{\gamma D(t)}\right)^{(1-\gamma)(1-\delta)} B' = \left(\frac{1-\gamma}{\gamma D(t)}\right)^{(1-\gamma)(1-\delta)} B(t)^{(1-\gamma)(1-\delta)}$$

This dynamic problem cannot be solved analytically. We derive the value function numerically by discretizing the state-space and the variables over which the choices are made, and by using binomial tree model to approximate the distributions of risky asset returns. The problem is then solved by standard backward recursion on the approximated value function. To do this, we discretize the lagged endogenous state variables, $X(t) = \{W(t)\}$, into a (1×100) grid in each period t. At the terminal date T, the investor's value function takes the value

$$V_T(X_T) = \frac{\beta(1-\beta^L)}{1-\beta} \frac{B''}{1-\delta} (A_L W_T)^{1-\delta} \tag{19}$$

at all points in the state space. The value function at date T is then used to solve for the optimal decision rules and value function for all points on the grid at date $T-1$. Th e procedure is repeated recursively for each time period until the solution for date $t=0$ is found. Bilinear interpolation is used to calculate the value function for points in the space that lie between the grid points.

3. Numerical Results

The base-case parameter values for our numerical analysis are summarized in Table I and discussed below.

Table I
Base-case Parameter Values

Parameters of the Model	Notation	Base-case Value
Cobb-Douglas parameter	γ	0.6
Expected return on the stock	μ_1	8.0%
Stock volatility	σ_1	15%~30%
Expected return on the mutual fund	μ_2	6.0%
Mutual fund volatility	σ_2	15%
Risk-free rate	r_f	2.0%
Discount factor	β	0.96
Beneficiary periods	L	15years
Endowment(initial) time	t_0	Age 20
Terminal time	T	Age 85
Retirement time	J	Age 65
Decreasing rate of labor efficiency at time J	θ	50%
Mutual fund fees	Ψ	2%

Parameters	Value			
	Our model	Benchmark I	Benchmark II	Benchmark III
CRRA: δ	2	2	15	2
Fixed time cost of stock: F	0.005			0.005
Variable time cost parameter: e	0.06			
Index of $\alpha(t)$ in $K(\cdot,\cdot):\zeta$	1.3			1.3
Index of $W(t)$ in $K(\cdot,\cdot):\xi$	0.5			0.5
Fixed time cost of mutual fund: E	0.01			

We assume a full labor income process over age as in Figure 1. Following Cocco, Gomes, and Manhout(2004), and Yao and Zhang(2005) studies, we assume that full labor income linearly increases till about age 50 and then decreases till retirement at age $J = 65$, which is consistent with real labor income process. Then, after retirement we assume that full labor income is constant and equal to $\theta = 50\%$ of his full labor income at age 64.

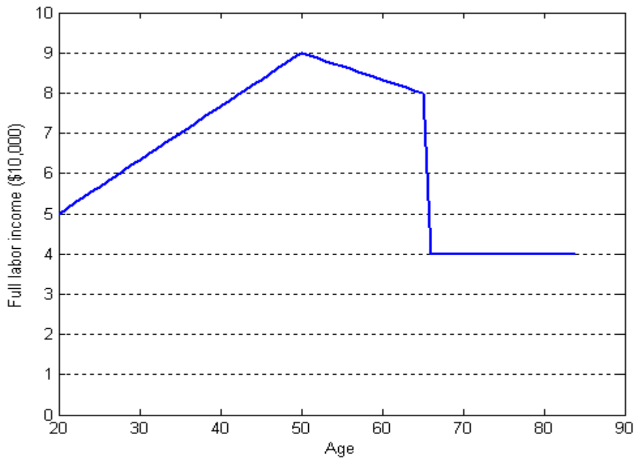


Figure 1
Full Labor Income
Since full labor income could not be measured exactly, we set the full labor income as a mixture of linear functions. Full labor income increases from $50,000 at age 20 to $90,000 at age 50, then decreases to $80,000 until the year before his retirement at age 65. A retiree's full labor income then becomes a fraction (1/2) of their full labor income they received when they were 65.

The participation cost structure of stock investment before retirement is described a s

$$m(t) = [0.005 + 0.06 \cdot \alpha(t)^{1.3} W(t)^{0.5}] \cdot M^{o}(t) \qquad (20)$$

The fixed time cost of stock investment is set at 0.005. The variable time cost is set to be the product of $K(\cdot,\cdot) = \alpha(t)^{\zeta} W(t)^{\xi}$ and its time sensitivity, where $K(\cdot,\cdot)$ is set at $\alpha(t)^{1.3} W(t)^{0.5}$ in (34) for calibrating historical data.

The fixed time cost of mutual fund investment is set at $E = 0.010$.

Over the past hundred years, the average return on equity was about 6 to 7 percent above the riskfree interest rate [See Mehra and Prescott (1985) and Mankiw and Zeldes (1991)]. Its volatility ranges historically between 15 and 20 percent. However, Claus and Thomas (2001), Fama and French (2002), and others have argued that the expected future equity fisk premium should be substantially lower than the historical average of 7-8%. Thus, we set the riskfree rate at $r_f = 2.0\%$, the expected return of stock at $\mu_1 = 8.0\%$ and the expected return of mutual fund at $\mu_2 = 6.0\%$. The standard deviation of the stock return is set to be the reciprocal function of the participation time cost associated with risky asset investment and the investor's age,

$$\sigma_1(m(t)) = \frac{1}{(5-0.075t)m(t)+4}, \quad (21)$$

which is shown in Figure 2. We can see that the stock return volatility is monotonicall y decreasing with $m(t)$ at each time period. Its value ranges between 15% and 25%. The first derivative is negative and the second derivative is decreased with the investo r's age. This assumption means that at the same age the more time an investor pays to monitor his stock, viz. $m(t)$ increases, the better he can control his stock by reducin g stock return volatility. Furthermore, an investor needs to pay more time cost in his s tock investment to reduce the same proportion of stock return volatility as he ages.

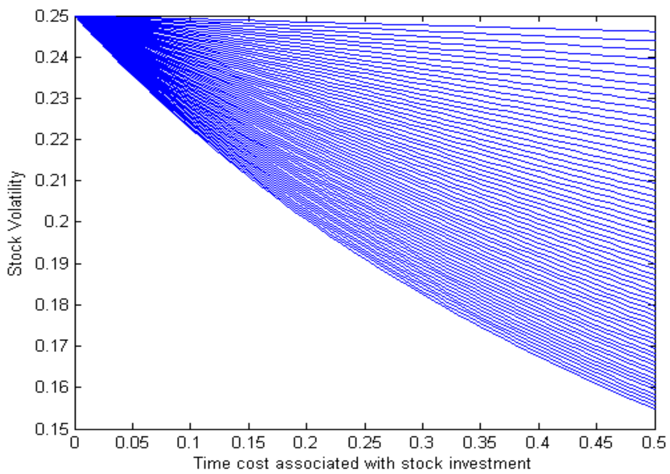


Figure 2
Stock Return Volatility
This Figure illustrates the relationship among stock return volatility, the participation time cost associated with stock investment, and the investor's age.

The standard deviation of the mutual fund return is set at $\sigma_2 = 15\%$. Finally since m utual funds provide professional management, an investor should pay for this service through a fee that is based on the total value of his account, we set this fees as the $\Psi = 2\%$ proportion of his mutual fund's assets.

Our discussion in this section focuses on the optimal consumption and investment decisions of the investors who can invest in the stock market both directly (through st ocks) and indirectly (through mutual funds) and can costlessly adjust his portfolio at e ach period.

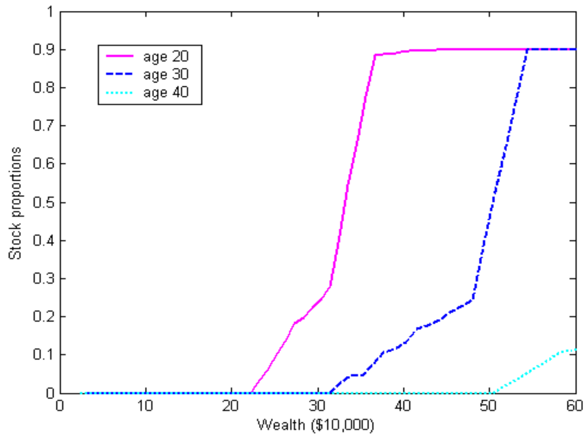
The optimal Stock Investment

Figure 3 illustrates the locus of the optimal stock proportions. Figure3-(a) shows the investor's optimal stock proportions at age 20, 30, and 40 as a function of the investor's beginning-of-period net-worth. These stock proportion curves are all upward slopping, which indicates that the investor increases his stock proportions with wealth after he participated in the stock market. This indicates that the wealth level has a significant impact on the investor's optimal stock proportions. In a standard consumption model (even in the presence of leisure and labor), we cannot expect any association between the wealth level and the optimal stock proportions. In addition, the existing transaction cost models such as Constantinides(1986) and Davis and Norman(1990) fail to generate the wealth effect since transaction costs therein are assumed to be a constant fraction of trading size, not the trader's wealth level. On the contrary, the stock investment time cost including monitoring time given in our model is dependent on the investor's wealth. Two reasons can be attributed to this wealth effect. Firstly, the existence of fixed time cost creates the economy of scale in terms of time management; Secondly, concavity between wealth and variable time cost also benefits the wealthier investor. Even though they should bear the higher time cost, its relative cost to their wealth decreases due to innate concavity. As a consequence, the wealthier can increase their proportions in the stock investment.

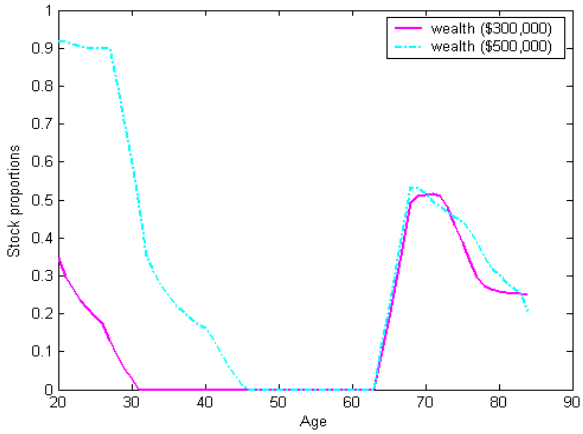
Figure 3-(b) depicts the investor's optimal stock proportions over age given wealth level \$300,000 and \$500,000. Given the wealth level, the optimal stock proportions varied inversely with the full labor income over age. This can be explained by the substitutability between labor and stock investment. At the first phase of life cycle (roughly until the investor's forties), the optimal stock proportions of two lines decreases in the age of the investor. During this part, the full labor income increases with the investor's age, the investor tends to spend more time on labor. Therefore, he prefers to decrease his investment in stock market. In midlife, the full labor income reaches its maximum part, so that the investor tilts his portfolio towards the mutual

fund investment, which only need a fixed time cost. After retirement the full labor income is cut in half, in order to accumulate wealth to insure his remaining life and bequest his children, the poorer investor slightly increases his stock proportions, since stocks offer the greater potential return than mutual funds. Then after 75 years old, the optimal stock proportions decline again. This can be explained by the following reasons. The volatility of the stock return is assumed to be the reciprocal function of the participation time cost associated with stock investment in our model, and the reciprocal function shifts upward as the investor ages. Thus the more time cost is used in stock management, the better the investor's stock return volatility can be controlled. Furthermore, an older investor should spend more time cost than a younger one to monitor and control his stock return volatility to be a significant level. For an investor after 75 years old, his time cost to stock investment is too high and he prefers leisure to make money.

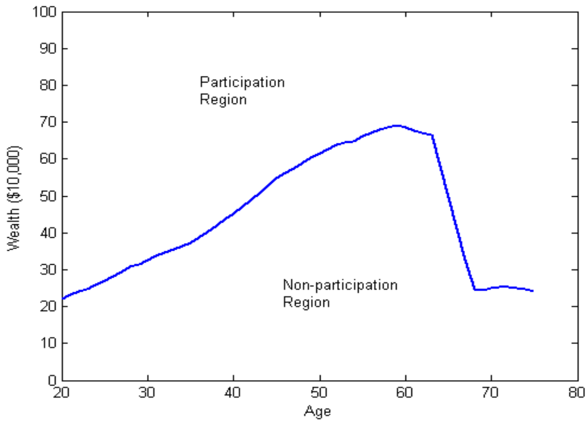
The investor's optimal stock investment participation choice as a function of the in vestor's beginning-of-period net-worth and the investor's age is described in Figure3-(c). It dichotomizes the entire spectrum of wealth into two regions: participation and non-participation. Since an investor in our model face variable time costs of stock



(a)



(b)



(c)

Figure 3
Optimal Stock Investment
This figure displays the relationship between stock investment, age and wealth. The stock proportions over wealth range from $0 to $600,000 at age 20, 30, and 40 are shown in (a), the stock proportions over age given wealth level $300,000 and $500,000 are shown in (b), and the stock investment participation region with respect to age is shown in (c).

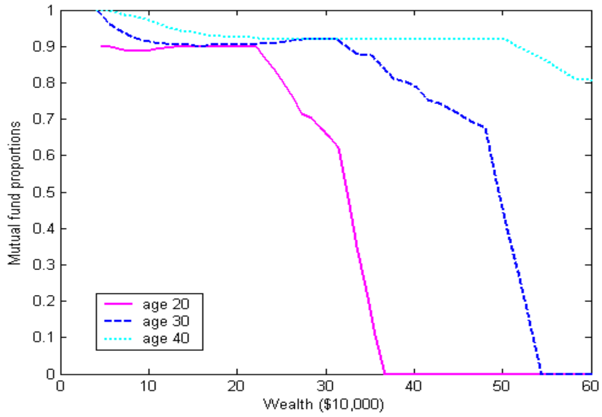
market participation; if these variable time costs exceed the benefit of participation, he may hold no stocks. Thus the investor above a certain threshold wealth level participates in the stock market. This threshold wealth level is age-dependent. At the age of twenty, only an investor with wealth above about $20,000 participates in the stock market. In contrast, at age fifty, investors whose wealth level is greater than about $55,000 invest in the stock. So we can know that the threshold level of wealth for stock market participation gradually increases before the investor reaches his late fifties and decreases thereafter.

The Optimal Mutual Fund Investment

Figure 4 illustrates the locus of the optimal mutual fund proportions. The investor's optimal mutual fund proportions at age 20, 30, and 40 as a decreasing function of the investor's beginning-of-period net-worth are shown in Figure 4-(a), which looks like a mirror image of Figure 3-(a). This means that an investor optimally chooses to decrease his investment in mutual funds and increase his investment in stocks as his net wealth increases. The following reasons can be attributed to this wealth effect. Through mutual fund an investor can access to the stock market without variable time cost. In such a case, he should pay managing fees for delegating his risky investment. Consequently, he trades the time cost for management fees. This mutual fund management fee has a linear relationship with the investor's wealth. But variable time cost associated with stock investment has a concave association with the investor's wealth. This concavity benefits the wealthier investor. The increasing of wealth leads the investor to increase his stock holdings and reduce his mutual fund holdings.

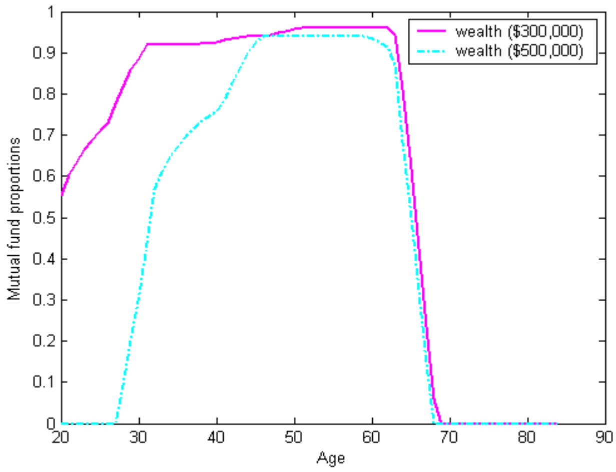
Interestingly, contrary to stock, given the wealth level, the variation of mutual fund proportion over age is similar with the variation of full labor income over age. This is clearly demonstrated in Figure 7-(b) in which we plot the investor's mutual fund proportions over age given wealth level$100,000 and $500,000. This can be explained by the fixed time cost of mutual fund investment. Once an investor enters mutual fund market, no matter how much he invests, his time cost will not be changed. Since an aged investor with higher full labor income tends to spend more time on labor, so he prefers to invest in the mutual fund investment rather than stock investment.

The investor's optimal mutual fund investment participation choice as a function of the investor's beginning-of-period net-worth and the investor's age is described in Figure4-(c). It dichotomizes the entire spectrum of wealth into two regions: participation and non-participation. Since an investor in our



Mutual fund proportions
Wealth ($10,000)
age 20
age 30
age 40

(a)



Mutual fund proportions
Age
wealth ($300,000)
wealth ($500,000)

(b)

(c)

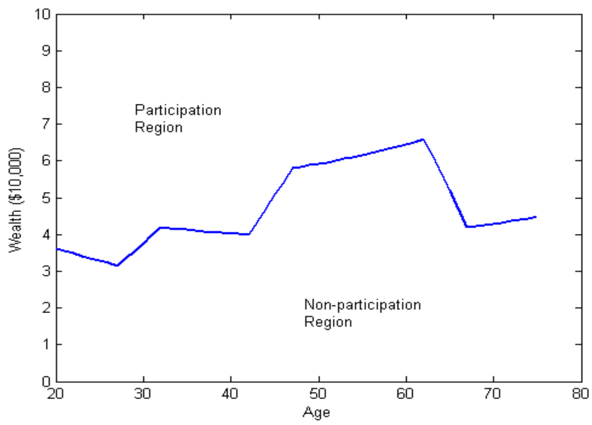
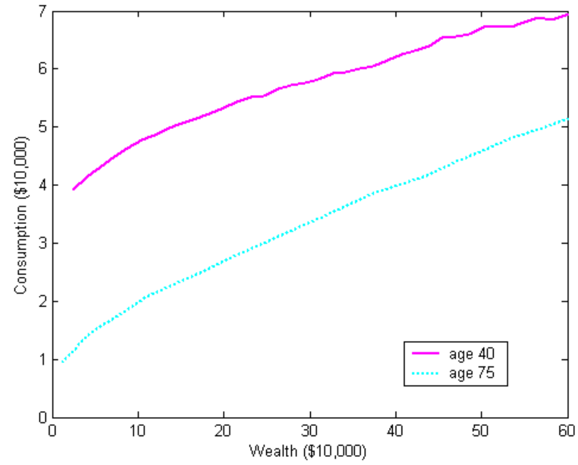


Figure 4
Optimal Mutual Fund Investment
This figure displays the relationship between mutual fund investment, age and wealth. The mutual fund proportions over wealth range from $0 to $600,000 at age 20, 30, and 40 are shown in (a), the mutual fund proportions over age given wealth level $300,000 and $500,000 are shown in (b), and the mutual fund investment participation region with respect to age is shown in (c).

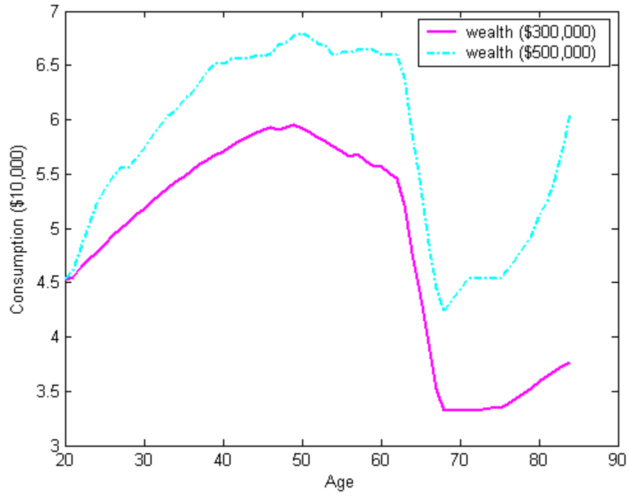
model face fixed time costs of mutual fund market participation; if this fixed time cost exceeds the benefit of participation, he may hold no mutual funds. Since there's no variable time cost associated with mutual fund investment, the threshold wealth level of participating in mutual fund investment is much lower than that of stock investment.

The Optimal Consumption

Having discussed the optimal investment policies, we now consider the optimal consumption policy. Figure 5-(a) shows the investor's optimal consumption choice at age 40 and 75 as a function of the investor's beginning-of-period net-worth. Optimal consumption is an increasing function of wealth, and its monotonic shape is close to linearity as consistent with the existing consumption-based model. However, its sensitivity to wealth, or equivalently the slope of optimal consumption, varies as age increases. A young agent's consumption is less sensitive to wealth since he takes into account his future labor income, though in a limited sense due to the borrowing constraint. In contrast, an older agent cannot expect a large amount of future income as he ages, and thus his consumption is more sensitive to his wealth level as he ages.



(a)



(b)

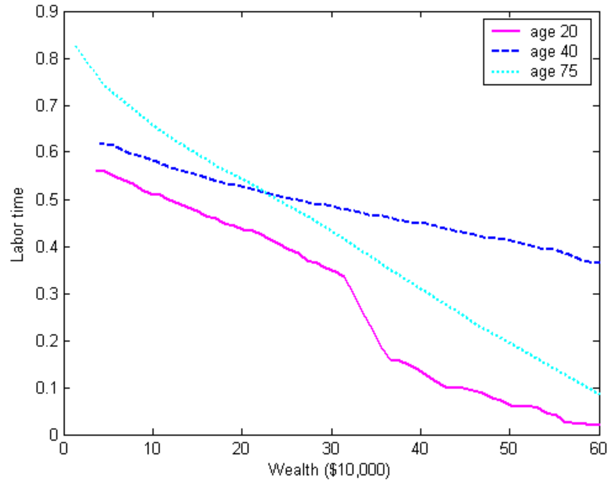
Figure 5
Optimal Consumption
This figure displays the relationship between consumption, age and wealth in our model. The consumption over wealth range from $0 to $600,000 at age 40 and 75 are shown in (a), and

the consumption path over age given the wealth level $300,000 and $500,000) are shown in (b).

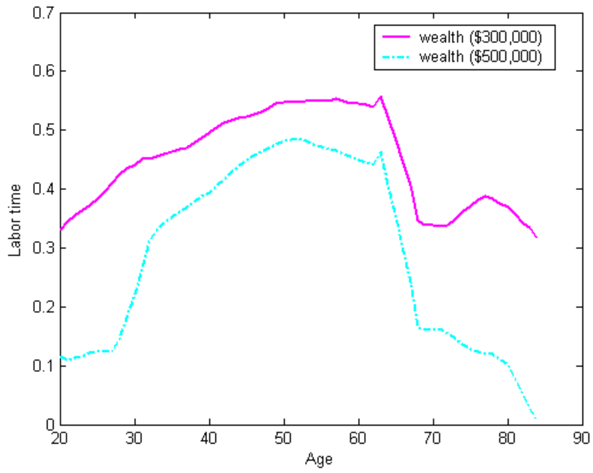
Figure 5-(b) shows the optimal consumption choice over the whole life cycle given wealth level$300,000 and $500,000. The investor's optimal consumption increases in the age of the investor before the investor reaches his late forties and decreases thereafter. It then sharply increases as the investor approaches retirement. The elderly seem to increase his consumption even at advanced ages, which is not consistent with the existing consumption-based model. The reasoning is that as mortality rates rise with age, the probability increases that a consumer will die with substantial wealth. The response is to increase consumption so that wealth declines with age. In the first phase of the life cycle (roughly until age 45), a young investor anticipate higher income in the future, and he is constrained by borrowing and short sales constraints. Therefore, a young investor want to consume, not save, he increases his optimal consumption choice as he ages. And then in middle age, spending reaches a maximum as family size increases and incomes peak. As investor approaches retirement, the mortality rate increases, then the investor increases his consumption choice because he is uncertain about how long he will live. However, there is a bequest motive in our model. In principle, bequests consistent with the life cycle hypothesis because with uncertainty about the date of death people will die with estates, and those estates will go to beneficiaries as accidental bequests. Furthermore, the inferences about a bequest motive are based on how bequests vary by the age of the deceased; yet, because the wealthy live longer than the poor, the wealthy holds precautionary savings to insure him against the risk of living too long. The upward slope of the optimal consumption choice of the wealthy is much smaller than that of the optimal consumption choice of the poor. This pattern is consistent with the existing model of labor income in a finite horizon as derived by Merton (1971). Since we introduce a variation of full labor income over age as in Figure 1, this is how the corresponding variation in optimal consumption is primarily induced. Overall, the above account suggests that investor try to maintain a relatively stable level of consumption.

The Optimal Labor Time Choice

The optimal labor time choice over age and wealth is illustrated in Figure 6. The labor input is inversely associated with wealth level as shown in Figure3-(a). The wealthier investor is more likely to participate in the stock investment, which provides an alternative source of income. The sensitivity of labor input to wealth level is higher for either a young investor or a retiree. This indicates



(a)



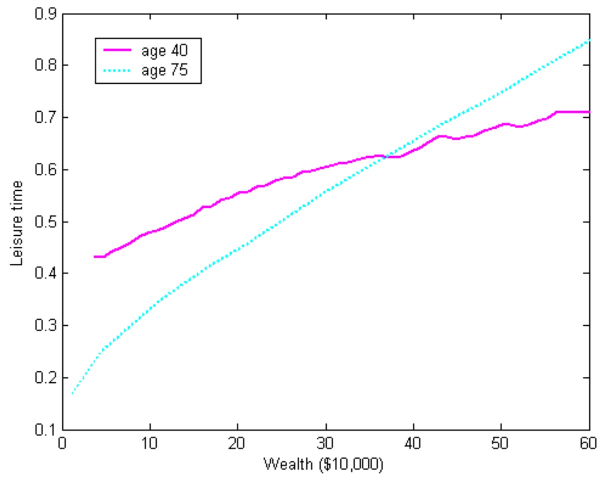
(b)

Figure 6
Optimal Labor Time
This figure displays the relationship between labor time, age and wealth in our model. The labor time over wealth range from $0 to $600,000 at age 40 and 75 is shown in (a), and the labor time path over age given the wealth level $300,000 and $500,000) is shown in (b).

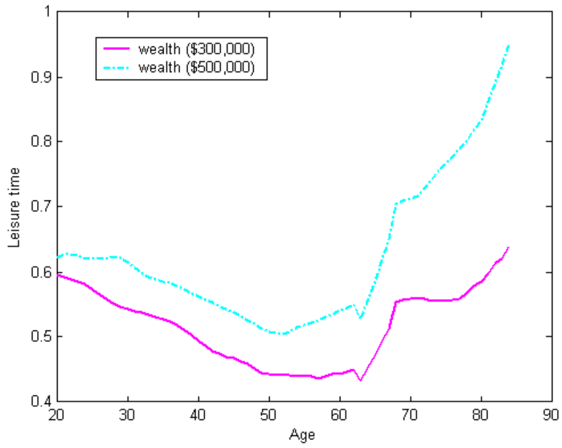
that when an investor suffers from low full labor income, his labor choice becomes more sensitive to his wealth. This result is attributed to the fact that wealthier economic agent is less willing to work for a low full labor income whereas a poor agent has no alternative but to increase his labor time due to limited access to the risky asset market. When his full labor income is high, he wants to fully utilize the higher labor income regardless of his wealth, which narrows the gap of labor input between the wealthy and the poor. This is also well demonstrated in Figure6-(b) in which we plot the optimal labor time choice over age given wealth level$300,000 and $500,000. The gap of these two lines in the first part (roughly until 40 years old) and the last part (after retirement) is wider than that in the middle part. And the inverted U-shape of labor input over age indicts that at the same level of wealth, the optimal labor time initially increases in the age of the investor before the investor reaches his fifties, and sharply decreases thereafter. It then smoothly declines as the investor approaches retirement. The overall pattern of the investor's optimal labor time is very similar to the full labor income. This reflects that the investor's optimal labor time choice is determined primarily by the variation of the full labor income. In addition, the labor input line over age shifts downward as given wealth level increases, which indicts at same age a wealthier investor has less labor input than a poor one.

The Optimal Leisure Time Choice

The optimal leisure consumption is illustrated in Figure 7. Figure 7-(a) shows the investor's optimal leisure time choice at age 40 and 75 as an increasing function of the investor's beginning-of-period net-worth. This indicates that the wealthier an investor, the more time he allocates to leisure. This positive association of leisure time to wealth is more evident for a retiree. At retirement, the opportunity cost of spending time in leisure declines, because during this part, the full labor income reaches its minimum value. Thus, for the same amount of increase in wealth, a retiree spends more time on leisure than a younger agent. In addition, the optimal leisure time represents a U-shaped relationship with age as shown in Figure7-(b). Investors prefer spending more time on labor when their full labor income is higher. The difference in leisure time between the wealthier and the poorer is smaller when the full labor income is higher. Overall these figures look like mirror images of Figure6, which illustrates the optimal labor input. This finding indicates a close substitutability between labor and leisure; at a low level of full labor income, the investor prefers leisure to labor and vice versa.



(a)



(b)

Figure 7
Optimal Leisure Time
This figure displays the relationship between leisure time, age and wealth in our model. The leisure time over wealth range from $0 to $600,000 at age 40 and 75 is shown in (a), and the leisure time path over age given the wealth level $300,000 and $500,000) is shown in (b).

5. Calibration

Our discussion so far has been focused on the optimal decision rules over the state space at different ages. Given the investor's optimal consumption, labor leisure and investment choices defined on the state space, we can obtain time-series profiles of consumption and portfolio choices using simulation. Following the optimal path, the investor's wealth level naturally increases up to a certain age and then decreases due to a decline in full labor income. Table 2 documents a change in wealth distribution for different age ranges in U.S. households. The overall wealth level increases till the age range from 55 to 64, then gradually decreases. Specifically, we first simulate stock return and mutual fund return based on a serially uncorrelated markov process with two outcomes for each variable. We then use the optimal policy rules from our state-space solution to calculate the investor's optimal numeraire good consumption, labor, leisure and portfolio choices. Stock proportions, mutual fund proportions, consumption, labor and leisure time choices are updated each period to determine the investor's optimal decisions for the next period. The time-series profiles of the optimal decisions are generated by repeating the calculation from $t = 0$ (age=20) to $t = 65$ (age=85). We also explore the relative performance of our model to benchmark models. And then we compare the investor's optimal portfolio decisions with the U.S. data such as SCF (2004), SIPP (2000), ATUS (2004), and CES (2004).

We assume that the investor's initially endowed wealth at age 20 is $10,000, which is below the average wealth of economic agents whose age range from twenty to thirty five in SCF (2004). Other baseline parameter values are identical to those set in section 4 as tabulated in Table 1.

To explore the relative performance of our model in explaining the historical data, we consider the following benchmark models:

1. Benchmark I (Quasi-Merton's Model with $\delta = 2$)
   $\delta = 2$
   $F = 0,$ and $e = 0$
2. Benchmark II (Quasi-Merton's Model with $\delta = 15$)
   $\delta = 15$
   $F = 0,$ and $e = 0$
3. Benchmark III (Fixed and Variable Time Cost Model with only Direct Stock Investment)
   $\delta = 2$
   $F = 0.005,$ and $e = 0.06$

Table 2
Percent Distribution of Household net worth by age, Survey of Income and Program Participation

Age of Householder	Percentage of Household Group(%)	Amount of net worth ($) Zero or negative	$1 to $4,999	$5,000 to $9,999	$10,000 to $24,999	$25,000 to $49,999	$50,000 to $99,999	$100,000 to $249,999	$250,000 to $499,999	$500,000 and over
Less than 35	21.37	28.7	15.5	9.8	13.9	11.6	9.2	7.7	2.2	1.3
35 to 44 years	23.62	16.2	9.2	5.6	9.2	12.4	14.8	18.8	8	5.7
45 to 54 years	20.40	11.7	6.9	3.9	6.6	10.2	15.2	22.8	13	9.7
55 to 64 years	13.51	9.2	6.5	2.9	6.1	8.3	13.8	22.9	15	15.3
65 years and over	21.1	6.7	6.6	3	5.2	8.4	17.3	26.5	14.6	11.6
65 to 69 years	5.38	7.1	6.2	2.3	5.3	7.8	17.2	25.1	14.6	14.2
70 to 74 years	5.46	6.8	6	2.9	5.1	7.6	15.5	27	16.3	12.7
75 and over	10.26	6.3	7.1	3.5	5.3	9.2	18.3	27	13.7	9.6

Source: U.S. Census Bureau (2000).

Table 3
Age-average net worth analysis

Age of head (years)	Average(Median) Wealth from SCF	Our Model	Benchmark I	Benchmark II	Benchmark III
Less than 35	$14,200	$16,227	$60,626	$100,800	$11,176
35 to 44 years	$69,400	$63,356	$197,184	$187,600	$63,146
45 to 54 years	$144,700	$12,785	$409,600	$302,900	$133,719
55 to 64 years	$248,700	$21,439	$686,000	$476,200	$238,155
65 to 74 years	$190,100	$26,138	$827,184	$492,400	$292,477
75 years or more	$163,100	$23,731	$775,660	$365,900	$242,400

This table reports the average (median) net worth. The second column shows the historical net worth of U.S. households reported in the 2004 Survey of Consumer Finance (SCF).

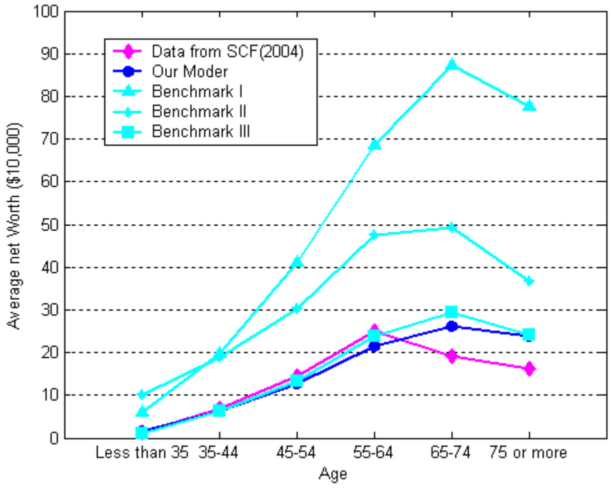


Figure 8
Average net Worth
This figure displays the average wealth of our model, benchmark models, and historical data reported in SCF (2004) with respect to the investor's age

Table 3 and Figure 8 summarize the performance of our model and benchmarks in explaining the optimal net-worth at each age range reported in SCF (2004). The investors' average wealth level p redicted by our model exhibits a hump shape in age, with increasing wealth level until investors reach their retirement and declining wealth level thereafter. This path of th e average wealth level is generally close to the median wealth level at each range of a ge documented in SCF (2004). But the theoretical net-worth slightly deviates from the historical counterpart after retirement owing to the o ver estimation of risky investment predicted by our model, which is generated by the i ncorporating of the mutual fund investment. In contrast, both Benchmark I and II sub stantially overestimate the average wealth levels. Since there is no time cost structure in those two models, as a consequence, the limited participation generated by time co st does not exist. Thus all investors participate in the stock market as shown in Table 9. This overestimated stock investment generated the error calibration of them. In add ition, since the investor is less risk averse in Benchmark I, his risky investment weigh t is higher, which on average results in greater wealth. Benchmark II can substantially reduce the wealth level toward the historical counterpart, but not sufficiently. The ov erall performance of Benchmark III is similar to our model, but its projection after ret irement is worsen.

Table 4
Age-Average consumption analysis

Age of head (years)	Average annual expenditures	Our model	Benchmark I	Benchmark II	Benchmark III
Less than 35	$42,701	$34,510	$32,431	$30,317	$33,253
35 to 44 years	$50,402	$43,600	$46,074	$46,550	$43,493
45 to 54 years	$52,764	$49,817	$58,963	$53,988	$47,906
55 to 64 years	$47,299	$50,497	$76,023	$54,499	$44,281
65 to 74 years	$31,104	$31,988	$63,383	$42,075	$33,981
75 years or more	$25,763	$29,846	$94,011	$40,964	$32,493

This table reports the average optimal consumption over age. The second column shows the consumption level reported in the 2004 Consumer Expenditure Survey (CES). The third column shows the amount of consumption predicted by our model, and the forth, fifth, sixth and seventh column show the consumption by Benchmark I, II, III and IV respectively. The stock investment time cost structure is assumed as follows: $m(t)=[0.005+0.06\cdot\alpha(t)^{1.3}W(t)^{0.5}]\cdot M^{o}(t)$; the mutual fund time cost structure is assumed as $E=0.01\cdot N^{o}(t)$. CRRA is set to 2 in our model, Benchmark I, III and IV but 15 in Benchmark II. And in each model the result is based on the assumption of initial wealth $10,000 at age 20.

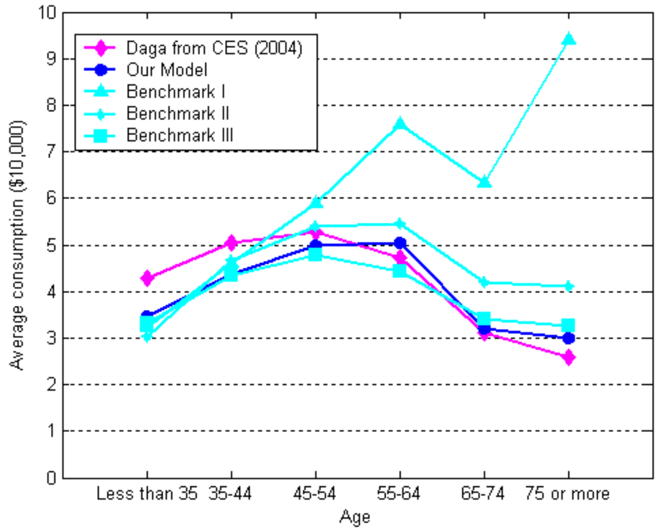


Figure 9
Average Consumption
This figure displays the average consumption of our model, benchmark models, and historical data reported in CES (2004)

Table 4 and Figure 9 present the average optimal consumption data of our model, benchmark models, and historical consumption data reported in CES (2004). The average consumption in our model exhibits an inverted U-shape in age, with increasing consumption until investors reach their fifties and declining consumption as they enter retirement. This is attributed to the time-series behavior of the investor's net-worth and labor income. Obviously, it successfully explains the variation of the optimal consumption reported in CES (2004) over the entire age range. In contrast, other benchmark models have difficulty in fitting the data. In Benchmark II wherein risk aversion is high, the investor's aversion to intertemporal substitution of consumption also increases, and he gradually increases (smoothes) his consumption over time despite a change in his full labor income.

In Table 5 and Figure 10, we present the optimal average labor income of our mode l, benchmark models, and historical labor income data reported in SCF (2004). The a verage labor income of our model exhibits a hump shape in age and reaches a peak m ean value of about $58,380 for economic agents between ages 45 and 55. This is large ly consistent with the SCF data, thus our model performs very well in explaining the observed variation of total income reported in SCF (2004). Even though other models can also capture its overall shape, they overestimate it by a large amount.

Table 5
Age-Average income analysis

Age of head (years)	Average(Median) Income from SCF	Our Model	Benchmark I	Benchmark II	Benchmark III
Less than 35	$32,900	$36,479	$37,749	$37,207	$33,545
35 to 44 years	$49,800	$48,846	$56,668	$47,780	$46,948
45 to 54 years	$61,100	$58,380	$74,826	$64,852	$55,287
55 to 64 years	$54,400	$57,909	$88,187	$60,400	$49,524
65 to 74 years	$33,300	$31,596	$64,139	$23,192	$23,392
75 years or more	$23,700	$26,931	$46,122	$16,955	$22,560

This table reports the average income over age. The second column shows the average income reported in the 2004 Survey of Consumer Finance (SCF).

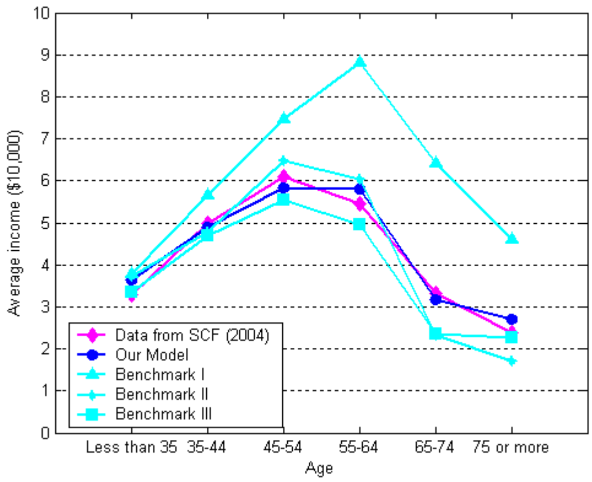


Figure 10
Average Income
This figure displays the average income of our model, benchmark models, and historical data reported in SCF (2004)

Table 6
Age-Average hours per day spent in activities for total population by age

Age of head (years)	All Time Except Subsistence Activities(hours)	Average Leisure (hours)	Percent of Average Leisure(%)	Average Labor (hours)	Percent of Average Labor(%)
Less than 35	9.8	4.4	44.8%	4.71	48%
35 to 44 years	9.82	4.15	42.2%	4.98	50.7%
45 to 54 years	10.13	4.51	44.5%	4.87	48%
55 to 64 years	9.86	5.45	55.2%	3.75	38%
65 to 74 years	8.98	7.31	81.4%	0.7	7.79%
75 years or more					

Source: Bureau of Labor Statistics, 2005
This table reports average hours per day spent in activities such as leisure and labor over age reported in the 2005 American Time Use Survey. The second column shows all the available hours per day except subsistence activities. The third and fourth columns show the leisure hours and percentages in all available time. The fifth and sixth columns show the labors hours and percentages in all available time.

Table 7
Average hours per day worked by employed persons

Employed Person	Average Hours per Day	Percent of Average Labor Time
Total Employed Person	7.59	78%
Full-Time Employed Person	8.08	83%
Part-Time Employed Person	5.58	57%

Source: Bureau of Labor Statistics, 2005
This table shows the average hours per day worked by an employed person reported in the 2005 American Time Use Survey. This employed person's labor hours are higher than the overall average in Table 6. Estimates of labor time in our model are usually higher than historical labor times for all ranges of age. However, if we consider only employed persons, our result is more consistent with historical data.

Table 8
Leisure and labor time analysis

Age of Householder	Percent of Leisure Time in a Day			
	Our model	Benchmark I	Benchmark II	Benchmark III
Less than 35	39%	38%	35%	40%
35 to 44 years	38%	41.3%	42%	39%
45 to 54 years	38%	44.4%	41.3%	36%
55 to 64 years	40%	46%	43%	35%
65 to 74 years	50%	79%	65%	55.4%
75 years or more	50%	99%	69%	54%

Age of Householder	Percent of Labor Time in a Day			
	Our model	Benchmark I	Benchmark II	Benchmark III
Less than 35	60%	61%	65%	59%
35 to 44 years	60%	58%	57.5%	60%
45 to 54 years	60%	55%	58%	64%
55 to 64 years	59%	53%	56%	64.5%
65 to 74 years	44%	15%	34%	31%
75 years or more	45%	0.02%	30%	32%

This table reports the leisure and labor time over age. The upper table shows the percentage of leisure time and the lower table shows the percentage of labor time. In both tables, the second column is the time predicted by our model. The third, forth, fifth, and sixth columns show the leisure and labor times from Benchmark I, II, III and IV respectively. The stock investment time cost structure is assumed as follows: $m(t) = [0.005 + 0.06 \cdot \alpha(t)^{1.3} W(t)^{0.5}] \cdot M^o(t)$; the mutual fund time cost structure is assumed as $E = 0.01 \cdot N^o(t)$. CRRA is set to 2 in our model, Benchmark I, III and I II but 15 in Benchmark II. And in each model the result is based on the assumption of initial wealth $10,000 at age 20.

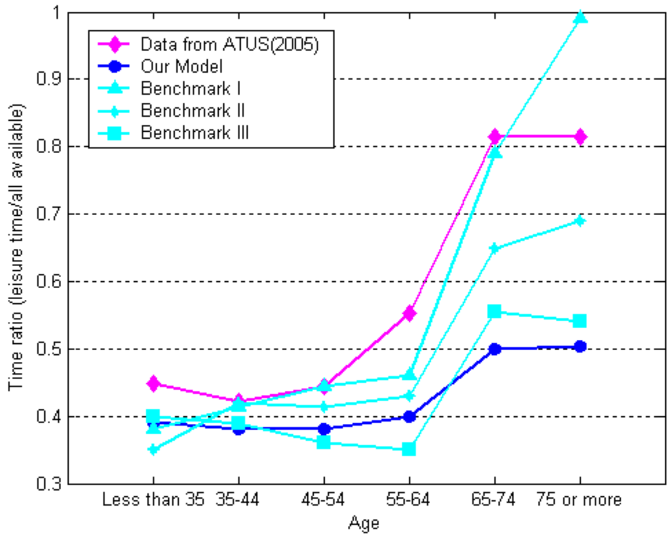


Figure 11
Average Leisure Time
This figure displays the average leisure time with respect to the investor's age. Each line indicates the leisure time per unit time from historical data provided by ATUS (2005), our model, benchmark I, II, and III.

Then we investigate the average labor and leisure time choices. Table 6 summarizes the average hours per day spent in leisure and labor of U.S. households reported in ATUS (2005). In Table 8 and Figure 11 and 12, we present the average labor and leisure time choices of our model and benchmark models. We find that the average labor time declines as the investor ages, but the average leisure time increases as the investor ages. Our model can capture the overall pattern of labor and leisure time over the entire age range better than other benchmarks, but it is outperformed by other benchmark models. However, this result is misleading since the data reported in ATUS (2005) is based on the entire population including the unemployed. Table 8 reports the average hours worked per day reported by only the employed in ATUS (2005). Comparing these features to those in Table 7 insinuates that the labor hours of the employed are much greater, and consequently leisure time consumed by them must be lower. Therefore, we can conjecture that our model may be more consistent with historical data when we focus only on the employed.

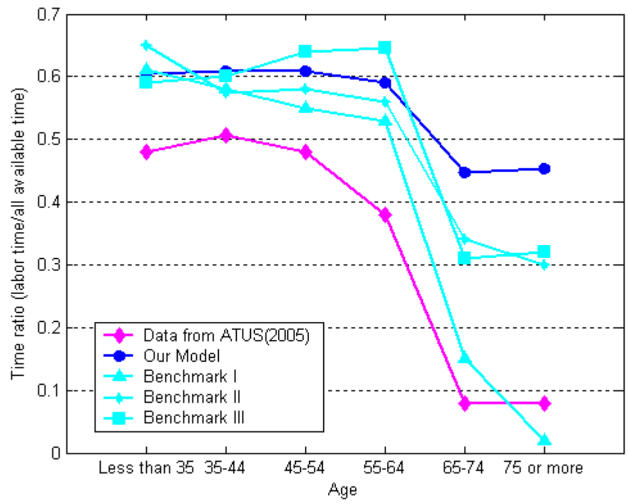


Figure 12
Average Labor Time
This figure displays the average labor time with respect to the investor's age. Each line indicates the leisure time per unit time from historical data provided by ATUS (2005), our model, benchmark I, II, and III.

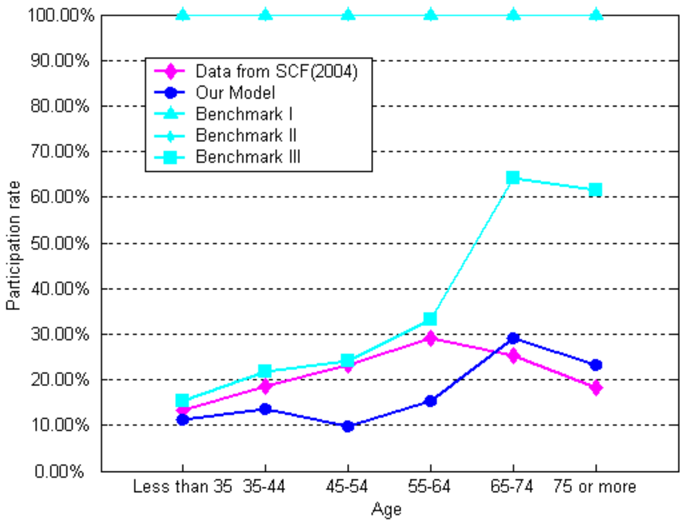


Figure 13
Stock Market Participation Rate
This figure displays the stock market participation rate with respect to the investor's age. Each line indicates participation rates of historical data reported in SCF (2004), our model, benchmark I, II, and III.

Table 9
Age-Stock Market Participation Rate

Age of head (years)	Participation rate in stock from SCF (2001)	Participation rate in stock and mutual fund from SIPP(2000)	Participation rate of our model	Participation rate of Benchmark I	Participation rate of Benchmark II	Participation rate of Benchmark III
Less than 35	13.3%	18.4%	11.2%	100%	100%	15.43%
35 to 44 years	18.5%	26.9%	13.7%	100%	100%	21.8%
45 to 54 years	23.2%	31.3%	9.7%	100%	100%	24.23%
55 to 64 years	29.1%	32.3%	15.3%	100%	100%	33.3%
65 to 74 years	25.4%	29%	29%	100%	100%	64.24%
75 years or more	18.4%	29.8%	23.3%	100%	100%	61.46%

This table presents the participation rate in the stock market average (median) net worth. The second column and third column show the participation rate provided by SCF (2001) and SIPP (2000). Data from SCF (2001) accounts only for `stock' participation ratio, whereas data from SIPP (2000) accounts for `mutual funds' as well as stock investment.

An important feature of portfolio choices is that a large fraction of individuals do n ot own stocks. Table 9 and Figure 13 documents the performance of our model in exp laining the stock participation rate at each age range reported in SCF (2004). Our mod el can basically fit the historical stock participation rate. Therefore, the theoretical sto ck participation rate deviates from the historical counterpart at age range from 45 to 6 0. This calibration error is owing to the incorporating of mutual fund investment. At h igh full labor income, investors tend to spend more time on labor, so they prefer mutu al fund investment than stock investment. In summary, our model captures the overall observed stock participation rate except for the age range from 45 to 60. In contrast, both Benchmark I and Benchmark II dictate 100 percent participation, which conflict s with the data. Benchmark II is able to generate the average risky investment weight given the equity premium. However, it does so under the condition that every single i nvestor participates in the stock market. Put differently, it does not generate cross-sectional heterogeneity across investors to be consistent with the data. Besides the we ll-known "risk-free rate puzzle", this is another critical drawback questioning the plausibility of the model. Benchmark III performs well before retirement, but is dramatically deviates fr om the historical counter after retirement, which is generated by the overestimation of risky investment for retiree.

Overall the classical models such as Benchmark I and II which do not incorporate any time cost of risky investment, suffer from an innate drawback; they do not simultaneously fit consumption, wealth and income level observed in the data. For example, if we increase the risk aversion of an investor, we can reduce risky investment, but simultaneously increase his incentive to cumulate in wealth more aggressively. Thus young economic agents consume less relative to the observed data. Alternatively if we change the risk aversion parameter or full labor income to fit the observed consumption, the model deviates more from the observed income and wealth. More importantly, these models are deficient of an ability to generate limited participation so that they cannot reproduce the observed heterogeneity across investors in their participation and stock holding over age.

Benchmark III introduced the fixed and variable cost for the stock investment. It is based on a constant full labor income unlike the calibration exercise to sort out the net impact of cost structures on participation and stock investment. In this model, variable time costs alone are able to engender limited participation. However, Benchmark III overestimated the risky investment for retirees, and this overestimation results in an increase in wealth and consumption due to the high expected return on the stock.

In our model, we incorporated the mutual fund investment and explored the impact of the relationship between stock and mutual fund investment on the optimal decisions. We find that mutual fund has a negative effect on investors' stock market participation decision. Be different from Benchmark III, this effect made our model perform very well in explaining the stock participation rate after retirement. An investor who can also invest in mutual fund market lowers the stock proportion in his net worth by substituting risky stocks by mutual funds to control the overall risk exposure. In the meantime, he also raises the equity proportion in his liquid financial portfolio to take advantage of the diversification benefit. Thus, our model slightly overestimates the wealth level after retirement. Even then, our model still outperforms Benchmark I, II and III.

6. Conclusion

In this paper, we analyzed the optimal dynamic consumption, leisure, and portfolio decisions for an investor who can invest in the stock market directly (through stocks) and indirectly (through mutual funds). Our results indicate that the investor optimally chooses to have more stock proportions than mutual fund proportions until the investor reaches his forties and then chooses to have more mutual fund proportions than stock proportions during the rest of his life. This roughly supports and rationalizes the in vestment advice given by popular finance books and financial counselors. Through mutual funds an investor can dramatically reduce the time cost associated with managing his stock investment. In such a case, he should pay managing fees for delegating his stock investment. Consequently, he trades the time cost for management fees. In equilibrium, this creates a clientele effect. If in 20s and 30s an investor has lower full labor income, opportunity time cost associated with stock investment is smaller than the management fees, thus he prefers to invest in stock market directly. In middle age, the full labor income reaches its peak, opportunity time cost increases and is greater than the management fees. After retirement, though the full labor income is cut in half, the investor can obtain more leisure utility from the same leisure time, opportunity time cost is still greater than the management fees, so he still prefers to invest in the stock market indirectly.

Mutual fund ownership has a significant impact on the investor's portfolio decision. The investor who can also invest in the stock market indirectly through mutual funds substitutes risky mutual funds for risky stocks in his net worth, yet increases the equity proportion in his liquid financial portfolio to take advantage of the diversification benefit.

An important goal of this paper, as stated before, was to compare model life cycle profiles with the historical data reported from SCF (2004), SIPP (2000), ATUS (2004), and CES (2004). Detailed analysis of the cross-sectional dispersion of consumption, wealth, income, labor time and leisure time shows that incorporating mutual fund investment the investor's theoretical portfolio choices under time constraints are closely consistent with the historical data. In contrast, benchmark models which are similar to the classical consumption-based models such as Merton (1971) and Luca (1976), do not incorporate any time cost of risky investment; they fail to simultaneously capture the aforementioned behavior of investors documented in the data. Most importantly, these models cannot generate limited participation. Consequently, they cannot reproduce the observed heterogeneity across investors in their participation and stock holding over age. Even though, Benchmark III which is based on the time cost of risky investment such as Ahn, D, In Joon Kim and S. Yoon (2006) approximately fit the historical data; without the possibility of indirect investment in the stock market through mutual funds it overestimates the participation rate and wealth level after age 55.

References

Abel, A., 2001, “The Effects of investing Social Security Funds in the Stock Market When Fixed Costs Prevent Some Households from holing Stocks,” *American Economic Review*, 91, 129-148.

Ahn, D. and S. Yoon, “Asset Pricing with Time Costs of Risky Asset Management and Predictable Stock Returns,” *Work-in-Process*, Seoul National University.

Ahn, D., Kim I. and S. Yoon, “Time Costs of Risky Asset Management: Dynamic Portfolio Choice and Limited Participation,” *EFA 2006 Zurich Meetings Paper*

Aizcorbe, A., A. Kennickell, and K. Moore, 2003, “Recent Changes in U.S. Family Finances: Evidence from the 1998 and 2001 Survey of Consumer Finances,” *Federal Reserve Bulletin, Survey of Consumer Finance(SCF)*, 89, 1-32.

Ameriks, J. and S. Zeldes, 2004, “How do Household Portfolio Shares Vary with Age?,” *working paper.*

Anderson, R., 2001, “United State Life Tables 1998,” *National Vital Statistics Reports*, 48, 1-40.

Basak, S. and D. Couco, 1998, “An Equilibrium Model with Restricted Stock Market Participation,” *Review of Financial Studies*, 11, 771-792.

Becker, G., 1965, “A Theory of the Allocation of Time,” *Economic Journal*, 75, 493-517.

Bodie, Z., C. Merton and W. Samuelson, 1992, “Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model,” *Journal of Economic Dynamics and Control*, 16, 427-449.

Brav, A., G. Constantinides and C. Geczy, 2002, “Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence,” *Journal of Political Economy*, 110, 793-824.

Breeden, D., 1979, “An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities,” *Journal of Financial Economics*, 7, 265-296.

Cao, H., T. Wang and H. Zhang, 2005, “Model Uncertainty, Limited Participation and Asset

Prices," *Review of Financial Studies*, 18, 1219-1251.

Claus, J., and Thomas, 2001, "Equity Premia as Low as Three Percent? Evidence from Analysts' Earnings Forecasts for Domestic and International Stock Markets," *Journal of Finance* 56, 1629-1666.

Cocco, J., F. Gomes and P. Maenhout, 2005, "Consumption and Portfolio Choice over the Life-Cycle," *Review of Financial Studies*, 18, 491-533.

Constantinides, G., 1986, "Capital Market Equilibrium with Transaction Costs," *Journal of Political Economy*, 94, 842-862.

Constantinides, G., J. Donaldson and R. Mehra, 2002, "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle," *Quarterly Journal of Economics*, 117, 269-296.

Cox, J. and C. Huang, 1989, "Optimal Consumption and Portfolio Policies when Asset Prices Follow a Diffusion Process," *Journal of Economic Theory*, 49, 33-83.

Curcuru, S., J. Heaton, D. Lucas and D. Moore, 2004, "Heterogeneity and Portfolio Choice: Theory and Evidence," *Working Paper*, University of Chicago and NBER.

Dammon, R., C. Spatt and H. Zhang, 2001, "Optimal Consumption and Investment with Capital Gains Taxes," *Review of Financial Studies*, 14, 583-616.

Dammon, R., C. Spatt and H. Zhang, 2004, "Optimal Asset Allocation with Taxable and Tax-Deferred Investing," *Journal of Finance*, 59, 999-1037.

Dumas, B. and E. Luciano, 1991, "An Exact Solution to a Dynamic Portfolio Choice Problem under Transaction Costs," *Journal of Finance*, 46, 577-595.

Davis, M. and Norman, A., 1990, "Portfolio Selection with Transaction Costs," *Mathematics of Operations Research*, 15, 676-713.

Epstein, L., S. Zin, 1989, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937-969.

Fama, E. F., and K. R. French, 2002, "The Equity Premium," *Journal of Finance,* 57, 637-659

Gomes, F. and A. Michaelides, 2005, "Asset Pricing with Limited Risk Sharing and

Heterogeneous Agents," *Working Paper*, London Business School.

Gomes, F. and A. Michaelides, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance*, forthcoming.

Gronau, R., 1977, "Leisure, Home Production, and Work-the Theory of the Allocation of Time Revisited," *Journal of Political Economy*, 85, 1099-1124.

Guiso, L., M. Haliassos, and T. Jappelli, 2003 "Household Stockholding in Europe: Where do We Stand and Where do We Go?" *Economic policy*, 36, 123-170.

Hansen, L., and K. Singleton, 1982, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica*, 50, 1269-1286.

Hansen, L., and R. Jagannathan, 1991, "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy*, 99, 225-262.

Heckman,J., 1974, "Shadow Prices, Market Wages, and Labor Supply," *Econometrica*, 42, 679-694.

Heaton, J. and D. Lucas, 2000, "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk," *Journal of Finance*, 55, 1163-1198.

Kleven, H., 2004, "Optimum Taxation and the Allocation of Time," *Journal of Public Economics*, 88, 545-557.

Lew, D., and D. Larson, 2005, "Accounting for Stochastic Shadowvalues of Time in Discrete Choice Recreation Demand Models," *Journal of Environmental Economics and Management*, 50, 341-361.

Mankiw, N., S. Zeldes, 1991, "The Consumption of Stockholders and Non-Stockholders," *Journal of Financial Economics*, 29, 97-112.

Mehra, R. and E. Prescott, 1985, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145-161.

Menahem E. Yaari, 1965, "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *The Review of Economic Studies*, Vol. 32, No. 2., pp. 137-150

Merton, R., 1971, "Optimal Consumption and Portfolio Rules in a Continuous Time Model," *Journal of Economic Theory*, 3, 373-413.

Merton, R., 1973, "An Intertemporal Capital Asset Pricing Model," *Econometrica*, 41, 867-887.

Merton, R., 1992, Continuous-Time Finance, *Blackwell Publishers*, Cambridge, MA

Nils H. Hakansson, 1969, "Optimal Investment and Consumption Strategies Under Risk, an Uncertain Lifetime, and Insurance," *International Economic Review*, Vol.10, No. 3., pp. 443-466.

Nils H. Hakansson, 1970, "Optimal Investment and Consumption Strategies Under Risk for a Class of Utility Functions," *Econometria*, Vol. 38, No. 5., pp. 587-607

Paul A. Samuelson, 1969, "Lifetime Portfolio Selection By Dynamic Stochastic Programming," *The Review of Economic and Statistics*, Vol. 51, No. 3. , pp. 239-246

Payne, J. and J. Bettman, 1996, "When Time is Money: Decision Behavior under Opportunity Cost Time Pressure," *Organizational Behavior and Human Decision Processes*, 66, 131-152.

Prescott, M., 1986, "Theory Ahead of Business Cycle Measurement," *Carnegie-Rochester Conference Series on Public Policy*, 25, 11-44.

Robert C. Merton, 1969, "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case," *The Review of Economic and Statistics*, Vol. 51, No. 3. , pp. 247-257

Santos, T. and P. Veronesi, 2006, "Labor Income and Predictable Stock Returns," *Review of Financial Studies*, 19, 1-44.

Shelley, K., 2005, "Developing the American Time Use Survey activity classification system." *Monthly Labor Review*, June 2005, 3-15.

Shiller, R., 1990, "Market Volatility and Investor Behavior," *American Economic Review*, 80, 58-62.

U.S. Census Bureau, 2003, "Net Worth and Asset Ownership of Households: 1998 and 2000," *Survey of Income and Program Participation(SIPP)*, Household Economic Studies.

U.S. Department of Labor, U.S. Bureau of Labor Statistics, 2005, "Consumer Expenditures in

2003," *Consumer Expenditure Survey(CES).*

U.S. Department of Labor, U.S. Bureau of Labor Statistics, 2005, "American Time Use Survey-2004 Results Announced by BLS," *American Time Use Survey(ATUS).*

Vissing-Jorgensen, A., 2002a, "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*, 110, 825-853.

Vissing-Jorgensen, A., 2002b, "Towards an Explaination of Household Portfolio Choice Heterogeneity: Non¯nancial Income and Participation Cost Structure," *Working Paper*, North-western University.

Weil, P., 1989, "The Equity Premium Puzzle and the Risk-Free Rate Puzzle," *Journal of Monetary Economics*, 24, 401-421.

Yao, R. and H. Zhang, 2005, "Optimal Consumption and Portfolio Choice with Risky Housing and Borrowing Constraints," *Review of Financial Studies*, 18, 197-239.