Admissibility Conditions in Risk Adjustment of Momentum Strategies

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Abstract

There are still ongoing debates on profitability of momentum strategies mainly due to its sensitivity to the choice of benchmark asset pricing model in its risk adjustment. If we assume an ideal benchmark, we can prove that the risk-return tradeoff still hold on average between winners and losers. And we use these admissibility conditions as diagnostic tool in risk adjustment of momentum strategies. Empirically, we examine if each benchmark satisfies our admissibility conditions.

1 Introduction

If there are momentums in stock prices, short selling losers and buying winners will be a profitable trading strategy. At first Jegadeesh and Titman(1993) made this momentus portfolios and assessed the performance using CAPM benchmark and argued that there are abnormal profits under these trading strategies. Meanwhile, Ahn, Conrad, and Dittmar (2003) proposed a non-parametric benchmark using 20 industry sorted portfolios as basis assets instead of a specific parametric benchmark. They explained almost all of the residual excess returns of momentum strategies.

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In this paper, we mainly focus on the industry momentum. Specifically, we construct industry momentum portfolios based on the 17 industry sorted portfolios. For the CAPM benchmark, we derive stochastic discount factor by using the return of market index and for the non-parametric benchmark, we set 17 industry sorted portfolios as basis asset. We can get the risk adjusted return of momentumportfolio by using these stochastic discount factors, namely alpha using the analogy of Jensen's alpha. The CAPM beta of each industry represents the sensitivity of the return of each industry to the market portfolio return and non-parametric risk measure represents the sensitivity to the return of hedge portfolio which we will define in Chapter 4.

In comparing the profitability assessment of our momentum portfolios (based on 17 industry sorted portfolios) against both benchmarks, we can identify the large difference as in the literature. Under the conditional CAPM benchmark, we find that 15 out of 16 strategies are profitable at the 5% significance level. However, we find only 2 out of 16 strategies are profitable against the conditional LOP(Law of One Price) non-parametric benchmark at the same significance level . Joint test of all 16 strategies reject the null hypothesis of no excess return at the 5% significance level against CAPM benchmark. But the joint test fails to reject the null hypothesis against conditional non-parametric benchmark.

In this paper, we focus whether the risk adjustment under each benchmark is admissibile. We proved that on average, the risk measure of an ideal benchmark represents the risk-return tradeoff between winners and losers. We set this as admissibility conditions and use it as a diagonostic tool for each benchmark. Empirically, we find that CAPM benchmark does not satisfy admissibility conditions. This means the market return does not span the payoffs from momentum strategies based on the returns of our basis assets. Meanwhile, the non-parametric risk measure satisfies these conditions.

The rest of this paper is outlined as follows. In Section 2, we describe the data. In Section 3 we briefly summarize the results of the performance assessments of ours and existing studies both against the

CAPM benchmark and the non-parametric benchmark. In Section 4 we formalize the theoretical link between past return information and risk. In Section 5 we simulate the relationship between the probability of an industry becoming an extreme performer and the ideal risk measure. In Section 6 we examine if each risk measure satisfies our admissibility condition. In Section 7 we discuss the interpretation of these results and briefly summarize.

2 Data

2.1 Data

In this paper, we use the return data of the equally weighted 17 and 48 U.S. industry-sorted portfolios to get the payoffs from the industry momentum trading strategies. In particular, we use 17 industry sorted portfolios as a basis assets for the non-parametric estimation. We obtained these data from Ken. French's data library over the sample period from 31 December 1962 to 28 February 2006, yielding 519 monthly observations. We use the risk free rate and market index of the same data source. We get the conditional factors, such as term spread and default spread from the Federal Reserve report H.15 over our sample periods; and dividend yield from Datastream International. Because dividend yield data starts from 28 February 1965, we make the momentum payoffs start from this point in time.

3 Sensitivities of Performance Assessment Against Each Risk Measure

3.1 Performance Assessments of Momentum Strategies

We implemented momentum portfolios on the 17 industry sorted portfolios. Our assessment of the performance against the CAPM benchmark is 83 basis points per month for the same strategy. When we used the conditional CAPM benchmark, 83 basis points abnormal returns still remain per month.

Ahn, Conrad and Dittmar (2003) set 20 U.S. industry sorted portfolios as basis asset and constructed non-parametric performance measures for the returns of momentum strategies based on the returns of individual stocks. From their unconditional and conditional LOP estimation, we can get α which represents average monthly excess performance over the returns of basis assets. We can also get p-values from a chi-squared test of the hypothesis of $H_0 : \alpha = 0$.

Like Ahn, Conrad and Dittmar (2003), we allowed investors to condition their positions on public information and assess the performance against conditional performance measure. In Ahn, Conrad and Dittmar's (2003) case, none of the 16 trading strategies earn a significant abnormal return under this conditional performance measure. In our case, two strategies continue to exhibit abnormal performance at the 5% significance level. For these two strategies, the abnormal profits fall, on average, by 46.7 basis points or 46% relative to the CAPM risk-adjusted profits. Table 1 shows our test results of conditional LOP (law of one price) estimation. The differences between these test results are due to the sample period and the characteristics of assets.

And we jointly tested the null hypothesis of no abnormal return of 16 momentum strategies. The momentum strategies are constructed by overlapping the return of momentum portfolios and this cause the serial correlation. We adjusted this serial correlation by Newey-West method in constructing F-test statistics. We can identify that the null hypothesis against unconditional and conditional non-parametric benchmark cannot be rejected at the 5% significance level. But the null hypothesis is rejected against unconditional and conditional CAPM benchmark.

In summary, we can identify the same large difference as in the literature on the profitability assessment of momentum strategies. Like the results of existing literature, our performance assessment on momentum strategies is highly sensitive to the benchmark. This situation require a criterion on which we can judge the credibility of each assessment. So, we develop a simple theoretical model of the risk measure based on a good benchmark.

4 The Risk Characteristics of Extreme Performing Industry

4.1 Return and Economic Risk

It is well known from Cochrane (2001) that the fundamental valuation equation can be written as:

$$E[d_{t+\tau}R_{i,t+\tau} \mid \mathcal{F}_t] = 1 \;\forall\; i, \tau. \qquad \text{(Eq.2.1)}$$

where, $R_{i,t+\tau}$ is gross return. All the stochastic discount factors of pricing models should satisfy the relationship that fundamental valuation equation represents. We can get this discount factor $d_{t+\tau}$ by using Hansen's (1982) Generalized Method of Moments(GMM). Unlike a specific parametric CAPM model, the performance evaluations of non-parametric model are based on the simple conditions. These measures are based on the minimum market equilibrium conditions such as law of one price(LOP) hypothesis. We can easily extend the non-parametric measure to the conditional measure which considers explicitly the time varying property of risk premium as argued by Chen and Knez(1997).

From Eq.2.1, we can get the return of the hedge portfolio or the constant consumption portfolio which is perfectly correlated with the stochastic discount factor d_t. Suppose $d_t = k_0 + k_1 R_d$ where R_d is the return on hedge portfolio and $k_1 \neq 0$. Since stochastic discount factor is an affine function of R_d, it is completely correlated with the return of hedge portfolios. The first and second moments of stochastic discount factor can be written as:

$$E(R_d) = \frac{E(d_t) - k_0}{k_1}, \quad \sigma^2_{R_d} = \frac{\sigma^2_d}{k_1^2} \qquad \text{(Eq.2.2)}$$

We can rewrite the fundamental valuation equation, Eq.2.1 as:

$$\begin{aligned} E(R_i) - \frac{1}{E(d_t)} &= -\frac{1}{E(d_t)} Cov(d_t, R_i) \qquad \text{(Eq.2.3)} \\ &= -\frac{1}{E(d_t)} k_1 \beta_{di} \sigma^2_{R_d} \end{aligned}$$

In Eq.2.3, $\beta_{di} = Cov(R_d, R_i)/\sigma^2_{R_d}$ represents the sensitivity measure of R_i with respect to R_d. The return of a hedge portfolio should also satisfy the fundamental valuation equation, we obtain the following relations.

$$E(R_d) - \frac{1}{E(d_t)} = -\frac{1}{E(d_t)} k_1 \sigma^2_{R_d} \qquad \text{(Eq.2.4)}$$

Because of $\beta_{dd} = 1$ in Eq.2.4. So, from Eq.2.3 and Eq.2.4, we can rewrite fundamental valuation equation as:

$$E[R_i] - R_f = \beta_{di}(E[R_d] - R_f) \qquad \text{(Eq.2.5)}$$

We write the sensitivity of each industry sorted portfolio to this return of the hedge portfolio as β_{di}. We derive the stochastic discount factor that constitutes risk factor from the market data itself.

4.2 Theoretical Model

To examine the risk characteristics of extreme performing industries, we assume the genuine risk measures of the 20 hypothetical industries such as:

$$\theta = (\beta_{d1}, \beta_{d2}, \cdots, \beta_{d19}, \beta_{d20})' \; where \; \beta_{d1} < \beta_{d2} < \cdots < \beta_{d19} < \beta_{d20} \qquad \text{(Eq. 3.1)}$$

Without loss of generality, we can set $\lambda_t \sim N(\overline{\lambda}, \sigma^2_\lambda)$ and $Var(\varepsilon_{it}) = \sigma^2_\varepsilon \; \forall i = 1, ..., 20$ as the distribution of risk premium and idiosyncratic risk. So, we can describe the joint distribution of industry returns as:

$$(R - R_f) \sim MVN_{20}(\theta\overline{\lambda}, \theta\theta'\sigma^2_\lambda + \sigma^2_{\varepsilon i} I) \qquad \text{(Eq.3.2)}$$

where, the volatility of the risk premium $\sigma_\lambda = \sigma_{R_d}$ and I denotes the identity matrix. We can define the probability of a particular industry being selected as an extreme performer as:

$$\Pr^w_i = \text{Prob}[industry \; i \in W]$$

Of the 20 industries, we define the top two industries as the winners and the bottom two industry as the losers. Clearly, the probability of an industry being selected as extreme performers will depend both on the return of that industry and the threshold value which is used as a criterion of the selection of extreme performers. $\delta_t^w(\lambda_t)$ denotes the threshold value which characterizes an industry as winners at time t. When the market moves in a positive direction and the risk premium has a positive value, $\delta_t^w(\lambda_t)$ is the $19th$ highest return when industries ranked in order of their return value at time t. Then we can define the probability of an industry being selected as a winner as:

$$\begin{aligned}
\Pr_i^w =& \text{Prob}[\beta_{di}\lambda_t + \varepsilon_t \geq \delta_t^w(\lambda_t) \ and \ \lambda_t > 0] \qquad \text{(Eq. 3.4)} \\
& where, \\
\delta_t^w(\lambda_t) =& 19th \ of \ Sort(R_{1t}, R_{2t}, ..., R_{19t}, R_{20t}) \\
=& 19th \ of \ Sort(\beta_{d1}\lambda_t + \varepsilon_{1t}, \beta_{d2}\lambda_t + \varepsilon_{2t}, ..., \beta_{d19}\lambda_t + \varepsilon_{19t}, \beta_{d20}\lambda_t + \varepsilon_{20t})
\end{aligned}$$

This definition also applies to the case where the market moves in a negative direction. In this case, the $19th$ highest return appears to be the threshold value. If we suppose that there is no idiosyncratic risk, the industries which have β_{di} of $1st$ and $2nd$ of θ will be selected as winners. So, the value of risk premium at each point in time determines which industries are included in the winner category. In any case, the $19th$ highest return will appear to be the threshold value of becoming a winner. The threshold value of the portfolios of winners can be:

$$\delta_t^w(\lambda_t) = \delta_t^{w0} = \begin{Bmatrix} \beta_{d19}\lambda_t & if & \lambda_t \geq 0 \\ \beta_{d2}\lambda_t & if & \lambda_t < 0 \end{Bmatrix} \qquad \text{(Eq. 3.6)}$$

In cases more relevant to the reality, where $\sigma_\varepsilon \neq 0$, we can write the threshold values as:

$$\delta_t^w(\lambda_t) = \delta_t^{w0} + f(\beta_{di}, \lambda_t, \sigma_\varepsilon) \qquad \text{(Eq.3.7)}$$

If the number of groups are relatively small, $f(\cdot)$ will have a second order effect in Eq.3.7. Since we divide 20 industries into 10 groups and the number of groups is relatively large compared to the whole industries, the risk measure of each industry and the risk premium at each time and the distributions of each industry's idiosyncratic risk will simultaneously influence the threshold value at each time. We can treat the threshold value of winner or loser as a stochastic variable. There are some needs to modify the constant volatility hypothesis of idiosyncratic risk according to the characteristics of industries. Particularly, Brown, Goezmann, Ibbotson, Ross (1992) suggested that the distribution of unsystematic risk depends on the risk measure itself. Following their discussion, we define $\sigma_{\varepsilon i}$ instead of σ_{ε} as $\sigma_{\varepsilon i} = \delta(\beta_{di} - \overline{\beta})^2 + \sigma$, where δ and σ are constant. In this case, not only the systematic risk but also the idiosyncratic risks influence the threshold value of being selected as a winner. For reference, we can empirically estimate this relationship by using 48 U.S. industry-sorted portfolios:

$$\begin{aligned} \widehat{\sigma}^2 &= 0.0046 + 0.001(\beta_{di} - \overline{\beta})^2 \\ t - value &: 14.78,\ 2.18,\ R^2 = 0.094,\ N = 48 \end{aligned}$$

The risk measures in this estimation are the sensitivity of the return of 48 industry-sorted portfolios to the return of the hedge portfolios based on 17 industry-sorted portfolios. Either the idiosyncratic risk of each industry follows the same distribution or is different from one industry to another, and change in value at every point in time, but the influences will be converged to zero on average.

Over the sample periods, β_{di} is given fixed to all industries but risk premium has the distribution of $\lambda_t \sim N(\overline{\lambda}, \sigma_{\lambda}^2)$. There is a need to consider the change of risk premium at every point in time. Moreover, if we consider the existence of idiosyncratic risk, it is not enough to classify the winner or loser by the systematic risk criterion alone. Therefore, the probability of an industry becoming an extreme performer depends on the unexpected part of industry return. The unexpected part of industry return is related to the variation of both idiosyncratic risk and risk premium at every point in time. The probability of a certain industry being included in the winner category is defined as:

$$\Pr_i^w \approx \text{Prob}[\varepsilon_{it} > \eta_i(sign(\lambda_t))\lambda_t] \qquad \text{(Eq. 3.8)}$$
$$\eta_i(sign(\lambda_t)) = \begin{Bmatrix} \beta_{d19} - \beta_{di} & if & \lambda_t \geq 0 \\ \beta_{d2} - \beta_{di} & if & \lambda_t < 0 \end{Bmatrix}$$

In mathematical form, the probability of being included in winner category can be represented as following Definition 1.

Definition 1 *Let $F_{\varepsilon i}$, F_λ and $F_{-\lambda}$ is cumulative distribution functions of ε_{it}, λ_t, and $-\lambda_t$. $G_{\varepsilon i} = 1 - F_{\varepsilon i}$ is reverse cumulant of ε_{it} and $\phi(\cdot)$, $\Phi(\cdot)$ are standard normal probability density function and cumulative density function. The approximated probability of being winner can be written as:*

$$\begin{aligned} Pr_i^h &\approx Probability[\varepsilon_{it} > \eta_i(sign(\lambda_t))\lambda_t] \qquad (Eq.3.9) \\ &= \int_0^\infty G_{\varepsilon i}[(\beta_{dN} - \beta_{di})u]dF_\lambda(u) + \int_0^\infty G_{\varepsilon i}[(\beta_{di} - \beta_{d1})v]dF_{-\lambda}(v) \\ &= \frac{1}{\sigma_\lambda}\int_0^\infty \Phi\left(\frac{-(\beta_{dN} - \beta_{di})u}{\sigma_{\varepsilon i}}\right)\phi\left(\frac{u - \overline{\lambda}}{\sigma_\lambda}\right)du \\ &+ \frac{1}{\sigma_\lambda}\int_0^\infty \Phi\left(\frac{-(\beta_{di} - \beta_{d1})u}{\sigma_{\varepsilon i}}\right)\phi\left(\frac{u + \overline{\lambda}}{\sigma_\lambda}\right)du \end{aligned}$$

Proof. For more complete proof, see Sam Ho Son's Phd. dessertation paper(SNU. 2007). ■

In Figure 1, we simulated the return of assets under particular set of parameters and calculate the probability of being chosen a winner. We can find a good approximation in that figure.

We formalize the probability of an asset being included in winners under two assumptions. Firstly, we consider the idiosyncratic risk that follows the probability distribution of $\varepsilon_t \sim N(0, \sigma_\varepsilon)$. Secondly, we consider the case where the idiosyncratic risk that follows the probability distribution of $\varepsilon_{it} \sim N(0, \delta(\beta_{di} - \overline{\beta})^2 + \sigma)$. The relationship between the past return information or the probability of a certain industry portfolio becoming an extreme performer and the risk measure can be formalized as Lemma 1.

Lemma 2 *First, the case of* $\varepsilon_t \sim N(0, \sigma_\varepsilon)$. *In this case, when the market moves in a positive direction* ($\lambda_t > 0$), *the probability of a certain asset being included in the winner/loser portfolio is increasing/decreasing function of* β_{di}. *In contrast, when the market moves in a negative direction* ($\lambda_t < 0$), *the probability of a certain asset being included in the winner/loser portfolio is decreasing/increasing function of* β_{di}. *Second, the case of* $\varepsilon_{it} \sim N(0, \delta(\beta_{di} - \overline{\beta})^2 + \sigma)$. *We can find the same relationship between the probability of a certain asset being included in the winner/loser portfolio and* β_{di} *for sufficiently large* β_{di}.

So we can write for sufficient large β_{di} as:

$$\frac{\partial \text{Prob}(r_{it} \in W | \lambda_t \geq 0)}{\partial \beta_{di}} > 0 \qquad \text{(Eq. 3.10)}$$
$$\frac{\partial \text{Prob}(r_{it} \in W | \lambda_t < 0)}{\partial \beta_{di}} < 0$$

Proof. For more complete proof, see Sam Ho Son's Phd. dessertation paper(SNU. 2007). ■

Lemma 1 shows that the intuition in the determination of the probability of a certain industry being included in extreme performers when $\sigma_\varepsilon = 0$ is still effective when we consider the case of $\sigma_\varepsilon \neq 0$. That is to say, if the idiosyncratic risk ε_t follows a distribution of $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ and the realized risk value of each industry is different to each other, it will be converged to zero on the long term average. Consequently, the idiosyncratic risk does not influence the probability of a particular industry being included in the extreme performing industry group in the long term. They only have second order affect when the number of the group is small relative to the number of industries, as mentioned above.

In this case, the systematic risk β_{di} has main affect whether a particular industry is included in the winner or loser category at each point in time. The risk premium has a positive or negative value at each point in time. As a result, the relationship between the probability of an industry being an extreme performer and the industry's β_{di} is not monotonic. In the determination of a certain industry being included in the extreme performing group, the effect of β_{di} is conditioned to the value of the risk premium.

When the distribution of idiosyncratic risk is different for all industries and follows the distribution of $\varepsilon_{it} \sim N(0, \delta(\beta_{di} - \overline{\beta})^2 + \sigma)$, we can find that the intuition in the determination of the probability of a particular industry being an extreme performer when $\sigma_\varepsilon = 0$ is not effective. In this case, we can assess that the probability of being in winner category increases with the sufficiently large systematic risk value β_{di}. The probability decreases with the sufficiently small β_{di}. As well as the realized value of risk premium, the systematic risk also affects the idiosyncratic risk and by this indirect path also influences the sign of the derivative of probability of an industry being included in the winners portfolios. Now we can characterize the relationship between the probability of an industry being selected as a winner and β_{di} as the following proposition.

Proposition 3 *The relationship between β_{di} and the probability of an industry being selected as a winner is dependent on the sign of the mean value of the market price of risk, $\overline{\lambda}$ and the value of parameter δ.*

- First, the case of $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ and $\overline{\lambda} = 0$: the probability of a particular industry being included in the winner portfolio has a positive/negative relationship with β_{di} of industries that is larger/smaller than the cross-section average beta $\overline{\beta}_d$.

$$\frac{\partial \mathrm{Pr}_i^w}{\partial \beta_{di}} \begin{Bmatrix} > 0 \ if \ \beta_{di} > \overline{\beta}_d \\ = 0 \ if \ \beta_{di} = \overline{\beta}_d \\ < 0 \ if \ \beta_{di} < \overline{\beta}_d \end{Bmatrix} \qquad \text{(Eq. 3.11)}$$

If $|\beta_{di} - \overline{\beta}_d| = |\beta_{dj} - \overline{\beta}_d|$ *and* $\beta_{di} > \overline{\beta}_d$ *and* $\beta_{dj} < \overline{\beta}_d$, then the distance between β_{di} and $\overline{\beta}$ is identical to the distance between β_{dj} and $\overline{\beta}$. Then, $\mathrm{Pr}_i^w = \mathrm{Pr}_j^w$. Pr_i^h as a functional of β_{di} displays a symmetric 'U' shape.

- Second, the case of $\varepsilon_{it} \sim N(0, \delta(\beta_{di} - \overline{\beta})^2 + \sigma)$ and $\overline{\lambda} = 0$: we can find that Pr_i^h as a functional of β_{di} does not display a symmetric 'U' shape. It is asymmetrically tilted to the right.

Proof. For more complete proof, see Sam Ho Son's Phd. dessertation paper(SNU. 2007). ■

- First, the case of $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ and $\overline{\lambda} > (<)0$: the probability of a certain industry being included in the winner portfolio has a positive/negative relationship with β_{di} of industries that is larger/smaller than the average beta $\overline{\beta}_d$. That relationship can be positive or negative near $\overline{\beta}_d$. However, for sufficiently large β_{di} and positive risk premium, they must have a positive relationship. This relationship can be represented as:

$$\frac{\partial \mathrm{Pr}_i^w}{\partial \beta_{di}} \begin{Bmatrix} > 0 \ (= \text{indeterminate}) \ if \ \beta_{di} \geq \overline{\beta}_d \\ = \text{indeterminate}(< 0) \ if \ \beta_{di} < \overline{\beta}_d \end{Bmatrix} \qquad \text{(Eq. 3.12)}$$

If $|\beta_{di} - \overline{\beta}_d| = |\beta_{dj} - \overline{\beta}_d| \ and \ \beta_{di} > \overline{\beta}_d \ and \ \beta_{dj} < \overline{\beta}_d$, then $\mathrm{Pr}_i^w > \mathrm{Pr}_j^w$. The shape of Pr_i^h as a functional of β_{di} tilts to the right.

- Second, the case of $\varepsilon_{it} \sim N(0, \delta(\beta_{di} - \overline{\beta})^2 + \sigma)$ and $\overline{\lambda} > (<)0$: In this case we can find the shape of Pr_i^h as a functional of β_{di} tilts to the right.

Proof. For more complete proof, see Sam Ho Son's Phd. dessertation paper(SNU. 2007). ■

Corollary 4 *First, the case of* $\varepsilon_t \sim N(0, \sigma_\varepsilon)$. *If* $\overline{\lambda} = 0$, *then* $Pr_{19}^w = Pr_2^w$. *If* $\overline{\lambda} > (<)0$, *then* $Pr_{19}^w > (<) Pr_2^w$. *As the risk premium becomes larger, the shape of* Pr_i^h *as the functional form of* β_{di} *becomes more asymmetric. Second, the case of* $\varepsilon_{it} \sim N(0, \delta(\beta_{di} - \overline{\beta})^2 + \sigma)$. *We can find that as the parameter* δ *becomes larger, the shape of* Pr_i^h *as the functional form of* β_{di} *becomes more symmetric.*

Proof. For more complete proof, see Sam Ho Son's Phd. dessertation paper(SNU. 2007). ■

Proposition 1 and Corollary 1 show that the functional relationship between β_{di} and $\Pr_i^w$ depends critically on the value of the risk premium and the parameter δ. If the risk premium is zero or the value of parameter δ is large, then the shape of that functional relationship is symmetric U shape. In contrast, if the risk premium has positive value or the value of parameter δ is small, the shape of that relationship tilts to the right, and for sufficiently large β_{di} that relationship will be a monotonically increasing function.

According to Proposition 1 and Corollary 1, we can conjecture the shape of Pr_i^h as a functional of β_{di} lies between symmetric U shape, and asymmetrically tilts to the right shape. If a certain risk measure satisfies the requirement of this ideal benchmark, the shape of Pr_i^h as a functional of β_{di} should show this shape. Therefore, we can call these conditions as the admissibility conditions in risk adjustment; i.e, when the average risk premium is positive, and the parameter δ has positive value, then the shape of Pr_i^h as a functional of β_{di} lies between symmetric U shape, and asymmetrically tilted to the right shape.

5 A Simulation Model

In this section, we assume that each industry has its genuine risk measure. We simulate the relationship between these genuine risk measures of the industries and the probability of an industry being included in extreme performers. We assume the genuine risk measures of the hypothetical 20 industries as in the Eq. 3.1. Also as in the previous section, we still assume that $\lambda_t \sim N(\overline{\lambda}, \sigma_\lambda^2)$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon)$. It is not reported the case of $\varepsilon_{it} \sim N(0, \delta(\beta_{di} - \overline{\beta})^2 + \sigma)$ because of its redundant characteristics.

In figure 1, we can identify the effect of systematic risk to the probability of an industry being included in the winner category. When the average risk premium is zero, the curve is symmetric around the average beta. When the beta of an industry is larger than the cross-sectional average beta, the probability of a particular industry being included in the winner group is an increasing function of that beta. And when the beta of an industry is smaller than the cross-sectional average beta, the probability of a particular industry being included in the winner group is a decreasing function of beta. Under the assumption of Case 1, the determining factor of that probability is the size of the beta relative to the cross-sectional average beta.

And more importantly, we can identify the curve tilts to the right as the average risk premium becomes positive. That is, when the average risk premium has positive value, the probability of the high risk industries being included in the winners group is larger than the low risk industries. And we can also define the probability as a weighted average concept. On average, if the return of an industry is nearer to the winners, the weighted probability is larger. If we use weighted probability we can identify more explicitly the risk-return tradeoff between winners and losers and we use this probability concept in empirical test of the admissibility of each benchmark. However, we can conjecture that the probability of becoming a winner for a risky industry will be higher than for a low risk industry.

In normal cases, the average risk premium has a positive value. As a result, risk-return tradeoff relationship is still hold between winners and losers. We define the property of this ideal benchmark as admissibility condition which rational benchmark should satisfy. To assess the profitability of momentum strategies, we should use the risk measure which satisfies this admissibility condition. If a risk measure satisfies this condition, we can credit the performance assessment of this benchmark.

6 Empirical Test of the Admissibility of Each Benchmark

We can use the analytical results of Section 4 and Section 5 as a diagnostic for each benchmark. In this section, we examine if the CAPM and non-parametric benchmark satisfy our admissibility conditions.

We define the probability in Figure 2 as the weighted average that measures, on average, how close an industry is to the winner. This figure illustrate the relationship between the probability as weighted average and the conditional non-parametric risk measure of each industry. We can identify that the conditional non-parametric risk measures satisfy our admissibility condition. The difference of the probability between the most risky industry and the safest industry is about 1%. So we can credit the performance assessment of momentum strategies using this risk measure.

We formally test the null hypothesis that the probability of becoming winner of high risk industry is equal to that of low risk industry. Because of the large sample size, we construct Z-statistics to test the null hypothesis. Since the orderings of becoming winner of each industry with respect to weighted probability are the same under unconditional and conditional benchmark, the test results are same in both cases. The differences in dummy probability cases are converged to the same ordering in weighted probability cases. We can identify the null hypothesis is rejected at 1% significance level against non-parametric benchmark with respect to the weighted probability of winner. In contrast, the p-value against CAPM benchmark is 0.2860 and the null hypothesis cannot be rejected. And with respect to the weighted probability of loser, the p-value against non-parametric benchmark is 0.0175 and the p-value against CAPM benchmark is 0.1033.

Figure 3 illustrate the relationship between the probability as weighted average of nearness to the winner and market beta of each industry. We can identify in this figure that the market beta does not satisfy our admissibility condition. So we cannot credit the results of the performance assessment of industry momentum strategies against the CAPM benchmark.

And we assessed the profitability of momentum strategies on 48 industry-sorted portfolios by using the non-parametric risk measure on the basis of 17 industry sorted portfolios. We found that there remain more unexplained abnormal returns of momentum strategies on 48 industry-sorted portfolios than the strategies on 17 industry-sorted portfolios. Out of 16 strategies, 9 are profitable at 5% significance level against the conditional non-parametric benchmark. Under the unconditonal non-parametric benchmark, all strategies are profitable at 5% significance level. Like the parametric risk measure, all strategies are profitable at 5% significance level under the conditional and unconditional CAPM measures.

However, in Table 2, we can identify that the CAPM risk measure is distorted. The conditional and unconditional CAPM beta of the loser of 48 industries in Table 2 is higher than those of the winner of 48 industries. The CAPM beta of the loser is too high and appears to be wrongly specified. Under the CAPM benchmark, the risk of winners minus losers appears to be negative. This negative CAPM risk measure corresponds to the negative risk premium contradicting to the CAPM theory itself. But the conditional and unconditional non-parametric risk measures appear to be appropriate in Table 2. From this fact, we can carefully conjecture the internal relationship between the distorted risk measure and the large abnormal returns of momentum strategies.

7 Conclusions and Extensions

In this paper, we develop a simple model of a risk measure. Our model shows that the genuine risk measure is increasing with risk premium. Proposition 1 and Corollary 1 in Section 4 suggest that on average, the risk-return tradeoff still holds between winners and losers. We take this relationship as a admissibility condition of a good benchmark. We find that the non-parametric risk measure satisfies our admissibility condition, but market beta does not.

Using these results, it might be possible to explain the difference in the results of the profitability assessment of momentum strategies. Market beta does not reflect the risk of each industry. If we accept the CAPM risk measure, the following results are inevitable. Since the payoffs from momentum strategies are winners minus losers, and momentum portfolios are implemented on the basis of past cumulative returns, winners must be safer assets than losers in the CAPM benchmark. As the risk adjusted residual return over benchmark return, alpha appears even larger than the average monthly return of momentum portfolios. Behind these large abnormal excess returns of momentum strategies, there is a distorted market beta. We cannot credit the profitability assessment of momentum strategies against CAPM benchmark.

Meanwhile, under the non-parametric risk measure, the relatively risky assets have higher frequency of being included in the winners and relatively safe assets have lower frequency of being included as winners. Winners must be riskier assets than losers against the non-parametric benchmark. We can credit the profitability assessment of momentum strategies against this benchmark. Under this benchmark, alpha appears much smaller than the average monthly return of momentum portfolios.

As a result, we can conjecture the causal relationship of the distorted risk measure to the large abnormal excess returns of momentum portfolios. If a certain risk measure is distorted, the profitability assessment of momentum strategies against this benchmark remains large abnormal excess returns. However, the inverse causal relationship is not clear. For example, we cannot explain the residual returns of the momentum portfolios based on the returns of 48 industry sorted portfolios by using our non-parametric risk measure on the basis of 17 industry sorted portfolios. In this case, it is not clear that the existence of this abnormal excess return is due to the fact that our non-parametric risk measure on the basis of 17 industry sorted portfolios is distorted or due to the fact that there are mis-priced industries in these 48 industry sorted portfolios. The identification of this mis-priced factor is left for the next research program.

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(J / K)	Average return	CAPM-alpha	P-value	IND17-alpha	p-value
(3/3)	0.0094	0.0095	7.75E-07	0.0045	0.0086
(3/6)	0.0075	0.0076	5.41E-06	0.0034	0.0362
(3/9)	0.0070	0.0070	4.34E-06	0.0029	0.0649
(3/12)	0.0062	0.0061	6.06E-06	0.0023	0.0725
(6/3)	0.0091	0.0092	1.78E-05	0.0031	0.1051
(6/6)	0.0083	0.0083	1.52E-05	0.0028	0.1424
(6/9)	0.0080	0.0080	1.37E-05	0.0030	0.1029
(6/12)	0.0058	0.0057	0.0005	0.0009	0.5273
(9/3)	0.0097	0.0098	1.01E-05	0.0033	0.1087
(9/6)	0.0083	0.0082	9.37E-05	0.0027	0.1788
(9/9)	0.0066	0.0064	0.0013	0.0014	0.5203
(9/12)	0.0043	0.0041	0.0209	-0.0005	0.7218
(12/3)	0.0097	0.0097	1.93E-05	0.0030	0.0667
(12/6)	0.0071	0.0070	0.0012	0.0008	0.6048
(12/9)	0.0049	0.0047	0.0179	-0.0007	0.6709
(12/12)	0.0029	0.0026	0.1412	-0.0022	0.1674

Table 1 shows the monthly mean returns to 16 momentum strategies to 17 industry-sorted portfolios. The portfolios are ranked on the basis of the cumulative returns in portfolio formation periods and held for holding periods as in Jegadeesh and Titman(1993). The Data covers the period from 31 December 1962 to 28 February 2006, yielding 519 monthly observations. This table also shows the risk adjusted performance measure against both of conditional non-parametric and conditional CAPM risk measure. Alpha represents average monthly excess return over the the portfolios of basis assets for the strategy. P-value represents the test results against the null hypothesis of zero abnormal returns.

[Table 1] Conditional Test Results of Equally Weighted 17 Industries

	Unconditional		Conditional	
Decile	CAPM beta	IND17 beta	CAPM beta	IND17
1	1.0791738	-0.59062624	1.0870857	0.34029588
2	1.0334936	-0.30042035	1.0464113	0.30018764
3	1.0427468	-0.11057745	1.0553586	0.19797869
4	1.0292807	-0.017991403	1.0415492	0.008734932
5	1.0203778	0.095550795	1.0314418	0.056490264
6	1.0291572	0.3965774	1.0371613	0.76602419
7	1.0425104	0.53664177	1.047776	1.7468181
8	1.0398339	0.96500861	1.0421715	3.589517

Table 2 shows relationship between the ordered returns momentum strategy of 6-month portfolio formation periods and 6-month holding period and the non-parametric and CAPM risk measure under unconditional and conditional conditions. We construct the risk measures to the ordered returns of momentum strategies against the hedge portfolios of each benchmark. We can identify that the CAPM risk measure is distorted. The conditional and unconditional CAPM beta of the loser of 48 industries in Table 5 is higher than those of the winner of 48 industries. In contrast, we find that the non-parametric risk measure ordered in the same way as the ordered returns of momentum strategy.

[Table 2] Conditional Test Result (Betas on the Portfolios of 6/6 Momentum Strategy)

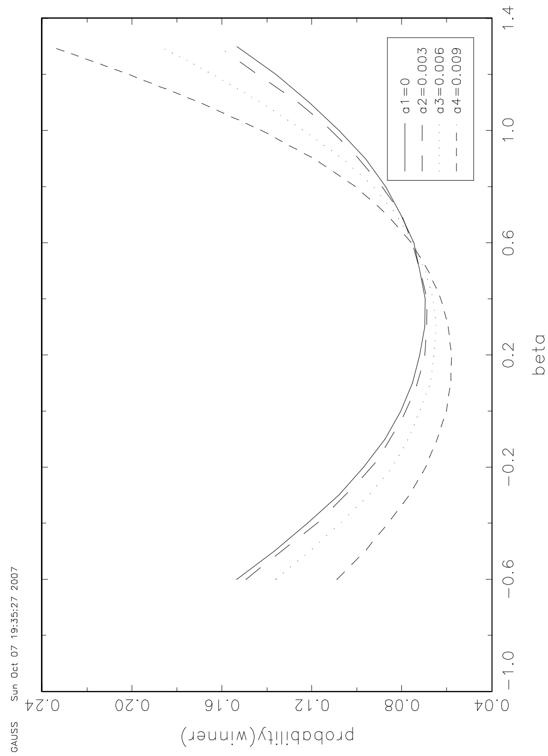

Figure 1: Case 1, The functional relationship between the risk measures and the probabilities of being included in winners of industries for different values of the average value of risk premium. We still assume that $\overline{\lambda} = 0.012$, $\sigma_\lambda = 0.012$, and $\sigma_\varepsilon = 0.01$

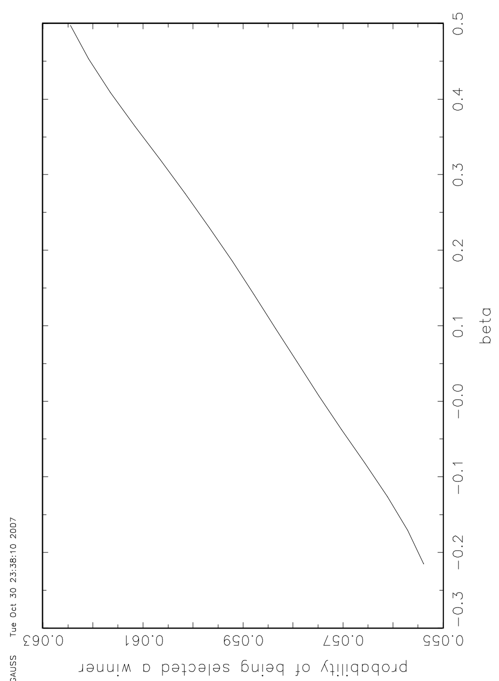

Figure 2: The nonparametric kernel estimates of the relationship between the probability as weighted average of winners and conditional non-parametric risk measure on the basis of 17 industries

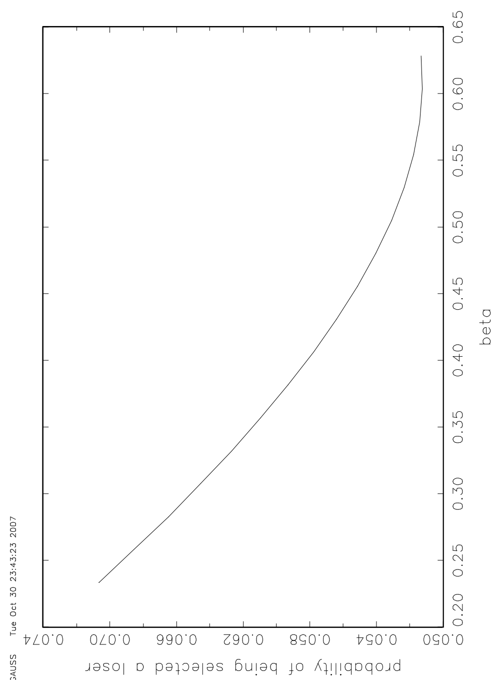

Figure 3: The nonparametric kernel estimates of the relationship between the probability as relative frequency of being included in losers and conditional market beta