A Model of the Consumer's Bid Price Determination with Adjustment Costs[1]

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Abstract

How do consumers make a bidding decision when they arrive at the market to realize their predetermined consumption plan, especially when there is uncertainty on the prospect for his/her order being matched at the market and costs are ensued from the matching failure? We developed a simpler model than the Nash equilibrium models by assuming the probability of bid matching depends on the market price. We showed, by a simulation with artificial data, that consumers can submit bids above the subjective value of the consumption with the introduction of the costs from the matching failure.

1. Introduction

How do consumers make a bidding decision when they arrive at the market to realize their predetermined consumption plan? Traditionally economists resort to the Warlasian tatônement process to give explanations for this. However the process will be different if we introduce uncertainty that some of consumer bids fail to be matched at the market and costs arise from the subsequent consumption adjustment.

Vickrey(1961) was the first who dealt with this issue. He formulated a non-cooperative Nash equilibrium model of bidding by risk neutral economic agents in single unit auctions. Later Harris and Raviv(1981) extended the Vickrey model to multiple unit auctions. Cox, Smith and Walker(1984) reported the results of various experiments to test the empirical properties of individual bidding behavior. While the game models use the interactive probability distributions considering all the other bidders in the auction, they introduced complicated assumptions to make the solution tractable, which restrict the extension of the auction models to the consumer's bid behavior in the market in general. For example, the model requires information on the total number of auction participants, N, and the quantity offered, Q, which is hardly available for most of the markets except a certain form of a specified auction markets.

These game models focus mainly on verifying the optimality of the market equilibrium and comparing the efficiency of the alternative auction systems such as Dutch auction and English auction. In addition, all of these auction models use the assumption that individual bids for a

[1] Comments from Jin Choi, Jaejoon Woo, and Larry Morgan have improved the earlier draft. Still all the remaining errors are the author's responsibility.

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given commodity can't exceed the predetermined values in the optimization process. This condition, which was initially introduced by Chamberlin(1948), has been consistently used in the experimental economics researches.

This paper attempts to develop a theoretical model to analyze the consumer's bidding behavior at the market when the consumer can adjust his/her consumption plan responding to the market price changes. Unlike the Nash game models, the probability of the bids being matched is assumed to depend upon the market price which follows a certain probability distribution. Our main interest lies specifically in the determination of the bid price when consumers are conscious of the costs when their bids are not matched in the market. We further look into the possibility that a bid higher than the reservation price produces more welfare to the buyer than a bid lower than the reservation price.

Our ultimate goal is to develop a bid-generating system, based on the traditional consumption theory, which can produce the humped shape of the limit order book observed in most of the exchange markets.

In section 2 we revisit the traditional consumer's problem and suggest an alternative way of modeling to introduce uncertainty about the bid matching. Section 3 introduces an alternative consumers' bidding model and section 4 presents the simulation results, using the artificial data to find out the properties of the optimum behavior derived from the model. Conclusions and suggestions for further research are given in the last section.

2. Consumer's problem revisited

Let's say there are n consumers who solve the following optimization problem simultaneously,

$$\text{Max } U(C_1, C_2: \Theta_i, \Sigma_i, \Omega), \quad \text{subject to } I_i \geq p_1C_1+p_2C_2, \qquad (1)$$

where C_1; commodity 1 that is concerned,

C_2; collection of all the other commodities,

p_1, p_2; prices of the corresponding commodities, respectively,

I_i; income of the i^{th} consumer,

Θ_i, Σ_i; preference and information sets of the i^{th} consumer, respectively,

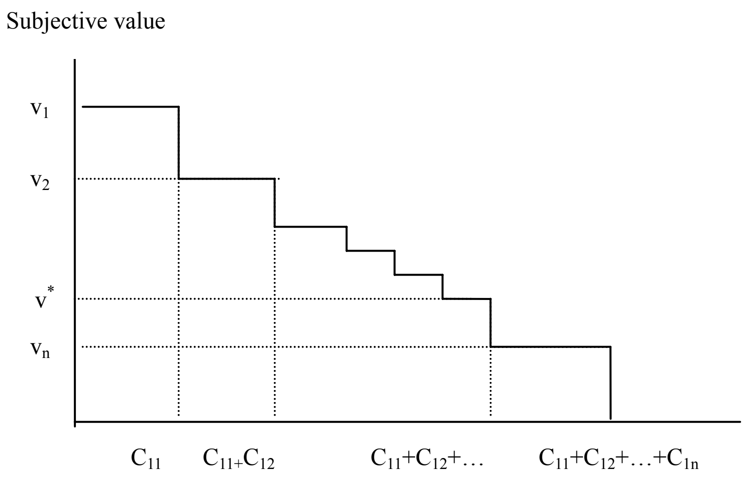
Ω; market matching system implied in Smith(1964,1982).

We assume no cross effect of consumption between two commodities. In addition, we assume $U(0,C_2)>0$ for $C_2>0$; namely, even no consumption of commodity 1 at all produces positive utility. This assumption helps the consumer to refrain from paying an extreme price for commodity 1 when the availability of commodity 1 gets very difficult.

Once we are given p_1, p_2, I_i, Θ_i, Σ_i, Ω, we can solve (1) in the straight forward way to get the optimum values for C_1^* and C_2^*. When we change p_1, with other variables fixed, the units of C_1 demanded and the units of C_2 demanded will change interactively. When there is no cost incurred in shifting around between C_1 and C_2, the value of one dollar spent either for C_1 or C_2 will be the same at the optimum.

Now assume the quantity of commodity 1 chosen by these n consumers is fixed at one unit for each consumer. That is, the marginal utility from the second unit of consumption of commodity 1 drops suddenly to zero. For n consumers, there will be n subjective values, v_1, v_2, - -, v_n, derived from the unit consumption of commodity 1. We can derive the market demand for commodity 1 by adding up the units at these values horizontally and the equilibrium price will be set once the market supply is given in the perfect information environment. As in <Figure1>, individuals who bid below v* will be denied the allocation of commodity 1 and have to adjust their consumption plans. The information on the level of v^* is not known a priori. Only consumers whose bid exceeds v* will be able to carry out the initial consumption plans.

<Figure 1> Market demand for C_1 and the allocation of individual bids.



C_{1i}; i consumer's quantity demanded for commodity 1.

Now what if our consumers are concerned about the matching probability of their bids? We can find a clue from Vickrey(1961). He formulated a non-cooperative Nash equilibrium model of bidding by risk neutral economic agents in single unit auctions. Later Harris and Raviv(1981) extended the Vickrey model to multiple unit auctions. Cox, Smith and Walker(1984) reported the results of various experiments to test the empirical properties of individual bidding behavior. The Nash gaming model summarized in Cox, Smith and Walker(1988) is as follows.

When there are N bidders, competing for Q units of a homogeneous good offered in perfectly inelastic supply, each bidder submits a bid for a single unit by maximizing the following objective function.

$$\text{Max } U(v_i\text{-}b_i,\theta_i)G(b_i) \qquad (2)$$

where b_i; individual I's bid price

v_i; monetary value for i,

θ_i; parameters representing the individual's characteristics,

$G(b_i)$; probability that all N-1 rivals of bidder i will bid prices less than or equal to b_i, i.e., probability that i's bid will be accepted.

When bidder i raises his/her bid, the probability that it will beat the rival's bids increases while the welfare value of the bid(=v_i-b_i) declines. Each bidder must thus balance these two factors in determining his/her bid, considering the probable bids of the others.

Since this is a multi-agent non-cooperative game, we need complicated assumptions to make the solution tractable, which restrict the extension of the auction models to the consumer's problem in the market in general. For example, the model requires information on the total number of auction participants, N, and the quantity offered, Q, which is hardly available for most of the markets except a certain form of specified auction markets. Also these game models used simpler distributions like uniform distribution for v_i and G to get the solution for real market application.

In addition, all of these auction models use the assumption that individual bids for a given commodity can't exceed the predetermined values in the optimization process. This assumption, which was initially introduced by Chamberlin(1948), has been consistently used in the experimental economics researches.

Chamberlin(1948) takes these values as 'the Marshallian demand price' which works as a limit price in the market[3]. Smith(1962) defined this price as the 'reservation price,' the maximum price that a buyer is willing to pay for one unit of the fictitious commodity, and the participants in his experiments are not allowed to place bids beyond the reservation prices. Individual buyers are assumed to determine their bids to maximize the difference between these reservation prices and the actual bids[4].

However, when we introduce uncertainty on the prospect individual's order being matched at the market and costs are ensued from the matching failure, there might be a possibility that a bid

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[3] Chamberlin(1948), p.96.

[4] Smith(1962), p.112.

higher than the reservation price produces more welfare to the consumer than a bid lower than the reservation price, where the benefit of raising the bid matching probability dominates. In this situation restricting the participants in the experiments not to make bids beyond their reservation prices may prevent them from revealing their economic motivations to the full extent.

In the real life we are often forced to accept the price higher than the price that we have set before we go on shopping. Other expenditures may have to be foregone thereupon. Also in the commodities and securities markets long hedgers often submit bids higher than the forward sold price to stop the loss of their open position in the volatile market.

3. Model of consumer's bid price determination with adjustment costs

In the following we develop a model of consumer's bid price determination where consumers face the uncertainty about their bids being matched and can adjust their consumption plan to accommodate this[5]. We assume the market to which our consumers submit their bids is a discrete call auction market so that they have to wait for a while between the sessions[6].

Unlike the Nash game models, the probability of the bids being matched is assumed to depend only upon the market price which follows a certain probability distribution. This assumption enables us to avoid the complexity of defining all the other participants' bidding strategy distributions. Especially in a dynamic market environment where it is difficult to obtain the information on the number of buyers and the total quantity available, it would be more economical and thus realistic for buyers to calculate their bid success probability simply from the current market price distribution, not from all the other buyers' bid distributions. In fact the information on the latter distribution can be said to be a subset of the information on the former distribution since all the other buyers' behavior will be reflected in the market price eventually.

With the introduction of the uncertainty about the bid matching, the representative consumer's problem (1) changes to determining the bid price, b, to maximize the following welfare function.

$$\underset{b}{Max} \quad W(b)=\int_0^b U(1,C_{21})\Psi'(p)dp+\int_b^\infty U(0,C_{20})\Psi'(p)dp \qquad (3)$$

where Ψ'; probability density function of p,

p; market price of commodity 1, $p\in(0,\infty)$,

C_{21}; consumption of commodity 2 when the bid is accepted,

C_{20}; consumption of commodity 2 when the bid is not accepted,

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5 Here consumers include traders who have long hedging needs, i.e., bona fide hedgers with forward sold positions in cash commodities.

6 According to Smith(1993), bidding behavior is affected by the matching system in the market.

b: bid price for commodity 1 and $b \in (0, \infty)$.

From the budget constraint, $C_{21}=(I-b)/p_2$ and $C_{20}=I/p_2$. Substitute these into (3) and introduce v, the reservation price of commodity 1, then (3) becomes;

$$W(b)=\int_0^b U(1, C_2^* + \frac{v-b}{p_2})\Psi'(p)dp + \int_b^\infty U(0, \frac{I}{p_2})\Psi'(p)dp, \qquad (4)$$

where $C_2^* = (I-v)/p_2$, the amount of consumption 2 when $p_1=v$. v can be defined as the maximum price to obtain one unit of consumption 1 without consideration of the bid matching probability, given p_2 and I. We assume the individual consumer's choice of b doesn't affect $\Psi(p)$.

Proposition I: At optimum $b=b^*$,

$$\frac{U(1, \frac{I-b^*}{p_2}) - U(0, \frac{I}{p_2})}{U_2 / p_2} = \frac{\Psi(b^*)}{\Psi'(b^*)}. \qquad (5)$$

where $U_2 = \partial U(1, \frac{I-v}{p_2}) / \partial C_2$.

Proof:

Since the consumer's choice of bid doesn't affect Ψ, (4) can be rewritten as follows.

$$W(b) = U(1, \frac{I-b}{p_2})\Psi(b) + U(0, \frac{I}{p_2})(1-\Psi(b))$$

$$\frac{\partial W}{\partial b} = U(1, \frac{I-b}{p_2})\Psi'(b) + U_2(1, \frac{I-b}{p_2})(-\frac{1}{p_2})\Psi(b) - U(0, \frac{I}{p_2})\Psi'(b) = 0$$

$$\frac{1}{p_2}U_2(1, \frac{I-b}{p_2})\Psi(b) = [U(1, \frac{I-b}{p_2}) - U(0, \frac{I}{p_2})]\Psi'(b)$$

The second order condition is;

$$U_{22}(1, \frac{I-b}{p_2})\Psi(b)p_2^{-2} - U_2(1, \frac{I-b}{p_2})(\frac{2}{p_2})\Psi'(b) + [U(1, \frac{I-b}{p_2}) - U(0, \frac{I}{p_2})]\Psi''(b) < 0.$$

<QED>

Condition (5) means that, at the optimum, the ratio of the marginal utilities of consumption 1 and consumption 2 equals to the contribution of the marginal density to the total probability. For the normal distribution, the RHS of (5) continues to increase as the bid prices are raised[7]. As the value of consumption 1 becomes more precious compared to consumption 2, consumers seek to raise the probability of bid matching.

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[7] See that Ψ/Ψ' rises as bids increase in <Figure-2>.

Corollary: From (5), $U(1, C_2^* + \frac{v-b^*}{p_2}) - U(0, I/p_2) > 0$.

This is readily obtained from (5) since $U_2 > 0$, $\Psi/\Psi'>0$. That is, allocating money b^* for consumption 1 is better than being enforced to spend all the money on consumption 2. This difference can be defined as an adjustment cost of the consumption plan since the first term represents the welfare achieved when the bid is accepted in the market and the second term is the welfare obtained when the bid is not accepted. The value of one dollar spending is different between those two states[8]. We denote this adjustment cost as Δ.

Proposition II:

When b*>v, W(b*)>W(v) if,

$$\Delta > [U(1, C_2^*) - U(1, C_2^* + (v-b^*)/p_2)]/\Gamma. \qquad (6)$$

And when b^*=v, W(b^*)=W(v). Here, $\Gamma = \frac{\Psi(b^*) - \Psi(v)}{\Psi(v)}$.

Proof: From (4), W(b*) - W(v)

$$= U(1, \frac{I-b^*}{p_2})\Psi(b^*) + U(0, \frac{I}{p_2})(1-\Psi(b^*)) - [U(1, \frac{I-v}{p_2})\Psi(v) + U(0, \frac{I}{p_2})(1-\Psi(v))]$$

$$= U(1, C_2^* + \frac{v-b^*}{p_2})\Psi(b^*) - U(1, C_2^*)\Psi(v) + U(0, \frac{I}{p_2})(\Psi(v) - \Psi(b^*))$$

$$= [U(1, C_2^* + \frac{v-b^*}{p_2}) - U(0, \frac{I}{p_2})]\Psi(b^*) - [U(1, C_2^*) - U(0, \frac{I}{p_2})]\Psi(v) > 0,$$

$$\text{if } \frac{\Psi(b^*)}{\Psi(v)} > \frac{U(1, C_2^*) - U(0, I/p_2)}{U(1, C_2^* + \frac{v-b^*}{p_2}) - U(0, I/p_2)}.$$

Subtract 1 from both sides to get (6).

When b^*=v, W(b^*)-W(v)=0. QED.

Proposition II implies that consumers may determine the bid level higher than the reservation price if the adjustment cost incurred with the bid failure exceeds the welfare loss from paying higher price for the consumption, discounted by the probability improvement from the higher bid submission. In (6), Γ represents the relative probability improvement in the bid acceptance and the numerator is the welfare loss from choosing b higher than v.

Next, we are interested in the properties of the optimum bid price, b*, of N consumers with

[8] The lump sum bonus paid when the bids are matched in the experiments of Smith(1993) can be included in these costs.

different levels of v when all the other variables are given. Problem (4) can be simplified as a function of (v-b) and Ψ when I, P_2, Θ_i, Σ_i, and Ω are given as in (7).

$$\underset{b}{Max} \quad H[(v-b), \Psi : I, p_2, \Theta_i, \Sigma_i, \Omega] \qquad (7)$$

A. Type I consumer

Here we develop the model only for risk neutral consumers to simplify the simulation work later. Following the game models, we first derive the optimum behavior of the consumers(called as 'bidders' in those game models) who try to maximize only the expected welfare surplus or return when their bids are successful. We name them as Type I consumers. Under this condition, the integrated utility function(***H***) in (7) will be increasing monotonically in the expected value of surplus or return(E(v-b)=E(R)). This can be expressed as the product of the position profit, (v-b), and the probability of the bid for commodity 1 being matched at the market, $\Psi(p_t|b)$, where the market price of commodity 1, p_t, is assumed to be uncertain until the end of the current period, t, as in (8).

$$E(R) = \Psi(p_t|b)(v-b). \qquad (8)$$

and $\Psi(p_t|b) = \int_0^b \Psi'(p_t)dp_t$, where Ψ' is the density function of the market price. We assume $\Psi(p_t|0)=0$, $\Psi(p_t|\infty)=1$. We also assume $\Psi(p_t)$ has the first and the second moments and $\Psi(p_t|b)$ is differentiable with respect to b. The shape of $\Psi(p_t|b)$ will depend upon each individual's experience or information on the characteristics of p_t changes and the order matching system of a given market. But we assume it is identical for all consumers in the market.

Equation (8) implies that a consumer places his/her bid considering the return (or welfare surplus) realized from the current position and the probability of his/her bid being hit at the market. As the consumer raises his/her bid, the return from consumption 1 will deteriorate while the probability of the order being hit will increase.[9] That is, $\partial\Psi / \partial b = \Psi' > 0$.

The optimal condition will be;

$$\frac{\partial E(R)}{\partial b} = \Psi'(v-b) - \Psi = 0 , \qquad (9)$$

$$\Psi'(v-b) = \Psi , \qquad (10)$$

$$(v-b) = \Psi / \Psi' , \qquad (10\text{-}1)$$

[9] As the consumer raises her bid above the reservation price, she still derives a positive utility since we assume $U(0, C_2)>0$. Therefore she may choose to stay out of market if she has to pay an unexpectedly high price which exceeds the reservation price by more than a certain amount.

The second order condition will be $\Psi''(v-b) \leq 2\Psi'$. Condition (10) implies that, at the optimum, the degree of the marginal return decrease (or loss increase) (LHS) should not exceed the cumulative probability of the bid being hit (RHS) as the bid price goes up.

From (10-1), (v-b)>0, and hence b<v, since $\Psi/\Psi'>0$. Here, Ψ/Ψ' measures the contribution of the marginal probability to the cumulative probability of a given bid price change. This optimum condition means that, if a consumer considers only to maximize the expected return, his/her bid price will be always less than the reservation price given by the initial consumption plan. Therefore, the condition presumed by Chamberlin(1948) and Smith(1962) can be justified.

In stock markets we can find traders who stick to the rule given by (10), that is, who never submit bids at a loss. One of the major risks accompanying this rule is the possibility that losses accumulate fast in a short period of time. For example, assume a trader's forward sold price is 90 and the market is fluctuating around 100. The trader can place his/her bid at 89 and keep waiting for a very thin opportunity of realizing a profit.

A more sensible trader would consider equally this risk of fast loss accumulation when his/her bid is left unmatched in a volatile market. The optimization problem (8) needs to be changed for this type of consumers since they are concerned about their bids not accepted at the market and costs arise in adjusting their consumption plans according to changing market conditions.

B. Type II consumer

To incorporate the cost of adjusting the consumption plan when the bid fails to be matched at the market, we make a change to the problem (8) as follows.

$$E(AR) = \int_0^b (v-b)\Psi'(p_t)dp_t - \int_b^\infty \Delta\Psi'(p_t)dp_t$$

$$= \Psi(p_t|b)(v-b)-(1-\Psi(p_t|b))\Delta. \qquad (11)$$

Here Δ represents the cost arising from the bid failure and is assumed to be non-negative[10]. It is assumed to be independent of the distribution function, $\Psi(p_t)$, and the choice of b. The first term in the RHS of (11) is the expected profit when his/her bid is successfully matched and the second term is the expected cost of the bid failure.

Accordingly, the optimum condition (10) changes to (12).

$$\Psi'(v-b+\Delta) = \Psi \qquad (12)$$

[10] To specify the relationship between Δ in (6) and Δ in (11), we need to introduce more complicated assumptions on the welfare function structure and the price distribution. In order to simplify the discussion, here we just assume that a fixed cost arises when the bid fails to be matched in the market.

Thus, v-b+Δ>0, and for some large Δ, b>v. This means that it may be in the buyer's interest to place the bid, b, above the reservation price, v, when the cost of his/her order not matched is considered. Even though the consumer has to pay the price higher than his/her planned reservation price, he/she can be happier with the success of her bid than losing the consumption opportunity at all.

4. Simulation Result

Rather than seeking analytical solutions to examine the properties of the optimum conditions (10) and (12), we develop a simulation method using the artificial data. The simulation is focused on how Type I and Type II consumers respond to changing market expectations, respectively, for a given range of the reference price.

The artificial data for the market price and the reference price are constructed as follows. The market price of commodity 1 is set at 100 at the beginning of the current period. However, the price at the end of the period is assumed to follow the normal distribution with mean μ and volatility σ. For the base case, μ=100 and σ=2% are considered. We assume consumers are concerned about the range of the market price change on a specific day. So σ represents the daily volatility. The daily volatility of 2% is equivalent to 32% of the annual volatility, which is twice larger than the level observed during the normal days of the stock markets in most of the developed countries[11]. The reservation prices are given as integers between 90 and 110.

The consumers are assumed to know the mean value and the volatility of the market price before they submit the bids. However, they don't expect their bids to change the market price distribution. We look at the effect of the changes in the mean and the volatility of the market price upon their optimum bid level.

<Figure 2> shows the optimum bid determination of Type I consumers whose reservation prices are given as 98, 100, 102, respectively. The optimum bid of the break-even consumer whose reservation price is 100 is calculated at 98.20[12]. The welfare surplus of this consumer would be 1.80 if this consumer's bid is accepted at the market. However, the probability of the bid acceptance is only 18%.

Consumers with higher reservation price who are expected to make profits on the average submit bids higher than consumers with lower reservation price. But the marginal increase of the bid price is less than the change in the reservation price and decreasing. For example, the marginal increase in the optimum bids declines from 1.65 (=98.20-96.55) to 1.39 (=99.61-

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[11] One reason to use a large volatility is to identify the properties of the optimum bids by magnifying the change in prices. We tried 0.2% of volatility and the major conclusions of the simulations were not significantly affected except generating long decimal points.

[12] This number is calculated by the interpolation method, using the values of (v-b) and Ψ/Ψ' at b=98 and 99.

98.20) as the reservation price rises from 98 to 100 and to 102, consecutively.

<Table-1> summarizes the optimum bids and their matching probabilities of consumers with reservation prices from 92 to 108[13]. As explained already, consumers with higher reservation prices tend to bid higher but at a decreasing rate. That is, the willingness to pay more to secure the commodity falls short of the reservation price increase. This is reflected in the decline of the bid ratio, which is defined as the ratio of the bid to the reservation price.

We can attribute this result to the assumptions of the risk neutral consumer and the normal distribution of the market price. For the normal distribution, the contribution of the marginal probability to the total probability, Ψ/Ψ', continues to rise as the bid price increases, and consumers become less willing to sacrifice the profit to raise the bid matching probability as in <Figure-2>. That is, consumers are getting more interested in raising profit (=v-b) than enhancing the bid matching probability(Ψ) as the reservation price goes up. Therefore, bids above the mean of the price distribution (100) tend to contract down toward the mean, and the bid function curves away from the reservation price when the reservation price gets higher .

This non-linearity of the bid function is similar with Vickrey (1961) who assumes risk neutral bidders and the uniform distribution of the bidding price. On the other hand, Cox, Smith and Walker(1988), using the uniform price distribution but the constant relative risk averse utility, show the linear relationship between the bidder's value(equivalent to the 'reservation price') and their bids in the first-price auction[14]. However, as Potters, Marc, Jean-Philippe Bouchaud(2006) report, the limit order books of most of the securities markets have a humped shape near the current price and thus it would be more realistic to use a distribution whose density has its peak at the mean such as normal distribution.

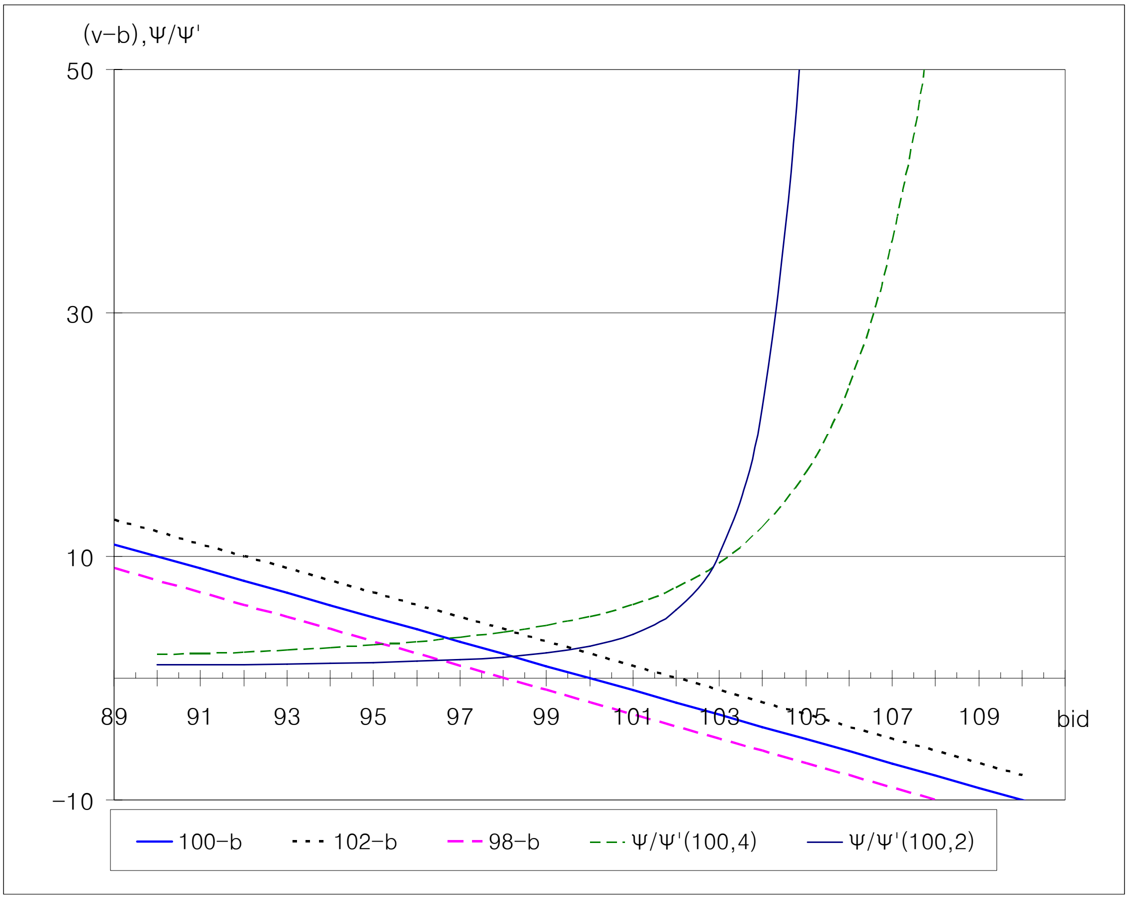
Nonetheless, for the risk-averse consumer, bid prices may go up further with the higher reservation price to protect the profit on hand.

In <Table-1> consumers with the reservation price far below the current market price stick to the low bids even with a very low chance of matching. They don't have any real interest in immediate bid matching. Potters, Marc, Jean-Philippe Bouchaud (2006) report the empirical evidence that there are market participants who believe that large jumps in the price are always possible. However, this result seems to show the limited rationality of type I consumer.

<Figure-2> Optimum bid determination of Type I consumers

[13] We confined the reservation prices range within 4 standard deviations from the mean to manage the size of the simulation.

[14] The first-price bid auction is the market in which the auctioned object is awarded to the bidder who submits the highest bid at a price that is equal to his or her bid. (Cox, Smith and Walker(1988), p.97)



<Table-1> Optimum bids and matching probability for Type I consumer

Reservation price	μ=100, σ=2,cost=0				μ=100, σ=4,cost=0			
	bid	bid ratio	bid distance	match probability	bid	bid ratio	bid distanc e	match probability
108.00	102.13	0.946	1.06	0.86	101.39	0.939	0.35	0.64
107.00	101.80	0.951	0.90	0.82	100.97	0.944	0.24	0.60
106.00	101.46	0.957	0.73	0.77	100.47	0.948	0.12	0.55
105.00	101.13	0.963	0.56	0.71	99.96	0.952	-0.01	0.50
104.00	100.69	0.968	0.35	0.64	99.39	0.956	-0.15	0.44
103.00	100.19	0.973	0.10	0.54	98.79	0.959	-0.30	0.38
102.00	99.61	0.977	-0.20	0.42	98.15	0.962	-0.46	0.32
101.00	98.96	0.980	-0.52	0.30	97.46	0.965	-0.63	0.26
100.00	98.20	0.982	-0.90	0.18	96.75	0.968	-0.81	0.21
99.00	97.40	0.984	-1.30	0.10	96.01	0.970	-1.00	0.16
98.00	96.55	0.985	-1.73	0.04	95.22	0.972	-1.20	0.12

97.00	95.66	0.986	-2.17	0.01	94.41	0.973	-1.40	0.08
96.00	94.74	0.987	-2.63	0.00	93.58	0.975	-1.61	0.05
95.00	93.80	0.987	-3.10	0.00	92.73	0.976	-1.82	0.03
94.00	92.85	0.988	-3.58	0.00	91.86	0.977	-2.03	0.02
93.00	91.88	0.988	-4.06	0.00	90.97	0.978	-2.26	0.01
92.00	90.91	0.988	-4.55	0.00	90.07	0.979	-2.48	0.01
100.00	97.53	0.98	-1.24	0.32	96.36	0.96	-0.91	0.26

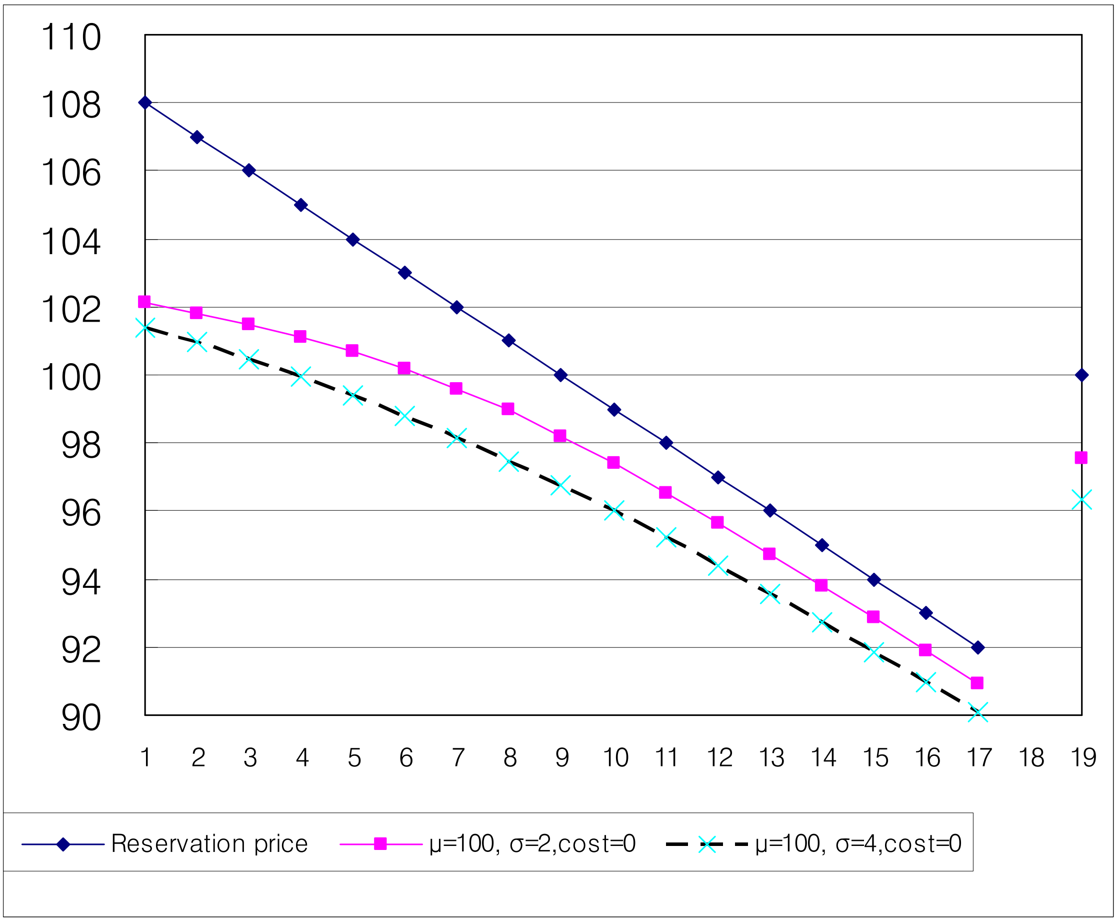
1) Numbers at the bottom row are the averages.

2) bid ratio(%)=bid/reservation price, bid distance=(bid-current price)/σ.

It would be interesting to see how far the consumers place their bids away from the current market price. This is represented by the bid distance, which measures the distance from the current price, standardized by the volatility. For example, the break-even consumers place bids at about 90% of the volatility below the current price. Consumers who anticipate the profit by one volatility (v=102) put their bids at 20% of the volatility below the current price. However, these figures vary with the daily volatility of the market.

The last column of <Table-1> shows the simulation result when the market volatility is raised to 4%, an extreme market day of a sudden panic or jump. Interesting to note is that consumers determine bids lower than before and the drop in the bids is most conspicuous with the consumer whose reservation price equals to the current price (100). We can visually confirm this from <Figure-3>. The curvature of the bid function becomes more linear. The reason is that, as the market becomes more volatile, the contribution of the marginal probability to the total probability of bid success, Ψ/Ψ', jumps up over the range of bids until the curve of Ψ/Ψ' with 4% volatility crosses the curve with 2% volatility. This time consumers have more room to raise profit (v-b) at a given probability than when the volatility was 2%. Further observation from <Table-1> is that consumers with reservation prices below 100 are blessed with higher matching probabilities even though they are lowering their bids as the market becomes more volatile.

<Figure-3> Comparison of the optimum bids with different volatilities with no cost of bid failure



Next we examine the case when consumers expect the market price change[15]. <Table-2> summarizes the simulation result when the mean of the market price is expected to move either up or down in parallel by one volatility.

<Table-2> Optimum bids with the expectation of price change for Type I consumers

Reservation price	μ=102, σ=2,cost=0					μ=98, σ=2,cost=0				
	bid	change	bid ratio	bid distance	probability	bid	change	bid ratio	bid distance	probability
108.00	103.46	1.33	0.958	1.73	0.77	100.43	-1.70	0.930	0.22	0.73
107.00	103.13	1.33	0.964	1.56	0.71	100.25	-1.55	0.937	0.12	0.71
106.00	102.69	1.23	0.969	1.35	0.64	100.13	-1.33	0.945	0.06	0.70
105.00	102.19	1.06	0.973	1.10	0.54	99.80	-1.33	0.950	-0.10	0.67
104.00	101.61	0.91	0.977	0.80	0.42	99.46	-1.23	0.956	-0.27	0.64

[15] We assume the consumers' expectation on market price change doesn't cause direct price change at the end of the period.

103.00	100.96	0.77	0.980	0.48	0.30	99.13	-1.06	0.962	-0.44	0.61
102.00	100.20	0.60	0.982	0.10	0.18	98.69	-0.91	0.968	-0.65	0.57
101.00	99.40	0.44	0.984	-0.30	0.10	98.19	-0.77	0.972	-0.90	0.52
100.00	98.55	0.34	0.985	-0.73	0.04	97.61	-0.60	0.976	-1.20	0.46
99.00	97.66	0.26	0.986	-1.17	0.01	96.96	-0.44	0.979	-1.52	0.40
98.00	96.74	0.19	0.987	-1.63	0.00	96.20	-0.34	0.982	-1.90	0.33
97.00	95.80	0.14	0.988	-2.10	0.00	95.40	-0.26	0.983	-2.30	0.26
96.00	94.85	0.11	0.988	-2.58	0.00	94.55	-0.19	0.985	-2.73	0.19
95.00	93.88	0.08	0.988	-3.06	0.00	93.66	-0.14	0.986	-3.17	0.14
94.00	92.91	0.06	0.988	-3.55	0.00	92.74	-0.11	0.987	-3.63	0.09
93.00	91.93	0.05	0.988	-4.03	0.00	91.80	-0.08	0.987	-4.10	0.06
92.00	90.95	0.04	0.989	-4.53	0.00	90.85	-0.06	0.987	-4.58	0.04
100.00	98.05	0.53	0.98	-0.97	0.22	96.81	-0.71	0.97	-1.59	0.42

1) Numbers at the bottom row are the averages.

2) bid ratio(%)=bid/reservation price, bid distance=(bid-reservation price)/σ.

3) Changes are calculated as the difference from the base case.

4) Probabilities are calculated from the normal distribution with mean=102 and 98 for each corresponding case, and volatility=2%.

When consumers expect the market price is rising, they would raise the bids to keep up with the matching probability. However, the magnitude of the bid increase is less than the expected price change because the desired change in the probability weight (Ψ/Ψ′) when the mean increases to 102 is smaller than the change in the expected price. Thus the probabilities of bid matching in the sixth column are smaller than those in the base case where the price is expected to remain unchanged. Note that the matching probabilities calculated now are lower than the previous case with mean 100. The probabilities will be different depending upon whether the price rise is expected only for an individual consumer under consideration or for all the market participants. Here we assume the price change is public information.

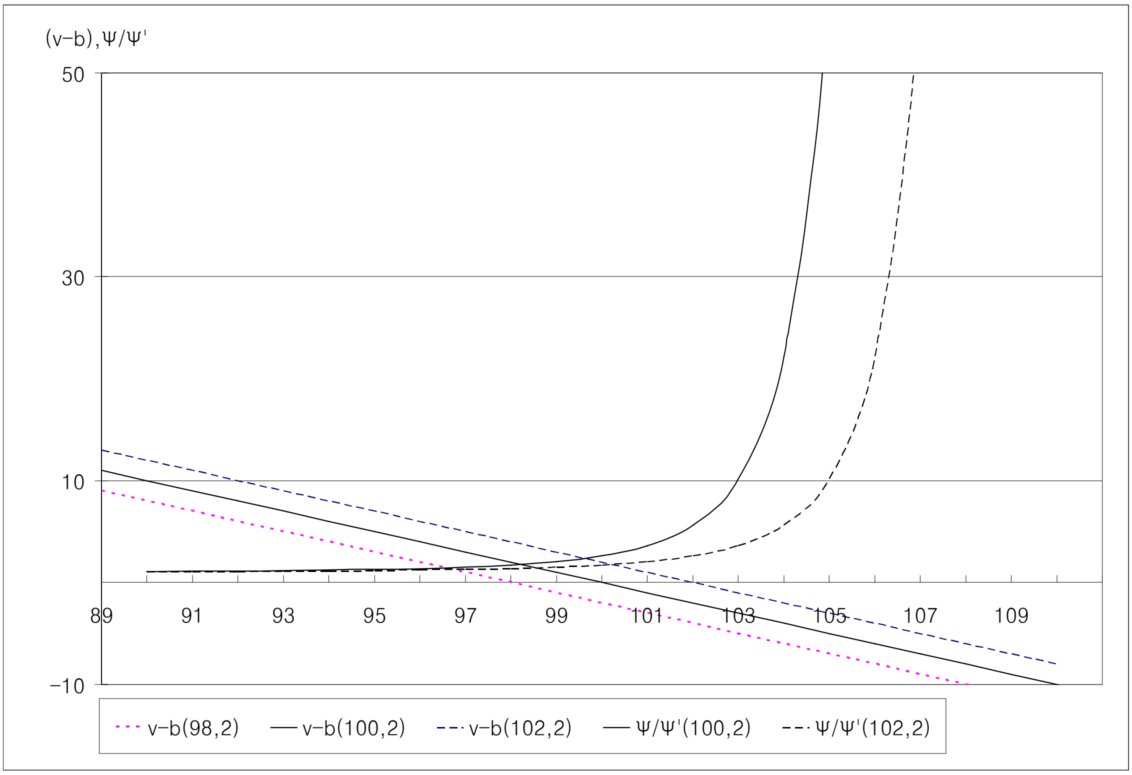
Also <Table-2> shows the bid response across different reservation prices. Consumers with higher reservation price raise bids relatively more compared with those with lower reservation price. This is because the probability of the bid matching is deteriorating fast as the mean of the price distribution approaches their bids from the left.

<Figure-4> explains the optimum bid response to the expected price rise of Type I consumers when the reservation prices are given at 98, 100, and 102, respectively

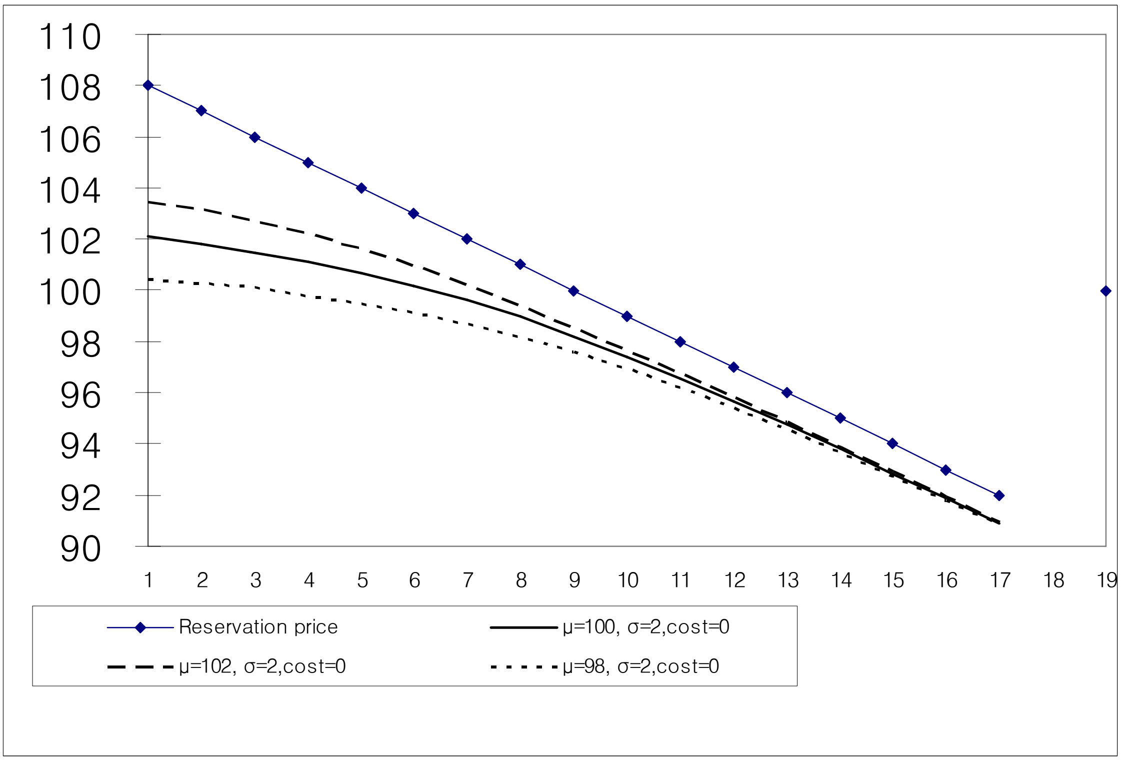
Symmetrically, when the price is expected to decline, consumers cut down their bids to secure more profit as the matching probability is improving, but at a decreasing rate.

In all the magnitude of the bid price adjustment is less than the expected market price change for Type I consumers. <Figure-5> compares bid responses to price changes across the different reference prices.

<Figure-4> Optimum bid response to the expected price rise of Type I consumer



<Figure-5> Bid distributions of Type I consumers when the market price is expected to move by one volatility



Now we look into the behavior of Type II consumers who are conscious of the costs arising from the bid failure. The difficulty here is to define the costs of the bid failure in the simulation. When a consumer's bid is not matched, he/she has two choices: one is to give up the purchase of commodity 1 and change the initial consumption plan, and the other is to wait for another match in the next period. In both cases costs tend to increase as the market price becomes more volatile. For example, traders maintain cash reserves in their transaction accounts to wait for the next trading opportunity, which tend to be linked with the market volatility.

Following this market convention, we assume the cost of bid failure is proportional to the daily volatility. We apply two different levels of costs, σ(=100%) and 2σ(=200%) to the optimum condition (12)[16]. The simulation results for each case are shown in <Table-3> and <Table-4>, respectively.

<Table-3> Optimum bids for Type II consumers when the market volatility=2%

Reservation price	μ =100, σ=2,cost=σ					μ =100, σ=2,cost=2σ				
	bid	change	bid ratio	bid distance	probability	bid	change	bid ratio	bid distance	probability

[16] To exactly quantify the costs, we need to select the specific form for the utility function.

108.00	102.43	0.30	0.948	1.22	0.89	102.79	0.66	0.952	1.40	0.92
107.00	102.25	0.45	0.956	1.12	0.87	102.61	0.81	0.959	1.31	0.90
106.00	102.13	0.67	0.963	1.06	0.86	102.43	0.97	0.966	1.22	0.89
105.00	101.80	0.67	0.969	0.90	0.82	102.25	1.12	0.974	1.12	0.87
104.00	101.46	0.77	0.976	0.73	0.77	102.13	1.43	0.982	1.06	0.86
103.00	101.13	0.94	0.982	0.56	0.71	101.80	1.60	0.988	0.90	0.82
102.00	100.69	1.09	0.987	0.35	0.64	101.46	1.86	0.995	0.73	0.77
101.00	100.19	1.23	0.992	0.10	0.54	101.13	2.17	1.001	0.56	0.71
100.00	99.61	1.40	0.996	-0.20	0.42	100.69	2.49	1.007	0.35	0.64
99.00	98.96	1.56	1.000	-0.52	0.30	100.19	2.79	1.012	0.10	0.54
98.00	98.20	1.66	1.002	-0.90	0.18	99.61	3.06	1.016	-0.20	0.42
97.00	97.40	1.74	1.004	-1.30	0.10	98.96	3.30	1.020	-0.52	0.30
96.00	96.55	1.81	1.006	-1.73	0.04	98.20	3.46	1.023	-0.90	0.18
95.00	95.66	1.86	1.007	-2.17	0.01	97.40	3.60	1.025	-1.30	0.10
94.00	94.74	1.89	1.008	-2.63	0.00	96.55	3.70	1.027	-1.73	0.04
93.00	93.80	1.92	1.009	-3.10	0.00	95.66	3.77	1.029	-2.17	0.01
92.00	92.85	1.94	1.009	-3.58	0.00	94.74	3.83	1.030	-2.63	0.00
100.00	98.81	1.29	0.99	-0.59	0.42	99.92	2.39	1.00	-0.04	0.48

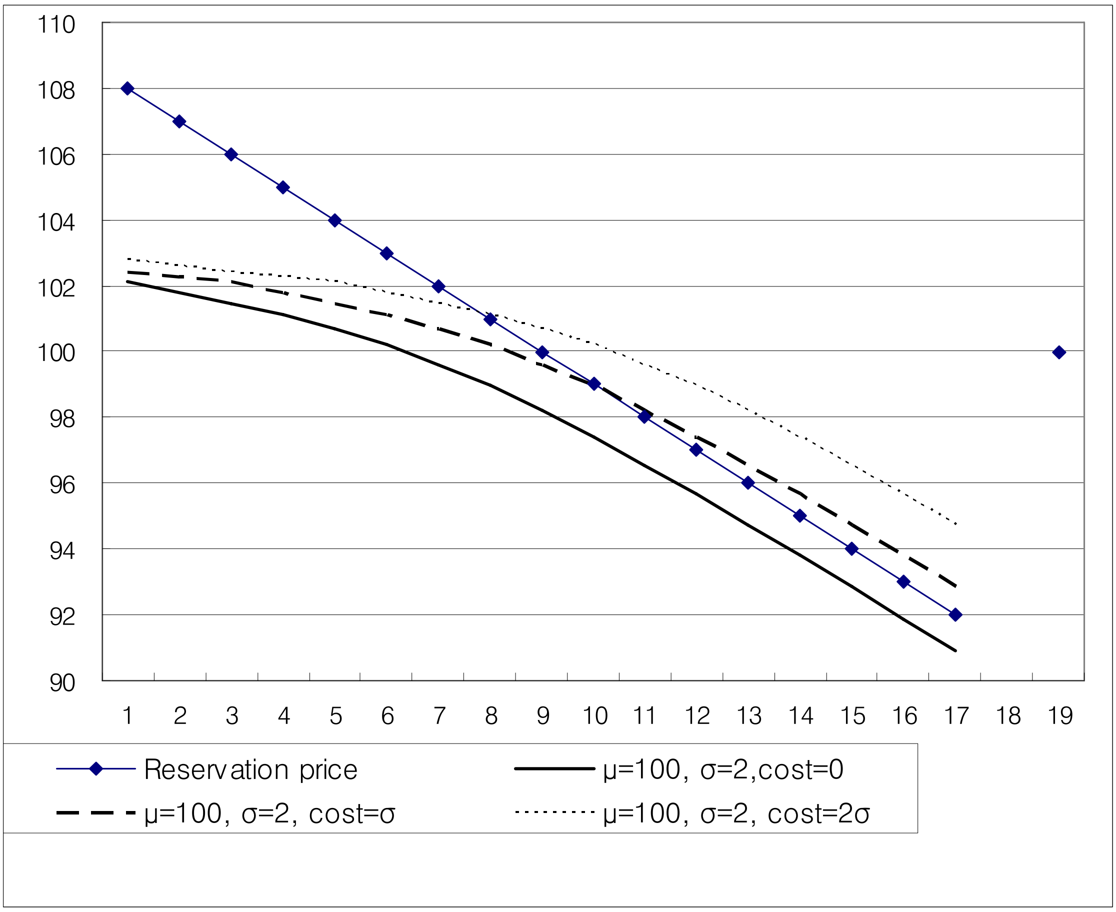
1) Numbers at the bottom row are the averages.

2) bid ratio(%)=bid/reservation price, bid distance=(bid-reservation price)/σ.

3) 'change' is the difference from the base case for Type I consumers.

When the bid failure costs are considered, consumers have to raise the bid to increase the matching probability above the level given for Type I consumer and thus avoid the costs incurred when their bids are not accepted. <Table-3> shows the result when the market volatility is 2% with the unmatched cost equal to 100% and 200% of σ, respectively. The bid prices of Type II consumers are higher than those of Type I consumers by 1.29 points on the average. This time consumers with lower reservation price are more eager to raise the bids. Furthermore, consumers with the reservation price below 98 even choose for their bids to exceed the reservation price, so are willing to accept the trading loss. However, the bid increase doesn't exceed the size of the unmatched cost. <Figure-6> compares the optimum bids for the three different levels of the unmatched cost at the same time; with the unmatched cost=0, 100%, and 200% of σ, respectively.

<Figure-6> Comparison of bids with the unmatched cost when the volatility=2%



This result is in contrast to the condition in the experiments by Chamberlin (1948) and Smith (1962, 1964, 1993). Especially in the experiments carried out by Smith participants were strictly forbidden to submit bids above the reservation price to obtain the lump sum bonus given to them when their orders are matched. The reservation prices in those experiments were given by the random selection of cards carrying the arbitrary numbers[17].

When the market volatility is given at 4%, an extreme market of a sudden panic or jump, bid prices rise steeply with the consideration of the unmatched cost, as shown in <Table-4>. This time the slope of the bidding function gets steeper with higher cost, meaning that consumers with higher reservation prices become more sensitive to the unmatched cost than when the volatility is given at 2%. We can see this from <Figure-7>. In a market panic, consumers who have large open position profits on hand are forced to respond to the higher unmatched costs.

In conclusion we can pursue numerous simulations by introducing various shapes of unmatched costs and price distributions, such as making the unmatched cost as a function of the

[17] This condition in those experiments might have been inevitable because they couldn't charge upon the participants, mostly college students, the loss amount when their bids exceeded the reference price.

market price as in (13). To find out the solution for (13), we may need to introduce more complicated assumptions.

$$E(AR) = \int_0^b (v-b)\Psi'(p_t)dp_t - \int_b^{\infty}\Delta(p_t)\Psi'(p_t)dp_t \qquad (13)$$

<Table-4> Optimum bids for Type II consumers when the market volatility=4%

Reservation price	μ =100, σ=4,cost=σ					μ =100, σ=4,cost=2σ				
	bid	change	bid ratio	bid distance	probability	bid	change	bid ratio	bid distance	probability
108.00	102.85	1.46	0.952	0.71	0.76	103.90	2.51	0.962	0.98	0.84
107.00	102.52	1.55	0.958	0.63	0.74	103.65	2.68	0.969	0.91	0.82
106.00	102.18	1.72	0.964	0.55	0.71	103.40	2.93	0.975	0.85	0.80
105.00	101.81	1.85	0.970	0.45	0.67	103.19	3.22	0.983	0.80	0.79
104.00	101.39	2.00	0.975	0.35	0.64	102.85	3.46	0.989	0.71	0.76
103.00	100.97	2.18	0.980	0.24	0.60	102.52	3.73	0.995	0.63	0.74
102.00	100.47	2.32	0.985	0.12	0.55	102.18	4.03	1.002	0.55	0.71
101.00	99.96	2.50	0.990	-0.01	0.50	101.81	4.34	1.008	0.45	0.67
100.00	99.39	2.64	0.994	-0.15	0.44	101.39	4.64	1.014	0.35	0.64
99.00	98.79	2.79	0.998	-0.30	0.38	100.97	4.96	1.020	0.24	0.60
98.00	98.15	2.93	1.002	-0.46	0.32	100.47	5.25	1.025	0.12	0.55
97.00	97.46	3.05	1.005	-0.63	0.26	99.96	5.55	1.031	-0.01	0.50
96.00	96.75	3.17	1.008	-0.81	0.21	99.39	5.81	1.035	-0.15	0.44
95.00	96.01	3.28	1.011	-1.00	0.16	98.79	6.06	1.040	-0.30	0.38
94.00	95.22	3.36	1.013	-1.20	0.12	98.15	6.29	1.044	-0.46	0.32
93.00	94.41	3.44	1.015	-1.40	0.08	97.46	6.49	1.048	-0.63	0.26
92.00	93.58	3.51	1.017	-1.61	0.05	96.75	6.68	1.052	-0.81	0.21
100.00	98.94	2.57	0.99	-0.27	0.42	100.99	4.63	1.01	0.25	0.59

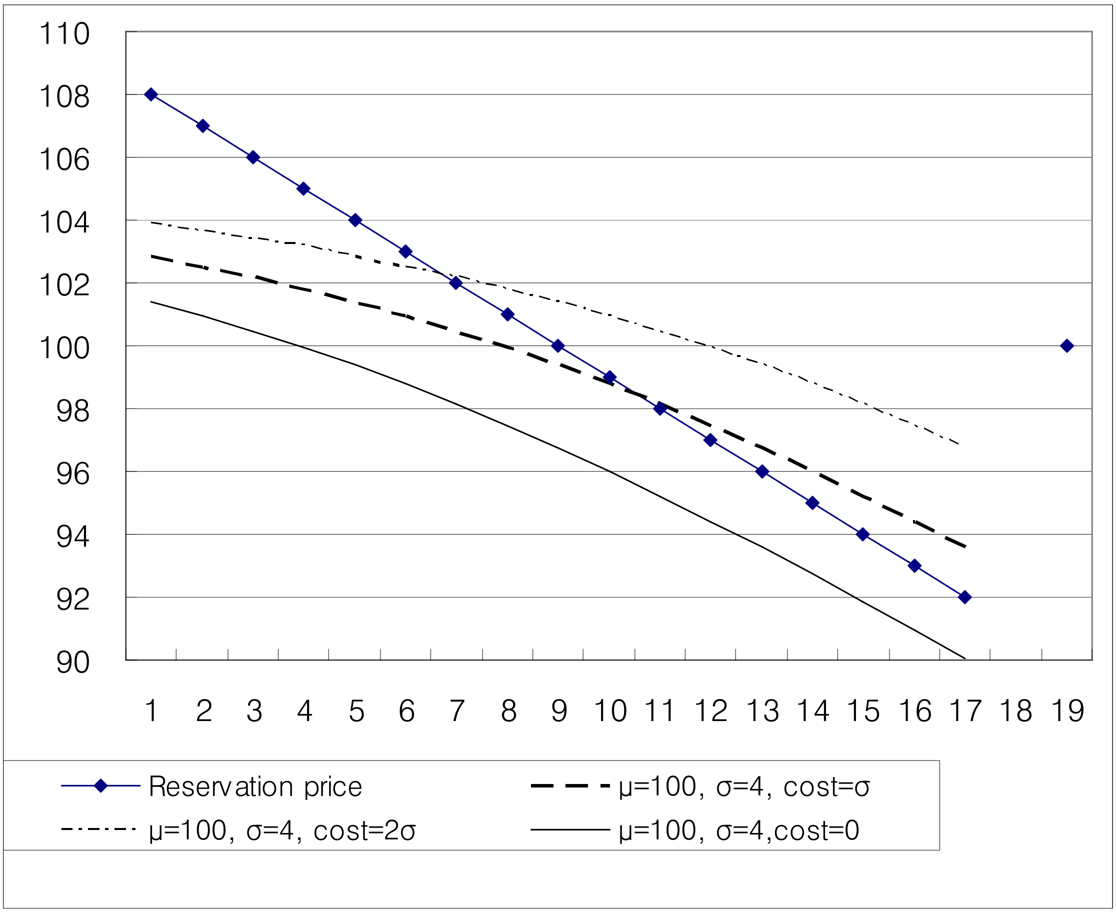
1) Numbers at the bottom row are the averages.

2) bid ratio(%)=bid/reservation price, bid distance=(bid-reservation price)/σ.

3) 'change' is the difference from the case when the volatility=4% and no unmatched cost.

4) Probabilities are calculated with the normal distribution with mean=100 and volatility=4%.

<Figure-7> Comparison of bids with the unmatched cost when volatility=4%



4. Conclusion and further research

This paper attempts to develop a theoretical model to analyze the consumer's bidding behavior at the market when the consumer can adjust his/her consumption plan responding to the market price changes. Our interest lies specifically in the determination of the bid price when consumers are conscious of the costs when their bids are not matched in the market.

Our ultimate goal is to seek the theoretical foundation for the bid generation in the market and develop a system simpler than the Nash equilibrium game models. While the game models use the interactive probability distributions considering all the other bidders in the market, our model employs only the probability distribution of the market price. In order to look into the economic outcomes derived from the model we resort to a simulation method with the artificial data rather than seeking analytical solutions and/or human experiments.

In the simulation we showed that bid prices can exceed the subjective value of the commodity when consumers are allowed to adjust the initial consumption plan and a certain amount of costs are incurred when their bids are not matched in the market. This is in contrast to the condition given in those experiments by Chamberlin (1948) and Smith (1962, 1964, 1993). In addition, we replicated the result of Vickrey (1961) that, with risk neutrality, the bid function curves away from the reservation prices and the bids above the mean of the price distribution contract down toward the mean, which can eventually produce the humped shape of the limit order book in the real market.

Even though our analysis provides the case where costs arise from the failure of bid matching from the consumer's perspective, no exact methodologies are provided to measure the size of the bid failure costs. Also we need to introduce various preference functions and market interactions into the simulation. In order to do this, we believe two further developments are necessary. One is a real market survey that investigates how the hedgers with bona fide interests determine the level of their bids, given the market price distribution and the open position profit and loss. The other is developing a simulation system equipped with the market matching operation, i.e., a system which can generate bids and offers from artificial traders and complete the order matching and clearing.

In the end the artificial market simulation method may be able to replace human experiments by introducing more conditions of the real economy into the model. For a theory testing purpose, human experiments are costly and constrained by the conditions of given resources. It is also difficult to replicate the experiment results by other researchers. The artificial simulation method has problems too. The most significant one is the difficulty in introducing market interactions. But this is becoming a matter of technical problem considering the current rapid technology improvement.

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