Law-of-One-Price in the Presence of Commodity Arbitrage: Theory and Evidence

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Abstract

This paper examines the stochastic trade cost model and analyzes the time series behavior of real exchange rate from both a theoretical and an empirical perspective. Motivated by the fact that deviations from the Law-of-One-Price (LOP) converge to trade costs once arbitrage occurs, we assume that LOP deviations are a two-sided censored random variable. We show that the large and volatile deviations from LOP can be explained by relevant real factors such as trade costs, output ratio volatility and intertemporal elasticity of substitution. We view our framework as complementary to those that emphasize the role of sticky prices.

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1. Introduction

The Law-of-One-Price (LOP) states that international relative price differentials should be arbitraged away so that identical goods in different countries should sell for the same price, when expressed in a common currency. Yet the evidence from the empirical literature shows that not only are relative prices quite different across countries, but also such deviations are highly volatile and persistent. These characteristics of LOP deviations have been the central puzzle in international macroeconomics literature, with the source of the puzzling behavior remaining unclear.

In this paper, we study the general equilibrium trade cost model to analyze the resulting equilibrium behavior of real exchange rate from both a theoretical and an empirical perspective. In particular, we explore the determinants of the time series behavior of real exchange rate in the context of a trade cost model and then discuss their implications for the puzzling behavior of the real exchange rate.

Recently, the trade cost models of Dumas (1992), Sercu et al.(1995), Obstfeld and Rogoff (2000), Betts and Kehoe (2001), O'Connell and Wei (2002), Burstein et al. (2003), and Crucini et al. (2005) have re-emerged as candidate models to explain the puzzling behavior of international relative prices and trade flows. The genesis of such models has derived from the recognition that sticky-price models, while useful in addressing international monetary policy questions, still lack the ability to explain the persistence and unconditional volatility of deviations from LOP and PPP (See, for example, Betts and Devereux (2000), and Chari et al. (2002)). With emphasis on the role of trade frictions and real shocks, the trade cost model shows that price deviations will be bounded by fixed limits of arbitrage, which are usually treated as proportional transportation costs. In the special case where preferences are separable across time and goods, the theory implies that LOP deviations will be a two-sided censored random variable. The probability that such censoring occurs will be a function of the distribution of the output ratio, the size of the trade costs, and the intertemporal elasticity of substitution in consumption.

In the context of this theory, we explore the determinants of time series behavior of LOP deviations. By establishing the comparative statics properties, we first show that the mean of LOP deviations is positively related to trade costs and mean output ratio, but negatively related to output ratio volatility. Second, the variance of LOP deviations is shown to be positively related to trade costs and output ratio volatility, but negatively associated with intertemporal elasticity of substitution. We then provide empirical evidence to support our model's predictions for the real exchange rate volatility.

The remainder of this paper is organized as follows. We begin in Section 2 by describing the economic environment and key predictions of the stochastic trade cost model by Sercu et al. In Section 3, we derive theoretical expressions for the mean and variance of the time-series distribution of the LOP deviations. We conduct comparative statics exercises to show how the mean and the variance are affected by changes in their determinants. The implication of censoring for the persistence of LOP deviations is also discussed. Section 4 presents empirical evidence that supports the predictions made from the theoretical propositions by using the implied trade costs estimated from a threshold autoregressive (TAR) model. Section 5 presents final discussion and conclusions.

2. The Theory of Goods Market Arbitrage

The theory described in this section builds upon the framework of Sercu, Uppal, and van Hulle (1995), which explores the quantitative implications of transaction costs for real and nominal exchange rate variability.

Each individual in the home and foreign country has identical time separable preferences and seeks to maximize lifetime utility over an infinite horizon. Every period, the home and foreign country are endowed with $y_j, y_j^* \geq 0$ units of good *j* respectively, which are stochastic and perishable. We assume that trade in physical goods is costly and of the iceberg variety. Thus, if x_j units of the good are shipped from the home to the foreign country, $x_j / (1+\tau_j)$ units are received. Finally, following Sercu et al., the individuals are assumed to pool all idiosyncratic risks associated with the random endowment processes. Thus, we solve the following hypothetical social planning problem[2]:

$$Max \ E_0 \sum_{t=0}^{\infty} \beta^t \left(\sum_{j=0}^{N} u(c_{jt}) + \sum_{j=0}^{N} u(^{*}_{jt}) \right)$$

such that, for every good j:

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[2] Sercu et al. assume financial markets are complete, so that the relative price of the same good across two locations is the ratio of the marginal utilities of the home and foreign agent evaluated at the endowment point (when the no-trade equilibrium occurs) or the point of arbitrage, in which case the relative price is determined by the trade cost (when the trade occurs). The case presented here assumes separability of utility across goods; the general case of non-separable utility is virtually intractable.

$$c_{jt} = y_{jt} - x_{jt} + \frac{x^*_{jt}}{1+\tau_j}$$

$$c^*_{jt} = y^*_{jt} - x^*_{jt} + \frac{x_{jt}}{1+\tau_j}$$

$$0 \le x_{jt} \le y_{jt}$$

$$0 \le x^*_{jt} \le y^*_{jt}$$

where x_{jt} is the export of the good from the home to the foreign country, and $x^*_{jt} \big/ (1+\tau_j)$ is the import of the good from the foreign to the home country.

We have assumed complete symmetry of countries in setting up the model. In the absence of trade costs, the equilibrium would involve an even split of the endowment of each good in each state of nature. The presence of trade costs alters the optimal allocations because it is less costly to provide utility to the home agents using goods that are available in the home country, thus avoiding deadweight losses associated with trade costs.

2.1 Implications

Assuming that utility takes the form $u(c_{jt}) = \frac{1}{1-\theta_j}(c_{jt})^{1-\theta_j}$, the quantity and relative price implications of the model are conveniently summarized as follows:

LOP deviations (Q_{jt})	Trade patterns	Condition
$\frac{1}{1+\tau_j}$	Home country exports	If $\frac{y^*_{jt}}{y_{jt}} < \left(\frac{1}{1+\tau_j}\right)^{\frac{1}{\theta_j}}$
$\left(\frac{y^*_{jt}}{y_{jt}}\right)^{\theta_j}$	No trade equilibrium	Otherwise
$1+\tau_j$	Foreign country exports	If $\frac{y^*_{jt}}{y_{jt}} > (1+\tau_j)^{1/\theta_j}$

According to this model, there are three possible trade patterns and corresponding relative prices:

● A no-trade equilibrium (when the output ratio is sufficiently close across countries, so that the gains from trade are not high enough to offset the trade costs). The implicit relative price is a matter of reading off the appropriate marginal valuations, expressed as the ratio of home and foreign marginal utility evaluated at autarkic output points. The implied price differential is not sufficient at these output levels to justify paying the trade cost[3].

● An equilibrium in which goods flow from the home to the foreign country (when the home output is sufficiently large relative to the foreign output); the price in the home country is τ less than the foreign price.

[3] The relative price formula will be familiar to readers of Backus, Kydland and Kehoe (1994) or Backus and Crucini (2000). In these studies, q is the terms of trade, the output ratio is replaced with the ratio of imports to exports, and the elasticity is across imports and exports, rather than the same good (although this interpretation may be adapted to the present setting quite easily). The difference is that in these models the terms of trade has an infinite support because unique goods have different prices at all output ratios.

● An equilibrium in which goods flow from the foreign to the home country (when the foreign output is sufficiently large relative to the home output); the price in the home country is τ greater than the foreign price.

2.2 Distributional Assumptions

The trade cost model with endowment shocks has a rich set of implications for the stochastic properties of LOP deviations. Because the distribution of the real exchange rate has a support bounded by the trade costs, the real exchange rate is a two-sided censored random variable with continuous and discrete parts. In our analysis it is useful to make some transformations of variables and time-series distributional assumptions. We thus work with the logarithm of LOP deviations and assume: $\log(y^*_{jt} / y_{jt}) \sim N(\mu_j, \sigma^2_j)$. Then, the distribution of $q_{jt} \{\equiv \log Q_{jt}\}$ is:

q_{jt}	$f(q_{jt})$	Trade patterns	Condition
$-\tau_j$	$\Phi(A_j)$	Home country exports	If $\frac{y^*_{jt}}{y_{jt}} < \left(\frac{1}{1+\tau_j}\right)^{\frac{1}{\theta_j}}$
$\theta_j \log \frac{y^*_j}{y_j}$	$\Phi(B_j) - \Phi(A_j)$	No trade equilibrium	Otherwise
τ_j	$1-\Phi(B_j)$	Foreign country exports	If $\frac{y^*_{jt}}{y_{jt}} > (1+\tau_j)^{1/\theta_j}$

where $A_j = \frac{-\tau_j - \theta_j \mu_j}{\theta_j \sigma_j}$, $B_j = \frac{\tau_j - \theta_j \mu_j}{\theta_j \sigma_j}$ and $\Phi(.)$ is the standard normal cdf.

The time-series distribution of q has a support bounded by the trade costs $(-\tau \ \tau)$. Thus, given that we are assuming the output ratio has a normal distribution, we just need to calculate the probability that a normal random variable exceeds (or falls short of) a particular threshold to compute the probability mass at these discrete thresholds. In general, the moments of q depend on the mean of the output ratio (μ), the variance of the output ratio (σ^2), the intertemporal elasticity of substitution (θ) and the trade costs (τ). In the sections that follow, we develop the implications of the trade cost model for the time-series properties of LOP deviations.

3. Time-Series Properties of LOP Deviations

3.1 The Mean of LOP Deviations

The time-series mean of q_{jt} is perhaps the most interesting moment; it tells us how large the absolute deviation from LOP is expected to be. The mean is given by:

$$
\begin{aligned}
E(q_{jt}) &= p(q_{jt} = -\tau_j)E(q_{jt} \mid q_{jt} = -\tau_j) + p(q_{jt} = \tau_j)E(q_{jt} \mid q_{jt} = \tau_j) \\
&\quad + p(-\tau_j < q_{jt} < \tau_j)E(q_{jt} \mid -\tau_j < q_{jt} < \tau_j) \\
&= \Phi(A_j)(-\tau_j) + (1 - \Phi(B_j))(\tau_j) \\
&\quad + (\Phi(B_j) - \Phi(A_j))\left[\theta_j \mu_j + \sigma_j \theta_j \frac{\phi(A_j) - \phi(B_j)}{\Phi(B_j) - \Phi(A_j)}\right] \qquad (1)
\end{aligned}
$$

where $\phi(.)$ is the standard normal pdf. Having determined the mean of LOP deviations, without loss of generality, we assume $\mu_j > 0$ and establish the comparative statics for the mean of LOP deviations.

Proposition 1.1. *Deviations from the Law-of-One-Price increase as trade costs increase.*

Proof. To investigate the effect of an increase in the trade cost, totally differentiate Equation (1) with respect to τ_j to obtain

$$\frac{dE(q_{jt})}{d\tau_j} = \underbrace{\left[1-\Phi(B_j)-\Phi(A_j)\right]}_{(a)} + \underbrace{\frac{\tau_j}{\sigma_j\theta_j}\left[\phi(A_j)-\phi(B_j)\right]}_{(b)}$$
$$+\underbrace{\left[\frac{\tau_j+\mu_j\theta_j}{\sigma_j\theta_j}\phi(A_j)-\frac{\tau_j-\mu_j\theta_j}{\sigma_j\theta_j}\phi(B_j)\right]}_{(c)}+\underbrace{\frac{\mu_j}{\sigma_j}\left[\phi(A_j)+\phi(B_j)\right]}_{(d)}>0 \qquad (2)$$

Consider following two cases: First, if $-\tau_j < 0 < \tau_j < \mu_j$, then $(a) > 0$, $(b) > 0$, $(c) > 0$, $(d) > 0$, and we have $dE(q_{jt})/d\tau_j > 0$. Second, if $-\tau_j < 0 < \mu_j < \tau_j$, then $(a) > 0$, $(b) > 0$, $(c) > 0$, $(d) > 0$, and we have $dE(q_{jt})/d\tau_j > 0$. Therefore $dE(q_{jt})/d\tau_j > 0$. ■

Suppose the home country has more output than the foreign country. In the absence of trade costs, the total world output would be evenly split between two countries, and resulting deviations from LOP would be zero. However, if trade costs are strictly positive, the no-trade equilibrium arises, and the resulting relative prices are determined purely by the relative output ratio. This causes the relative price to deviate from the parity, and therefore, deviations from LOP become larger with the larger trade costs.

Next, we show the effects of a change in the mean output ratio.

Proposition 1.2. *Deviations from the Law-of-One-Price increase as the supply of perishable goods rises in one country relative to the other country.*

Proof. Totally differentiating Equation (1) with respect to μ_j, we obtain

$$\frac{dE(q_{jt})}{d\mu_j} = \theta_j\left[\Phi(B_j) - \Phi(A_j)\right] \tag{3}$$

Because $B_j = \dfrac{\tau_j - \theta_j\mu_j}{\theta_j\sigma_j} > A_j = \dfrac{-\tau_j - \theta_j\mu_j}{\theta_j\sigma_j}$, we have $\Phi(B_j) - \Phi(A_j) > 0$ and hence $dE(q_{jt})/d\mu_j > 0$. ■

Proposition 1.2 implies that as the mean of the output ratio converges to infinity (zero) - the log of the output ratio converges to infinity (negative infinity) -, the real exchange rate converges to -τ (τ). In other words, Proposition 1.2 simply states the conditions for stable trading patterns in this endowment economy. If the endowment of the good is always arbitrarily larger at home than abroad, the home country will export, and the price will be τ higher for foreign consumers relative to domestic consumers. Thus, for traded goods with distinct and stable comparative advantage, the distribution of q_{jt} would be discrete with all of the mass on the arbitrage limit.

Turning to the effects of a change in the volatility of output ratio, we have the following.

Proposition 1.3. *Deviations from the Law-of-One-Price decrease as the volatility of output ratio increases.*

Proof. Totally differentiating Equation (1) with respect to σ_j yields

$$\frac{dE(q_{jt})}{d\sigma_j} = \theta_j \left[\phi(A_j) - \phi(B_j)\right] \tag{4}$$

If $-\tau_j < 0 < \tau_j < \mu_j$, then $\phi(A_j) - \phi(B_j) < 0$ and we have $dE(q_{jt})/d\sigma_j < 0$. If $-\tau_j < 0 < \mu_j < \tau_j$, then $\phi(A_j) - \phi(B_j) < 0$ and we have $dE(q_{jt})/d\sigma_j < 0$. Therefore $dE(q_{jt})/d\sigma_j < 0$. ■

As the volatility of the output ratio increases, trade reversals become more frequent, and the probability mass converges on the limit points. Thus, holding fixed trade costs, the mean of LOP deviations is decreasing in the output ratio volatility.

3.2 The Variance of LOP Deviations

In this subsection, we explore the determinants of relative price variability over time. The variance of q_{jt} is generally quite complicated, so we focus on the symmetric case for our analytical results. The only additional symmetry assumption we need is $\mu_j = 0$ (since we have been assuming symmetric arbitrage costs and a symmetric distribution for the output ratio). As shown in detail in the Technical Appendix, the variance of LOP deviations is:

$$Var(q_{jt}) = 2\Phi(A_j)\tau_j^2 + [\Phi(B_j) - \Phi(A_j)]\theta_j^2\sigma_j^2\left[1 + \frac{A_j\phi(A_j) - B_j\phi(B_j)}{\Phi(B_j) - \Phi(A_j)}\right] \quad (5)$$

where $A_j = \frac{-\tau_j}{\theta_j\sigma_j}, B_j = \frac{\tau_j}{\theta_j\sigma_j}$ assuming $\mu_j = 0$. As is evident in Equation (5), the volatility of LOP deviations is affected by trade costs (τ), the elasticity of substitution (θ), and the volatility of the output ratio (σ^2).

Having determined the volatility of LOP deviations, we now turn to comparative statics to examine how the volatility responds to each of its determinants. From Equation (5), we can derive the following results:

Proposition 2.1. *Volatility of Law-of-One-Price deviations increases as trade costs increase.*

Proof. From totally differentiating Equation (5) with respect to τ_j, we obtain:

$$\frac{dVar(q_{jt})}{d\tau_j} = 4\tau_j\Phi(A_j) \quad (6)$$

Because τ_j>0 and $\Phi(A_j) > 0$, we have $dVar(q_{jt})/d\tau_j > 0$. ■

Proposition 2.1 states that when the preference parameter and output ratio volatility remain constant, the trade costs have a positive impact on the volatility of real exchange rate. This occurs because higher trade cost makes goods less likely to be traded and hence increases the limit for exchange rate fluctuations. Consider a good, say a haircut, with $\tau \approx 1$ and subject to prohibitive trade costs. Obviously the volatility of the relative prices of a haircut will be larger than that of the relative prices of goods with smaller trade costs.

The effect of the elasticity of substitution on the real exchange rate volatility can be established as follows.

Proposition 2.2. *Volatility of Law-of-One-Price deviations increases as the intertemporal elasticity of substitution decreases.*

Proof. Totally differentiating Equation (5) with respect to θ_j yields

$$\frac{dVar(q_{jt})}{d\theta_j} = 2\theta_j \sigma_j^2 \left[(\Phi(B_j) - \Phi(A_j)) - 2B_j \phi(B_j) \right] \tag{7}$$

Because $\Phi(B_j) - \Phi(A_j) - 2B_j\phi(B_j) > 0$, we have $dVar(q_{jt})/d\theta_j > 0$. ■

The parameter θ measures the extent to which individuals are willing to substitute consumption over time. As θ gets large, it takes larger changes in relative prices to get individuals to alter their consumption plans over time. We refer to this as the direct effect: Less substitutability (that is large θ) directly leads to a higher volatility of relative prices. However, it is obvious from the equilibrium condition that an increase in θ also decreases the no-trade zone, which in turns reduces the volatility of relative prices. We refer to this as the indirect effect. Thus the overall impact of an increase in θ depends on the relative size of direct and indirect effects. Proposition 2.2 shows that the direct (negative) effect dominates the indirect (positive) effect, and hence the elasticity of substitution has a negative impact on the volatility of LOP deviations.

Finally the impact of the volatility of endowment ratio on the real exchange rate volatility can be analyzed as in Proposition 2.3.

Proposition 2.3. *Volatility of Law-of-One-Price deviations increases as the volatility of output ratio increases.*

Proof. Totally differentiating Equation (5) with respect to σ_j yields

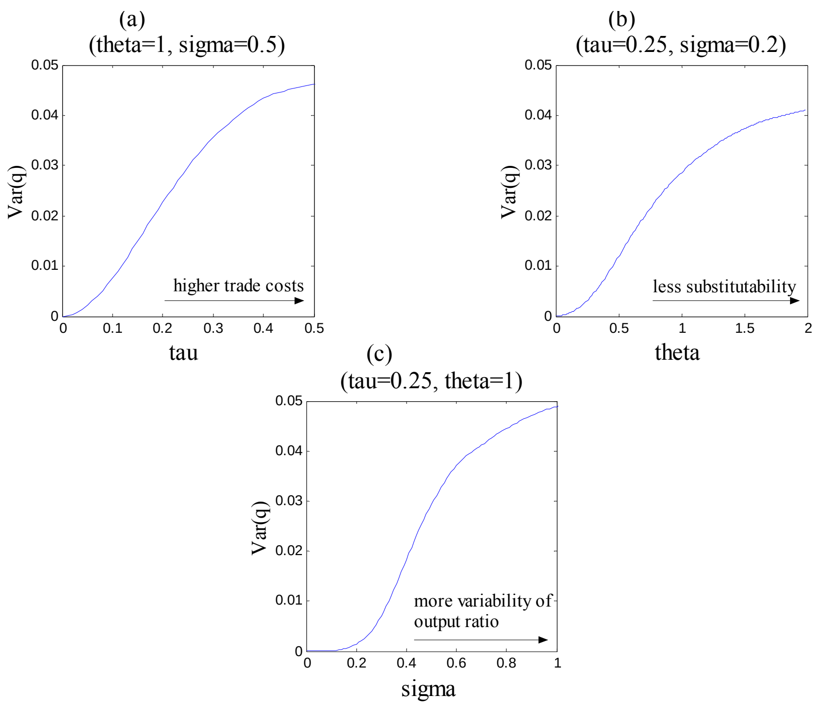
$$\frac{dVar(q_{jt})}{d\sigma_j} = 2\theta_j^2 \sigma_j \left[(\Phi(B_j) - \Phi(A_j)) - 2B_j \phi(B_j) \right] \tag{8}$$

Because $\Phi(B_j) - \Phi(A_j) - 2B_j\phi(B_j) > 0,$ we have $dVar(q_{jt})/d\sigma_j > 0.$ ■

Here again, we have two opposing effects. The direct effect of a rise in the output ratio volatility is to increase the variability of q since within the arbitrage bands, the two are log-linearly related. On the other hand, as the distribution of output ratio becomes flatter due to its increased variability, the chance that the output ratio will fall outside of the no-trade zone (i.e., censoring points) also rises. Thus, as the volatility of output ratio becomes arbitrarily large, the probability mass converges on the limit points, which reduces the real exchange rate volatility conditional on being at one of the limit points. We refer to this as the indirect effect. Proposition 2.3 shows that the direct (positive) effect dominates the indirect (negative) effect, implying that the output ratio volatility has a positive impact on the volatility of the real exchange rate.

Figures 1(a) through 1(c) illustrate the results of the Monte Carlo experiment for the volatility of LOP deviations. The lines are the time-series variance of LOP deviations. Figure 1(a) examines the impacts of changing trade costs (τ) on the volatility of LOP deviations, assuming that $\theta = 1.0$ and $\sigma = 0.5$.. The result confirms what Proposition 2.1 suggests. As the trade costs increase, the LOP deviations become more volatile. In Figure 1(b), we assume $\tau = 0.25$ and $\sigma = 0.2$, and examine the impacts of changing intertemporal elasticity of substitution on the volatility of LOP deviation. The experimental result shows that the volatility of LOP deviations increases in less substitutability, and hence it is negatively related to intertemporal elasticity of substitution, as suggested by Proposition 2.2. Likewise, Figure 1(c) shows that, given $\tau = 0.25$ and $\theta = 1.0$, a rise in the output ratio volatility has a positive effect on the volatility of LOP deviations, again as suggested by Proposition 2.3.

Figure 1. Monte Carlo Simulations for Time-Series Volatility of LOP Deviations



3.3 The Persistence of LOP Deviations

A conventional way of examining the persistence of the real exchange rate is to consider an AR(1) process:

$$q_t = \rho q_{t-1} + \varepsilon_t \tag{9}$$

where ε_t is assumed to be N(0,σ^2). Of course, in the context of the trade cost model, this statistical model is mis-specified because the q_t process is non-linear: it is a two-sided censored variable that is a mixture of discrete and continuous distributions. Motivated by the fact that real exchange rates follow the nonlinear process around the thresholds characterized by the trade costs, we relax the assumption of instantaneous adjustment by allowing for movement outside the arbitrage bands. This approach basically emphasizes that price differentials decay slowly within arbitrage bands, but rapidly outside the bands as international trade takes place (see Michael et al. (1997), Obstfeld and Taylor (1997), and O'Connell (1998)). In this context, we adopt the threshold autoregressive (TAR) process in which trade costs lead to a nonlinear adjustment by triggering faster reverting behavior for larger deviations from LOP. Because the trade cost model predicts that deviations from LOP revert to the threshold points, we consider the BAND-TAR model that can be written as:

$$\Delta q_t = \left\{ \begin{array}{lll} \beta^{out}(q_{t-1} - \tau) + e_t^{out} & if & q_{t-1} > \tau \\ \beta^{in} q_{t-1} + e_t^{in} & if & -\tau < q_{t-1} < \tau \\ \beta^{out}(q_{t-1} + \tau) + e_t^{out} & if & q_{t-1} < -\tau \end{array} \right\} \tag{10}$$

where $e_t^{out} \sim N(0, \sigma^{out^2})$, $e_t^{in} \sim N(0, \sigma^{in^2})$ and τ represents the threshold point (i.e., trade costs). β^{out} and β^{in} refer to the convergence speed outside and inside the band respectively. Following Obstfeld and Taylor (1997), we assume $\beta^{in} = 0$ such that the real exchange rate follows a random walk inside the band. This is because LOP deviations that lie within the band do not exhibit mean-reversion as arbitrage does not take place due to trade costs. Moreover, the trade cost model predicts that LOP deviations within the band follow a process determined by the output ratio, so we have $q_t = \log(y_t^* / y_t)^{\theta}$. Thus, our assumption of $\beta^{in} = 0$ can also be justified by the recognition that the output ratio within the no-trade zone may follow a random walk.

The proposed TAR model (Equation (10)) is used to find optimal threshold points in the presence of trade costs. By using this model, we are able to examine the empirical relationship between the real exchange rate volatility and TAR-implied trade costs.[4] The optimal threshold value is estimated using a best-fit grid search for an optimal threshold (τ) that maximizes the log-likelihood ratio.[5]

4. Empirical Analysis

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[4] In this paper, the relationship between β^{out} and τ is beyond our consideration because there is no rationale for any significant cross-sectional relationship between persistence `outside the band' and size of the threshold. One extension of this study would be to explore the overall persistence parameter both inside and outside the bands and its relationship to the size of the threshold.

[5] See Obstfeld and Taylor (1997) for detailed descriptions of this estimation methodology.

The theoretical propositions discussed in the previous section provide testable predictions regarding the effects of trade costs, elasticity of substitution, and output ratio volatility on time-series behavior of deviations from LOP. In this section, we test those predictions using aggregate level data.

4.1 Data

According to the propositions developed in Section 3, the volatility of real exchange rates is expected to be positively related to the trade costs and output ratio volatility, and negatively related to the intertemporal elasticity of substitution. We use aggregate level data to examine these propositions. While using aggregate data limits the researchers to moments other than the mean, it provides longer time horizon data needed to calculate second moment or business cycle-type properties (e.g. standard deviation or persistence over time).

With the exception of the measures of trade costs, the data are constructed using the IMF's International Financial Statistics for 47 countries from 1973 1/4 to 1998 4/4.[6] We measure the real exchange rate volatility as the standard deviation of bilateral real exchange rates using the CPI and nominal exchange rate data. The output ratio volatility is measured as the standard deviation of output ratios using real GDP data.

The simplifying assumptions we made in specifying the utility function (separability across goods with a common elasticity for all goods) direct us to think about intertemporal

[6] The countries include 23 developed countries and 24 developing countries. Country income groups are taken from the World Development Indicators.

elasticities of substitution in consumption. The importance of the elasticity for the volatility of relative prices was made obvious by Backus et al. (1992). They derived an Euler equation linking the ratio of traded quantities to the terms of trade:

$$p_t = \varphi \log(\frac{x_t}{m_t})$$

where p_t is the terms of trade between domestic (x_t) and foreign (m_t) goods and φ is the elasticity of substitution across them. In the context of the trade cost model studied here, the relationship within the no-trade zone is mathematically identical:

$$q_t = \theta \log(\frac{y_t^*}{y_t})$$

where q_t is now the relative valuation of the same good across two locations and the quantity ratio is the production allocation of that good across locations. This equation suggests a method for estimating the intertemporal elasticity of substitution in our study:

$$\widehat{\theta} = \frac{std(q_t)}{std(\log(\frac{y_t^*}{y_t}))} \tag{11}$$

The crucial variable we construct is the trade costs. Direct measures of trade costs are known to be difficult to measure due to inconsistencies across countries in bilateral value and quantity data for trade, cross-hauling of goods, and aggregation bias. Motivated by the difficulties of the standard method in measuring trade costs, we estimate the trade costs in the threshold autoregressive model (TAR) by searching for the optimal threshold value that maximizes the log-likelihood ratio. In this paper, for the purpose of robustness check, we also consider geographic distance (in kilometers) between country capitals as an alternative measure for the trade costs.

4.2 Implied Trade Costs

In this subsection, we apply the BAND-TAR model to search for the optimal threshold value using real exchange rates for 47 countries with Belgium being the numeraire country. Table 1 provides a summary of our findings on the optimal thresholds. The estimated thresholds (e.g. cost as a percentage of trade) are heterogeneous across countries, ranging from a low of 0.6% between Belgium and Netherlands to a high of 57.1% between Belgium and Indonesia. It is worth noting that implied trade costs are lower between Belgium and other European countries (0.6%~13.5%) than they are between Belgium and countries outside Europe (4.4%~57.1%). This result is expected since European countries are both geographically close and economically integrated.

One would expect higher trade costs for Europe with respect to some distant locations. This is examined by using the U.S. as the numeraire location. According to Table 1, the implied trade costs for Europe vis-à-vis the United States is higher. The trade costs from the U.S. ranges from a low of 7.3% between the U.S and Canada to a high of 46.0% between the U.S and Sri Lanka.

4.3 Regression Results

In order to examine the effects of trade costs, intertemporal elasticity of substitution and output ratio volatility on the real exchange rate volatility, we consider two main regression specifications: One with only the trade costs, intertemporal elasticity of substitution and output ratio volatility, and the other controlling for nominal exchange rate volatility. We also estimate each specification using geographic distance as a proxy for the trade costs. We choose Belgium as the numeraire country in our empirical work:

(1a) $\sigma_q = \alpha_0 + \alpha_1 \log \tau + \alpha_2 \theta + \alpha_3 \sigma_y + \varepsilon$

(2a) $\sigma_q = \delta_0 + \delta_1 \log \tau + \delta_2 \theta + \delta_3 \sigma_y + \delta_4 \sigma_s + \upsilon$

(1b) $\sigma_q = \gamma_0 + \gamma_1 \log D + \gamma_2 \theta + \gamma_3 \sigma_y + \eta$

(2b) $\sigma_q = \lambda_0 + \lambda_1 \log D + \lambda_2 \theta + \lambda_3 \sigma_y + \lambda_4 \sigma_s + \varphi$

where σ_q is a standard deviation of log real exchange rate, τ is the implied trade costs estimated from the TAR model, θ denotes the intertemporal elasticity of substitution, σ_y is a standard deviation of log output ratio, D is a geographic distance in kilometers measured as the greater circle distance from the numeraire country (Belgium), and σ_s is a standard deviation of first difference in log nominal exchange rate. To avoid multicollinearity, we demean and detrend real exchange rates and output ratios in calculating the intertemporal elasticity of substitution.

Table 2 presents cross-sectional regression results for Specification (1a)~(2b). As discussed in Section 3, an increase in trade costs or volatility of output ratio is expected to increase the volatility of the real exchange rate, while an increase in the intertemporal elasticity of substitution is likely to decrease the volatility of the real exchange rate. The trade cost model also implies that, within the arbitrage bands, changes in the nominal exchange rate lead to a one-to-one relationship with changes in the real exchange rate. Therefore we expect $\delta_1 > 0$, $\delta_2 > 0$, $\delta_3 > 0$, and $\delta_4 > 0$ in Specification (2a). Note that the coefficient for the elasticity is also expected to be positive because the larger value of the parameter θ indicates a lower elasticity of substitution (that is less substitutability).

The result in Table 2 supports the model's predictions. The point estimates of $\delta_1, \delta_2, \delta_3$, and δ_4 are positive and statistically significant, as expected in Specification (2a). This finding is consistent with both the idea of price rigidity and the trade cost model. Note that the Specification (1a) includes only implied trade costs, elasticity of substitution and output ratio volatility as explanatory variables, omitting the nominal exchange rate volatility. Unless the covariance between σ_S and other explanatory variables is equal to zero, the point estimates of α_1, α_2, and α_3 would be biased in the direction of the correlation between σ_S and other regressors times the direction of the effect of σ_S. Table 2 shows that omitting the effect of nominal exchange rate causes an upward bias in the magnitude of the effect of trade costs, elasticity of substitution and output ratio volatility.

Turning to the geographic distance, the estimation result is similar to the previous result, with all the coefficients being positive as expected. Again, Specification (2b) implies that locations farther apart have more volatile real exchange rates, even when controlling for the nominal exchange rate.

The economic interpretations of coefficients in the context of the trade cost model are as follows: After controlling for elasticity of substitution, output ratio variability and nominal exchange rate volatility, if trade costs increase from 10% to 11%, then the time-series standard deviation of real exchange rate is expected to increase by 0.31. If the distance between locations increases from 1000 kilometers to 1,200 kilometers, the expected increase is 0.18. These are large effects, considering that σ_q ranges from 0.0129

(Belgium-Luxembourg) to 0.622 (Belgium-Nigeria).

Finally it is interesting to examine the effect of trade costs on the real exchange rate volatility conditional on income groups. Bravo Ortega and Giovanni (2004) showed that the degree of nontradability caused by trade costs is more apparent for more developed countries due to diverse and dynamic industrial structures across sectors of the economy. Table 3 reports estimates of the parameters for developed and developing countries. We see that the coefficient for the trade costs is larger and more significant for developed countries, as expected. Moreover, the regression function explains a greater portion of the cross-sectional variation in our data for developed countries.

5. Conclusions

Trade costs drive a natural wedge between relative prices in different locations, leading to deviations from LOP for traded goods. Although an expanding body of literature has documented this effect across space and time, the channel through which trade costs affect the time series behavior of the real exchange rates remains yet unclear. To address the economic significance of arbitrage costs, this paper answers two questions in the context of a trade cost model: (1) What are the determinants of the time series behavior of the real exchange rates? and (2) Can trade costs and other determinants explain why real exchange rates behave in such a volatile manner?

To answer these questions, this paper examines the stochastic general equilibrium trade cost model and analyzes the resulting equilibrium behavior of real exchange rate from both a theoretical and an empirical perspective. We show that the mean of LOP deviations is positively related to trade costs and mean output ratio, but negatively related to output ratio volatility. More importantly, we show that the variance of LOP deviations is positively associated with trade costs and output ratio volatility, but negatively associated with intertemporal elasticity of substitution. These results suggest that it is possible to generate realistic real exchange rate movements within a flexible price framework if relevant real factors, such as trade costs, output ratio volatility and intertemporal elasticity of substitution, are introduced. Our framework complements those that emphasize the role of sticky prices.

In this paper, we also provide empirical evidence to test the model's predictions for real exchange rate volatility. Motivated by the fact that deviations from LOP are expected to converge to the trade costs once arbitrage occurs, we use the threshold autoregressive model to estimate the optimal threshold value as a measure for trade costs. Evidence from cross-sectional regression supports the main predictions of the trade cost model. First, real exchange rate volatility has a significant positive correlation with trade costs and output ratio volatility, but is negatively related to elasticity of substitution even after controlling for the nominal exchange rate effect. Second, this finding also holds when we use geographic distance as an alternative measure for trade costs. Finally, the effect of trade costs on real exchange rate volatility is more apparent for developed countries than for developing countries.

The trade cost model implies, in the absence of trade costs, that the time series mean and variance of LOP deviations over the sample period will be zero. This finding is not controversial, and the choice of these two metrics is largely determined by the availability of absolute versus index number data. Of course in the case where absolute price data is available, one might be concerned about the power of the first versus the second moment in detecting the deviations from LOP. Thus, our future work should concentrate on assessing the theoretical propositions that use absolute price data.

Technical Appendix

This appendix derives the variance of log real exchange rate, q_t (a two-sided censored random variable). The variance is written as:

$$Var(q_t) = E(conditional\ \ \text{var}iance(q_t)) + Var(conditional\ \ mean(q_t))$$

Beginning with the first term, we have:

$$\begin{aligned} E(\text{conditional variance}(q)) &= \Pr(q = -\tau) Var(q \mid q = -\tau) \\ &+ \Pr(q = \tau) Var(q \mid q = \tau) \\ &+ \Pr(-\tau < q < \tau) Var(q \mid \text{-}\tau < q < \tau) \end{aligned}$$

Recall that $Var(q \mid q = -\tau) = Var(q \mid \mathrm{q} = \tau) = 0$. Since we are assuming $\mu = 0$ and the real exchange rate inherits the stochastic properties of output ratio, $\log(y^* / y) \sim N(0, \sigma^2)$, the conditional variance within the censoring points can be computed as follows:

$$\begin{aligned} Var(q \mid = \tau < q < \tau) &= Var(\theta \log(\frac{y^*}{y}) \mid \text{-}\tau < \theta \log(\frac{\mathrm{y}^*}{y}) < \tau) \\ &= \theta^2 \sigma^2 \left[1 + \frac{A\phi(A) - B\phi(B)}{\Phi(B) - \Phi(A)} - (\frac{\phi(A) - \phi(B)}{\Phi(B) - \Phi(A)})^2 \right] \\ &= \theta^2 \sigma^2 \left[1 + \frac{A\phi(A) - B\phi(B)}{\Phi(B) - \Phi(A)} \right] \end{aligned}$$

where $A = \dfrac{-\tau}{\theta\sigma}$, $B = \dfrac{\tau}{\theta\sigma}$, $\Phi(.)$ is the standard normal cdf, and $\phi(.)$ is the standard normal pdf. Substituting the conditional variance expression, we have:

$$E(\text{conditional variance}(q)) = [\Phi(B) - \Phi(A)] \theta^2 \sigma^2 \left[1 + \frac{A\phi(A) - B\phi(B)}{\Phi(B) - \Phi(A)} \right]$$

Next, the second term can be computed as:

$$
\begin{aligned}
Var(conditional\ \ mean(q_t)) &= E(conditional\ \ mean^2(q_t)) \\
&\quad -\left[E(conditional\ \ mean(q_t))\right]^2 \\
&= \left\{\Phi(A)\tau^2 + (1-\Phi(B))\tau^2 + (\Phi(B)-\Phi(A))0\right\} - 0^2 \\
&= 2\Phi(A)\tau^2
\end{aligned}
$$

and therefore we have:

$$
Var(q) = 2\Phi(A)\tau^2 + \left[\Phi(B)-\Phi(A)\right]\theta^2\sigma^2\left[1+\frac{A\phi(A)-B\phi(B)}{\Phi(B)-\Phi(A)}\right]
$$

which is the expression found in the text of the paper.

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Table 1. Estimation results for trade costs (TAR τ)

Developed countries	Numeraire : Belgium	Numeraire : U.S.	Developing countries	Numeraire : Belgium	Numeraire : U.S.
Australia	0.167	0.168	Chile	0.547	0.163
Austria	0.062	0.110	Colombia	0.506	0.179
Belgium		0.119	Cote d'Ivoire	0.097	0.149
Canada	0.222	0.073	Ecuador	0.562	0.190
Denmark	0.080	0.180	Egypt	0.367	0.174
Finland	0.190	0.114	Guatemala	0.498	0.189
France	0.018	0.142	India	0.494	0.178
Germany	0.020	0.123	Indonesia	0.571	0.372
Greece	0.042	0.131	Kenya	0.029	0.207
Ireland	0.184	0.149	Malaysia	0.326	0.309
Israel	0.134	0.250	Mexico	0.161	0.117
Italy	0.084	0.242	Morocco	0.186	0.266
Japan	0.336	0.205	Nigeria	0.260	0.358
Korea	0.089	0.197	Pakistan	0.393	0.386
Luxembourg	0.021	0.120	Panama	0.310	0.235
Netherlands	0.006	0.125	Paraguay	0.051	0.127
New Zealand	0.214	0.197	Philippines	0.184	0.202
Norway	0.045	0.179	Senegal	0.044	0.187
Portugal	0.053	0.268	South Africa	0.161	0.190
Singapore	0.152	0.155	Sri Lanka	0.290	0.460
Spain	0.067	0.263	Thailand	0.304	0.172
Sweden	0.135	0.282	Venezuela	0.526	0.132
Switzerland	0.041	0.184			
U.K.	0.054	0.178			
U.S.	0.119				
Average	0.106	0.173	Average	0.312	0.225

Table 2. Determinants of Real Exchange Rate Volatility: Whole Sample

Explanatory variable	(1a)	(1b)	(2a)	(2b)
Intercept	0.159	0.096	-0.164	-0.068
	(3.115)	(2.004)	(1.945)	(0.886)
Trade costs	0.047	0.031		
	(3.651)	(2.554)		
Distance			0.026	0.009
			(2.114)	(0.813)
Intertemporal elasticity of substitution	0.026	0.022	0.030	0.025
	(4.521)	(4.254)	(4.891)	(4.654)
Output ratio volatility	0.361	0.274	0.381	0.317
	(2.276)	(1.930)	(2.070)	(1.989)
Nominal exchange rate volatility		0.385		0.449
		(3.607)		(3.921)
R-squared	0.653	0.737	0.587	0.669
Number of observations	46	46	46	46

Absolute t-statistics are in parentheses.

Table 3. Determinants of Real Exchange Rate Volatility: Developed vs. Developing Countries

Explanatory variable	Developed countries	Developing countries
Intercept	0.135	0.114
	(4.895)	(1.365)
Trade costs	0.028	0.023
	(4.449)	(1.001)
Intertemporal elasticity of substitution	0.014	0.022
	(3.545)	(2.875)
Output ratio volatility	0.082	0.185
	(1.163)	(0.622)
Nominal exchange rate volatility	0.099	0.417
	(1.394)	(2.467)
R-squared	0.832	0.571
Number of observations	24	22

Absolute t-statistics are in parentheses.