Does the Liquidity Effect Guarantee a Positive Term Premium?

Abstract

This paper examines the liquidity effect and the term structure in two versions of the limited participation model — an imperfect information model and a transaction cost model. With a discrete state solution approach, I find a striking contrast: while the imperfect information model successfully generates the liquidity effect and the positive term premium seen in the data; the transaction cost model replicates only the liquidity effect. This is because the adjustment cost that drives the liquidity effect in the transaction cost model also creates an adjustment cost effect, which leads to a negative term premium.

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1 Introduction

As discussed in Blinder (1998) and Walsh (2003), monetary policy has become the central tool of macroeconomic stabilization, and most central banks use interest rates as their policy instruments. This interest rates-oriented monetary policy directs us to put more emphasis on the liquidity effect and the term structure. This paper examines the relationship between the liquidity effect and the term structure in two versions of the limited participation model — an imperfect information model due to Christiano, Eichenbaum, and Evans (1997a) and a transaction cost model by Dotsey and Ireland (1995). According to the previous studies, if a monetary model is successful to generate the liquidity effect,[1] the model also captures the positive term premium seen in the data.[2] For example, Backus, Gregory, and Zin (1989) and Salyer (1990) show that the positive term premium can not be replicated in cash-in-advance (CIA) models which fail to capture the liquidity effect. Instead, Coleman, Gilles, and Labadie (1992) find that a limited participation model (LPM) which succeeds to reproduce the liquidity effect also generates the positive term premium.

While the limited participation model (LPM) successfully captures the liquidity effect, there are two sub-models in that category. The original limited participation models studied by Lucas (1990), Christiano and Eichenbaum (1991), and Fuerst (1992) impose a restriction on households' decision timing for cash holding used for consumption. Unlike the timing of the CIA model in which all the decisions are made *after* the money shock, households are assumed to make their cash holding decisions

[1] Among many empirical literature showing the liquidity effect, Leeper and Gordon (1992), Strongin (1995), Leeper, Sims, and Zha(1996), Christiano, Eichenbaum, and Evans (1999) are traditional samples.
[2] Regarding the positive term premium literature, refer to Backus, Gregory, and Zin (1989), Salyer (1990), den Haan (1995), Bansal and Coleman (1996), Cochrane (1999) and Dai and Singleton (2002).

before observing the current money growth realization in the original limited participation models. Consequently, the total amount of savings, the households' initial money holding minus the cash holding for consumption, is also determined before the money shock. Under this condition, a new money injection by a central bank is the only *variable source* of loanable funds in a loan market in which nominal interest rates are determined. Therefore, the new money injection leads to the increase of loanable funds, and results in the fall of loan rates (nominal interest rates), the liquidity effect. Since then, this modeling strategy is widely used by Christiano and Eichenbaum (1992), Dow (1995), Christiano, Eichenbaum, and Evans (1997a), and others.[3]

On the other hand, Dotsey and Ireland (1995) argue that it is more realistic to assume that households can adjust, at some cost, their cash balances *after* observing the true money growth realization. According to their paper, when the adjustment cost parameter is sufficiently large, their model captures the liquidity effect. However, for parameter values less than this amount, nominal interest rates are primarily determined by the expected inflation effect. They also insist that their model nests the CIA model and the original limited participation model: when the parameter is zero, their model shows the CIA model-behaviors implying no liquidity effect; as the size of adjustment cost parameter gets bigger, it succeeds in generating the liquidity effect; when the parameter is infinite, it is exactly the same as the original limited participation model. They presume that the original limited participation model's assumption of one-period-ahead cash holding decision stems from the infinite adjustment cost.[4] Cooley and

[3] Refer to Chari, Christiano and Eichenbaum (1995), Schlagenhauf and Wrase (1995), Christiano, Eichenbaum, and Evans (1997b), and Evans and Marshall (1998).

[4] Their original intention is not to advocate their model. Rather, they insist to explore another model to generate the liquidity effect other than the original limited participation model. They think that the liquidity effect in the original limited participation model is dependent on the size of adjustment cost parameters, and therefore it is not reliable.

Quadrini (1999), Hendry and Zhang (2001), and Keen (2004) follow this line assuming that households' cash holding decision occurs after the money growth realization.

For the standardization of our notation, from now on, I name the original limited participation model studied by Christiano, Eichenbaum, and Evans (CEE,1997a) an imperfect information model, and the variants of the limited participation model initially discussed by Dotsey and Ireland (1995) a transaction cost model. Though both models are based on the limited participation model setting, the imperfect information is the critical factor in the imperfect information model, whereas the adjustment cost is in the transaction cost model. My study is motivated by this point. While both models show the liquidity effect, do they still show the positive term premium? In other words:

Does the liquidity effect guarantee a positive term premium?

To analyze this, I adopt Evans and Marshall's (1998) modeling strategy: I introduce one- and two-period bonds into the original CEE (1997a) model along the line discussed by Evans and Marshall (1998) (their strategy is described in detail in section 3). I, however, depart from Evans and Marshall (1998) by using a discrete-state solution approach. The main reason for adopting this approach is that it allows us to capture the exact behavior of time-varying term premia, which are approximated in the linearization method employed by Evans and Marshall (1998). In this paper, I set up two limited participation models under the imperfect information and the transaction cost settings and compare the behavior of interest rates and term premia.

My results show a striking contrast: while the imperfect information model successfully replicates the liquidity effect and the positive term premium seen in the data; the transaction cost model generates the liquidity effect but fails to capture the positive term premium. The main reason for these results is originated from the fact that the term premium in the transaction cost model is determined by the interaction of the liquidity effect and the adjustment cost effect explained right after. *The liquidity effect* in the transaction cost model is driven by *the adjustment cost*. In addition, the adjustment cost of one-period bond is always bigger than that of two-period bond, which is named *the adjustment cost effect*. This is because a one-period bond return, which is related to firms' labor demand via their borrowing of wage payment from financial intermediaries, is determined by the condition of bonds market and labor market; whereas a two-period bond return is only affected by the bonds market situation. While the liquidity effect, through a term premium, makes the difference between the two-period bond return and the one-period bond return, $R_t^{II} - R_t^{I}$, more positive, the adjustment cost effect works the opposite way as the liquidity effect. The reason is that the higher adjustment cost of one-period bond induces investors to require a more return on the one-period bond to compensate for the bigger adjustment cost of it.

Given these two conditions, the increase of adjustment cost creates two effects: on the one hand, the liquidity effect is strengthened as the adjustment cost parameter gets bigger; on the other hand, the larger parameter raises the adjustment cost of one-period bond much more. This means that if the parameter is small, the liquidity effect dominates the adjustment cost effect; but the parameter is large, vice versa. As a whole, to generate the liquidity effect, a large adjustment cost parameter is needed. However, with this big parameter value, the adjustment cost effect dominates the liquidity effect, which leads to a negative term premium.

To check the robustness of my result, I tested different utility functions and a different location of an adjustment cost term.[5] Though the results are not reported here,

[5] The adjustment cost term can be put in a utility function; or a budget constraint and a CIA constraint.

they show the same negative term premium. Despite the virtue of the transaction cost model, generating the liquidity effect without the strong assumption of imperfect information, it has a critical weak point in capturing the correct term premium sign.

In section 2, we will look at the simplest theory of term premium. The model set-up of the imperfect information and the transaction cost are in section 3. Simulated results will be described in section 4. Conclusion closes this paper.

2 A Basic Framework of Term Premium Theory

2.1 A Characterization of Uncertainty

In order to expedite our discussion in later sections, first I need to introduce the term premium determination theory in a simple setting. In this paper, I only consider a money shock. Thus the only exogenous uncertainty comes from the random growth rate of money, x_t. It is also assumed that the money growth rate follows a three-state Markov process. The realizations of x_t can take on low, middle, or high values, denoted s = 1, 2, and 3, respectively.

$$x_t = \begin{cases} x_3 = \mu_x + \delta \\ x_2 = \mu_x \\ x_1 = \mu_x - \delta \end{cases} \tag{2-1}$$

Hence, a transitional probability matrix, which shows the probability that the money growth rate moves from the state of t-1(t) period to the state of t(t+1) period, is a three by three matrix. For example, in the probability matrix (2-2), π_{11} and π_{22} show the conditional probability that money growth rate moves from a low state to a low state, a middle state to a middle state, respectively. Consequently, π_{12} represents the conditional probability that the money growth rate moves from the low state to the middle state. Under the assumption of uniform ergodic distribution of money growth rates, the unconditional probabilities (p1, p2, p3) = (1/3, 1/3, 1/3). If I assume normal ergodic distribution, (p1, p2, p3) depend on the serial correlation parameter of money growth rates, as we will see in section 4.

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{pmatrix} \tag{2-2}$$

Following this characterization of uncertainty, the movement from state one to state two or state three means an expansionary money shock and vice versa. By comparing the value of nominal interest rates or other economic variables such as inflation rates in each state, we can check each economic variable's response to a money shock.

2.2 The Liquidity Effect

Before exploring the term premium sign, let's check the liquidity effect in a CIA model. The presence of the liquidity effect is essential to determine the term premium sign. In a traditional CIA model, the FOC's for one- and two- period bonds can be expressed like below:

$$\beta R_t^I E_t\left[\frac{U'_{C,t+1}}{P_{t+1}}\right] = \frac{U'_{C,t}}{P_t} \tag{2-3}$$

$$\beta (R_t^{II})^2 E_t\left[\left(\frac{U'_{C,t+1}}{P_{t+1}}\right)\frac{1}{R_{t+1}^I}\right] = \frac{U'_{C,t}}{P_t} \tag{2-4}$$

Here, β means a representative agent's subjective discount rate. R_t^I and R_t^{II} are the gross returns of one- and two- period bonds at period t, respectively. $U'_{C,t}$ implies the marginal utility of consumption, and P_t is the price level.

The right-hand sides of above equations show the marginal cost of saving one dollar which is used to purchase one- or two-period bond, the sacrifice of my utility caused by a decreased consumption by one dollar. The left-hand sides of both equations express the marginal benefit of extra one dollar invested to one- or two-period bond. If we buy a one-period bond with one dollar, they generate R_t^I dollars of return at the end of that period. This amount of money can be used to purchase $\frac{R_t^I}{P_{t+1}}$ units of consumption at t+1 period. The left-hand side of equation (2-3) denotes the discounted expected benefit of that consumption. By the same analogue, the left-hand side of equation (2-4) implies the marginal benefit of consumption which comes from the gross return of two-period bonds. Note that the two-period bonds are liquidated after one period. This fact is shown as $(R_t^{II})^2 E_t[\frac{1}{R_{t+1}^I}]$ term in equation (2-4).

From equation (2-3), I can derive an equation which determines the movement of the gross return of one-period bonds:

$$(R_t^I)^{-1} = \beta E_t[\frac{U'_{C,t+1}}{U'_{C,t}} \frac{P_t}{P_{t+1}}] \tag{2-5}$$

As is clear in equation (2-5), the movement of one-period nominal bond return is determined by intertemporal marginal utilities of consumption and expected inflation effects. We call this relationship *Fisherian Fundamentals*.

If we recall the previous assumption that there is only a money shock—consumption growth is zero, then equation (2-5) implies that one-period nominal bond return is entirely determined by the expected inflation effect:

$(R^{I}_{t})^{-1} = \beta E_{t}[\frac{P_{t}}{P_{t+1}}] = \beta E_{t}[1+\pi_{t+1}]^{-1}$.[6] Under the assumption of a positive serial correlation of money growth rates ($\pi > 1/3$), a realization of high money growth in this period implies that a high money growth state in next period is more likely. This raises the expected inflation. These whole mechanisms hint that the nominal bond return in the high state of money growth is bigger than that in the low state, $R^{I}_{1} < R^{I}_{2} < R^{I}_{3}$ when $\pi > 1/3$.[7] Thus, the CIA model does not show the liquidity effect. With the same logic, I can express the movement of two-period nominal bond return like below by using equation (2-3) and (2-4):

$$(R^{II}_{t})^{-2} = \beta E_{t}[\frac{P_{t}}{P_{t+1}} \frac{1}{R^{I}_{t+1}}] \qquad (2\text{-}6)$$

As is clear from equation (2-5) and $(R^{I}_{t})^{-1} = \beta E_{t}[1+\pi_{t+1}]^{-1}$, two-period nominal bond return and an expansionary money shock show a positive relationship due to the assumption of the positive serial correlation of money growth rates, $\pi > 1/3$. This implies there is no liquidity effect in the CIA model.

2.3 Term Premia

A term premium (conditional on the state of time t) is defined as the difference between the expected nominal return of two-period bond liquidated after one period and the certain nominal return of one-period bond:

$$TP_{t} = (R^{II}_{t})^{2} E_{t}[\frac{1}{R^{I}_{t+1}}] - R^{I}_{t} \qquad (2\text{-}7)$$

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[6] As long as nominal bond returns are positive, the CIA constraint is binding, $M_{t} = P_{t}C_{t}$. Therefore, under the assumption of constant consumption, money growth rate, x_{t}, is equal to inflation rate, π_{t}.

[7] If we extend the CIA model to a production economy, introducing leisure into the utility function additionally, money has a real effect in the short-run. The inflation tax on consumption which stems from a CIA constraint reduces consumption and increases a demand for leisure. This leads to the fall of current consumption and labor supply. Therefore, a positive money shock results in a *fall* of output, which is also contrary to the liquidity effect. For more detailed discussions, refer to Walsh (2003) Ch. 3.

As I mentioned in section 1, the empirical studies show that the sign of term premium is positive.[8]

To check the sign of term premia, equation (2-7) can be transformed into $TP_t = (R_t^{II})^2[E_t(\frac{1}{R_{t+1}^I}) - \frac{R_t^I}{(R_t^{II})^2}]$ for a computational convenience. Now, I can transform $\frac{R_t^I}{(R_t^{II})^2}$ into $\frac{E_t[(\frac{U'_{C,t+1}}{P_{t+1}})\frac{1}{R_{t+1}^I}]}{E_t[\frac{U'_{C,t+1}}{P_{t+1}}]}$ by using equation (2-3) and (2-4). Finally the term premium equation is simplified like below:[9]

$$TP_t = (R_t^{II})^2\{-\frac{Cov_t[\frac{U'_{C,t+1}}{P_{t+1}}, \frac{1}{R_{t+1}^I}]}{E_t[\frac{U'_{C,t+1}}{P_{t+1}}]}\} \qquad (2\text{-}8)$$

It is obvious that the conditional covariance in equation (2-8) determines the term premium signs. Let's assume that a high money growth realization in t+1 period. This implies that the first term of covariance, $\frac{U'_{C,t+1}}{P_{t+1}}$, is a low state. Since money growth rates are positively correlated, the money growth rate in t+2 period is also high, which leads to a low $\frac{1}{R_{t+1}^I}$ as confirmed in equation (2-5). Consequently, the covariance is positive

[8] Equation (2-7) means the term premium is defined as a holding premium. If we define the term premium as a rolling premium, $TP_t = (R_t^{II})^2 - E_t[R_{t+1}^I]R_t^I$. In any case, data show that the term premium is positive.

[9] The intermediate processes are following: $E_t(\frac{1}{R_{t+1}^I}) - \frac{R_t^I}{(R_t^{II})^2} = E_t(\frac{1}{R_{t+1}^I}) - \frac{E_t[(\frac{U'_{C,t+1}}{P_{t+1}})\frac{1}{R_{t+1}^I}]}{E_t[\frac{U'_{C,t+1}}{P_{t+1}}]}$ $= E_t(\frac{1}{R_{t+1}^I}) - \frac{E_t[\frac{U'_{C,t+1}}{P_{t+1}}]E_t[\frac{1}{R_{t+1}^I}] + Cov_t[(\frac{U'_{C,t+1}}{P_{t+1}}), \frac{1}{R_{t+1}^I}]}{E_t[\frac{U'_{C,t+1}}{P_{t+1}}]} = E_t(\frac{1}{R_{t+1}^I}) - E_t[\frac{1}{R_{t+1}^I}] - \frac{Cov_t[(\frac{U'_{C,t+1}}{P_{t+1}}), \frac{1}{R_{t+1}^I}]}{E_t[\frac{U'_{C,t+1}}{P_{t+1}}]}$

and the term premium is negative. This is another defect of the CIA model other than the lack of showing the liquidity effect.

3 Model Set-Ups

3.1 The Imperfect Information Model

The model I want to use here is originated from Christiano, Eichenbaum, and Evans (1997a), but their model does not consider the term premium. On the other hand, Evans and Marshall (1998) explicitly introduce a bond market into their model and analyze the term structure of nominal interest rates. Here, I adopt Evans and Marshall (1998)'s modeling strategy. While they analyze various maturities of bond yields by using a linearization method, I focus on only one- and two-period bond returns with the discrete-state solution approach.

Like a standard limited participation model, the model economy is composed of four representative agents: households, firms, financial intermediaries, and a central bank. The important rigidities that this model has are 1) Households' cash holding decision occurs before the realization of money shock. 2) The monetary injection from the central bank is solely delivered to the financial intermediaries. 3) Firms have to finance their wage bills from the financial intermediaries in advance of production.

In addition to these standard limited participation model set-ups, I introduce a bonds market, in which households trade one- and two- period nominal bonds, denoted B_t^I and B_t^{II}, after observing the true money growth realization. The gross returns of above two bonds are expressed as R_t^I, R_t^{II}. Like Evans and Marshall (1998), I assume one-period bonds purchased at t-1 period are paid off at the end of t-1 period, not the beginning of t period. I adopt this timing convention because the bond payoff is known with certainty and this ensures that all cash are kept in households at the end of each period.

Regarding the two-period bonds, I assume that two period-bonds bought at t-1 period are liquidated after one period. I also assume that the bonds market precedes the goods market. The timing of events for the whole economy and the sequential flow of funds for each economic agent are depicted in Figure 3-1 and Figure 3-2.

3.1.1 Households

Households enter each period with M_t units of money, K_t units of capital, and B^{II}_{t-1} units of two-period bonds purchased at t-1 period.[10] To focus on the bond returns' behavior, I assume that all households own the same amount of capital, one unit. Therefore, through all the periods, K_t=1. Before observing the money growth shock, households decide how much amount of cash, Q_t, they will carry to purchase consumption. Then households take the remaining cash balance, $M_t - Q_t$, and two-period bonds to financial intermediaries. At this point, money growth shocks are realized. Upon observing the real money growth, households adjust their portfolios in the bonds market by liquidating two-period bonds bought at t-1 period, then buy new one- and two-period bonds by using the cash balance, $M_t - Q_t$, and the liquidated two-period bond return, $B^{II}_{t-1}(\frac{R^{II\,2}_{t-1}}{R^{I}_{t}})$. This portfolio constraint is:

$$B^{I}_{t} + B^{II}_{t} \leq (M_t - Q_t) + B^{II}_{t-1}(\frac{R^{II\,2}_{t-1}}{R^{I}_{t}}) \tag{3-1}$$

[10] Actually, households enter each period with their labor in addition to money holding, capital and bonds. These are expressed as M_t and (B^{II}_{t-1}, N_t, K_t) at households' {Inflow} row in Figure 3-2.

Figure 3-1: Timing of Events
(The Imperfect Information Model)

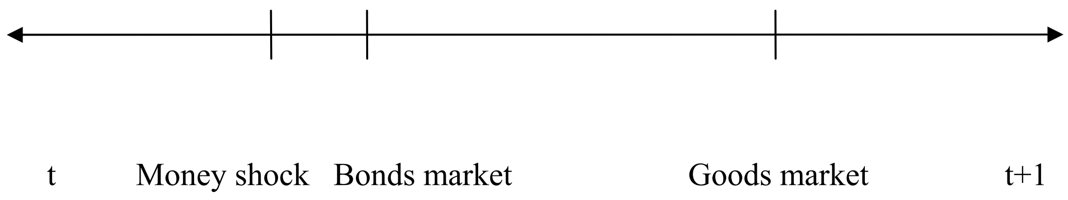
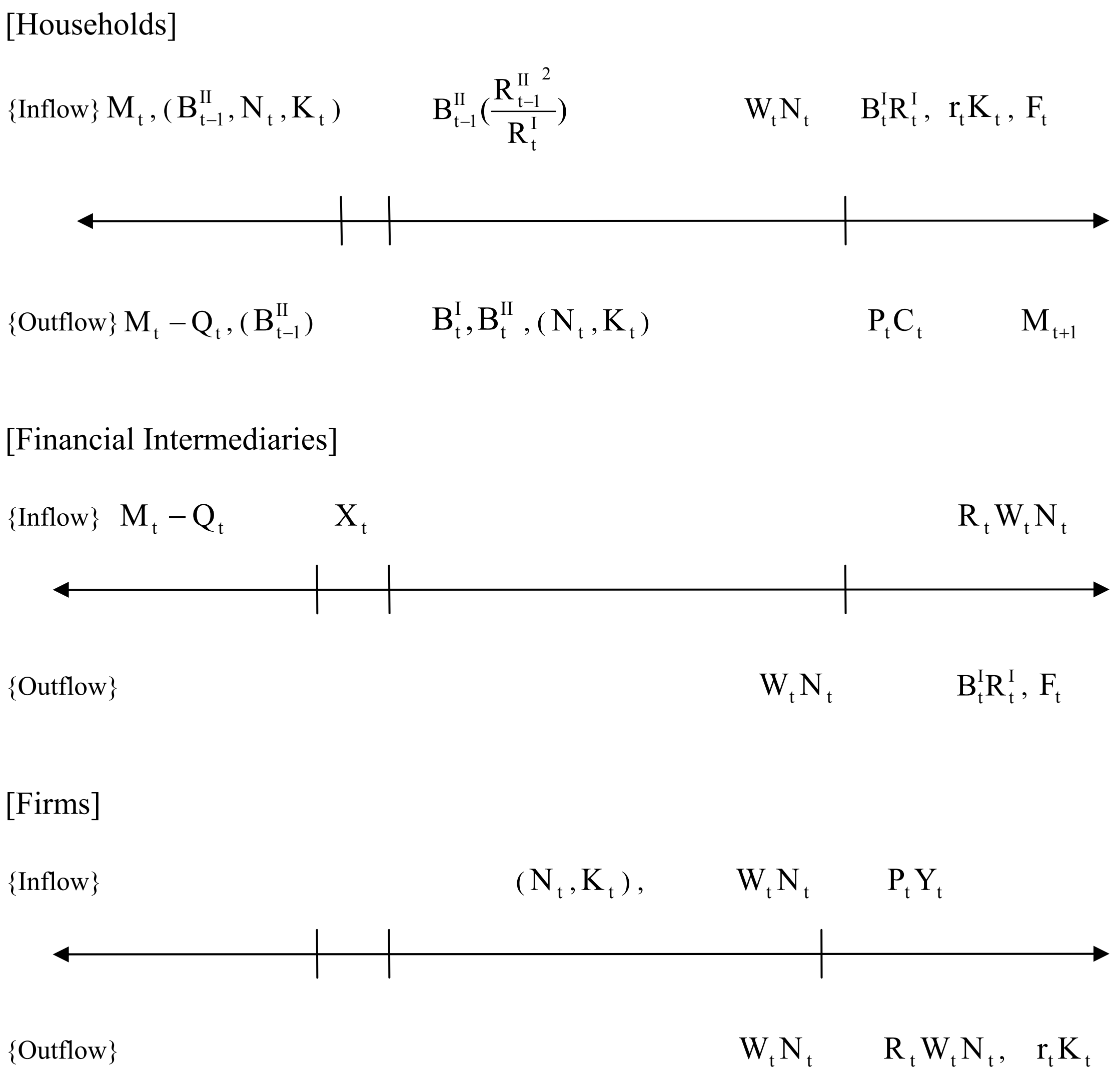


Figure 3-2: Flow of Funds
(The Imperfect Information Model)



After completing the bonds market transactions, households choose their consumption, C_t, and labor, N_t, to maximize their expected life time utility:

$$E_{-1}\sum_{t=0}^{\infty}\beta^t[U(C_t,N_t)] \tag{3-2}$$

It is assumed that the utility function has the following form:[11]

$$U(C_t,N_t) = [C_t - \frac{\psi_0}{1+\psi}N_t^{1+\psi}]^{1-\gamma} \Big/ (1-\gamma) \tag{3-3}$$

Here, ψ_0 is a scaling parameter, ψ is the inverse of labor supply elasticity, and γ is a risk-aversion parameter. At this point, firms pay their wage bills, W_tN_t, in advance of production, and this labor income increases households' total amount of money that will be used for the consumption purchase in the goods market. This flow of funds for households' consumption is expressed as the following CIA constraint:

$$P_tC_t \le Q_t + W_tN_t \tag{3-4}$$

At the end of each period, households earn the rental profits from the firms, r_tK_t, the return of one-period bonds, $B_t^IR_t^I$, and total profits of the financial intermediaries and the firms, denoted F_t and D_t [12], respectively. The following budget constraint shows households' total flow of funds:

$$M_{t+1} \le Q_t + W_tN_t - P_tC_t + B_t^IR_t^I + r_tK_t + F_t + D_t \tag{3-5}$$

Households' optimal choice of Q_t, C_t, N_t, M_{t+1}, B_t^I, and B_t^{II} must satisfy the following first order necessary conditions (FOC's). Here, λ_t, υ_t, and ξ_t express

[11] This utility function ensures a constant labor supply elasticity, $\frac{1}{\psi}$, with respect to a real wage.

[12] In equilibrium, the revenue of firms equals to factor payments, $P_tY_t = R_tW_tN_t + r_tK_t$, implying $D_t = 0$.

multipliers for the portfolio constraint, equation (3-1), the CIA constraint, equation (3-4), and the budget constraint, equation (3-5).

$$Q_t : E_{t-1}[\frac{\lambda_t}{P_t}] = E_{t-1}[\frac{\upsilon_t + \xi_t}{P_t}] \tag{3-6}$$

$$C_t : U'_{C,t} = \upsilon_t + \xi_t \tag{3-7}$$

$$N_t : U'_{N,t} + (\upsilon_t + \xi_t)\frac{W_t}{P_t} = 0 \tag{3-8}$$

$$M_{t+1} : \beta E_t[\frac{\lambda_{t+1}}{P_{t+1}}] = \frac{\xi_t}{P_t} \tag{3-9}$$

$$B^I_t : \beta R^I_t E_t[\frac{\lambda_{t+1}}{P_{t+1}}] = \frac{\lambda_t}{P_t} \tag{3-10}$$

$$B^{II}_t : \beta E_t[\frac{\lambda_{t+1}}{P_{t+1}}\frac{R^{II^2}_t}{R^I_{t+1}}] = \frac{\lambda_t}{P_t} \tag{3-11}$$

Equation (3-10) is originally $R^I_t \frac{\xi_t}{P_t} = \frac{\lambda_t}{P_t}$. This implies that the discounted expected value of money is determined by the budget constraint multiplier. If I combine this original one and equation (3-9), I can get equation (3-10). Above FOC's can be further simplified like below by using equation (3-6) and (3-7):

$$N_t : U'_{N,t} + U'_{C,t}\frac{W_t}{P_t} = 0 \tag{3-12}$$

$$B^I_t : \beta E_{t-1}\{R^I_t E_t[\frac{U'_{C,t+1}}{P_{t+1}}]\} = E_{t-1}[\frac{U'_{C,t}}{P_t}] \tag{3-13}$$

$$B^{II}_t : \beta \frac{(R^{II}_t)^2}{R^I_t} E_t[E_{t+1}[\frac{U'_{C,t+2}}{P_{t+2}}]] = E_t[\frac{U'_{C,t+1}}{P_{t+1}}] \tag{3-14}$$

Equation (3-12) shows that households' marginal rate of substitution between consumption and labor should be equal to the real wage. Households' labor supply decision is determined by this relationship. The most important characteristic of the imperfect information model appears in equation (3-13) and (3-14). These two equations are the mirror images of equation (3-10) and (3-11). The left-hand sides of equation (3-10) and (3-11) again denote the marginal benefit of one dollar in t+1 period, and the right-hand sides of them imply the marginal cost of saving one dollar in t period.

Unlike the CIA model in section 2, the marginal utility of consumption, $U'_{C,t}$, is replaced by the portfolio constraint multiplier, λ_t. The reason is that the extra one dollar relaxes the portfolio constraint instead of being used for consumption in this limited participation model. However, due to the households' cash holding decision timing which occurs before the realization of the money shock, the final FOC's are expressed as equation (3-13) and (3-14). In equation (3-13), the expectation operator is associated with t-1. Therefore, while equation (2-3) in the CIA model always holds, equation (3-13) in the imperfect information model holds on average. It implies that R_t^I in the CIA model and the same one in the imperfect information model might show different behaviors in response to a money shock.

Note that the two-period bond pricing FOC, equation (3-14), is also slightly different from the one in the CIA model case. Since the information timing of one-period bonds is associated with t-1 period, the two-period bond formula is written as a one-period-ahead relationship from equation (3-13). In this model, the gross return of one-period bonds, R_t^I, is known to us with certainty. That is the reason why the one-period bond return appears as R_t^I, not R_{t+1}^I in equation (3-14).

3.1.2 Firms

Every period, firms hire capital and labor in order to maximize their profit. The maximization problem is:

$$\max_{K_t, N_t} P_t Y_t - R_t W_t N_t - r_t K_t \tag{3-15}$$

$$\text{s.t. } Y_t = K_t^{\alpha} N_t^{1-\alpha} \tag{3-16}$$

According to my assumptions for the firms' flow of funds, firms have to pay their wage bills in advance of production. Hence, they have to borrow their wage payment, $W_t N_t$, from the financial intermediaries. At the end of each period, firms have to pay the borrowed wage bills multiplied by the gross nominal interest rates, denoted $R_t W_t N_t$, back to the financial intermediaries and the rent for capital, $r_t K_t$, to the households.

Firms' optimal choice of N_t and K_t must satisfy the following FOC's:

$$R_t \frac{W_t}{P_t} = (1-\alpha)(\frac{K_t}{N_t})^{\alpha} \tag{3-17}$$

$$\frac{r_t}{P_t} = \alpha(\frac{K_t}{N_t})^{\alpha-1} \tag{3-18}$$

Equation (3-17) and (3-18) show the traditional micro economic principles that factor incomes are determined by their marginal products.

3.1.3 Financial Intermediaries

In every period, perfectly competitive financial intermediaries trade bonds with households at price unity. In addition to the bond trading, financial intermediaries inelastically supply their funds, $M_t - Q_t + X_t$, to the firms in the loan market. Here, $M_t - Q_t$ is the net amount of money transferred from households to financial intermediaries and X_t is the new money injection from the central bank. At the end of each period, financial intermediaries get revenues, $R_t W_t N_t$, and pay households one-period bond returns, $B_t^I R_t^I$. A profit for financial intermediaries, F_t, and a loan market clearing condition are below:

$$F_t = M_t - Q_t + X_t - W_t N_t + R_t W_t N_t - B_t^I R_t^I \tag{3-19}$$

$$W_t N_t = M_t - Q_t + X_t \tag{3-20}$$

From equation (3-19), we know that the monetary injection, X_t, is solely distributed to the financial intermediaries and households are excluded from this money injection. The left-hand side of equation (3-20) represents the demand for funds, and the right-hand side of it does the supply of funds. If we combine equation (3-19) and (3-20), we can derive $F_t = R_t W_t N_t - B_t^I R_t^I$. A perfect competition among financial intermediaries ensure that one-period nominal lending rates earned by the intermediaries, R_t, is equal to the gross nominal return of one-period bonds paid by the intermediaries to the households, R_t^I. This is:

$$R_t = R_t^I \tag{3-21}$$

3.1.4 Central Bank

The central bank plays only as a money supplier to the financial intermediaries in this model. The evolution of money growth is defined as:

$$M_{t+1} = (1 + x_t) M_t = M_t + X_t \tag{3-22}$$

As is assumed in the CIA model in section 2, the money growth rate, x_t, follows a three-state Markov process—a low, middle or high state. A transitional probability matrix is also the same as the previous one in section 2.

3.1.5 Market Clearing Conditions

We can derive the market clearing conditions for the bonds market, the goods market, the labor market, and the loan market. From the portfolio constraint, we can derive the net amount of one-period bonds bought at t period. Generally, in equilibrium, the net amount of one- or two-period bond is equal to zero. However, in this limited participation model, due to the households' fund sources for buying new bonds--

$M_t - Q_t$ and $B^{II}_{t-1}(\frac{R^{II\ 2}_{t-1}}{R^{I}_{t}})$, and the pay-off timing of one-period bonds-- the end of this period, the equilibrium amount of one-period bonds is $M_t - Q_t$, not zero:

$$B^{I}_{t} = M_t - Q_t \tag{3-23}$$

On the other hand, the equilibrium amount of two-period bonds is still zero. In this sense, two-period bonds are redundant assets. They have no effect on the equilibrium prices or quantities.

The budget constraint implies the following goods market clearing condition:

$$P_tC_t = P_tY_t \tag{3-24}$$

The total amount of funds in the goods market is derived from the CIA constraint, equation (3-4), like below:

$$P_tC_t = M_t + X_t \tag{3-25}$$

From equation (3-12), (3-17), and (3-21), the labor market clearing condition is derived:

$$R^{I}_{t} = \frac{(1-\alpha)}{\psi_0} N_t^{-\alpha-\psi} \tag{3-26}$$

For the convenience, the loan market clearing condition is replicated here from equation (3-20):

$$W_tN_t = (M_t - Q_t) + X_t \tag{3-20}$$

If we combine equation (3-20) and (3-25), we can derive the following expression:

$$\frac{W_tN_t}{P_tC_t} = \frac{(M_t - Q_t) + X_t}{M_t + X_t} \tag{3-27}$$

Equation (3-27) implies that the ratio of funds in the loan market to the funds in the goods market. This equation is the most important one in the imperfect information model to derive the liquidity effect. The wedge between the value of money in the loan market and the goods market is the critical source generating the liquidity effect. This will be discussed in detail in section 4. Here, I want to briefly show how it is related to the liquidity effect. From equation (3-17), $\frac{W_t}{P_t} = \frac{(1-\alpha)}{R_t}(N_t^{-\alpha})$ [13]. If we combine equation (3-16) and equation (3-24), we can get $C_t = N_t^{1-\alpha}$. Now, substituting $\frac{W_t}{P_t} = \frac{(1-\alpha)}{R_t}(N_t^{-\alpha})$ and $C_t = N_t^{1-\alpha}$ into the left-hand side of equation (3-27) yields:

$$\frac{(1-\alpha)}{R_t^I} = \frac{(M_t - Q_t) + X_t}{M_t + X_t} \tag{3-28}$$

Here, Q_t is the households' money set aside for consumption before the money shock. Hence, $M_t - Q_t$ is fixed before the new money injection, X_t, is realized. Clearly, nominal interest rates and the new money injection show a negative relationship.[14]

3.2 The Transaction Cost Model

In the transaction cost model, households make their cash holding decision after observing the realized money growth state. However, there is a cost to adjust their cash holding decision. This adjustment cost is expressed as a certain amount of time, which is proportionate to the scaled cash balance. The critical difference between the imperfect information model and the transaction cost model lies in this timing of households' cash holding decision. This new timing of events and households' flow of funds are depicted in Figure 3-3 and Figure 3-4. Note, in Figure 3-4, $M_t - Q_t$ occurs after the money growth realization.

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[13] Here, I use $K_t = 1$.

[14] $M_t - Q_t$ in the numerator in the right-hand side of equation (3-28) is less than M_t in the denominator. Under this condition, the increase of X_t results in the rise of whole terms in the right-hand side of equation (3-28).

Figure 3-3: Timing of Events
(The Transaction Cost Model)

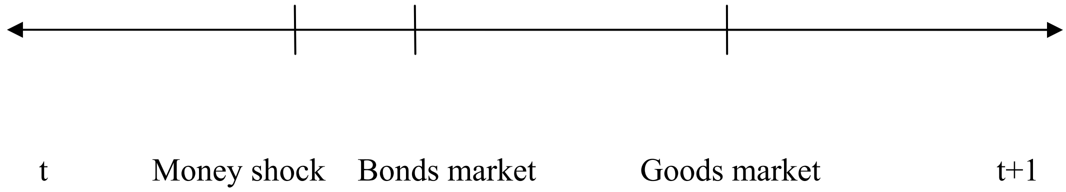
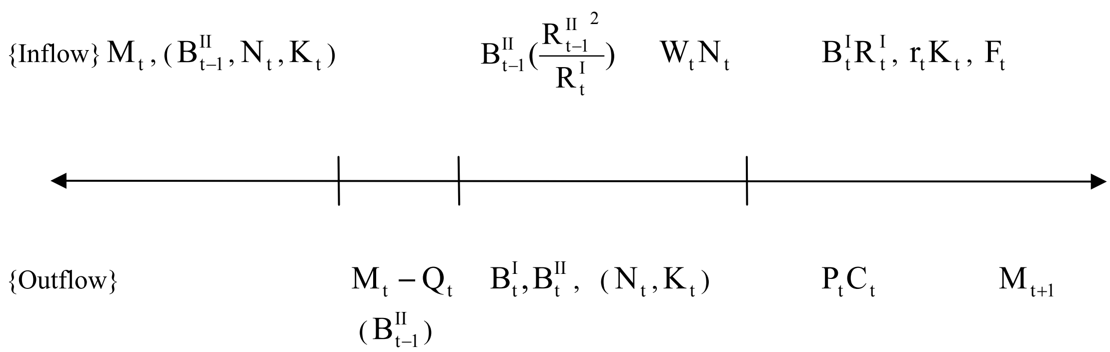


Figure 3-4: Households' Flow of Funds
(The Transaction Cost Model)



3.2.1 Households

Like the imperfect information model set-up, the households choose C_t and N_t in every period to maximize their expected lifetime utility. However, the cash holding decision which occurs after the money growth realization makes the households spend some time for the portfolio adjustment. This time cost of portfolio adjustment enters into the households' utility function, and the increase of this time cost has a negative effect on the households' utility. Hence, the households' new utility function is as follows[15]:

$$U(C_{t,} N_t, \frac{Q_t}{M_t}) = \left[C_t - \frac{\psi_0}{1+\psi}\{N_t + \frac{\phi}{2}(\frac{Q_t}{M_t} - \overline{q})^2\}^{1+\psi}\right]^{1-\gamma} \Big/ (1-\gamma) \qquad (3\text{-}29)$$

[15] I follow the quadratic adjustment cost term that Dotsey and Ireland (1995) use.

Here, ϕ is an adjustment cost parameter and $\overline{q}$ is a steady-state value of cash balance. This implies that Q_t / M_t grows at $\overline{q}$ rate in the steady state. Hence, in the steady state or under the condition that ϕ is zero, the adjustment cost term vanishes. Households' portfolio constraint, CIA constraint, and budget constraint are the same as the imperfect information model's. They are replicated below:

$$\lambda_t : B_t^I + B_t^{II} \le (M_t - Q_t) + B_{t-1}^{II}(\frac{R_{t-1}^{II\,2}}{R_t^I}) \tag{3-1}$$

$$\upsilon_t : P_t C_t \le Q_t + W_t N_t \tag{3-4}$$

$$\xi_t : M_{t+1} \le Q_t + W_t N_t - P_t C_t + B_t^I R_t^I + r_t K_t + F_t + D_t \tag{3-5}$$

Here, λ_t, υ_t, and ξ_t denote each constraint's multiplier.

To maximize their expected life time utility, the households' decisions must satisfy the FOC's below:[16]

$$Q_t : \frac{\lambda_t}{P_t} = U'_{Q,t} + \frac{U'_{C,t}}{P_t} \tag{3-30}$$

$$N_t : U'_{N,t} + U'_{C,t}\frac{W_t}{P_t} = 0 \tag{3-31}$$

$$M_{t+1} :\ \beta E_t[U'_{M,t+1} + \frac{\lambda_{t+1}}{P_{t+1}}] = \frac{\xi_t}{P_t} \tag{3-32}$$

$$B_t^I : \beta R_t^I E_t[U'_{M,t+1} + \frac{\lambda_{t+1}}{P_{t+1}}] = \frac{\lambda_t}{P_t}\ ^{17} \tag{3-33}$$

$$B_t^{II} : \beta E_t[\frac{\lambda_{t+1}}{P_{t+1}}\frac{R_t^{II2}}{R_{t+1}^I}] = \frac{\lambda_t}{P_t} \tag{3-34}$$

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[16] $U'_{Q,t} = U'_{C,t}(-\psi_0)\{N_t + \frac{\phi}{2}(\frac{Q_t}{M_t} - \overline{q})^2\}^{\psi}\{\phi(\frac{Q_t}{M_t} - \overline{q})\frac{1}{M_t}\}$, $U'_{M,t} = U'_{C,t}(\psi_0)\{N_t + \frac{\phi}{2}(\frac{Q_t}{M_t} - \overline{q})^2\}^{\psi}\{\phi(\frac{Q_t}{M_t} - \overline{q})\frac{Q_t}{M_t^{\,2}}\}$

[17] This equation is originally $R_t^I \frac{\xi_t}{P_t} = \frac{\lambda_t}{P_t}$. Like the imperfect information model case, the discounted expected value of money is determined by the budget constraint multiplier. If we combine this original one and equation (3-32), we can get equation (3-33).

The reason why $U'_{M,t+1}$ appears in equation (3-33) results from the presence of adjustment cost term, especially $\frac{Q_t}{M_t}$, in the utility function. While Q_t affects on $\frac{\lambda_t}{P_t}$ as a $U'_{Q,t}$, M_t affects on $\frac{\xi_t}{P_t}$ as a $U'_{M,t+1}$.

Bond pricing FOC's above can be further simplified like below by using equation (3-30) and (3-32):

$$B_t^I : \beta R_t^I E_t[\frac{U'_{C,t+1}}{P_{t+1}} + U'_{Q,t+1} + U'_{M,t+1}] = \frac{U'_{C,t}}{P_t} + U'_{Q,t} \tag{3-35}$$

$$B_t^{II} : \beta (R_t^{II})^2 E_t[(\frac{U'_{C,t+1}}{P_{t+1}} + U'_{Q,t+1})\frac{1}{R_{t+1}^I}] = \frac{U'_{C,t}}{P_t} + U'_{Q,t} \tag{3-36}$$

The critical difference between the FOC's of one-period bond return in the imperfect information model, equation (3-13), and that in the transaction cost model, equation (3-35), is that while R_t^I in the former case is random, R_t^I in the latter case is not random. It implies that while the former R_t^I is affected by the imperfect information; the latter R_t^I is distorted by the adjustment cost term.

4 Results

4.1 Parameterization

To compute the equilibrium behaviors, parameter values must be calibrated. There are three kinds of parameters: preferences (β, ψ, ψ_0), technology (α), and monetary policy (μ_x, δ, π). Since our interests are mainly in the bond return behaviors in both models, I adopt parameter values from CEE (1997a) that is the most famous and standard limited participation model.

Economic agents' discount factor, β , is set to 0.9926. The elasticity of labor supply with respect to the real wage, $\frac{1}{\psi}$, is equal to 1. The parameter ψ_0 is calculated so that the steady state value of labor is 1. I use 0.36 as the capital share value of α . Monetary policy behavior is characterized by μ_x, δ, π . I assume that the money growth rate follows the three-state Markov process $(\mu_x - \delta, \mu_x, \mu_x + \delta)$ as in section 2. In an i.i.d case, I set $\mu_x = 0.02$, and $\delta = 0.017$. When there is a positive correlation in money growth rates, I set $\mu_x = 0.02$, and $\delta = 0.02$. This implies that the autocorrelation coefficient of money growth rates is 0.5. The transitional probability $\pi_{j,k} = \text{prob}\{x_{t+1} = x(k) \mid x_t = x(j)\}$, for j, k =1, 2, 3, and x (j) corresponds to $\mu_x - \delta, \mu_x, \mu_x + \delta$ respectively for j=1, 2, 3. In the i.i.d case, I set $\pi_{j,k} = 0.33$.[18]

4.2 Equilibrium

Before defining the equilibrium, I need to scale all nominal variables. In both models, $P_t, W_t, r_t, Q_t, X_t, B_t^I$, and B_t^{II} are scaled by the beginning of period money stock, M_t . The real variables (C_t, N_t, K_t, Y_t), nominal gross returns of one- and two-period bonds (R_t^I, R_t^{II}), and the term premium (TP_t) are not scaled.

4.2.1 The Imperfect Information Model

As I assumed before, the money growth rate follows the three-state Markov process. In this discrete-state solution approach, I need to clearly define each unknown's state

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[18] Following the CEE (1997a), I use $\pi_{j,k} = \begin{pmatrix} 0.58 & 0.34 & 0.08 \\ 0.08 & 0.84 & 0.08 \\ 0.08 & 0.34 & 0.58 \end{pmatrix}$ in the positive correlation case. This transition probability matrix implies that the ergodic distribution follows a normal distribution. For more detailed discussion of the Markov process, refer to Christiano (1990).

dependency. Currently, there are five unknowns, $q_t, N_t, R_t^I, R_t^{II}$, and TP_t. Here, q_t denotes the scaled amount of households' cash holding-$\frac{Q_t}{M_t}$. The decision to choose q_t is associated with t-1 period information. Therefore, q_t takes on three states, q_j, where j= 1,2,3. All the other variables are associated with both the results of cash holding decision that occurs before the money growth realization (j = 1, 2, 3) and the condition of the bonds market or the goods market which open after the money growth shock (k = 1, 2, 3). Hence, each N_t, R_t^I, R_t^{II}, and TP_t takes on nine values, such as $N_{j,k}, R_{j,k}^I, R_{j,k}^{II}$, and $TP_{j,k}$, where j=1, 2, 3, and k=1, 2, 3. [19] To get these 39 solutions, I construct 39 equations: [20]

$$\beta E_j[R_{j,k}^I E_k[\frac{U'_{C;k,l}}{P_{k,l}}]] = E_j[\frac{U'_{C;j,k}}{P_{j,k}}] \tag{4-1}$$

$$\beta \frac{(R_{j,k}^{II})^2}{R_{j,k}^I} E_k[E_l[\frac{U'_{C;l,m}}{P_{l,m}}]] = E_k[\frac{U'_{C;k,l}}{P_{k,l}}] \tag{4-2}$$

$$R_{j,k}^I = \frac{(1-\alpha)}{\psi_0} N_{j,k}^{-\alpha-\psi} \tag{4-3}$$

$$N_{j,k}^{\alpha+\psi} = \frac{1}{\psi_0}(\frac{1-q_j+x_k}{1+x_k}) \tag{4-4}$$

$$TP_{j,k} = (R_{j,k}^{II})^2 E_k[\frac{1}{R_{k,l}^I}] - R_{j,k}^I \tag{4-5}$$

Equation (4-3) is the labor market clearing condition. As is clear from this equation, the one-period bond return has an effect on the real economic activity through the relationship $M_t - Q_t = B_t^I$ and $R_t = R_t^I$. The two-period bond return, however, only appears in the bonds pricing formula equation (4-2) implying R_t^{II} has no real effect. Equation (4-4) is derived from the ratio of funds equation (3-27). To simplify the left-

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[19] Here, j denotes t-1 period information, and k denotes t period information.
[20] The subscript l denotes t+1 period information and m denotes t+2 period information.

hand side of equation (3-27), we have to use equation (3-12) which implies labor supply curve, $\frac{W_t}{P_t} = \psi_0 N_t^{\psi}$. If we substitute the labor supply curve and the goods market clearing condition, $C_t = N_t^{1-\alpha}$, into the left-hand side of equation (3-27), and scale the right hand side of it, then, we get equation (4-4). Equation (4-5) is the definition of term premium.

4.2.2 The Transaction Cost Model

Unlike the imperfect information model, all the decisions occur after the money growth realization in the transaction cost model. Hence, each unknown takes on only three values in this case (k = 1, 2, 3).

Five unknowns take on three values, k=1, 2, 3. Therefore, fifteen non-linear equations are needed to get those solutions:

$$\beta R_k^I E_k\left[\frac{U'_{C,l}}{P_l} + U'_{Q,l} + U'_{M,l}\right] = \frac{U'_{C,k}}{P_k} + U'_{Q,k} \tag{4-6}$$

$$\beta (R_k^{II})^2 E_k\left[\left(\frac{U'_{C,l}}{P_l} + U'_{Q,l}\right)\frac{1}{R_l^I}\right] = \frac{U'_{C,k}}{P_k} + U'_{Q,k} \tag{4-7}$$

$$R_k^I = \frac{(1-\alpha)}{\psi_0} N_k^{-\alpha}\left[N_k + \frac{\phi}{2} H_k^{\,2}\right]^{-\psi} \tag{4-8}$$

$$N_k^{\alpha}\left[N_k + \frac{\phi}{2} H_k^{\,2}\right]^{\psi} = \frac{1}{\psi_0}\left(\frac{1 - q_k + x_k}{1 + x_k}\right) \tag{4-9}$$

$$TP_k = (R_k^{II})^2 E_k\left[\frac{1}{R_l^I}\right] - R_k^I \tag{4-10}$$

Here, $H_k = (\frac{Q_k}{M_k} - \bar{q})$ which means a relative cash balance compared to the steady-state cash balance, and $\frac{\phi}{2}(H_k)^2$ implies the portfolio adjustment cost term. Note that the adjustment cost term in the utility function has an effect on the total labor supply equation. Unlike the imperfect information model case in which $\frac{W_t}{P_t} = \psi_0 N_t^{\ \psi}$, the labor supply curve is expressed as $\frac{W_t}{P_t} = \psi_0 (N_t + \frac{\phi}{2} H_k{}^2)^{\psi}$ in the transaction cost model. Since the portfolio adjustment process takes households' time, the total amount of labor is modified to include the adjustment cost. This modified labor supply equation changes the labor market clearing condition, equation (4-8), and the ratio of funds in the loan market to the funds in the goods market, equation (4-9). Equation (4-1) to (4-5) and (4-6) to (4-10) are exactly symmetric except the state dependent subscripts and the adjustment cost term in the total labor supply. Therefore, the derivation processes for (4-8) and (4-9) are the same as those of (4-3) and (4-4). Note that in the steady state, the adjustment cost is zero and each time subscript vanishes. This implies that equation (4-1) to (4-5) and equation (4-6) to (4-10) share the same equilibrium points.

4.3 The Liquidity Effect and Term Premia

The response of each variable to a contractionary money shock is calculated via the following elasticity concepts:

$$dv = \log(\frac{v_{t+1}}{v_t}) \Big/ \log[\frac{(1+\mu_x)}{(1+\mu_x-\delta)}] \tag{4-11}$$

[21]

$$dw = w_{t+1} - w_t \Big/ \log[\frac{(1+\mu_x)}{(1+\mu_x-\delta)}] \tag{4-12}$$

Here v = (N_t), w = (R_t^{I}, R_t^{II}, INF_t). Hence, dv represents a percentage change of each variable with respect to a one percent change of unanticipated money shock, and dw denotes the simple change of each variable which is scaled by $\log[\frac{(1+\mu_x)}{(1+\mu_x-\delta)}]$.

[21] Theoretically, $dv = \log(\frac{v_{t+1}}{v_t}) \Big/ \log[\frac{(1+\mu_x-\delta)}{(1+\mu_x)}]$, when we analyze the responses of v to a contractionary money shock. However, to get a negative sign in this case, I change the denominator to $\frac{(1+\mu_x)}{(1+\mu_x-\delta)}$. Instead, in an expansionary money shock case, we have to use $\frac{(1+\mu_x+\delta)}{(1+\mu_x)}$.

In addition to the each variable's response to the contractionary money shock, term premia themselves are calculated. In the imperfect information model, the term premium takes on nine values, and in the transaction cost model, it does three values.

4.3.1 The Imperfect Information Model

Economic variables' responses to the money contraction in this model are summarized in Table-1. The first table shows the case in which the labor-supply elasticity ($\frac{1}{\psi}$) is unity under different risk aversion parameters (γ). An i.i.d case and a positive correlation case show the same pattern. To the contractionary money shock, one- and two-period bond returns (R_t^{I}, R_t^{II}) rise—this demonstrates the presence of the liquidity effect. Obviously, labor (N_t) falls and an inflation rate (INF_t) decreases. Notably, the response of R_t^{I} is bigger than that of R_t^{II}. Also the responses of R_t^{I}, R_t^{II}, and N_t in the i.i.d case are greater than those in the correlation case. The reason is that the expected inflation effect is bigger in the correlation case. Unlike the CIA model in section 2, the responses of R_t^{I}, R_t^{II}, and N_t in this limited participation model are determined by the interaction of the liquidity effect which stems from the wedge of money values between money market and goods market and the expected inflation effect. In the i.i.d case, the expected inflation effect is smaller than that in the correlation case. This causes the bigger responses of R_t^{I}, R_t^{II}, and N_t in the i.i.d case. Consequently, the response of INF_t is larger in the correlation case. Risk aversion parameter (γ) has not much effect on each economic variable's response in both cases.

The second table exhibits the simulation results under different labor supply elasticities, $\frac{1}{\psi}$. When the labor supply elasticity is bigger ($\psi = 0.1$), N_t and INF_t

respond more aggressively. However, there is no qualitative difference between the benchmark case ($\psi = 1, \gamma = 1$) and the various simulation cases.

Term premium signs are shown in Table-2. The first table is the bench mark case and it shows that all term premia are positive. Due to the imperfect information setting, the term premium takes on nine values. The second and third tables are various simulations under different labor supply elasticities. Like Table-1, there is no significant difference between the bench mark case and the various simulations. Obviously, the average term premium is positive in either the i.i.d or the correlation case as we can see in Table-1.

To see the liquidity effect and the term premium signs more deeply, we have to explore the FOC's, equation (4-1) to (4-5) thoroughly. In equation (4-4), q_j is predetermined in each period due to the households' portfolio decision timing that occurs before the money shock. Under this condition, a new money shock, x_k, causes the increase of the right-hand side of equation (4-4). Accordingly, the amount of labor, the left-hand side of (4-4), also goes up. This increased labor supply results in the decrease of one-period bond return, $R^I_{j,k}$, via equation (4-3). Hence, the liquidity effect is accomplished through equation (4-3) and (4-4).

Based on the liquidity effect, the one-period bond return and the two-period bond return are determined along with equation (4-1) to (4-3). Finally, the term premium definition, equation (4-5), produces the positive term premium result. One notable feature of this model is that the one-period bond return is affected by the bond market condition, equation (4-1), and the labor market condition, equation (4-3); while the two-period bond return is only determined by the bond market condition, equation (4-2). At this point, we need to look at the labor market condition further. In the imperfect information setting, the key interest rate that is affected by the availability of money market funds is a lending rate. This lending rate is tied to the firms' marginal labor cost, $\frac{W_t}{P_t}R_t$. As is explained in section 3, the perfect competition in the financial sector ensures $R_t = R_t^I$.

Despite the disparity between the one-period bond and the two-period bond determination procedure, the liquidity effect triggered by the wedge of money value in both markets consistently generates the positive term premium in the imperfect information model. On the other hand, in the transaction cost model, a new effect called an adjustment cost effect is created. This causes an undesirable result, a negative term premium. More detailed explanation will be given later.

4.3.2 The Transaction Cost Model

As I mentioned earlier, the transaction cost model nests the CIA model and the imperfect information model, which depends on the size of adjustment cost parameters, ϕ. From equation (3-35) and (3-36), we can easily see that when ϕ is zero, FOC's for the one- and the two-period bonds in the transaction cost model are exactly the same as those in the CIA case. Table-3 and Table-4 show the economic variables' responses to a money contraction and the term premium signs in the CIA model (when ϕ is zero in the transaction cost model). As we expect, they do not show the liquidity effect and the positive term premia.

When I increase the ϕ up to 15, the transaction cost model reveals the meaningful liquidity effect. These results are shown in Table-5. Though the magnitudes are slightly different from the Table-1 of the imperfect information model case, all the signs are consistent with the Table-1. The most surprising part in this paper is found in Table-6. Despite the liquidity effect in Table-5, all term premium signs are negative except the high money growth state under the assumption of a positive serial correlation. Moreover, while the average term premium in the imperfect information model shows a positive sign; the same one in the transaction cost model exhibits a negative sign. This is the critical defect of the transaction cost model.

To confirm my findings, I can explore the results in a different angle. In the transaction cost model, the liquidity effect is driven by the adjustment cost as already mentioned. Graph-1 shows the degrees of liquidity effect when the adjustment cost parameter varies from 0 to 20. As is clear in the graph, the degrees of liquidity effect increase as the adjustment cost parameter gets bigger. Note that while the liquidity effect in the i.i.d case appears with relatively small parameters such as 1, the same effect in the positive correlation case reveals with larger parameters such as 6. Therefore, in order to see the liquidity effect in R_t^I and R_t^{II} regardless of the money growth correlation, a somewhat larger parameter like 7 is required.

The next point that we should pay attention to is that the adjustment cost of one-period bond is always bigger than that of two-period bond. As acknowledged in the previous sub-section, the one-period bond return is affected by both the bonds market and the labor market. More specifically, the introduction of adjustment cost causes the distortion of marginal utility in equation (4-6) and the change of labor supply in equation (4-8) in one-period bond case; whereas the adjustment cost affects the two-period bond only through equation (4-7). I name this effect *the adjustment cost effect*. As we already saw, the liquidity effect induces a positive term premium. In other words, the liquidity effect makes the difference of average returns, $R_t^{II} - R_t^I$, more positive.

However, the adjustment cost effect works the opposite way as the liquidity effect. It makes the $R_t^{II} - R_t^{I}$ more negative. The intuition behind this result is that the bigger adjustment cost of one-period bond leads investors to require higher return on the one-period bond to compensate for the loss of adjustment cost. Hence, unlike the imperfect information model in which there is no adjustment cost channel, here is a new distortion in the transaction cost model.

Therefore, bond returns are determined by not only the liquidity effect but also the adjustment cost effect. This implies the term premium is also directed by the interaction of the liquidity effect and the adjustment cost effect. As a logical consequence, the liquidity effect dominates the adjustment cost effect within a small adjustment cost parameter region; while the adjustment cost effect dominates the liquidity effect within a large parameter region.

Graph-2 shows the changes of average returns. In the i.i.d case, there is no gap between R_t^{I} and R_t^{II} initially. However, as the parameter gets bigger, the gap becomes wider, which indicates the dominance of adjustment cost effect. In the positive correlation case, R_t^{I} is higher than R_t^{II} at first, reflecting the lack of the liquidity effect. Note that this gap is gradually narrower in the small parameter region, which indicates the faster growth of the liquidity effect. However, once the parameter exceeds 4, the gap again gets wider due to the dominance of the adjustment cost effect. Graph-2 clearly shows the interaction of the liquidity effect and the adjustment cost effect.

The movement of average term premia is shown in Graph-3. Regardless of the money growth correlation, the term premia grow toward a positive direction at first, and then goes down. This is another reflection of the competition between the liquidity effect and the adjustment cost effect. When the liquidity effect grows faster, the term premium goes up; and while the adjustment cost effect is dominant, it goes down. Note that the term premium in the i.i.d case shows a positive sign within a small parameter region. However, the term premium in the positive correlation case does not show a positive sign within that reason due to the lack of the liquidity effect.

In conclusion, to generate the liquidity effect, somewhat larger parameter values are needed in the transaction cost model. Ironically, even though these larger parameter values give us the liquidity effect, they create a side effect called the adjustment cost effect. This adjustment cost effect even dominates the liquidity effect and results in the negative term premium.

To check the robustness of this result, I tried different utility functions and put the adjustment cost in a budget constraint and a CIA constraint instead of a utility function. These different model settings still show a negative term premium. Also the argument that the adjustment cost of one-period bond is always greater than that of two-period bond is immune to the definition of adjustment cost such as an adjustment cost defined by the growth rate of cash balances, as long as the lending rate is connected to the one-period bond return, R_t^I. In other words, the one-period bond return has a real effect on the economic activity, whereas the two-period bond does not.

5 Conclusion

The results of this paper provide us with several interesting implications. First of all, I confirm that despite the strong assumption of the one-period-ahead decision making, the imperfect information model guarantees the liquidity effect and the positive term premium seen in the data. On the other hand, the transaction cost model successfully captures the liquidity effect under the after-shock decision making with the portfolio adjustment cost. Unfortunately, however, it fails to generate the positive term premium.

This paper shows that an adjustment cost that drives the liquidity effect in the transaction cost model also creates an adjustment cost effect, which works the opposite way as the liquidity effect. To generate the liquidity effect, a large adjustment cost is needed. However, with this big cost, the adjustment cost effect dominates the liquidity effect, which leads to a negative term premium.

Based on these findings, I conclude that it is needed to be cautious to use the transaction cost model in the term structure studies. As mentioned in section 1, Dotsey and Ireland (1995) point out that the transaction cost setting may not be relevant for the asset pricing study, but relevant for the business cycle modeling. Nonetheless, there has been no discussion on the ability of the transaction cot model to replicate the positive term premium.

This paper clearly shows the difference between the imperfect information model and the transaction cost model, thus provide essential guidance on using two monetary models for counterfactual policy analyses. Further study should be directed to invent another modeling strategy to guarantee the liquidity effect and the positive term premium without the strong assumption of the one-period-ahead decision-making. This study will make monetary models much richer.

Table 1 - Responses to a Money Contraction in the Imperfect Information Model

ψ =1 / Vabls.	i.i.d		Corr. (Coef. = 0.5)	
	$\gamma = 1$	$\gamma = 5$	$\gamma = 1$	$\gamma = 5$
R^{I}_{t}	***0.63***	0.63	***0.61***	0.61
R^{II}_{t}	***0.32***	0.32	***0.20***	0.20
N_t	-0.45	-0.45	-0.44	-0.44
INF_t	-0.29	-0.29	-0.89	-0.90

γ =1 / Vabls.	i.i.d		Corr. (Coef. = 0.5)	
	ψ =0.1	ψ =8	ψ =0.1	ψ =8
R^{I}_{t}	0.63	0.63	0.61	0.61
R^{II}_{t}	0.32	0.32	0.22	0.18
N_t	-1.33	-0.07	-1.30	-0.07
INF_t	-0.86	-0.05	-1.60	-0.57

Note: N_t is a percentage change, the other variables are absolute changes.

Table 2 - Term Premia in the Imperfect Information Model
(Basis points)

$\gamma = 1$, $\psi = 1$	s(t) / s(t-1)	1	2	3	Average
i.i.d	1	***1.85***	***1.83***	***1.81***	**1.83**
	2	***1.85***	***1.83***	***1.81***	
	3	***1.85***	***1.83***	***1.81***	
Corr.	1	2.10	0.83	2.16	**1.26**
	2	2.13	0.84	2.18	
	3	2.15	0.85	2.20	

$\gamma = 1$, $\psi = 0.1$	s(t) / s(t-1)	1	2	3	Average
i.i.d	1	*1.85*	*1.83*	*1.81*	*1.83*
	2	*1.85*	*1.83*	*1.81*	
	3	*1.85*	*1.83*	*1.81*	
Corr.	1	2.22	0.87	2.26	1.32
	2	2.24	0.88	2.28	
	3	2.26	0.89	2.30	

$\gamma = 1$, $\psi = 8$	s(t) / s(t-1)	1	2	3	Average
i.i.d	1	*1.85*	*1.83*	*1.81*	*1.83*
	2	*1.85*	*1.83*	*1.81*	
	3	*1.85*	*1.83*	*1.81*	
Corr.	1	2.03	0.81	2.09	1.22
	2	2.05	0.81	2.12	
	3	2.07	0.82	2.14	

Table 3 - Responses to a Money Contraction in the CIA Model
(ϕ = 0 in the Transaction Cost Model)

ψ =1 / Vabls.	i.i.d		Corr. (Coef. = 0.5)	
	γ = 1	γ = 5	γ = 1	γ = 5
R_t^I	-0.0	-0.0	***-0.42***	***-0.43***
R_t^{II}	-0.0	-0.0	***-0.32***	***-0.32***
N_t	-0.0	-0.0	0.30	0.31
INF_t	-0.0	-0.0	-0.41	-0.40

Note: Refer to the note in table 1.

Table 4 - Term Premia in the CIA Model
(ϕ = 0 in the Transaction Cost Model, Basis points)

γ = 1 ψ = 1	s(t) = 1	s(t) = 2	s(t) = 3
i.i.d	-0.0	-0.0	-0.0
Corr.	***-0.78***	***-0.31***	***-0.79***

Table 5 - Responses to a Money Contraction in the Transaction Cost Model (ϕ = 15) *

Vabls. \ $\psi=1$	i.i.d		Corr. (Coef. = 0.5)	
	$\gamma = 1$	$\gamma = 5$	$\gamma = 1$	$\gamma = 5$
R_t^{I}	***0.51***	0.51	***0.38***	0.39
R_t^{II}	***0.25***	0.26	***0.15***	0.15
N_t	-0.36	-0.37	-0.27	-0.28
INF_t	-0.24	-0.24	-0.59	-0.60

Vabls. \ $\gamma=1$	i.i.d		Corr. (Coef. = 0.5)	
	$\psi=0.1$	$\psi=8$	$\psi=0.1$	$\psi=8$
R_t^{I}	0.45	0.53	0.35	0.39
R_t^{II}	0.23	0.27	0.11	0.17
N_t	-0.97	-0.06	-0.75	-0.05
INF_t	-0.62	-0.04	-0.75	-0.52

Note: Refer to the note in table 1.

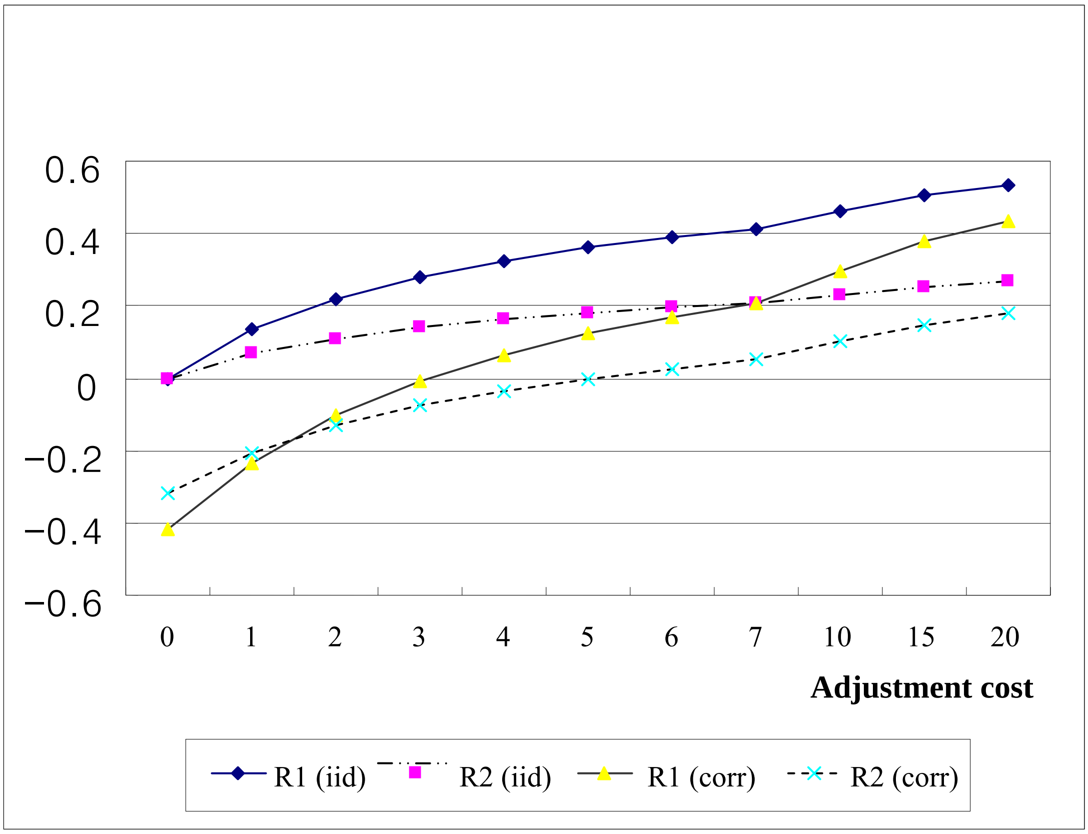
Table 6 - Term Premia in the Transaction Cost Model
(ϕ = 15, Basis points)

$\gamma = 1$ $\psi = 1$	s(t) = 1	s(t) = 2	s(t) = 3	Average
i.i.d	***-1.07***	***-1.06***	***-1.05***	**-1.06**
Corr.	-55.1	-1.03	***<u>52.4</u>***	**-1.12**

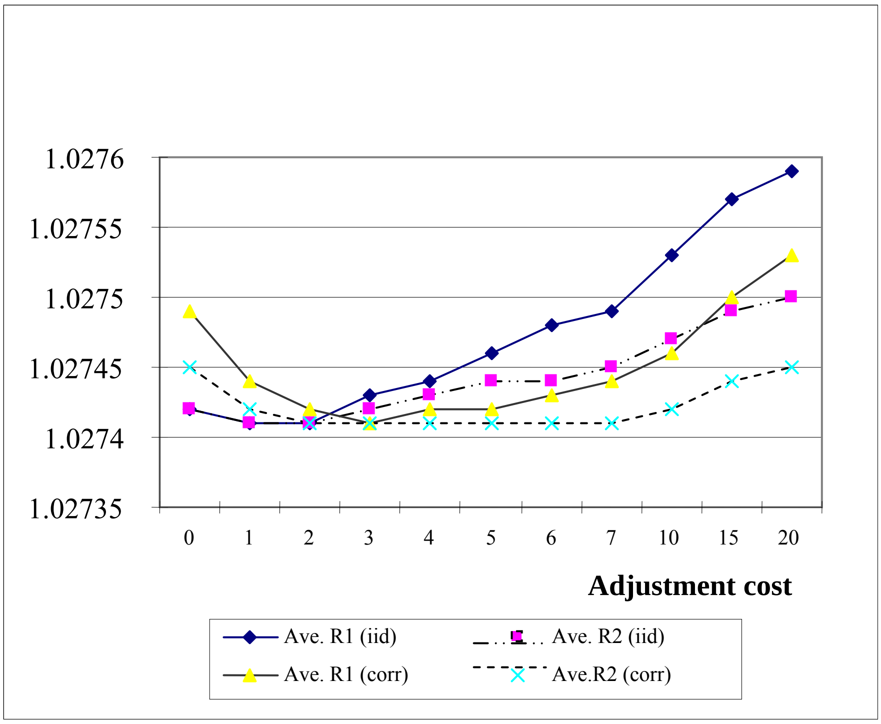
$\gamma = 1$ $\psi = 0.1$	s(t) = 1	s(t) = 2	s(t) = 3	Average
i.i.d	-1.07	-1.06	-1.05	-1.06
Corr.	-61.2	-1.02	59.1	-1.03

$\gamma = 1$ $\psi = 8$	s(t) = 1	s(t) = 2	s(t) = 3	Average
i.i.d	-1.08	-1.07	-1.06	-1.07
Corr.	-52.0	-1.05	49.1	-1.17

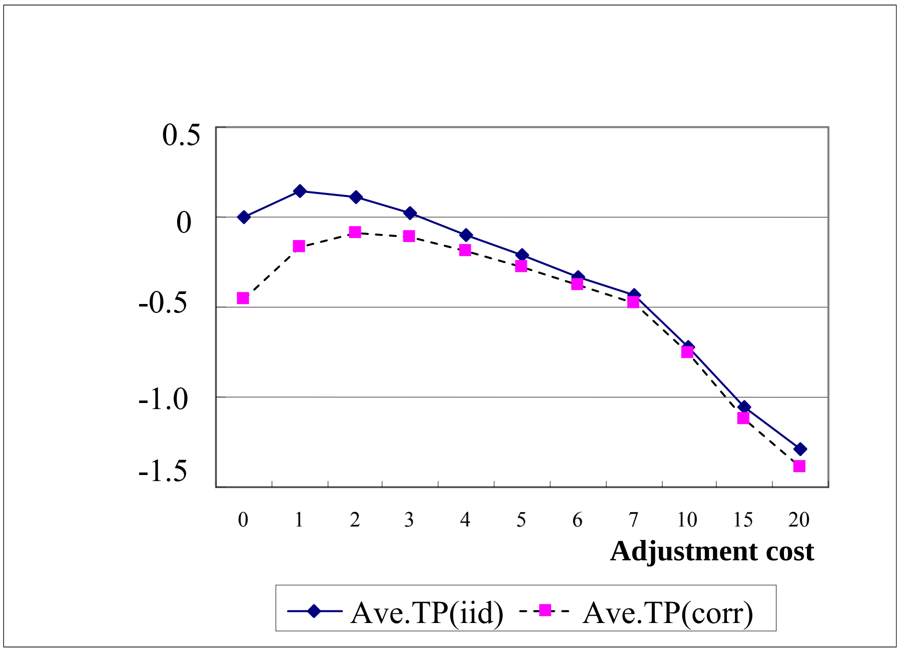
Graph 1 – Degrees of Liquidity Effects



Graph 2 - Changes of Average Returns



Graph 3 - Changes of Average Term Premia
(Basis Points)



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