Disclosure Manipulation and Elasticity of Intertemporal Substitution

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Abstract

This paper is concerned with the effect of disclosure manipulation in earnings reporting on the stock market and the elasticity of intertemporal substitution (EIS) in consumption. We try to quantify the effects of fraud and speculation on EIS, and develop a new theory of "noise-corrected" EIS. In the study of U.S. quarterly data, the effect is found to be so severe that the statistical properties of the estimated elasticity are heavily noised. The noise-corrected EIS is estimated of 0.51 similar to that of Kydland and Prescott (1982). Mehra and Prescott (1985, 2003) and Hall (1988) found that EIS is essentially zero, while Arrow (1971) predicts as '1' in theory, an obvious contraction between theory and empirical studies (the elasticity puzzle). Our result implies that economic behaviors may be working as predicted in theory even though the statistical significance is noised due to the effect, hence we need to be careful in interpreting the results. It is also found that under fraudulent earnings reporting, the asset market's channelling role for consumers' decisions and for government policies is thin and reversed.

Key Words: Disclosure manipulation, Residual uncertainty, Law of small numbers, Noise-corrected elasticity of intertemporal substitution, Relative risk aversion.

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1 Introduction

> If we have better information about our corporations, our society will benefit in many ways by preventing (financial) fraud and reducing the likelihood of speculative bubbles. (Shiller, The New Financial Order, 2003, p.197.)

In the aftermath of a number of high-profile scandals within the space of one year including Enron, WorldCom and Imclone, an investor interviewed by NBC nightline on August 12, 2002 said that "...I do not know the true value of stock. I just do not trust the people behind the stock market." Remarkably, on the mornings after the scandals broke, the stock prices of Enron and Worldcom plummeted to less than 10 cents, whereas they had previously traded as high as $84 and $64, respectively. Enron filed for bankruptcy because of its financial fraud and document shredding on December 2, 2001, and Worldcom filed after having overstated income by approximately $ 9 billion. The CEO of Imclone was charged for insider trading.

These examples are enough to remind us of the 'credibility problem' in the stock market. Levitt (1998), then the chairman of SEC (Securities and Exchange Commission), points out serious outright fraud in earnings reporting, and states that "wishful thinking may be winning the day over faithful representation on Wall Street." According to Shin (2003), such a credibility problem has obtained because "the arrival of public information influencing asset prices is provided by the interested parties who have a material interest in the reactions of the market to such news." For example, managers often choose to make partial disclosures rather than full disclosures, in which only successes are disclosed, whereas no failures are fully disclosed. Even though it is mandatory to periodically disclose verifiable information, therefore, the market participants seldom take the disclosure at face value: They discount it unless the disclosure

is guaranteed to be completely verified.

In the paper, our objective is to quantify the effects of fraud and speculation on the EIS to get ideas as to how fraud affects stock market behavior and how to make the market fair and efficient. Here, *fair* denotes correctness in the pricing process. The elasticity of intertemporal substitution is known to be close to zero in recent empirical studies. Hall (1988) finds the value close to zero, and Mehra and Prescott (1985, 2003) find the value to be 0.048, essentially zero, even though Arrow (1971) suggests '1' in theory, and Kydland and Prescott (1982) imply 0.66. According to the empirical findings, the stock market is not a useful instrument for the intertemporal behaviors of consumers and government policies, e.g., tax cuts for dividends, whereas the latter economic theory entails the effective channelling of the asset market between current consumption and future consumption. However, we doubt that empirical findings may be gleaned by the effect of market credibility problems. What happens if the effect is properly quantified and handled? We hypothesize that as long as noise is properly handled, we may get a non-zero sensible value of EIS consistent with economic theory.

In section 2, Bayesian pricing is formulated when there is residual uncertainty, and in section 3, the maximization problem is studied. Section 4 investigates the law of small numbers as a regulation theory under residual uncertainty. Some properties of equilibrium price are outlined in section 5. Section 6 examines the elasticity with a time-separable utility function under residual uncertainty. Section 7 and 8 apply theoretical and empirical studies on econometric tests to quantify the effects of fraudulent earnings management, and noise-corrected EIS is discussed. Section 9 concludes.

2 Pricing under Residual Uncertainty

According to the efficient market hypothesis (EMH) where everyone is a rational optimizer, stock price change must be unforecastable since the market fully incorporates the expectations and information of 'all' market participants, and only unpredictable parts of "errors" dominate price movements. Under the hypothesis, it is well known that the law of iterated expectation is satisfied across individuals because of information homogeneity. EMH assumes the restrictions: (1) each individual has the same information set (information homogeneity) (2) each individual is rational (rationality), and (3) forecast error$^i \sim N(0, \sigma^2)$ for all investor i where $\sigma^2 \in R^+$ (homoscedasticity).

However, if the market is dominated by increasing "residual uncertainty remaining after the self-interested disclosure"[1] (strategically noisy information), then stock price changes also can be unpredictable, and the market does not fully and correctly incorporate all relevant information in the market. There is a large volume of literature on the hypothesis including 'The Numbers Game' by Levitt (1998), 'Disclosure Game' by Shin (2003) and 'Stock Recommendation Game' by Morgan and Stocken (2003). Under this hypothesis, the law of iterated expectations is not necessarily satisfied since the restrictions (homogeneity, rationality, and homoscedasticity) are violated. Therefore, market participants do not take the market price as fair, rather they subjectively discount the market price so that

$$p_t^i = \frac{p_t(dis_t)}{\delta^i} \tag{1}$$

where p_t^i is subjectively discounted price, p_t is the market price, dis_t is disclosure and $\delta^i = 1+\epsilon^i \geqslant 1$, $\epsilon^i \geqslant 0$ is subjectively estimated degree of residual uncertainty by investor i. If $\epsilon^i = 0$, the subjectively discounted price is the same as the

[1] Shin (2003), p.106

market price.

Suppose market information is noisy and uneven across individuals. Residual uncertainty ϵ after disclosure, for example, the number of undisclosed failures of projects, is assumed to be $\epsilon_t|\epsilon_{t-1} \sim N(\epsilon_{t-1}, \triangle t)$ and each investor i observes a signal $\epsilon_t^i = \epsilon_t + \varepsilon^i$ where $\varepsilon^i \sim N(0, \sigma_i^2)$. Then, in the spirit of Allen et al (2004), the logged posterior density function of ϵ_t is proportional to

$$log\ h(\epsilon_t|\epsilon_{t-1}, \epsilon_t^i) \propto -\frac{1}{2}\left(\frac{1}{\triangle t}(\epsilon_t - \epsilon_{t-1})^2 + \frac{1}{\sigma_\varepsilon}(\epsilon_t - \epsilon_t^i)^2\right) \tag{2}$$

since 'posterior $\propto$ prior$\times$sample information' where ϵ_{t-1} is prior and ϵ_t^i is sample information for individual i. The logged posterior would be zero under the EMH. From the first order condition with respect to ϵ_t, we get

$$\begin{aligned} E_t^i(\epsilon_t) &= \frac{\sigma_\varepsilon \epsilon_{t-1} + \triangle t \epsilon_t^i}{\sigma_\varepsilon + \triangle t} \\ \bar{E}_t(\epsilon_t) &= \frac{\sigma_\varepsilon \epsilon_{t-1} + \triangle t \epsilon_t}{\sigma_\varepsilon + \triangle t} \end{aligned} \tag{3}$$

where the second is the average expectation in market. Then, the iterated expectation on the average expectation, suppressing subscripts, is

$$\begin{aligned} E^i(\bar{E}(\epsilon_t)) &= \frac{\sigma_\varepsilon \epsilon_{t-1} + \triangle t E^i(\epsilon_t)}{\sigma_\varepsilon + \triangle t} \\ &= \left(1 - \left(\frac{\triangle t}{\sigma_\varepsilon + \triangle t}\right)^2\right)\epsilon_{t-1} + \left(\frac{\triangle t}{\sigma_\varepsilon + \triangle t}\right)^2 \epsilon_t^i. \end{aligned} \tag{4}$$

Iterating finite n times because of the law of small numbers (discussed in a later section),

$$E^n(\bar{E}(\epsilon_t)) = \left(1 - \left(\frac{\triangle t}{\sigma_\varepsilon + \triangle t}\right)^n\right)\epsilon_{t-1} + \left(\frac{\triangle t}{\sigma_\varepsilon + \triangle t}\right)^n \epsilon_t^i \tag{5}$$

from which, it is certain that expectation is biased toward individual information.

In particular, under the data generating process such as the residual uncertainty, the stock price formed by Bayes rule is

$$\tilde{p}_t = \left(1 - \left(\frac{\triangle t}{\sigma_\varepsilon + \triangle t}\right)^n\right) p_{t-1} + \left(\frac{\triangle t}{\sigma_\varepsilon + \triangle t}\right)^n p_t^i \tag{6}$$

which is biased toward p_t^i. Note that T is stochastic that might be geometrically short but psychologically long. If $\triangle t$ is long enough, i.e., stochastic T is long, then $\tilde{p}_t \simeq p_t^i$ so that Bayesian price is proximately the same as the individual valuation.

3 The Law of Small Numbers: "Individual decision does matter"

To understand the behavioral properties of the equilibrium price under increasing residual uncertainty, we need to develop a regulation theory regarding the data generating process of the residual uncertainty and subjective expectation formation. This is the law of small numbers, developed by Kahneman and Tversky (1978). The law says that "there are hardly enough small numbers to satisfy all the demands placed on them." Based on Kahneman and Tversky (1978), we develop a more thorough law of small samples. The implications of the law of small numbers are that investors couldn't get correct distributional specifications on the behaviors of the residual uncertainty in the short- and long-period of time, and so that the market price of a stock could be biased from the fundamental value and individual decisions would affect the market price. The three properties are as follows.

Proposition 1. *(LSN I, Imperfection in Expectation)*
Suppose Ω_t is information set at t. $\epsilon_t - E_t^i \bar{E}(\epsilon_t|\Omega_t^i) = \epsilon_t - \delta_t^i \sim N(\psi, \rho\triangle t)$ where $\rho \in R^+$, $\psi \neq 0$ and $\delta_t^i = E_t^i \bar{E}(\epsilon_t|\Omega_t^i)$.

Proof: From the Chebyshev's inequality, $P(|\delta_t^i - \epsilon_t| < \kappa) \geqslant 1 - \frac{var(\delta_t^i)}{\kappa^2} \neq 1,\ \forall t$ because $\delta_t^i \rightarrow \epsilon_t^i$ as $\triangle t$ gets large (note section 2). Then, since $\epsilon_t|\epsilon_{t-1} \sim N(\epsilon_{t-1}, \triangle t)$, $\epsilon_t - \delta_t^i \sim N(\psi, \rho\triangle t),\ \psi = \epsilon_{t-1} - E(\delta_t^i),\ \forall t$.

The distributional specification of $\epsilon_t - \delta_t^i \sim N(\psi, \rho\triangle t)$ implies that the expectation couldn't realize stationary relations with true residual uncertainty and fails to get consistent estimators for the values over time. The following means the instability in the expectation formation over time.

Proposition 2. *(LSN II, Instability in Expectation)*
Suppose $E_t^i\bar{E}(\epsilon_t|\Omega_t^i) = \delta_t^i,\ E_{t-1}^i\bar{E}(\epsilon_{t-1}|\Omega_{t-1}^i) = \delta_{t-1}^i$. *Then, for investor i,* $E_t^i\bar{E}(\epsilon_t|\Omega_t^i) - E_{t-1}^i\bar{E}(\epsilon_{t-1}|\Omega_{t-1}^i) = \delta_t^i - \delta_{t-1}^i \sim N(\nu, \zeta\triangle t),\ \forall t$.

Proof: Since $\epsilon_t - \delta_t^i \sim N(\nu_1, \iota\triangle t)$ and $\epsilon_{t-1} - \delta_{t-1}^i \sim N(\nu_2, o\triangle t)$, $(\epsilon_t - \delta_t^i) - (\epsilon_{t-1} - \delta_{t-1}^i) \sim N(\nu_1 - \nu_2, \varrho\triangle t),\ \varrho = \iota + o$. That is, $(\epsilon_t - \epsilon_{t-1}) - (\delta_t^i - \delta_{t-1}^i) \sim N(\nu_1 - \nu_2, \varrho\triangle t)$. Since $\epsilon_t - \epsilon_{t-1} \sim N(0, \triangle t)$, then $\delta_t^i - \delta_{t-1}^i \sim N(\nu, \zeta\triangle t),\ \nu = \nu_1 - \nu_2,\ \zeta = \varrho - 1$.

Proposition 3. *(LSN III, Heterogeneity in Expectation)*
Suppose that the information sets of the investors are $\Omega_t^i \neq \Omega_t^j$ *for all* $i \neq j$. *Then,* $\delta_t^i - \delta_t^j \rightarrow N(\mu, \varsigma\triangle t)$ *for investors* $i \neq j$.

Proof: Since $\epsilon_t - \delta_t^i \sim N(\nu^i, \rho^i\triangle t),\ \rho^i \in R^+$ and $\epsilon_t - \delta_t^j \sim N(\nu^j, \rho^j\triangle t),\ \rho^j \in R^+$, then, since $\epsilon_t - \delta_t^i - (\epsilon_t - \delta_t^j) = \delta_t^i - \delta_t^j$, $\delta_t^i - \delta_t^j \sim N(\mu, \varsigma\triangle t),\ \mu = \nu^i - \nu^j,\ \varsigma = \rho^i + \rho^j$.

Because of the properties, the classical restrictions such as rationality and competitiveness would be violated. The rationality requires full specifications on the residual uncertainty, and the competitiveness implies that individual

couldn't affect market.

4 Equilibrium and Implied Interest Rates

Let $u(c_t)$ be the strictly increasing concave period-utility function with $u' > 0, u'' < 0$, and $u \in \mathcal{C}^1$, and β be the utility discount factor over time horizon. Then, the investor's problem is

$$\max_{c_s} u(c_t) + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s) \tag{7}$$

subject to

$$\begin{aligned} c_s + \frac{p_s}{\delta_s} a_s &= \left(\frac{p_s}{\delta_s} + \frac{d_s}{\delta_s} \right) a_{s-1} \\ \lim_{T \to \infty} \prod_{s=0}^{T} \frac{p_t}{\delta_{t+s-1}} a_{t+T} &\geqslant 0 \end{aligned} \tag{8}$$

where a_s is asset holding at s and d_s dividend. Note that the constraint is equivalent to the one in Deaton (1992) where asset stochastically depreciate.

According to the classical idea, stock valuation and asset holding are determined simultaneously where the market price is the 'right' price. The classical budget constraint stands on two strong restrictions. First, rationality; it is too strong in the sense that it requires full specifications on the distributions of the residual uncertainty in a short period of time. Second, competitiveness; it is also too strong in the sense that an individual behavior's behavior may still affect the market in the short run.

From the first-order condition of the investor's period-utility maximization problem, we get the following Euler equation:

$$1 = E_t \left(\xi^h_{t,t+1} \cdot (1 + r_{t+1}) \cdot \frac{\delta_t}{\delta_{t+1}} \right) \tag{9}$$

where $\xi^h_{t,t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$, $1 + r_{t+1} = 1 + \frac{p_{t+1}+d_{t+1}-p_t}{p_t}$, r_{t+1} is return of stock.

From the equilibrium condition, the so-called 'implied interest rate' of Alvarez and Jermann (2000) can be derived:

$$r^*_t = \frac{1}{\xi^h_{t,t+1} \cdot \tilde{\delta}} - 1 \tag{10}$$

where $\tilde{\delta} = \frac{\delta_t}{\delta_{t+1}}$.

5 Equilibrium Price and Price Distribution

We should discuss two properties of the equilibrium price. First, the price is strategically recognized for arbitrage opportunities by market participants. When risks are perceived or seen to be increasing, arbitrageurs do not necessarily take action instantaneously to reflect the true risk because arbitrage entails risks such as synchronization risk as in Abreu et al (2002). In that case, competitive equilibrium may be defined as a situation in which prices are such that all arbitrage profits are not eliminated under increasing or decreasing risks.

Proposition 4. *When residual uncertainty increases whose true movements are not known to market, but known to some individuals, the market price is such that some arbitrage opportunities remains.*

Proof: Let $p(\epsilon_t, dis) = \frac{p}{\epsilon}$ be the *fair* price, and $\tilde{p} = \frac{p}{\delta}$ be the equilibrium price, where δ is randomly chosen out of $(\delta^1, \delta^2, ..., \delta^i)$, $i = 1, 2, ..., n$. Since $\frac{\partial \tilde{p}}{\partial \delta} < 0$ with increasing risks, price $\tilde{p}(\delta_t|dis)$ is strictly superior to $p(\epsilon_t, dis)$, i.e., $\tilde{p}(\delta_t|dis) - p(\epsilon_t, dis) \geqslant 0$ for $\epsilon_t - \delta^i_t \geqslant 0$ where $p(\epsilon_t, dis)$ is the fully informative price under increasing residual uncertainty.[2]

[2] In the spirit of Grossman and Stiglitz (1980), the price difference is a compensation for the informed trader for their expending resources to obtain information.

Second, the market price is a function of the joint distribution of disclosure and subjective expectation formation, i.e., $(E^i\bar{E}(\epsilon_t|dis), dis)$. From the LSN I, however, the market price is not a 'statistical equilibrium' of Grossman and Stiglitz (1980):

Proposition 5. *$\tilde{p}(E^i\bar{E}(\epsilon_t|dis), dis)$ is hardly a 'statistical equilibrium' in that consumers can't learn the joint distribution of (ϵ_t, p_t^*) for all t where $p_t^* = p_t(\epsilon_t, dis)$.*

Proof: Since $\epsilon_t - E^i\bar{E}(\epsilon_t|dis) \sim N(\nu, f \cdot \triangle t)$ by LSN I, and $\tilde{p}(E^i\bar{E}(\epsilon_t|dis) - p(\epsilon_t, dis) = \left(\frac{1}{\delta_t} - \frac{1}{\epsilon_t}\right) p(dis) \simeq (2 - \delta_t - (2 - \epsilon_t))p(dis) = (\epsilon_t - \delta_t)p(dis)$ using Taylor expansion around $\delta = 1 = \epsilon$, then, $(\tilde{p}(E^i\bar{E}(\epsilon_t|dis) - p(\epsilon_t, dis)) \cdot \frac{1}{p(dis)} \sim N(\nu, f \cdot \triangle t)$.

Therefore, investors couldn't get a time persistent true stationary joint distribution, and the market price can hardly come about with a fully specifiable joint distribution. See Grossman and Stiglitz (1980, p.396) and Chamberlain (2000)[3] for detail.

6 Elasticity of Intertemporal Substitution

Suppose a time-separable power utility function with constant relative risk aversion is

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1 - \gamma} \tag{11}$$

[3]Since the joint distribution is only partially specifiable, i.e., the conditional predictive distribution for ϵ_t follows a non-ergodic Markov process, it is not possible for any decision rule to achieve 'posterior robustness' in the conditioning information in the spirit of Chamberlain (2000, p.260). In that sense, the risk premium can be compensation for this magnitude of unspecifiable distribution causing failure to achieve the robustness. Shin (2003) also has the same implication for the risk premium.

where $\frac{1}{\gamma}$ is the elasticity of intertemporal substitution. Let the discount factor $\beta = \frac{1}{1+\rho}$ where ρ is the rate of time preference. Suppose $\frac{\delta_t}{\delta_{t-1}} = 1 + \phi$ where $\phi = \frac{\delta_t - \delta_{t-1}}{\delta_{t-1}}$. Then, from the equilibrium conditions (suppressing subscripts),

$$1 + \rho = E\left((1+r)(1+g)^{-\gamma}(1+\phi)^{-1}\right) \tag{12}$$

where $g = \frac{c_{t+1} - c_t}{c_t}$, the growth rate of consumption from t to $t+1$.

To see the implication of the condition, we take a second-order Taylor expansion of the right-hand side around $g = r = \phi = 0$. Since the first-order term is $r - \gamma g - \phi$, and the second-order term is $-2(\gamma gr + r\phi - \gamma g\phi) + 2\phi^2 + \gamma(1+\gamma)g^2$, the Taylor approximation is,

(13)

$$((1+r)(1+g)^{-\gamma}(1+\phi)^{-1}) \simeq 1 + (r - \gamma g - \phi) + (\gamma g\phi - r\phi - \gamma gr) + \phi^2 + \frac{1}{2}\gamma(1+\gamma)g^2$$

Thus,

(14)

$$\begin{aligned}\rho &= E(r) - \gamma E(g) - E(\phi) - [E(r)E(\phi) + cov(r,\phi)] - \gamma[E(g)E(r) + cov(g,r)] \\ &\quad + \gamma[E(g)E(\phi) + cov(g,\phi)] + [E(\phi)^2 + var(\phi)] + \frac{1}{2}\gamma(1+\gamma)[(E(g))^2 + var(g)]\end{aligned}$$

When the time period involved is short, $E(r)E(\phi), E(g)E(r), E(g)E(\phi), E(\phi)^2$ and $E(g)^2$ are small relative to other terms, so, by omitting these terms,

$$\begin{aligned}E(r) &= \rho + \gamma E(g) + E(\phi) + cov(r,\phi) + \gamma cov(g,r) - \gamma cov(g,\phi) \\ &\quad - var(\phi) - \frac{1}{2}\gamma(1+\gamma)var(g)\end{aligned} \tag{15}$$

If the asset is risk free, $cov(r,\phi) = cov(g,r) = 0$, the equation simplifies to

$$r^{rf} = \rho + \gamma E(g) - \gamma cov(\phi, g) + E(\phi) - var(\phi) - \frac{1}{2}\gamma(1+\gamma)var(g) \tag{16}$$

Finally, the risk premium is

$$E(r) - r^{rf} = \gamma cov(g,r) + cov(r,\phi) \tag{17}$$

where the first term in the right-hand side of $\gamma cov(g, r)$ is the actuarially fair risk premium when there is no residual uncertainty problem and consumption is a homogeneous degree of 1 with production of firms as in Mehra and Prescott (1985). However, the second term in the bracket is the additional risk premium for the residual uncertainty. The value of $cov(r^E, \phi)$ should be positive since higher risk for the risk-averse investor should be offset by higher expected returns. If the risk aversion is stronger, then this covariance should be greater.

The second term $cov(r, \phi)$ requires further attention. Since $\delta_t^i - \delta_{t-1}^i \sim N(\nu, g \cdot \triangle t)$ by LSN II, the second term can be rewritten as

$$\begin{aligned} cov(r, \phi) &= \frac{1}{\delta_{t-1}} cov(r, \delta_t - \delta_{t-1}) \\ &= \frac{1}{\delta_{t-1}} E(r \cdot (\delta_t - \delta_{t-1})) - \frac{1}{\delta_{t-1}} E(r) E(\delta_t - \delta_{t-1}) \end{aligned} \tag{18}$$

whose sample analog would follow

$$\frac{1}{\delta_{t-1}} \cdot N(\varpi, \rho \cdot \triangle t) \tag{19}$$

which implies higher volatility of the risk premium where $\varpi \in R$ and $\rho \in R^+$.

7 Econometric Tests

The test equation for C-CAPM starts with a Euler equation for the consumer's decision in section 11.2. Assuming a power utility function, the Euler equation is

$$E_t \left(\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \cdot (1 + r_{t+1}) \cdot \frac{1}{1 + \phi} \right) = 1. \tag{20}$$

If we take the log for linearization,

$$log(\beta) - \gamma E_t log \left(\frac{c_{t+1}}{c_t} \right) + E_t(r_{t+1}) - E_t(\phi) + \frac{1}{2} var(r_{t+1} - \phi) = 0. \tag{21}$$

Suppose $var(\frac{c_{t+1}-c_t}{c_t}) \simeq 0$, $var(r_{t+1}) = E(r_{t+1}^2) - E(r_{t+1})^2 \simeq E(r_{t+1})^2$, $var(\phi) = E(\phi^2) - E(\phi)^2 \simeq E(\phi)^2$ and $cov(r_{t+1}, \phi) = E(r_{t+1} \cdot \phi) - E(r_{t+1})E(\phi) \simeq E(r_{t+1})E(\phi)$. Then,

$$\begin{aligned} var(r_{t+1} - \phi) &= var(r_{t+1}) - 2cov(r_{t+1}, \phi) + var(\phi) \\ &\simeq E(r_{t+1})^2 - 2E(r_{t+1})E(\phi) + E(\phi)^2 \\ &= \frac{(E(r_{t+1}) - E(\phi))^2}{E(r_{t+1})^2} \cdot E(r_{t+1})^2 \\ &\simeq k^2 \cdot var(r_{t+1}) \end{aligned} \tag{22}$$

where $k = \frac{E_t(r_{t+1}) - E(\phi)}{E_t(r_{t+1})}$. The objective of the test is to test whether the expected risk factor ϕ has any role in asset pricing. We can rewrite the equation (22) as follows. Since

$$E_t(r_{t+1}) - E_t(\phi) = \frac{E_t(r_{t+1}) - E_t(\phi)}{E_t(r_{t+1})} \cdot E_t(r_{t+1}) = k \cdot E_t(r_{t+1}), \tag{23}$$

the equation (22) is

$$E_t(r_{t+1}) = -\frac{log(\beta)}{k} - \frac{k^2}{2} var(r_{t+1}) + \frac{\gamma}{k} E_t log\left(\frac{c_{t+1}}{c_t}\right). \tag{24}$$

If $k = 1$, then the expected risk ϕ has no role, i.e., it is insignificant in determining asset price. Therefore, the hypotheses for the test are

$$\begin{aligned} H_0 :&\ \ k = 1 \\ H_1 :&\ \ k \neq 1. \end{aligned} \tag{25}$$

Under the assumption of homoscedasticity in stock returns, equation (20) simplifies to

$$E_t(r_{t+1}) = b_0 + b_1 E_t log\left(\frac{c_{t+1}}{c_t}\right) \tag{26}$$

where

$$b_0 = -\frac{log\beta}{k} - \frac{k}{2}\sigma_r^2 \tag{27}$$

where $\sigma_r^2 = var(r_{t+1})$ and

$$b_1 = \frac{\gamma}{k}. \tag{28}$$

Note that the variance of b_1 is

$$var(b_1) = \sigma_u^2 \left(log\left(\frac{\hat{c_{t+1}}}{c_t}\right)' log\left(\frac{\hat{c_{t+1}}}{c_t}\right)\right)^{-1} \tag{29}$$

where $log\left(\frac{\hat{c_{t+1}}}{c_t}\right)$ is instrumented regressor. The t-value is, therefore,

$$t = \frac{\gamma}{k} \cdot \frac{1}{\sqrt{var(b_1)}}. \tag{30}$$

The two important results are,

$$\frac{\partial b_1}{\partial k} < 0,\ b_1 \to 0 \tag{31}$$

and,

$$\frac{\partial t}{\partial k} < 0,\ t \to 0. \tag{32}$$

This implies that to estimate the coefficient of risk aversion, i.e., the elasticity of intertemporal substitution, the noise parameter should be properly handled. Otherwise, the estimates could be seriously misleading in its interpretations. Since $-k = \frac{E_t(\phi)-E_t(r_{t+1})}{E_t(r_{t+1})}$ measures units of risk per return, if $k = 1$, there is no effect of the risk on the elasticity, while there is significant effect if $k < 1$. $-b_1 = -\frac{\gamma}{k}$ measures constant relative risk aversion per risk contained in a unit of return.

Table 1. Relative Risk Aversion per Risk

	OLS	2SLS
$(\hat{\frac{\gamma}{k}})$	2.605	1.970

8 Econometric Test Results

We test the hypotheses in (31) using the data of Campbell (2003) during the period of 1960.1 - 1999.4. The OLS estimation result is as follows:

$$\begin{aligned} \textit{stock return} &= 0.054 - 2.605 \cdot \textit{consumption growth rate} \\ &\quad\;\; (2.428) \quad (-1.039) \\ R^2 &= 0.14 \end{aligned} \tag{33}$$

where the numbers in the parenthesis are t-values. The t-value for the structural parameter indicates that $\frac{\gamma}{k}$ is zero when in fact it is non-zero, implying that the noise effect of manipulation is serious.

A limitation of OLS is that it is biased in the structural model because of the simultaneous relationship between stock returns and optimal consumption decisions. For this reason, we adopt instrumental variables estimation using two-stage least square (2SLS) estimation. We choose twice lagged stock returns and consumption as instrument variables. The reduced form equation estimation shows that the first-stage R^2 is 0.12 and F-value is 10.18. The result of 2SLS estimation is as follows:

$$\begin{aligned} \textit{stock return} &= 0.049 - 1.970 \cdot \textit{consumption growth rate} \\ &\quad\;\; (2.186) \quad (-0.786) \\ R^2 &= 0.14 \end{aligned} \tag{34}$$

which is almost equivalent to the OLS results even though the absolute magnitude of the structural parameter estimator is smaller than that of OLS.

Table 2. $\hat{k}$ and EIS

	OLS	2SLS
$\hat{k}$	-20	-11.85
$\hat{\gamma}$	52.1	23.34
t	20.78	9.314
EIS	0.38	0.51

The parameter of relative risk aversion, γ, is assumed as positive for the risk averters. Therefore, the negative value of the estimator implies that k should be negative. Recall that

$$k = \frac{E_t(r_{t+1}) - E(\phi)}{E_t(r_{t+1})}. \tag{35}$$

Thus, the negative value of k means that

$$E_t(r_{t+1}) - E(\phi) < 0. \tag{36}$$

This result is important since it shows that the expected growth rate of risk is a significant determinant of stock returns, which strongly predicts rejection of the null hypothesis in (26).

The estimated value of k using OLS is, assuming $log(\beta) \simeq 0$, i.e., $\beta \simeq 1$,

$$\hat{k}_{OLS} = -20 \tag{37}$$

and that of 2SLS is

$$\hat{k}_{IV} = -11.85. \tag{38}$$

These figures are significantly different from the value of 1, implying the rejection of the null hypothesis in (26). Therefore, using OLS and IV estimations, we have found that there is a strong possibility that the stock return for the given data is heavily dependent on investors' expectations on residual uncertainty.

Table 3. EIS in the literature

	EIS
Hall (1988)	[0, 0.2]
Mehra and Prescott (1985, 2003)	0.048
Lucas (1990)	0.50
Kydland and Prescott (1982)	0.66
Arrow (1971)	1

It should be noticed that since $\hat{k}_{IV} = -11.85$,

$$\hat{\gamma} = \hat{k} \cdot (-1.97) = 23.34 \tag{39}$$

which is equivalent to the estimated coefficient of relative risk aversion of 25 by Mehra and Prescott (1985), and

$$t = \hat{\gamma} \cdot \frac{1}{\sqrt{var(b_1)}} = (-11.85) \cdot (0.786) = 9.314. \tag{40}$$

This implies that the estimated coefficient by Mehra and Prescott (1985) is the total amount of risk aversion, while ours is the amount of risk aversion for a unit of risk. Therefore, the elasticity of intertemporal substitution is

$$EIS = 0.51 \tag{41}$$

which is less than 1 in Arrow (1969), but is greater than 0.2 in Hall (1988).

9 Conclusion

This paper has attempted to analyze the effect of disclosure manipulation on the stock market and elasticity of intertemporal substitution in consumption. Analyzing U.S. quarterly data of $1960.1 - 1999.4$, we found that the noise effect of manipulation is so serious, and the estimated noise-corrected elasticity is 0.51 the same as that of Lucas (1990) and similar to that of Kydland and Prescott

(1982), however, very different from the values of Mehra and Prescott (1985, 2003) and Hall (1988). Our result implies that economic behavior may be working as predicted in theory even though statistical significance is affected by the residual uncertainty. Another noticeable fact is that the estimated coefficient is negative, which is completely contrary to the theoretical prediction. This means that the channelling of financial policies and consumer's intertemporal decisions is reversed and thin.

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