Strategic futures trading in oligopoly

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October 15, 2001

Keywords: Commitment; Futures contracts; Observability; Speculation

JEL classification: D82; G14; L13

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1. Introduction

It is well known that investors participate in futures market for a couple of different reasons: to hedge risk and uncertainties, to gain arbitrage profit by taking advantage of futures and/or spot market mispricing or to earn trading profit from speculation. Allaz (1992) and Allaz and Vila (1993) find that risk neutral producers use futures contract strategically to improve their competitive positions on the oligopolistic spot market. In the context of two-stage model of futures contracting followed by production in oligopolistic spot market, they demonstrate that in the absence of any uncertainty, futures contracts are employed by risk neutral firms as a credible strategic device in an attempt to commit themselves to more aggressive output decision later at the production stage.[1] Hughes and Kao (1998) shows that observability of futures positions is crucial for futures contracts to function as an effective commitment device, and in case of unobservable futures contracts, risk aversion of firms or cost uncertainties can restore the strategic usefulness of futures contracts.

Allaz (1992), Allaz and Vila (1993) and Hughes and Kao (1998) all assume perfect competition of futures market, where futures price is always an unbiased estimator of spot price at which futures contract will be settled. This paper considers a futures market where strategic tradings are conducted by information-based traders including producing firms. Unlike the models of Allaz (1992), Allaz and Vila (1993) and Hughes and Kao (1998) in which firms decides their optimal futures position taking the futures price as given, we extend strategic trading models of Kyle (1985) and Admatti and Pfleiderer (1988), and analyze optimal futures trading strategy adopted by producing firms which take the effect of their futures trading on the transaction price of futures contract as well as the spot price at which futures contract will be settled.

[1] Allaz and Vila (1993) extend the basic two-stage model to repeated rounds of futures trading.

If producing firms are risk neutral and have perfect information about the product market conditions, risk hedging cannot be the valid incentive for their participation in futures market any more. There are two reasons that prompt producing firms to take futures contract. First, as shown in Allaz (1992), Allaz and Vila (1993) and Hughes and Kao (1998), producing firms can use futures trading strategically to improve their positions on the product market competition. Second, they participate in futures market as information-based traders to gain trading profit. What sets this paper apart from standard market microstructure framework is that producing firms not only have access to the private information on the spot price at which futures contract will be settled, but also they can directly affect it since their product market decision determines spot price of product market.[2]

Depending on whether futures positions taken by producing firm are observable or not, qualitatively different results emerge. If futures positions taken by producing firms are observable, their motivation of adopting futures trading as a credible device to commit themselves to more aggressive product market strategy outweighs the incentive to earn positive trading profit from futures trading. In fact, producing firms expect to lose money from futures trading although they possess utmost advantage against other participants of futures market. However, if futures transactions are not observable at all, effectiveness of futures trading as a commitment device in the product market competition disappears, and they trade in the futures market just to earn trading profit. Unlike Hughes and Kao (1998) in which firms are not engaged in any futures trading in case of unobservability, we show that producing firms still participate in the futures market to earn trading profit even when usefulness

[2]Kyle (1985), Admatti and Pfleiderer (1998) and other market microstructure models typically assume that orders submitted by traders only affect transaction price, not the fundamental payoff of the security.

of futures position as a commitment device in the product market disappears due to unobservability.

This paper also raises the possibility of non-existence of equilibrium in the futures market. This is mainly due to the producing firms' ability of affecting transaction price at which futures contracts are traded as well as the spot price at which futures positions are settled. As transaction price of futures contracts become less sensitive to the orders submitted by traders due to sufficiently large number of information-based traders including producing firms, substantial noise in the futures market or competition among market makers, producing firms might adopt extremely aggressively trading strategy in the futures market to fully take advantage of their position affecting both futures and spot prices.

Section 2 presents basic model in a two-stage imperfect information game. Benchmark case of producing firms not being engaged in the futures trading is shown in Section 3. Section 4 and 5 are the main parts of this paper in which the cases of observable and unobservable futures positions taken by producing firms are analyzed. Section 5 concludes this paper and all the proofs are given in the Appendix.

2. The model

Consider a homogeneous commodity which has a futures market and a spot market. There is one period with two dates, 0 and 1. The commodity is produced by k firms, indexed by $i = 1, \cdots, k$ at date 1, and is sold in the spot market according to the following inverse industry demand function:

$$p = a + \theta - \sum_{i=1}^{k} q_i$$

where p is the spot market price of the commodity, q_i is the output of firm i, a is a positive constant, and θ is an industry-wide demand shock normally distributed with mean zero and variance σ_θ^2. The k firms compete in quantities and produce at zero marginal costs.[3]

The futures market for the commodity opens at date 0 and trades infinitely divisible futures contracts, each of which calls for delivery of one unit of the commodity at date 1 at a pre-specified futures price, p^f. There are four types of risk-neutral agents populated in the futures market: the k producing firms, n informed traders (indexed by $j = 1, \cdots, n$), a group of noise traders who trade for exogenous reasons, and a competitive market maker. The trading mechanism is that of Kyle (1985) and Admati and Pfleiderer (1988) in which k firms, the n informed traders, and the group of noise traders submit market orders to the market maker who takes a position that balances supply and demand.

Prior to trading in the futures market at date 0, k firms and the n informed traders receive a private signal which perfectly reveals the date 1 demand parameter, θ,[4]and the price schedule set by the market maker becomes common knowledge. Based on this private signal and the price schedule, firm i and informed trader j submit market orders to purchase x_i and y_j futures contracts to the market maker, respectively. Noise traders trade for liquidity reasons. They, as a group, submit a market order to purchase u futures contracts to the market maker, where u is a normally distributed random variable independent of θ, with mean zero and variance σ_u^2. The aggregate order flow is thus given by

$$z = \sum_{i=1}^{k} x_i + \sum_{j=1}^{n} y_j + u.$$

[3]The results of this paper do not change as far as marginal cost is constant and positive, and the slope of linear demand function is negative.

[4]Making the private signal imperfectly reveal θ affects none of the qualitative results.

Each participant in the futures market observes only his or her own market order. The market maker sets the futures price, p^f, according to the pre-announced price schedule of $F(z)$ efficiently (in the semi-strong form) conditional on the aggregate order flow such that she earns zero expected date 1 profits for each realization of z. It is assumed that market is able to observe only the aggregate net trading order, not individual orders separately. The properties of random variables and the numbers of informed traders and producing firms are public knowledge.

The set-up is a two-stage imperfect information game. The first stage of the game occurs at date 0 in which the four types of agents trade in the futures market and the futures price is determined by the market maker. The second stage of the game occurs at date 1. We consider two different scenarios in the second stage: One in which the futures positions of the k firms are publicly revealed (the observable case), and the other in which the futures positions of the k firms remain private information (the unobservable case). In either case, the k firms compete in a Cournot fashion from which the spot market price of the commodity is determined. Futures contracts are then settled at this price level.

3. The observable case

In this section, we consider the observable case where the futures positions of k firms become publicly known at date 1 prior to the firms making their production decisions.

As a benchmark case, if the futures market for the commodity is not assesible by the firms so that $x_i = 0$, we have following product market equilibrium.

$$q_i = p = \frac{1}{k+1}(a + \theta).$$

In this benchmark case, we have following equilibrium in the futures market where each informed trader submits market order of $Y_j(\theta)$ and price schedule set by the market maker is $F(z)$.[5]

Lemma 1. *If firms are absent in the futures market, then there exists a unique symmetric, linear, rational expectations equilibrium in the first stage in which*

$$F(z) = \frac{a}{k+1} + \lambda z, \quad \textit{where} \quad \lambda = \frac{\sqrt{n}\sigma_\theta/\sigma_u}{(k+1)(n+1)}$$
$$\textit{and} \quad Y_j(\theta) = \frac{1}{\lambda(n+1)}\frac{\theta}{k+1} = \frac{\theta}{\sqrt{n}\sigma_\theta/\sigma_u} \tag{1}$$

and each informed trader's expected trading profit is

$$\frac{\sigma_\theta\sigma_u}{\sqrt{n}(k+1)(n+1)} \tag{2}$$

To solve for a rational expectations equilibrium in which firms are allowed to trade in futures market, we use backward induction. Suppose firm i has the futures position of x_i. $x_i > 0$ represents long position taken by producing firm i while $x_i < 0$ implies that it takes short position in the futures market. In the second stage at date 1, for a given set of futures positions, futures price, p^f, and realization of demand parameter, θ, firm i's decision problem is given by

$$\max_{q_i} \ (a + \theta - \sum_{h=1}^{k} q_h)q_i + (a + \theta - \sum_{h=1}^{k} q_h - p^f)x_i,$$

where the first term is profit from selling in the spot market, and the second term is the gain or loss due to the futures position in the futures market. From Allaz (1992), the Cournot-Nash equilibrium yields

$$q_i = \frac{a + \theta - kx_i + \sum_{h \neq i} x_h}{k+1}. \tag{3}$$

[5]We follow the literature in this area and solve for the unique linear equilibrium. This does not preclude the possibility of non-linear equilibrium.

Substituting equation (3) into the inverse industry demand function, we obtain the spot market price of the commodity:

$$p = \frac{a+\theta}{k+1} + \frac{\sum_{i=1}^{k} x_i}{k+1}. \tag{4}$$

In the first state at date 0, firm i, anticipating that the spot market equilibrium is given by equations (3) and (4) in the second stage, chooses a market order, x_i, so as to maximize its expected date 1 profits from the spot and futures markets:

$$\begin{aligned}\max_{x_i} \quad & \mathrm{E}\Big\{\frac{1}{(k+1)^2}(a+\theta+x_i+\sum_{h\neq i} x_h)(a+\theta-kx_i+\sum_{h\neq i} x_h)\\ + \quad & \Big[\frac{1}{(k+1)}(a+\theta+x_i+\sum_{h\neq i} x_h) - p^f\Big]x_i\Big|\theta, x_i\Big\}.\end{aligned}$$

The optimal trading strategy is a function, X_i, such that $x_i = X_i(\theta)$.

Similarly, informed trader j, anticipating that the spot market price at date 1 is given by equation (4) in the second stage, chooses a market order, y_k, so as to maximize his or her expected date 1 profits:

$$\max_{y_j} \ \mathrm{E}\Big\{\Big[\frac{a+\theta+\sum_{i=1}^{k} x_i}{k+1} - p^f\Big]y_j\Big|\theta, y_j\Big\}.$$

The optimal trading strategy is a function, Y_j, such that $y_j = Y_j(\theta)$.[6]

The market maker, anticipating that the spot market price at date 1 is given by equation (4) in the second stage, observes the aggregate order flow, $z = \sum_{i=1}^{k} x_i + \sum_{j=1}^{n} y_j + u$, but not x_i, y_j, or u separately. She sets a single futures price, p^f, at which she will execute z. Conditional on observing z, the market maker earns zero expected date 1 profits so that

$$p^f = \mathrm{E}\Big[\frac{a+\theta}{k+1} + \frac{\sum_{i=1}^{k} x_i}{k+1}\Big| z\Big].$$

[6]Both producing firms and informed traders determine their optimal trading strategy in the futures market given the price schedule set by the market maker being fully aware of the way in which their trading order submitted to the market maker would affect the futures price.

Her pricing rule is a function, F, such that $p^f = F(z)$.

As is commonly assumed, we consider only the set of symmetric, linear, rational expectations equilibria. That is, $X_i(\theta) = \alpha + \beta\theta$, $Y_j(\theta) = \gamma + \delta\theta$, and $F(z) = \mu + \lambda z$.

Proposition 1. *If* $\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} > n(k^2-k+1)-k+1$, *then there exists a unique symmetric, linear, rational expectations equilibrium in the first stage in which*

$$X_i(\theta) = -\frac{a(k+n+1)(k-1)}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} + k(k^2+1)} - \frac{nk-n-2}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}}\theta, \quad (5)$$

$$Y_k(\theta) = \frac{k^2+1}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}}\theta, \quad (6)$$

$$\begin{aligned} F(z) \quad &= \quad \frac{a(k^2+1)(\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} - k(nk-n-2)}{(k+1)^2(k^3+k+\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u})} \\ &+ \quad \frac{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} - k(nk-n-2)}{(k+1)^2(k+n+1)}z. \end{aligned} \quad (7)$$

Proof. See the appendix. □

Since producing firms are risk neutral and have perfect information on the spot price, they will not do any risk hedging. There are two reasons that lead producing firms to being engaged in futures trading. First, they take positions in futures market to earn trading profit from their private information on the spot price at which futures contracts will be settled in the following period. In the futures market, producing firms compete against informed traders who also have private access to the information on θ. What distinguishes producing firms from informed traders is that trading orders submitted by producing firms affect not only the transaction price of futures market, $F(z)$, but also the following period's spot price at which all of the futures contracts will be settled.[7]

Secondly, and more importantly, producing firms use futures transactions strategically to improve their positions on the spot market. In fact, the second reason for producing firms' futures transaction dominate the first. As we can see in Proposition 1, as demand condition in spot market improves i.e. θ increases, producing firms are more likely to take short positions which is exactly opposite to informed traders' trading strategy in the futures market. Firm i's expected trading profit is given in following equation.

$$
\begin{aligned}
& E[X_i(\theta)\{\frac{1}{k+1}(a+\theta+\sum_{i=1}^{k} X_i(\theta)) - F(z)\}] \\
= \quad & \frac{\sigma_u^2(n+2-nk)}{(k+1+n)(k+1)^2(nk+2k+n)}(\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} - k(nk-n-2))
\end{aligned}
$$

It is interesting to notice that although firms have perfect information on θ, it is very likely that firms trade in the futures market expecting to *lose money.* The condition for earning negative expected profit is $n+2-nk<0$, which holds if $n>2$ and $k>2$, and we can see that only when the number of participants in futures market is very limited, firms expect to earn positive trading profit from futures trading based on their perfect information on θ.

The analysis of product market equilibrium reveals that a producing firm's futures position is a credible commitment device allowing it to take more aggressive product market strategy. Equation (3) demonstrates the role of futures trading as a commitment device. Compared with the case of no futures transaction, short position taken by a firm would lead to higher level of output while reducing competing firms' level

[7]This feature of the model is very different from typical market microstructure models which assume that orders submitted by traders do not affect the fundamental payoff of the security. Please refer to Kyle (1985) and Admatti and Pfleiderer (1988) for details on market microstructure model with strategic informed traders.

of output. This is because short position of futures contract renders the producing firm's output decision less price sensitive by improving marginal revenue, and firms are engaged in Cournot competition.

Each informed trader is expect earn following trading profit.

$$\begin{aligned} & E[Y_i(\theta)\{\frac{1}{k+1}(a+\theta+\sum_{h=1}^{k}X_i(\theta))-F(z)\}] \\ = & \frac{\sigma_u^2(k^2+1)}{(k+1+n)(k+1)^2(nk+2k+n)}(\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}-k(nk-n-2)) \end{aligned}$$

and it is a decreasing function of n. Put differently, as the competition among informed trader increases, each informed trader expects to smaller trading profit.

Since the market maker sets the price schedule such that he expects to earn zero trading profit for each realization of futures price, the expected trading loss of liquidity traders is always equal to the combined trading profit earned by firms and informed traders, and it is given in following equation.

$$\lambda\sigma_u^2 = \frac{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}-k(nk-n-2)}{(k+1)^2(k+n+1)}\sigma_u^2$$

One important feature of this model is the possibility of non-existence of equilibrium in the futures market. The condition for the existence of equilibrium given in Proposition 1 is that the numbers of both informed traders and producing firms do not exceed a certain level.[8] Both informed traders and producing firms strategic traders in the futures market in that they decides optimal trading strategy being fully aware of the effect of their trading on the futures price. We can find from $X_i(\theta)$ and $Y_j(\theta)$ given in Proposition 1 that informed traders and producing firms

[8]The condition for the existence of equilibrium is $\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} > n(k^2-k+1)-k+1>0$. Given $\frac{\sigma_\theta}{\sigma_u}$, $\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}-n(k^2-k+1)+k-1$ decreases in n and k, and it is more likely that equilibrium in the futures market do not exist at all.

take opposition positions in the futures market. Suppose, for instance, the number of producing firms increases. Then, total size of position taken by becomes larger, which provides the informed traders with chance to increase their positions with less impact on the futures. Therefore, increases in the numbers of informed traders and producing firms have escalating effect of both groups of traders taking ever increasing futures positions, which would possibly lead to the collapse of futures market.[9]

Now we move on to the analysis of product market equilibrium. As firms are engaged in futures trading, we have following equilibrium price and quantity of product market.

$$
\begin{aligned}
p &= \frac{1}{k+1}(a+\theta+kX_i(\theta)) \\
&= \frac{1}{k+1}[a\{1-\frac{k(k+n+1)(k-1)}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}+k(k^2+1)}\} \\
&\quad +\theta\{1-\frac{k(nk-n-2)}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}}\}] \\
q_i &= \frac{1}{k+1}(a+\theta-X_i(\theta)) \\
&= \frac{1}{k+1}[a\{1+\frac{(k+n+1)(k-1)}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}+k(k^2+1)}\} \\
&\quad +\theta\{1+\frac{(nk-n-2)}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}}\}]
\end{aligned}
$$

Following corollary given without proof summarizes the properties of equilibrium price and quantity.

Corollary 1. *As firms are engaged in futures trading, expected price becomes lower but its variance decreases while expected level of output increases and its variance goes down.*

[9] Allaz (1992), Allaz and Vila (1993) and Hughes and Kao (1997) do not have this problem of non-existence of equilibrium since they assume competitive futures market with non-strategic traders.

If firms participate in the futures market, then the expected consumers' surplus is given by

$$\begin{aligned} CS &= \frac{1}{2}E[(a+\theta-p)(kq_i)] \\ &= \frac{1}{2}E[(\frac{k(a+\theta)}{k+1}-\frac{kX_i(\theta)}{k+1})(\frac{k(a+\theta)}{k+1}-\frac{kX_i(\theta)}{k+1})] \\ &= \frac{k^2}{2(k+1)^2}E[(a+\theta)^2]+\frac{k}{2(k+1)^2}\{kE[X_i(\theta)^2]-2kE[X_i(\theta)(a+\theta)]\} \\ &= CS^0+\frac{k}{2(k+1)^2}\{kE[X_i(\theta)^2]-2kE[X_i(\theta)(a+\theta)]\} \end{aligned}$$

where CS^0 is the expected consumer surplus if firms are not engaged in futures trading. From Proposition 1, we have

$$\mathrm{E}[X_i(\theta)^2]-2\mathrm{E}[X_i(\theta)(a+\theta)]>0.$$

and therefore consumers are better off as firms participate in the futures market.

As firms are engaged in the futures trading, the expected producers' surplus is given by

$$\begin{aligned} PS &= E[kpq_i] \\ &= E[(\frac{(a+\theta)}{k+1}+\frac{kX_i(\theta)}{k+1})(\frac{k(a+\theta)}{k+1}-\frac{kX_i(\theta)}{k+1})] \\ &= \frac{k^2}{(k+1)^2}E[(a+\theta)^2]+\frac{k}{(k+1)^2}\{-kE[X_i(\theta)^2]+(k-1)E[X_i(\theta)(a+\theta)]\} \\ &= PS^0+\frac{k}{(k+1)^2}\{-kE[X_i(\theta)^2]+(k-1)E[X_i(\theta)(a+\theta)]\} \end{aligned}$$

where PS^0 is the producers' surplus if firms do not have access to the futures market. From Proposition 1, we can see that producers are worse off.

The possibility of futures trading prior to production stage allows producing firms to adopt futures trading as commitment device of taking more aggressive output decision. This leads to the product market equilibrium in which more output is produced at a lower price. Welfare implication of permitting producing firms to trade in the futures market is clear: consumers are better off while producing firms are worse off ironically due to the enlarged set of strategies to be adopted in product market.

Now we move on to the issue of how informative the futures price is about the price of product market. Since $p = (a + \theta + kX_i(\theta))/(k+1)$ and θ is the random variable that firms and informed traders have information about, we have following two measures for the informativeness of futures price, which turn out to same.

Corollary 2.

$$\frac{Var(p|p^f)}{Var(p)} = \frac{Var(\theta|p^f)}{Var(\theta)} = 1 - \frac{nk+n+2k}{(k+1)(n+k+1)}$$

Proof. See the appendix. □

As we can see from this corollary, as the number of informed traders increases, the futures price becomes more informative in that variance of p and θ conditional on futures price goes down. This concurs with the standard result of market microstructure model with more private information owned informed traders reflected in the transaction price as the the number of informed traders increases and thereby they submit larger trading order to the market maker. This is not necessarily true for the case of increasing number of producing firms. Since producing firms futures trading not only affect the future price but also the spot price at which futures contract will be settled, increasing number of producing firms weakens the informativeness of futures price.[10]

[10]In the context of standard market microstructure model, greater number of producing firms essentially increases the variance of fundamental payoff of the futures contracts, and thereby informativeness of futures price deteriorates.

4. The unobservable case

In this section, we consider the unobservable case where the futures positions of firms remain private information at date 1 prior to the firms making their production decisions.

Given futures price, p^f, set of conjectured futures positions, and realization of demand parameter, θ, firm i conjectures that firm h's output level is given by

$$q_h = \frac{1}{k+1}(a + \theta - kx_h^c + \sum_{l\neq h} x_l^c). \tag{8}$$

from the analysis in the previos section. Based on this conjecture, firm i's optimal output is given by

$$\begin{aligned} q_i &= \frac{1}{2}(a + \theta - \sum_{h\neq i} q_h - x_i) \\ &= \frac{1}{k+1}\Big(a + \theta - \frac{k+1}{2}x_i + \sum_{h\neq i} x_h^c - \frac{k-1}{2}x_i^c\Big), \end{aligned} \tag{9}$$

where the second equality follows from equation (8). From firm i's point of view, the spot market price of the commodity is given by

$$p = \frac{1}{k+1}\Big(a + \theta + \frac{k+1}{2}x_i + \sum_{h\neq i} x_h^c - \frac{k-1}{2}x_i^c\Big), \tag{10}$$

where the equality follows from equations (8) and (9).

Proposition 2. *If $4\sqrt{n+k}\sigma_\theta/\sigma_u > (k-1)^2 + n(k+1)$, then there exists a unique symmetric, linear, rational expectations equilibrium in the first stage in which*

$$X_i(\theta) = Y_i(\theta) = \frac{\theta}{\sqrt{n+k}\sigma_\theta/\sigma_u}, \tag{11}$$

$$F(z) = \frac{a}{k+1} + \frac{(\sqrt{n+k}\sigma_\theta/\sigma_u + k)z}{(k+1)(n+k+1)}. \tag{12}$$

Proof. See the appendix. □

The results from the unobservable futures positions are qualitatively different from those from the case of observable futures contracts. In the unobservable case, we are essentially back to the Kyle (1985) and Admati and Pfleiderer (1988) framework with $n+k$ informed traders without anyone participating in product market. We can make two distinctions from the case of observable futures positions. Firstly, both informed traders and producing firms take exactly same futures trading strategy. Secondly, as demand condition in the product market improves, *i.e.* θ becomes larger, producing firms are more likely to take greater long positions in futures transaction.

When futures positions taken by producing firms are observable, futures transactions are adopted as credible commitment device of being engaged in more aggressive output decision: as a firm takes short position of futures contract observed by other firms credibly signals that it would produce higher level of output, which, in Cournot competition, leads other firms to scaling down their production level. When a producing firm decides its trading strategy in the futures market, it takes into account this strategic effect on other firms' output decision.

But if producing firms cannot observe other firm's futures positions, this strategic effect of futures contract on other firms disappear. In case of unobservable futures positions, firms simultaneously determine their output levels based on *conjectured levels of other firms' futures positions.* Even if a firm deviates from the futures position conjectured by other firms, it cannot affect other firms production decisions since it is not observable at all, and therefore strategic effect of futures position in the product market no longer exists at all.[11]

[11]If we denote the firm i's expected profit Π_i, then it is $E[\ (a+\theta-\sum_{h\neq 1}^{k} q_h - q_i(x_i))q_i(x_i) + (a+\theta-\sum_{h\neq 1}^{k} q_h - q_i(x_i) - p^f)x_i]$. Other firms' output is not affected by x_i in case of unobservable

As is shown in Proposition 2, as the numbers of informed traders and producing firms increase, it is more likely that there will be no equilibrium in the futures market. It is due to the producing firms' ability to affect through their futures trading not only the transaction price of futures market but also the spot price at which the futures contract will be settled. As more informed traders and producing firms compete in the futures market, futures price becomes less sensitive to aggregate order submitted by traders, which consequently leads producing firms take more aggressive trading strategy particularly due to their ability to affect spot price in the following period, and the breakdown of futures market is even possible.[12]

Expected trading profit earned by each informed trader and firm expects is given in the following equation.

$$\frac{(\sqrt{n+k}\sigma_\theta/\sigma_u + k)\sigma_u^2}{(k+1)(n+k+1)(n+k)}$$

Unlike the observable case, firms are expected to earn positive profit from futures trading.

In case of product market equilibrium, we have following equilibrium price and quantity.

$$\begin{aligned} p &= \frac{1}{k+1}\Big(a + \theta + kX_i(\theta)\Big) \\ &= \frac{1}{k+1}\Big(a + \theta(1 + \frac{k}{\sqrt{n+k}\sigma_\theta/\sigma_u}))\end{aligned}$$

futures contract, and therefore, only firm i's output is affected by x_i. Thanks to envelope theorem, $\frac{d\Pi_i}{dx_i} = \frac{\partial \Pi_i}{\partial q_i}\frac{\partial q_i}{\partial x_i} + \frac{\partial \Pi_i}{\partial x_i} = \frac{\partial \Pi_i}{\partial x_i}$. This is essentially equivalent to producing firms determining futures trading strategy without any consideration on its effect on product market output.

[12]Equilibrium λ set by the market maker equates the expected trading loss incurred by liquidity traders with combined expected trading profits earned by informed traders and producing firms. As more informed traders and producing firms are engaged in futures trading, their competition results in smaller total profit earned by them, and thereby lower equilibrium *lambda*. As the detailed analysis in Appendix shows, if *lambda* is sufficiently small, then producing firms is taking infinitely large positions since their orders substantially improves spot price with much less impact on the futures price.

$$
\begin{aligned}
q_i &= \frac{1}{k+1}\Big(a+\theta-X_i(\theta)\Big) \\
&= \frac{1}{k+1}\Big(a+\theta(1-\frac{1}{\sqrt{n+k}\sigma_\theta/\sigma_u}))
\end{aligned} \tag{13}
$$

The properties of product market equilibrium is summarized in the following corollary.

Corollary 3. *As firms are engaged in futures trading, expected price and level of output remain the same regardless of number of informed trdaers, but the variance of the price becomes higher while the variance of the output goes down.*

Similar to the analysis in previous section, we have following corollary on informativeness of futures price.

Corollary 4.

$$
\frac{Var(p|p^f)}{Var(p)} = \frac{Var(\theta|p^f)}{Var(\theta)} = 1 - \frac{n+k}{n+k+1}
$$

Proof. See the appendix. □

Contrary to the observable case, as the number of informed traders and firms increases, the futures price becomes more informative in that variance of p and θ conditional on futures price goes down since in the unobservable case, both informed traders and firms take same trading strategy in the futures market.

Now we move on the welfare analysis. In this case, consumers are worse off because the expected consumers' surplus is reduced as compared to the case without the futures market. To see this, note that

$$
\begin{aligned}
CS &= CS^0 + \frac{k^2}{2(k+1)^2}\{\mathrm{E}[X_i(\theta)^2] - 2\mathrm{E}[X_i(\theta)(a+\theta)]\} \\
&= CS^0 + \frac{k^2}{2(k+1)^2}\Big[\frac{1}{\sqrt{n+k}\sigma_\theta/\sigma_u}(\frac{1}{\sqrt{n+k}\sigma_\theta/\sigma_u} - 2)\Big]\sigma_\theta^2.
\end{aligned}
$$

From the sufficient condition for the existence of a rational expectations equilibrium, we can see that as far as equilibrium in the futures market exists, consumers are worse off as producing firms are engaged in futures trading but their futures positions are not observable . Thus, we have $CS < CS^0$. Similarly, we can see that producers are better off as they are able to trade in futures market.

5. Conclusion

In this paper, we have analyzed incentives for firms to be engaged in futures trading in the context of oligopolistic product market and strategic trading in the futures market. If producing firms are risk neutral and have perfect information about the product market conditions, risk hedging cannot be the incentive for their participation in futures market any more. There are two reasons that prompt producing firms to take futures contract. First, producing firms can use futures trading strategically to improve their positions on the product market competition. Second, they participate in futures market as information-based traders to gain trading profit. As information-based traders, producing firms not ony have access to the private information on the spot price at which futures contract will be settled, but also they can directly affect it since their product market decision determines spot price of product market.

Depending on whether futures positions taken by producing firm are observable or not, we have qualitatively different results. If futures positions taken by producing firms are observable as they make product market decisions, their motivation of adopting futures trading as a credible commitment device to take more aggressive product market strategy outweighs the incentive to earn trading profit from futures trading as information-based trading. In fact, producing firms expect to lose money from futures trading although they possess utmost advantage against other participants of futures market. However, if futures transactions of producing firms are not observable at all, effectiveness of futures trading as a commitment device in the product market competition disappears, and they trade in the futures market just to earn trading profit.

We can draw a couple of interesting policy implications from this paper. First, as firms are allowed to be trading in the futures market prior to making product market decision, disclosure policy on their futures positions result in vastly different welfare effect. If firms are required to publicly disclose their futures contract, thereby every participant in the market are able to observe them, then as the analysis of this papers shows, consumers are better off while producing firms are worse off. But no disclosure requirement on the firms' futures contract is advantageous to producing firms but disadvantageous to consumers.[13] Second issue is on the stability of futures market. Analysis in this paper demonstrate that if futures price is set by risk neutral market maker who earns zero expected profit, then future market equilibrium may not exist at all with large number of information-based traders. This is due to the presence of producing firms in the futures market who can affect both futures price and the spot price at which futures contract will be settled. The assumption of risk neutral market maker earning zero expected profit is valid if sufficiently large number of market makers are competing to facilitate the futures trading.[14] In this case, restricting the competition among market makers are likely to ensure the existence of futures market equilibrium even in the presence of large number of information-based traders.[15]

[13] United States Statements of Financial Accounting Standards (SFAS) and International Accounting Standards Committee (IASC) do not have specific clauses on the disclosure of futures positions. For more details, please refer to Hughes and Kao (1998).

[14] Low entry barrier into market making of futures market is enough to squeeze incumbent market makers' profit to a very low level.

[15] In the context of model presented in this paper, sufficiently large λ prevents producing firms from taking infinite futures position and thereby breaking down market equilibrium . If the market

This paper assumes that all the market participants including producing firms are risk neutral. Extending the current model to the case of risk aversion of producing firms can reveal different motivations for futures trading including risk hedging, which is left for further research.

maker sets the price schedule with greater λ than the ones given in Propositions 1 and 2, then she is able to earn positive expected profit, and expected loss incurred by liquidity traders will increase although market equilibrium might be restored. One solution for the welfare loss of liquidity traders is to charge fixed entry fee to market makers to make up increased trading loss of liquidity traders.

Appendix

Proof of Lemma 1. Suppose the market maker uses a linear pricing rule of $\mu + \lambda z$, and that informed trader j conjectures that each of the other informed traders will submit a market order of $\gamma^c + \delta^c\theta$. The expected date 1 profits of informed trader j are given by

$$\left\{\frac{1}{k+1}[a+\theta] - \mu - \lambda[y_j + (n-1)(\gamma^c + \delta^c\theta)]\right\} y_j.$$

Solving the first-order condition yields

$$y_j = \frac{1}{2(k+1)\lambda}[a + \theta - (k+1)\mu - (k+1)(n-1)\lambda(\gamma^c + \delta^c\theta)].$$

The second-order condition requires that $\lambda > 0$. Let $y_k = \gamma + \delta\theta$. We can solve for the symmetric Nash equilibrium by setting $\gamma^c = \gamma$ and $\delta^c = \delta$:

$$\gamma = \frac{a}{(k+1)(n+1)\lambda} - \frac{\mu}{(n+1)\lambda}, \quad \delta = \frac{1}{(k+1)(n+1)\lambda}$$

By the project theorem, we have

$$E[\frac{a+\theta}{k+1}|z = n(\gamma+\delta\theta)+u] = \frac{a}{k+1} + \frac{n\delta\sigma_\theta^2}{(k+1)(n^2\delta^2\sigma_\theta^2 + \sigma_u^2)}(z - n\gamma),$$

from which we obtain

$$\mu = \frac{a}{k+1} - \frac{n\gamma\lambda}{k+1}, \quad \lambda = \frac{n\delta\sigma_\theta^2}{(k+1)(n^2\delta^2\sigma_\theta^2 + \sigma_u^2)}.$$

By solving the simulataneous equatios. we derive

$$\mu = \frac{a}{(k+1)}, \quad \lambda = \frac{\sqrt{n}\sigma_\theta/\sigma_u}{(k+1)(n+1)}.$$

Each informed trader expects to earn

$$\Pi^{ib} \equiv \frac{\sigma_\theta\sigma_u}{(k+1)(n+1)\sqrt{n}},$$

and since the market maker sets the price such that he expects to earn zero profit for each realization of price, liquidity traders' expected loss is equal to

$$L^{ib} \equiv \lambda\sigma_u^2 = \frac{\sqrt{n}\sigma_\theta\sigma_u}{(k+1)(n+1)}$$

and the result follows. $\square$

Proof of Proposition 1.

Suppose that firm i conjectures that firm j will submit a market order of $\alpha^c + \beta^c\theta$, that each informed trader will submit a market order of $\gamma + \delta\theta$, and that the market maker uses a linear pricing rule of $\mu + \lambda z$. The expected date 1 profits of firm i are given by

$$\frac{1}{(k+1)^2}[a+\theta+x_i+(k-1)(\alpha^c+\beta^c\theta)][a+\theta-kx_i+(k-1)(\alpha^c+\beta^c\theta)]$$

$$+\left\{\frac{1}{k+1}[a+\theta+x_i+(k-1)(\alpha^c+\beta^c\theta)]-\mu-\lambda[x_i+(k-1)(\alpha^c+\beta^c\theta)+n(\gamma+\delta\theta)]\right\}x_i.$$

Solving the first-order condition yields

$$x_i = \frac{2(a+\theta)+(k-1)(2-\lambda(k+1)^2)(\alpha^c+\beta^c\theta)-\mu(k+1)^2-\lambda n(k+1)^2(\gamma+\delta\theta)}{2(\lambda(k+1)^2-1)}.$$

The second-order condition requires that $\lambda > 1/(k+1)^2$. Let $x_i = \alpha + \beta\theta$. We can solve for the symmetric Nash equilibrium by setting $\alpha^c = \alpha$ and $\beta^c = \beta$:

$$\alpha = \frac{2a-\mu(k+1)^2-\lambda n\gamma(k+1)^2}{(k+1)^3\lambda-2k}, \qquad \beta = \frac{2-\lambda n\delta(k+1)^2}{(k+1)^3\lambda-2k}. \tag{A.1}$$

Suppose that informed trader j conjectures that each of the other informed traders will submit a market order of $\gamma^c + \delta^c\theta$, that each of the firms will submit a market order of $\alpha + \beta\theta$, and that the market maker uses a linear pricing rule of $\mu + \lambda z$. The expected date 1 profits of informed trader j are given by

$$\left\{\frac{1}{k+1}[a+\theta+k(\alpha+\beta\theta)]-\mu-\lambda[k(\alpha+\beta\theta)+y_j+(n-1)(\gamma^c+\delta^c\theta)]\right\}y_j.$$

Solving the first-order condition yields

$$y_j = \frac{a+\theta-\mu(k+1)+k(\alpha+\beta\theta)(1-(k+1)\lambda)-\lambda(\gamma^c+\delta^c\theta)(n-1)(k+1)}{2(k+1)\lambda}.$$

The second-order condition requires that $\lambda > 0$. Let $y_k = \gamma + \delta\theta$. We can solve for the symmetric Nash equilibrium by setting $\gamma^c = \gamma$ and $\delta^c = \delta$:

$$\gamma = \frac{a-\mu(k+1)+k\alpha(1-(k+1)\lambda)}{(k+1)\lambda(n+1)}, \qquad \delta = \frac{1+k\beta(1-(k+1)\lambda)}{(k+1)\lambda(n+1)}. \tag{A.2}$$

Suppose that the market maker conjectures that each of the two firms will submit a market order of $\alpha+\beta\theta$, and that each informed trader will submit a market order of $\gamma+\delta\theta$. The aggregate order flow, $z = k(\alpha+\beta\theta)+n(\gamma+\delta\theta)+u$, is normally distributed. By the projection theorem, we have

$$\begin{aligned} F(z) &= \mathrm{E}\left\{\frac{1}{k+1}[a+\theta+k(\alpha+\beta\theta)]\,\middle|\, z = k(\alpha+\beta\theta)+n(\gamma+\delta\theta)+u\right\} \\ &= \frac{1}{k+1}(a+k\alpha)+\frac{(1+k\beta)(k\beta+n\delta)\sigma_\theta^2}{(k+1)[(k\beta+n\delta)^2\sigma_\theta^2+\sigma_u^2]}(z-k\alpha-n\gamma). \end{aligned}$$

Let $p^f = \mu + \lambda z$. Thus, we have

$$\mu = \frac{1}{k+1}[a + k(1-(k+1)\lambda)\alpha - (k+1)n\lambda\gamma], \quad \lambda = \frac{(1+k\beta)(k\beta+n\delta)\sigma_\theta^2}{(k+1)[(k\beta+n\delta)^2\sigma_\theta^2+\sigma_u^2]}. \quad \text{(A.3)}$$

If we can solve for a unique solution of equations (A.1), (A.2), and (A.3), we are done.

Substituting δ in equation (A.2) into β in equation (A.1) and rearranging terms yields

$$\beta = -\frac{nk-n-2}{(k+1)^2(n+k+1)\lambda + k(nk-n-2)}. \quad \text{(A.4)}$$

Substituting equation (A.4) into δ in equation (A.2) and rearranging terms yields

$$\delta = \frac{k^2+1}{(k+1)^2(n+k+1)\lambda + k(nk-n-2)}. \quad \text{(A.5)}$$

Substituting equations (A.4) and (A.5) into λ in equation (A.3) and rearranging terms yields

$$\lambda = \frac{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} - k(nk-n-2)}{(k+1+n)(k+1)^2}. \quad \text{(A.6)}$$

Since λ is required to exceed $1/(k+1)^2$, we need

$$\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} > n(k^2-k+1) - k + 1$$

Substituting equation (A.6) into equations (A.4) and (A.5) yields

$$\beta = -\frac{nk-n-2}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}}, \quad \delta = \frac{k^2+1}{\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}}.$$

Substituting μ in equation (A.3) into γ in equation (A.2) yields

$$\gamma = \frac{n}{n+1}\gamma,$$

which implies that $\gamma = 0$. Substituting $\gamma = 0$ into α in equation (A.1) and μ in equation (A.3) yields

$$\alpha = \frac{1}{(k+1)^3\lambda - 2k}(2a - (k+1)^2\mu), \quad \mu = \frac{1}{k+1}[a + k(1-(k+1)\lambda)\alpha].$$

Solving the above equation for α and μ and using equation (A.6) yields

$$\begin{aligned}
\alpha &= -\frac{a(k+n+1)(k-1)}{k(k^2+1) + \sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u}}, \\
\mu &= \frac{a(k^2+1)(\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u} - nk^2 + nk + 2k)}{(k^2+2k+1)(k^3+k+\sqrt{(k^2+1)(nk+2k+n)}\frac{\sigma_\theta}{\sigma_u})}.
\end{aligned}$$

This completes our proof. □

Proof of Corollary 2. From the conditional variance of normally distributed random variables, we have

$$Var(p|p^f) = Var(p)(1 - \frac{Cov(p, p^f)^2}{Var(p)Var(p^f)}).$$

From the proof of Proposition 1, we have

$$\lambda = \frac{Cov(p, k\beta\theta + n\delta\theta + u)}{Var(k\beta\theta + n\delta\theta + u)}$$

and following equalities can be obtained from equations (A.4) and (A.5).

$$\begin{aligned}
\frac{Cov(p,p^f)^2}{Var(p)Var(p^f)} &= \frac{Cov(\frac{1}{k+1}(\theta + k\beta\theta), k\beta\theta + n\delta\theta + u)^2}{Var(\frac{1}{k+1}(\theta + k\beta\theta))Var(k\beta\theta + n\delta\theta + u)} \\
&= \lambda \frac{Cov(\frac{1}{k+1}(\theta + k\beta\theta), k\beta\theta + n\delta\theta + u)}{Var(\frac{1}{k+1}(\theta + k\beta\theta))} \\
&= \lambda \frac{Cov(\frac{\lambda(k+1)(n+k+1)}{(k+1)^2(n+k+1)\lambda + k(nk-n-2)}\theta, \frac{(nk+2k+n)}{(k+1)^2(n+k+1)\lambda + k(nk-n-2)}\theta + u)}{Var(\frac{\lambda(k+1)(n+k+1)}{(k+1)^2(n+k+1)\lambda + k(nk-n-2)}\theta)} \\
&= \frac{nk+n+2k}{(k+1)(n+k+1)}.
\end{aligned}$$

We follow the similar steps to show that $\frac{Cov(p,p^f)^2}{Var(p)Var(p^f)} = \frac{Cov(\theta,p^f)^2}{Var(\theta)Var(p^f)}$ which completes the proof. □

Proof of Proposition 2. Suppose each informed trader will submit a market order of $\gamma + \delta\theta$, and that the market maker uses a linear pricing rule of $\mu + \lambda z$. The expected date 1 profits of firm i are given by in symmetric rational expectation equilibrium.

$$\begin{aligned}
&\frac{1}{(k+1)^2}\left[a + \theta + \frac{k+1}{2}x_i + \frac{k-1}{2}(\alpha^c + \beta^c\theta)\right]\left[a + \theta - \frac{k+1}{2}x_i + \frac{k-1}{2}(\alpha^c + \beta^c\theta)\right] \\
+ &\left\{\frac{1}{k+1}\left[a + \theta + \frac{k+1}{2}x_i + \frac{k-1}{2}(\alpha^c + \beta^c\theta)\right] - \mu - \lambda[x_i + (k-1)(\alpha^c + \beta^c\theta) + n(\gamma + \delta\theta)]\right\}x_i.
\end{aligned}$$

Solving the first-order condition yields

$$\begin{aligned}
x_i = \frac{1}{(k+1)(4\lambda - 1)} &[\quad 2(a+\theta) + (k-1)(1 - 2(k+1)\lambda)(\alpha^c + \beta^c\theta) - 2(k+1)\mu \\
&- \quad 2(k+1)n\lambda(\gamma + \delta\theta)].
\end{aligned}$$

The second-order condition requires that $\lambda > 1/4$. Let $x_i = \alpha + \beta\theta$. We can solve for the symmetric Nash equilibrium by setting $\alpha^c = \alpha$ and $\beta^c = \beta$:

$$\alpha = \frac{1}{(k+1)^2\lambda - k}(a - (k+1)\mu - (k+1)n\lambda\gamma), \quad \beta = \frac{1}{k+1)^2\lambda - k}(1 - (k+1)n\lambda\delta). \quad \text{(A.7)}$$

Since the unobservable case is observationally equivalent to the observable case for the n informed traders and the market maker, we have γ and δ given in equation (A.2) and μ and λ given in equation (A.3). If we can solve for a unique solution of equations (A.7), (A.2), and (A.3), we have following solutions.

Substituting δ in equation (A.2) into β in equation (A.7) and rearranging terms yields

$$\beta = \frac{1}{(k+1)(n+k+1)\lambda - k}. \tag{A.8}$$

Substituting equation (A.8) into δ in equation (A.2) and rearranging terms yields

$$\delta = \frac{1}{(k+1)(n+k+1)\lambda - k} = \beta. \tag{A.9}$$

Substituting equations (A.8) and (A.9) into λ in equation (A.3) and rearranging terms yields

$$\lambda = \frac{\sqrt{n+k}\sigma_\theta/\sigma_u + k}{(k+1)(n+k+1)}. \tag{A.10}$$

Since λ is required to exceed 1/4, we need $4\sqrt{n+k}\sigma_\theta/\sigma_u > (k-1)^2 + n(k+1)$. Substituting equation (A.10) into equations (A.8) and (A.9) yields

$$\beta = \delta = \frac{1}{\sqrt{n+k}\sigma_\theta/\sigma_u}.$$

Substituting $\gamma = 0$ into α in equation (A.7) and μ in equation (A.3), and solving the above equation for α and μ yields

$$\alpha = 0, \quad \mu = \frac{a}{k+1}.$$

This completes our proof. $\square$

Proof of Corollary 4.

By following the similar steps as in the proof of Corollary 2, and from equations (A.8) and (A.9), we derive the result. $\square$

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