Optimal Implementable Monetary Policy in a DSGE Model with a Financial Sector

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Abstract: We build a simple New Keynesian DSGE model with a financial sector and examine if the Central Bank(henceforth CB)'s responding to shocks originating in the housing and/or financial markets improves social welfare. In order to do the analysis, we consider a Taylor-type rule and do simulations to find the coefficients of the rule that maximizes social welfare, that is, the weighted average of the welfares of the patient and impatient households. Based on the simulations, we find that (i) the CB's response to shocks originating in the housing market improves welfare by a (very) small amount, while (ii) the CB's response to shocks originating in the financial market improves social welfare significantly; more specifically, social welfare increases by a factor of about four when the shocks originate in the lending rate and by more than 5 percent when the shocks originate in the deposit rate.

Key Words: DSGE model, Taylor-type rule, optimal policy, financial sector

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1 Introduction

The financial market crisis in 2008 provoked, to say the least, some skepticism against main stream macroeconomics. Main stream macroeconomics had not paid much attention to various aspects of the financial and/or banking sectors. Reacting to the neglect of the financial market, many researchers are now trying to introduce the various aspects of the financial and/or banking sectors into a standard DSGE model and examine its implications for economic fluctuations and/or policy issues.

This paper sets up a simple New Keynesian DSGE model with a financial sector and examines if the Central Bank (henceforth CB)'s responses to asset and/or financial market movements improve social welfare. We extend Iacoviello (2005) to incorporate a financial market along the lines of Gerali et al. (2010, henceforth GNSS), Aslam and Santoro (2009), and Totzek (2009), which emphasize the supply side of the credit markets. For policy analysis, we examine optimal implementable rules along the lines of Schmitt-Grohe and Uribe (2004) rather than the Ramsey optimal policy rule. Monacelli (2007) examines the Ramsey policy in a DSGE model similar to that of Iacoviello (2005). However, the Ramsey rule does not deliver an operational rule linking the policy instrument to endogenous variables. For practical purposes, we thus instead examine Taylor-type implementable rules.

This paper focuses in spirit on the same issue as Curdia and Woodford (2010). However, there are several differences. Firstly, we pay more attention to the banking sector, rather than the financial intermediaries' market-based intermediation. This is because not many countries other than the most advanced ones such as the U.S. and the U.K have well developed financial intermediaries, and the roles of their banking sectors are still relatively more important in most countries. Secondly, we consider the changes in spreads due to the shocks both to the lending and to the deposit rate and see how big the welfare gain is in the case of responding to the spreads relative to the case where there is no response. Finally, we also examine what happens to social welfare if the CB responds to shocks originating in the housing market.

We find that the CB's responses to shocks originating in the housing market improves social welfare by a (very) small amount, whereas its response to shocks originating in the financial market improves social welfare dramatically. In the latter case, the CB's responses to fluctuations in spread due to shocks to the lending rate improve social welfare by a factor of about four while its responses to fluctuations in spread due to shocks to the deposit rate improves social welfare by more than 5 percent.

We set up a simple New Keynesian DSGE model with a financial sector in Section 2. Section 3 contains some discussions about the parameter calibration and the model characteristics. We do simulations based on a grid search to find the optimal implementable Taylor rule and report the results in Section 4. Section 5 then briefly concludes.

2 Model

The model economy consists of heterogeneous households: that is, patient and impatient households. It also consists of firms, the financial sector, and the monetary authority.

2.1 Households

2.1.1 Patient Households

A representative patient household chooses consumption, housing services, money holding, leisure and deposits to maximize its lifetime objectives given by [1], [2]

$$\max\ E_0 \sum_{t=0}^{\infty} (\beta^P)^t \left[\ln(C_t^P) + z_t^h \ln h_t^P + z^M \ln\left(\frac{M_t^P}{P_t}\right) - z^N \frac{(N_t^P)^{1+\sigma_N}}{1+\sigma_N} \right] \tag{1}$$

subject to the intertemporal budget constraint

$$C_t^P + q_t^h (h_t^P - h_{t-1}^P) + m_t^P + D_t^P \leq \tag{2}$$
$$w_t^P N_t^P + \frac{m_{t-1}^P}{\pi_t} + \frac{\delta D_{t-1}^P}{\pi_t} + \frac{(1-\delta) R_{t-1}^D D_{t-1}^P}{\pi_t} + \psi_t^G + \frac{\psi_{t-1}^B}{\pi_t} + T_t^P$$

where β^P and σ_N represent the discount factor of a representative patient household and the inverse of the elasticity of work effort with respect to the real wage, respectively. Here C_t is consumption, h_t housing services[3], M_t nominal money, m_t real money, N_t work hours, D_t one-period deposits in real terms, R_t^D the one-period gross nominal deposit rate, P_t the price level, π_t the inflation rate, w_t the real wage, q_t^h the real housing price, ψ_t^G and ψ_t^B profits remitted to patient households by the goods and the banking sectors, respectively, T_t the real lump-sum transfer from the government at time t, and z^M and z^N the parameters. The lump-sum transfer from the government is

1) We do not use an index for an individual household, since all households solve the identical optimization problem facing the same aggregate variables.

2) Here and in the following subsection, the superscripts P and I denote the patient and the impatient households, respectively.

3) We assume that the housing services are proportional to the housing stock.

given by $T_t^P = \dfrac{(M_t^P - M_{t-1}^P)}{P_t} = m_t^P - \dfrac{m_{t-1}^P}{\pi_t}$. The parameter δ represents the known probability of early deposit withdrawal and will be explained in more detail later. The housing service preference shock z_t^h follows an AR(1) process given by

$$\log(z_t^h) = (1-\rho_{z^h})\log\bar{z}^h + \rho_{z^h}\log(z_{t-1}^h) + \epsilon_{z^h,t}, \quad -1 < \rho_{z^h} < 1, \tag{3}$$

where $E(\epsilon_{z^h,t}) = 0$, and $\epsilon_{z^h,t}$ is i.i.d. over time.

The first order necessary conditions for the problem are

$$muc_t^P = \frac{1}{C_t^P} \quad , \quad muh_t^p = \frac{z_t^h}{h_t^P} \tag{4}$$

$$muc_t^P = \beta^P E_t\left(\frac{muc_{t+1}^P\left(\delta + (1-\delta)R_t^D\right)}{\pi_{t+1}}\right) \tag{5}$$

$$z^N (N_t^P)^{\sigma_N} = muc_t^P w_t^P \tag{6}$$

$$muc_t^P q_t^h = \frac{z_t^h}{h_t^P} + \beta^P E_t(muc_{t+1}^P q_{t+1}^h) \tag{7}$$

$$\frac{z^M}{m_t^P} + \beta^P E_t\left[\frac{muc_{t+1}^P}{\pi_{t+1}}\right] = muc_t^P, \tag{8}$$

where muc_t and muh_t denote the marginal utilities from consumption and housing services, respectively. Equation (5) is the modified Euler equation derived from the first order conditions for consumption and deposit holdings. If $\delta = 0$, it reduces to the standard Euler equation. Equation (6) relates the marginal utility of labor hours to the real wage rate. Equation (7) states that the marginal utility of current consumption equals the sum of the direct marginal utility gain from housing services and the expected utility from future consumption due to the capital gains on housing purchased during this period. Equation (8) says that the marginal rate of substitution between real money balances and consumption equals the opportunity cost of holding money.

2.1.2 Impatient Households

A representative impatient household chooses consumption, housing services, money holding, leisure and borrowings to maximize its lifetime objectives given by

$$\max\ E_0 \sum_{t=0}^{\infty} (\beta^I)^t \left[\ln(C_t^I) + z_t^h \ln h_t^I + z^M \ln\left(\frac{M_t^I}{P_t}\right) - z^N \frac{(N_t^I)^{1+\sigma_N}}{1+\sigma_N}\right] \tag{9}$$

subject to

$$c_t^I + q_t^h (h_t^I - h_{t-1}^I) + \frac{R_{t-1}^L b_{t-1}^I}{\pi_t} + m_t^I \leq w_t^I N_t^I + b_t^I + \frac{m_{t-1}^I}{\pi_t} + T_t^I, \tag{10}$$

where β^I represents the discount factor of a representative impatient household, b_t^I the one-period borrowing in real terms, and R_t^L the gross borrowing (lending) rate to the impatient household. The lump-sum transfer from the government is given by $T_t^I = \frac{(M_t^I - M_{t-1}^I)}{P_t} = m_t^I - \frac{m_{t-1}^I}{\pi_t}$.

The impatient household faces the borrowing constraint

$$R_t^L b_t^I \leq \chi_t^I E_t (q_{t+1}^h h_t^I \pi_{t+1}), \tag{11}$$

where χ_t^I denotes the loan-to-value (LTV) ratio and follows an AR(1) process given by

$$\log(\chi_t^I) = (1 - \rho_{\chi^I}) \log \overline{\chi}^I + \rho_{\chi^I} \log(\chi_{t-1}^I) + \epsilon_{\chi^I, t}, \quad -1 < \rho_{\chi^I} < 1, \tag{12}$$

where $E(\epsilon_{\chi^I,t}) = 0$, and $\epsilon_{\chi^I,t}$ is i.i.d. over time.

The first order necessary conditions for the problem are

$$muc_t^I = \frac{1}{C_t^I} \quad , \quad muh_t^I = \frac{z_t^h}{h_t^I} \tag{13}$$

$$muc_t^I = \mu_t^I R_t^L + \beta^I E_t \left(\frac{muc_{t+1}^I R_t^L}{\pi_{t+1}} \right) \tag{14}$$

$$z^N (N_t^I)^{\sigma_N} = muc_t^I w_t^I \tag{15}$$

$$muc_t^I q_t^h = \frac{z_t^h}{h_t^I} + \beta^I E_t (muc_{t+1}^I q_{t+1}^h) + \mu_t^I \chi_t^I E_t q_{t+1}^h \pi_{t+1} \tag{16}$$

$$\frac{z^M}{m_t^I} + \beta^I E_t \left[\frac{muc_{t+1}^I}{\pi_{t+1}} \right] = muc_t^I, \tag{17}$$

where μ_t^I denotes the Lagrange multiplier associated with the borrowing constraint. Note that equation (16) has an extra term (the third term on the right hand side of the equation) compared with the corresponding equation (7) for the patient households. This reflects the marginal value to impatient households of relaxing the borrowing constraint.

2.2 Goods Sector

The goods sector is composed of a continuum of monopolistically competitive intermediate firms and a competitive final good sector.

2.2.1 Final Good Producers

The aggregate final good is assembled by perfectly competitive firms. The final good producers purchase differentiated intermediate goods and bundle them into a single final good, which they sell in a perfectly competitive market. They assemble a continuum of intermediate goods via

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon^P - 1}{\epsilon^P}} di \right]^{\frac{\epsilon^P}{\epsilon^P - 1}}, \tag{18}$$

where ϵ^P denotes the elasticity of substitution among differentiated goods. Intratemporal cost minimization for achieving Y_t yields the demand for each intermediate good

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon^P} Y_t, \tag{19}$$

and the aggregate price level is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon^P} di \right)^{\frac{1}{1-\epsilon^P}}. \tag{20}$$

2.2.2 Intermediate Goods Production

We suppose that there is a continuum of firms producing differentiated goods, and that each firm indexed by i, $0 \le i \le 1$, produces its product with a constant return to scale technology:

$$Y_t(i) = A_t (N_t^P(i))^{\gamma} (N_t^I(i))^{1-\gamma}, \tag{21}$$

where A_t is the technology process at period t and follows an AR(1) process

$$\log(A_t) = (1-\rho_A)\log \overline{A} + \rho_A \log(A_{t-1}) + \epsilon_{A,t}, \quad -1 < \rho_A < 1. \tag{22}$$

$N_t^P(i)$ and $N_t^I(i)$ are the firm-specific demands for patient and impatient households′ labor, respectively.

2.2.3 Pricing of Intermediate Goods

We consider a canonical Rotemberg (1982)-type sticky price model. Each monopolistic competitive firm faces quadratic adjustment costs, $\frac{\kappa_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 Y_t$, in changing the price of its product, and the parameter κ_P measures the degree of nominal price rigidity. Each intermediate good producer chooses its optimal price for maximizing profits (accruing to patient households) in nominal terms given by

$$\max_{P_t(i), N_t^P(i), N_t^I(i)} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[P_t(i) Y_t(i) - w_t^P N_t^P(i) - w_t^I N_t^I(i) - \frac{\kappa_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 P_t Y_t \right], \tag{23}$$

where $\Lambda_{0,t}^P = (\beta^P)^t \frac{P_t \, muc_t^P}{P_0 \, muc_0^P}$ denotes the nominal stochastic discount factor, and the terms in squared brackets represent the nominal profits which will be transferred to patient households in the form of dividends.

Maximization is subject to (19) and (21), taking the real wages for the patient and impatient households, w_t^P and w_t^I, as given in perfectly competitive factor markets. The first order conditions for profit maximization yield

$$w_t^P = mc_t A_t \gamma (N_t^P)^{\gamma - 1} (N_t^I)^{1-\gamma} = mc_t \gamma \frac{Y_t}{N_t^P} \tag{24}$$

$$w_t^I = mc_t A_t (1-\gamma)(N_t^P)^{\gamma} (N_t^I)^{-\gamma} = mc_t (1-\gamma) \frac{Y_t}{N_t^I} \tag{25}$$

$$1 - \epsilon^P + \epsilon^P mc_t - \kappa_P(\pi_t - \pi)\pi_t + \beta^P E_t \left\{ \frac{muc_{t+1}^P}{muc_t^P} \kappa_P(\pi_{t+1} - \pi)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right\} = 0, \tag{26}$$

where mc_t is the Lagrange multiplier on the demand constraint.

2.3 Financial Intermediation

The financial sector is composed of a continuum of monopolistically competitive intermediate banks and a competitive final banking sector.

2.3.1 Final Banking Sector

The final banking sector acts as the ultimate loan provider to impatient households and is composed of perfectly competitive banks. It purchases differentiated loans from intermediate banks and aggregates them into a single composite loan. The composite loan index is given by

$$L_t = \left[\int_0^1 L_t(j)^{\frac{\epsilon^L - 1}{\epsilon^L}} dj \right]^{\frac{\epsilon^L}{\epsilon^L - 1}}, \tag{27}$$

where $L_t(j)$ represents the loan offered by an intermediate bank j and L_t is the Dixit-Stiglitz type composite index of loans to the impatient households. The parameter $\epsilon^L > 1$ represents the interest rate elasticity of the loan demand. Intertemporal cost minimization for achieving L_t implies that the optimal loan demand is given by

$$L_t(j) = \left(\frac{R_t^L(j)}{R_t^L} \right)^{-\epsilon^L} L_t. \tag{28}$$

2.3.2 Intermediate Banking Sector

Intermediate banks provide differentiated loans for the final banking sector and act under monopolistic competition caused possibly by the different specifications of commercial banks in terms of types of lending or geographical space.

The representative monopolistic bank maximizes the following profit in nominal terms:

$$\max E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^{P} \left[R_t^L(j) L_t(j) - (1-\delta) R_t^D D_t - \delta D_t - R_t^{IB} B_t^{IB} - \frac{\kappa_B}{2} \left(\frac{R_t^L(j)}{R_{t-1}^L(j)} - 1 \right)^2 L_t \right], \qquad (29)$$

where $\Lambda_{0,t}^{P} = (\beta^P)^t \frac{P_t\, muc_t^P}{P_0\, muc_0^P}$ denotes the nominal stochastic discount factor. R_t^D and R_t^{IB} are the gross deposit and interbank borrowing rate (or the policy rate), respectively, and regarded as given by the banking sector. The parameter κ_B represents the adjustment costs for changing the loan rate.

Maximization is subject to the following balance sheet constraint:

$D_t + B_t^{IB} + X_t^{CB} = L_t + Res_t$,

where B_t^{IB}, X_t^{CB} and Res_t represent the interbank borrowing, cash transfers from the Central Bank, and reserves deposited at the Central Bank, respectively. Banks must hold reserves to be ensured against early withdrawals. We interpret the early withdrawal rate δ as the required reserve ratio and suppose that $Res_t = \delta D_t$.[4] We assume that reserves earn no interest. The balance sheet constraint can be rewritten as

$$(1-\delta) D_t + B_t^{IB} + X_t^{CB} = L_t. \qquad (30)$$

Maximization of equation (29) subject to (28) and (30) results in the following first order efficiency conditions:

$$D_t:\ (1-\delta) R_t^D + \delta = \mu_t^B (1-\delta) \qquad (31)$$

$$B_t^{IB}:\ R_t^{IB} = \mu_t^B \qquad (32)$$

$$R_t^L:\ R_t^L(j) = \frac{\epsilon^L}{\epsilon^L - 1} \left[\frac{(1-\delta) R_t^D + \delta}{1-\delta} \right] \qquad (33)$$
$$- \frac{\kappa_B}{\epsilon^L - 1} \left\{ \left(\frac{R_t^L(j)}{R_{t-1}^L(j)} - 1 \right) \frac{R_t^L(j)}{R_{t-1}^L(j)} - \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \left(\frac{R_{t+1}^L(j)}{R_t^L(j)} - 1 \right) \frac{R_{t+1}^L(j)}{R_t^L(j)} \frac{L_{t+1}}{L_t} \right\},$$

where μ_t^B is the Lagrange multiplier (the nominal marginal cost) associated with the balance sheet constraint. Following Aslam and Santoro (2009), we suppose that the monetary authority injects the reserves as $X_t^{CB} = \delta D_t$.

2.4 Monetary Policy

Monetary policy follows the familiar Taylor-type rule:

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4) After many simulations, we have found that the model does not converge as δ becomes larger than 0.05. In some sense, this is what we expect from Diamond and Dybvig (1983). It is optimal for a patient household to withdraw deposits early as she expects others to withdraw their deposits early. As the early withdrawal rate rises, bank runs are more likely, and the model does not converge. Multiple equilibria exist.

$$\log(R_t^{IB}) = \rho_R \log(R_{t-1}^{IB}) + (1-\rho_R)\left[\rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \rho_y \log\left(\frac{Y_t}{Y^*}\right) + \overline{R}^{IB}\right] + \epsilon_{R,t}, \tag{34}$$

where π and $\overline{R}^{IB}$ represent the steady state values of the corresponding variables, and Y^* denotes the output level when prices are flexible. The policy rate shock $\epsilon_{R,t}$ is i.i.d. over time, and $E(\epsilon_{z^h,t})$ = 0.

2.5 Aggregate Resource Constraint

Substitution of the real profits of banks and firms into the aggregate budget constraints of both types of households yields

$$Y_t = C_t + \frac{\kappa_P}{2}[\pi_t - \pi]^2 Y_t + \frac{\kappa_B}{2}\left[\frac{R_t^L}{R_{t-1}^L} - 1\right]^2 \frac{L_t}{P_t}, \tag{35}$$

where $C_t = C_t^P + C_t^I$ is the aggregate consumption. In the loan market, $\frac{L_t}{P_t} = b_t^I$.

For the housing market, the equilibrium condition is given by

$$\overline{h} = h_t^P + h_t^I, \tag{36}$$

where $\overline{h}$ denotes the exogenous fixed housing stock.

3 Calibration and Model Characteristics

3.1 Calibration

For most parameter values, we just borrow the values used in previous research. Iacoviello (2005), Monacelli (2007) and Aslam and Santoro (2009) set β^P at 0.99, implying an annual real interest rate of 4%, while GNSS (2010) set it at 0.9943. We set β^P at 0.99. For β^I, Iacoviello (2005), Monacelli (2007), Aslam and Santoro (2009) and GNSS (2010) set it at 0.95, 0.98, 0.97 and 0.975, respectively. We set it at 0.97.

We set the average LTV ratio($\overline{\chi}^I$) at 0.75, following Monacelli (2007) and Aslam and Santoro (2009). We set $\overline{z}^h$=0.2 following Monacelli (2007) and GNSS (2010). The parameter values $\epsilon^L = 150$ and $z^M = 0.6$ are from Aslam and Santoro (2009). We set $\epsilon^P = 6$, implying a mark-up rate of 20 percent. This value is also used in Totzek (2009).

For the period between 1970 and 2005 in the U.S., the average interest rate on a one-month CD is 6.89%, the average Federal Funds rate 6.91%, and the average prime loan rate 8.95%. In the base-line model, we set $\beta^P = 0.99$, $\delta = 0.01$ and $\epsilon^L = 150$. With these parameter values, $R^D = 1.0101$, $R^{IB} = 1.0203$ and $R^L = 1.0272$, implying a steady state deposit rate, policy rate and lending rate of about 4%, 8.1%, 10.8%, respectively. With regard to the parameter values chosen here, we admit that the policy rate is a bit high relative to the values most frequently used. However, the qualitative results reported in this manuscript do not change when we try different parameter values that also make the model converge.

It is well known that the slope of the Phillips curve in the Calvo-type New Keynesian model is given by $\frac{(1-\xi)(1-\beta\xi)}{\xi}$, where ξ represents the probability of the price not changing, whereas the slope of the Phillips curve normalized by the steady state value of output in the Rotemberg-type New Keynesian model is given by $\frac{\epsilon^P - 1}{\kappa_P}$. We thus set κ_P to match the slope of the Phillips curve with ξ = 0.75, implying an average period of four quarters before a price change. For κ_B, we set κ_B = 10 similar to the 9.69 used in Totzek (2009). We set the persistent parameters and the standard deviations of various shocks at the values that have been frequently used. The parameter values calibrated for the baseline model are reported in Table 1.

3.2 Model Characteristics

3.2.1 Deterministic Steady State

We get the steady state value of the deposit rate from equation (5), which is given by

$$R^D = \frac{1}{1-\delta}\left(\frac{1}{\beta^P} - \delta\right). \tag{37}$$

This is bigger than the normal steady state value of interest rates, $\frac{1}{\beta^P}$, in models without a banking sector. Savers (or patient households) know in advance that they will withdraw their deposits early at the rate of δ, in which case they earn no interest, and they accordingly require that the banks should pay higher than the normal deposit rate, $\frac{1}{\beta^P}$, in cases without a banking sector. The spread between R^D and $\frac{1}{\beta^P}$ gets bigger as the early withdrawal rate increases, since $\frac{d\left(R^D - \frac{1}{\beta^P}\right)}{d\delta} = \frac{\frac{1}{\beta^P} - 1}{(1-\delta)^2} > 0$.

The steady state values of the policy rate and the lending rate are given by

$$R^{IB} = \frac{(1-\delta)R^D + \delta}{(1-\delta)} \tag{38}$$

and

$$R^L = \frac{\epsilon^L}{\epsilon^L - 1} R^{IB}, \quad (39)$$

respectively. The lending rate is the mark-up over the policy rate, which is the weighted average of the bank's marginal cost in the cases with and without early withdrawals. The steady state value of the policy rate is greater than the deposit rate and less than the lending rate. The lending rate is greater than the deposit rate for two reasons. First, for each dollar of deposits, the bank should keep fraction δ of it as reserves, which do not earn interest. In order to compensate for this loss of interest, the bank charges a higher rate on its lending. Second, it has market power and can thus charge an additional mark-up over its marginal costs.

The spread, $\frac{\delta}{1-\delta}$, between the steady state values of the policy rate and the deposit rate is also an increasing function of the early withdrawal rate, δ. As the early withdrawal rate rises, the loss of foregone interests to the banks grows, and the marginal value of bank balance sheet constraints (which equals the policy rate) accordingly gets higher.

3.2.2 Technology Shock

With higher productivity, output rises, the inflation rate falls, and interest rates fall. A positive productivity shock is beneficial to both patient and impatient households, in the sense that the consumption of both types of household rises. With higher productivity, the impatient households want to increase their consumption and do so to a larger extent than the patient households. In equilibrium, patient households lend resources to impatient households, thereby financing the rise in consumption of the latter.

A fall in lending rates loosens the borrowing constraint of impatient households, and encourages them to borrow more. Note that in the model the supply of housing is fixed. If the demand for housing of one group of households rises, then that of the other group of households must fall. A fall in deposit rates generates a negative income effect on the patient households' consumption and discourages them from increasing their demand for housing services. As a result, the demand of impatient households for housing services rises and that of patient households falls. If the rise in demand for housing services of impatient households dominates the fall in demand for housing services of patient households, the relative housing price rises. This rise in the relative price of housing further encourages consumption by impatient households.

3.2.3 Monetary Tightening

When the monetary authority raises the policy rate, both inflation and output fall. As the policy rate rises, the deposit and the lending rates also rise. Note that in equilibrium patient households are savers and impatient households borrowers. A rise in the deposit rate combined with a fall in the inflation rate produces a positive wealth effect on patient households, whereas a rise in the lending rate together with a fall in the inflation rate produces a negative wealth effect on impatient households. When the monetary authority raises the policy rate, the consumption and demand for housing services of the latter group of households thus fall. Impatient households bear most of the burden of the rate raise. In the housing market, as the fall in demand for housing services of impatient households dominates the rise in demand for housing services of patient households, the relative price of housing goes down.

3.2.4 Rise in Loan-to-Value Ratio

A rise in the loan-to-value ratio loosens the borrowing constraint of impatient households and encourages both their consumption and their demand for housing services. This means that the demand for housing of patient households falls. Since the rise in demand for housing services of impatient households dominates the fall in demand for housing services of patient households, the housing price rises. Consumption of both the patient and impatient households rises. Output and inflation also rise. The monetary authority raises the policy rate in response to the increases in both output and the inflation rate, and other interest rates follow the policy rate increase. As far as the consumption of patient households is concerned, there are two forces working in opposite directions. Recall that the efficiency condition for patient households requires that the marginal utility of consumption equal the sum of the direct marginal utility gain from housing services and the expected utility from future consumption due to the capital gains on housing purchased during this period. As the demand for housing services of patient households falls, the marginal utility of housing services rises, which initially causes consumption of patient households to fall. In contrast, the rise in housing price generates a positive wealth effect on the consumption of the patient households. If the first effect dominates, the consumption of patient households falls, and if the second effect dominates their consumption rises. Given the parameter values chosen here, the consumption of patient households turns out to rise.

4 Optimal Implementable Taylor Rule

Monacelli (2007) examines the Ramsey policy in a DSGE model similar to that of Iacoviello (2005). One disadvantage of the Ramsey rule is that it delivers the optimal path of the variables but not an operational rule linking the policy instrument to the endogenous variables. The policy instrument is typically a function of both the exogenous forces driving the economy and all the endogenous predetermined variables. Among the latter are the past values of the Lagrange multipliers associated with the constraints of the Ramsey problem, which are not directly observable by the policy authority. Another disadvantage is that the second order conditions of optimality cannot be easily checked. We must rely on saddle point optimality, which is not always guaranteed. Even if the policymaker could observe all of the state variables, using the equilibrium process of the policy variables could give rise to multiple equilibria.

Here we consider an optimal implementable Taylor rule rather than the Ramsey policy. By an implementable rule, we mean an operational rule along the lines of Schmitt-Grohe and Uribe (2004). By an optimal rule, we mean a rule that maximizes the following social welfare:

$$W_0 \equiv \theta E_0 \sum_{t=0}^{\infty} (\beta^P)^t\, U\left(C_t^P, h_t^P, \left(\frac{M_t^P}{P_t}\right), N_t^P\right) + (1-\theta) E_0 \sum_{t=0}^{\infty} (\beta^I)^t\, U\left(C_t^I, h_t^I, \left(\frac{M_t^I}{P_t}\right), N_t^I\right), \qquad (40)$$

where θ denotes the weight placed on the lifetime utility of patient households. In order to find the optimal implementable Taylor rule, we do simulations of 200 periods for each shock. Following the suggestions in Kim and Kim (2003), we do simulations based on the second order approximations of the equilibrium conditions. The simulation is performed in Dynare Version 3.065. The range of simulations for each parameter in the Taylor rule is reported in Table 2.

Table 3 reports the Taylor rule that maximizes the social welfare specified in equation (40).[5] For a one percent technology shock, the coefficient for the policy response to the output gap is about 0.3. Recall that $\rho_\pi \to \infty$ under pure inflation targeting, and $\rho_\pi = 1$, $\rho_y = 0$, and $\rho_R = 0$ under real interest rate targeting. In the model economy therefore, the optimal implementable rule when a technology shock hits the economy turns out to be neither inflation nor real interest rate targeting. The optimal rule responds to both the inflation rate and output gap, with a bigger response to the inflation rate. However, compared with the case of a policy rate shock, the monetary authority should respond more to the

5) One thing we should note is that social welfare increases as the weight on the patient households gets bigger. This is because their welfare is greater than that of the impatient households.

output gap when a technology shock hits the economy.

For a 0.15 percent interest rate shock, ρ_y is close to zero, implying that the optimal rule is to not respond to the output gap when the source of the shock is the policy rate itself. One thing that should be noted is that the smoothing parameter is close to one. This implies that the CB should respond in such a way that the policy rate shock becomes more persistent.

Cecchetti et al. (2000) argue for the Central Bank's response to asset price movements especially when the shocks originate in the asset market and asset price movements affect the economy significantly. They argue that the policy response to asset price movements in this case can prevent asset bubbles and stabilize the economy. Bernanke and Gertler (2001) on the other hand argue that it is dangerous for the Central Bank to respond to asset price movements independently in addition to the inflation rate and the output gap. They argue that the Central Bank should respond to asset price movements only to the extent that these movements affect the inflation rate (forecast), since it is very difficult to know if there actually is a bubble in the asset market.

We examine if a policy rate response to movements in housing prices due to a housing preference shock is welfare improving. The bottom three rows of Table 3 report the parameter values of the Taylor rule and the level of welfare. We see that the coefficient of the response to housing price movements is positive and that responding to housing price movements improves social welfare to only a slight extent. It is also shown that the response to housing price movements substitutes in part for the CB's response to the output gap.

Recently, especially since the financial crisis in 2008, we have observed increasing interest in adjusting for financial sector stress in setting the Taylor rule -- both from practitioners in the market and from academia. For example, McCulley and Toloui (2008) suggest making a financial conditions-adjustment in setting the policy rate. Taylor (2008) argues for responding to the spread between Libor at 3-month maturity and an index of overnight federal funds rates expected for the same period (the Overnight Index Swap (OIS)). Curdia and Woodford (2010) set up a DSGE model and show that it is desirable for the Central Bank to respond to the spread between the lending rate and the policy rate when the shock originates in the financial sector.

We examine if the response of the policy rate to the movements in spreads due to the lending rate shock improves social welfare. For the analysis, we consider the following shock to the lending rate:

$$\log(\epsilon_t^S) = (1-\rho_{\epsilon^S})\log\bar{\epsilon}^S + \rho_{\epsilon^S}\log(\epsilon_{t-1}^S) + \epsilon_{\epsilon^S,t}, \quad -1 < \rho_{\epsilon^S} < 1. \qquad (41)$$

The spread between the lending rate and the deposit rate is defined as either

$$S_t = \exp(\epsilon_t^S) R_t^L - R_t^D \tag{42}$$

or

$$S_t = R_t^L - \exp(\epsilon_t^S) R_t^D, \tag{43}$$

depending upon where the shocks to the spread originate. We admit that this approach does not tell us what the real sources of the shocks are, but we believe it is a simple way to capture the various sources of shocks occurring in the financial market. We will call the former the spread due to shocks to the lending rate, and the latter the spread due to shocks to the deposit rate.

Table 4 reports the parameter values of the Taylor rule and the level of social welfare. The upper half of the table shows that the coefficient of the response to the spreads due to shocks to the lending rate is −0.1, and that responding to the spreads improves welfare significantly. When the CB does not respond to the movements in spreads, social welfare is in the range of 42 to 80 depending upon the weight placed on the patient households. It increases dramatically when the CB responds to the spread movements. Now, it is in the range of 190 to 300. This complements the findings in Curdia and Woodford (2010), that the CB should lower the policy rate when the spread between the lending and the deposit rates becomes larger than the corresponding steady state value.

The lower half of the table shows the simulation results when the shocks to the spreads originate in the deposit market. Again, the optimal policy is shown to be to lower the policy rate when the spread goes up relative to the corresponding steady state value. It further shows that responding to the movements in spreads improves welfare quite a lot, although the improvement is not as large as in the case of shocks to the lending rate. The level of welfare is in the range of 42 to 80 when the CB does not respond to the spread, and in the range of 46 to 84 when the CB responds. The welfare gain is more than 5 percent.

Table 4 also shows that the optimal response to the movements in spread is not as large as what has been suggested in previous research. It turns out to be −0.1 when the shocks originate in the lending rate and −0.15 when they originate in the deposit rate. This finding also complements the findings in Curdia and Woodford (2010).[6]

6) We examine if there is any change in the optimal implementable rule when the early withdrawal rate, δ, rises. We do simulations for δ = 0.005, 0.01, and 0.02. We find that there is no significant change in the optimal rule and to save space do not report the results.

5 Concluding Remarks

We set up a simple DSGE model with a financial sector and examine the optimal implementable rule that maximizes social welfare, that is, the weighted average of the welfares of the patient and impatient households. We find that responding to housing price movements does not improve social welfare very much. However, responding to the spreads between the lending and the deposit rates improves social welfare dramatically; social welfare is increased by a factor of about four when the shocks originate in the lending rate, and by more than 5 percent when they originate in the deposit rate.

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Appendix: the Deterministic Steady State

$\pi = 1$

$\overline{h} = 1$

$$R^D = \frac{1}{1-\delta}\left(\frac{1}{\beta^P} - \delta\right)$$

$$R^{IB} = \frac{(1-\delta)R^D + \delta}{(1-\delta)}.$$

$$R^L = \frac{\epsilon^L}{\epsilon^L - 1}\left[\frac{(1-\delta)R^D + \delta}{1-\delta}\right]$$

$Res = \delta D$

$B^{IB} = 0$

$$mc = \frac{\epsilon^P - 1}{\epsilon^P}$$

$$\psi^G = (1-mc)Y$$

$$\zeta_1 = \frac{z^H}{\left(1 - \beta^I - \frac{(1-\beta^I R^L)}{R^L}\chi^I\right)}$$

$$\zeta_3 = 1 + (R^L - 1)\frac{\chi^I}{R^L}\zeta_1$$

$$\zeta_5 = \frac{\chi^I}{R^L}\frac{\zeta_1(1-\gamma)mc}{\zeta_3} = \frac{L}{Y}$$

$$\zeta_7 = \frac{C^P}{Y} = \left[(1-(1-\gamma)mc) + (R^L - 1)\zeta_5\right]$$

$$\zeta_8 = \frac{z^H}{1-\beta^P}$$

$$N^P = \left(\frac{mc \cdot \gamma}{z^N \zeta_7}\right)^{\frac{1}{\sigma_N + 1}}$$

$$N^I = \left(\frac{\zeta_3}{z^N}\right)^{\frac{1}{\sigma_N + 1}}$$

$$Y = (N^P)^{\gamma}(N^I)^{1-\gamma}$$

$$h^P = \frac{\zeta_7\zeta_8}{\zeta_7\zeta_8 + \dfrac{\zeta_1(1-\gamma)mc^G}{\zeta_3}}$$

$$h^I = \frac{\dfrac{\zeta_1(1-\gamma)mc^G}{\zeta_3}}{\zeta_7\zeta_8 + \dfrac{\zeta_1(1-\gamma)mc^G}{\zeta_3}}$$

$$\frac{q^h h^P}{Y} = \zeta_7\zeta_8$$

$$C^I = Y - C^P$$

$$w^P = mc \cdot \gamma \frac{Y}{N^P}$$

$$w^I = mc(1-\gamma)\frac{Y}{N^I}$$

$$\mu^I = \frac{(1-\beta^I R^L)}{R^L C^I}$$

$$m^P = \frac{z^M C^P}{(1-\beta^P)}$$

$$m^I = \frac{z^M C^I}{(1-\beta^I)}$$

$$L = D$$

$$\psi^B = R^L L - (1-\delta)R^D D - \delta D \ = L(R^L - R^D) + \delta D(R^D - 1)$$

Table 1 Parameter Values

Parameter	
β^P	0.99
β^I	0.97
δ	0.01
γ	0.6
σ_N	2
ϵ^P	6
ϵ^L	150
κ_P	adjusted to match slope of Phillips curve with $\xi = 0.75$
κ_B	10
$\overline{z}^h$	0.2
z^N	1
z^M	0.6
$\overline{\chi}^I$	0.75
ρ_z	0.95
ρ_m	0
ρ_{χ^I}	0
ρ_{z^h}	0.5
ϵ_z	0.03
ϵ_m	0.0015
ϵ_{χ^I}	0.02
ϵ_{z^h}	0.04

Table 2 Range of Simulations for Each Parameter in Taylor Rule

$$\log(R_t^{IB}) = \rho_R \log(R_{t-1}^{IB}) + (1-\rho_R)\left[\rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \rho_y \log\left(\frac{Y_t}{Y^*}\right) + \overline{R}^{IB}\right] + \epsilon_{R,t}$$

Coefficient	ρ_R	ρ_π	ρ_y	ρ_{q^h}	ρ_S
Range (interval)	0.5 ~ 0.9 (0.05)	1.1 ~ 2.5 (0.1)	0.0 ~ 0.3 (0.05)	0.0 ~ 0.3 (0.05)	-0.15 ~ 0.0 (0.05)

Table 3 Optimal Implementable Taylor Rule: the Case of a Housing Preference Shock

$$\log(R_t^{IB}) = \rho_R \log(R_{t-1}^{IB}) + (1-\rho_R)\left[\rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \rho_y \log\left(\frac{Y_t}{Y^*}\right) + \overline{R}^{IB}\right] + \epsilon_{R,t}$$

1% technology shock					
θ	ρ_R	ρ_π	ρ_y		W_0
0.4	0.5	1.1	0.3		412.9
0.5	0.55	1.1	0.3		504.7
0.6	0.5	1.1	0.3		624.5
0.15% policy interest rate shock					
θ	ρ_R	ρ_π	ρ_y		W_0
0.4	0.9	1.1	0.0		44.94
0.5	0.9	1.1	0.0		64.18
0.6	0.9	1.1	0.0		83.47
1% LTV shock					
θ	ρ_R	ρ_π	ρ_y		W_0
0.4	0.7	1.1	0.25		41.89
0.5	0.5	1.1	0.3		60.75
0.6	0.5	1.1	0.3		79.61
1% housing preference shock					
θ	ρ_R	ρ_π	ρ_y		W_0
0.4	0.5	1.1	0.2		41.83
0.5	0.6	1.1	0.3		60.75
0.6	0.5	1.1	0.3		79.55
1% housing preference shock					
θ	ρ_R	ρ_π	ρ_y	ρ_{q^h}	W_0
0.4	0.5	1.1	0.05	0.3	42.36
0.5	0.6	1.1	0.1	0.3	61.3
0.6	0.6	1.1	0.1	0.3	80.3

Table 4 Optimal Implementable Taylor Rule: the Case of a Spread Shock

$$\log(R_t^{IB}) = \rho_R \log(R_{t-1}^{IB}) + (1-\rho_R)\left[\rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \rho_y \log\left(\frac{Y_t}{Y^*}\right) + \rho_S \log\left(\frac{S_t}{S^*}\right) + \overline{R}^{IB}\right] + \epsilon_{R,t}$$

$S_t = \exp(\epsilon_t^S) R_t^L - R_t^D$					
0.1% spread shock					
θ	ρ_R	ρ_π	ρ_y		W_0
0.4	0.5	1.1	0.3		41.94
0.5	0.5	1.1	0.15		60.79
0.6	0.5	1.1	0.2		79.64
0.1% spread shock					
θ	ρ_R	ρ_π	ρ_y	ρ_S	W_0
0.4	0.9	1.1	0.0	-0.1	193.2
0.5	0.9	1.1	0.0	-0.1	246.5
0.6	0.9	1.1	0.0	-0.1	299.8
$S_t = R_t^L - \exp(\epsilon_t^S) R_t^D$					
0.1% spread shock					
θ	ρ_R	ρ_π	ρ_y		W_0
0.4	0.9	1.1	0.0		42.05
0.5	0.9	1.1	0.05		60.93
0.6	0.9	1.1	0.0		79.83
0.1% spread shock					
θ	ρ_R	ρ_π	ρ_y	ρ_S	W_0
0.4	0.5	1.1	0.0	-0.15	45.80
0.5	0.5	1.1	0.0	-0.15	65.38
0.6	0.5	1.1	0.0	-0.15	85.0

Figure 1 Impulse Responses of Various Variables with Respect to a Technology Shock

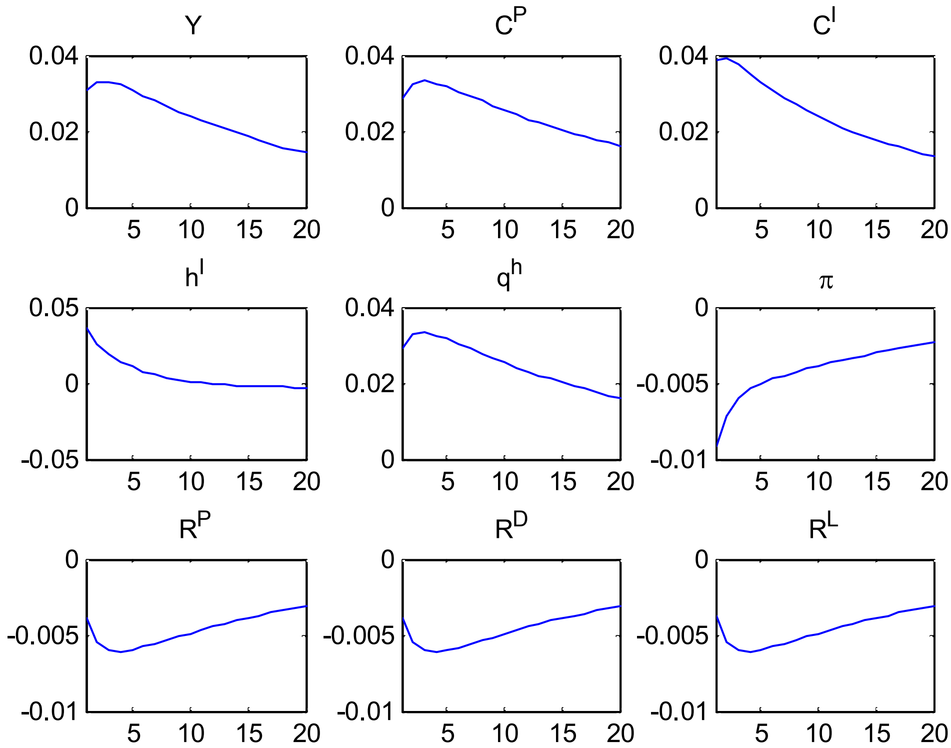

Figure 2 Impulse Responses of Various Variables with Respect to Monetary Tightening

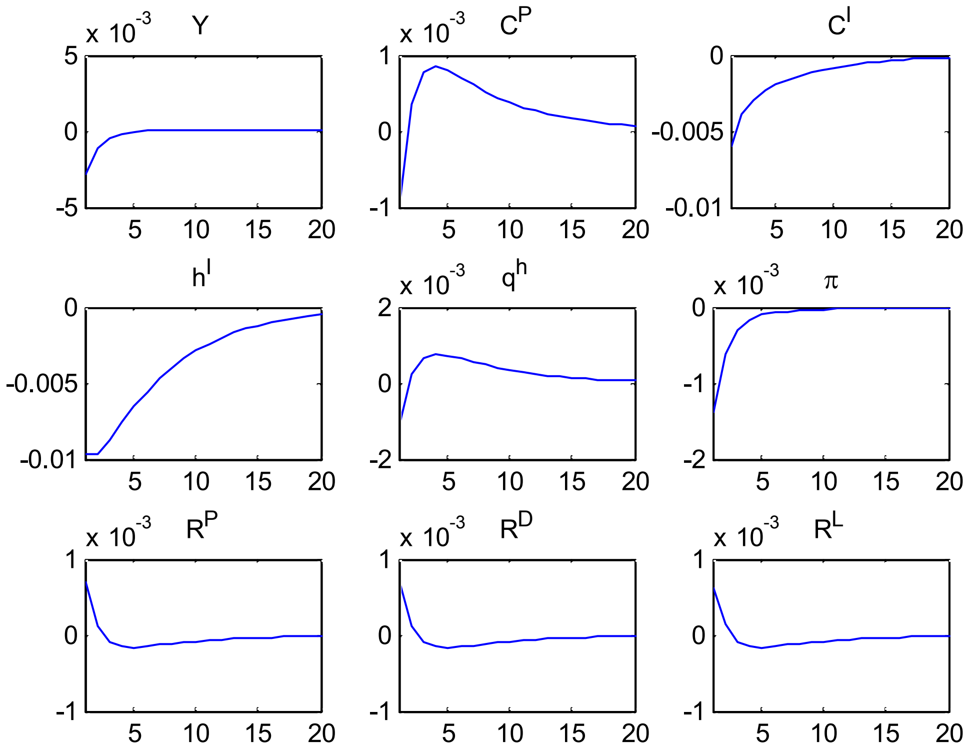

Figure 3 Impulse Responses of Various Variables with Respect to an LTV Shock