# Measuring Core Inflation under Price Divergence

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**Abstract**

Core inflation is commonly understood as the component in headline inflation that is expected to persist over the medium term. Timely conventional measures include limited influence estimators (e.g. trimmed means and medians) and excluded-item measures (e.g. headline inflation excluding energy and/or food items). These measures are appropriate when a single component underlies persistent price changes in the basket of consumption items. However, in this paper we present statistical evidence suggesting that (i) prices of consumer items are diverging over time, and (ii) several sub-convergent clubs underlie this overall price divergence. Under price divergence, the conventional core inflation measures become inaccurate, and are likely to be biased over longer time intervals. However, because price changes within each convergence club share the same persistent component, we can construct a more accurate measure of core inflation based on these groupings. Indeed, we show that our new proposed measure is better able to predict headline inflation than extant core inflation measures.

*Keywords*: Core Inflation, log $t$ regression, Time varying common factor representation, median estimator.

*The views expressed herein are those of the authors and not necessarily those of the Bureau of Economic Analysis or the Department of Commerce.

# 1 Introduction

Inflation is a fundamental indicator of both the health of the macro-economy and consumer welfare. While U.S. statistical agencies publish several measures of consumer price inflation, such as the Consumer Price Index-Urban Cities (CPI-U) and the Personal Consumption Expenditure (PCE) price index, policy-makers and analysts also pay attention to less volatile measures of inflation. These measures are conventionally referred to as "core inflation," and are designed to isolate the component of headline consumer inflation that is expected to persist for several years. Thus while a singular, preferred definition of core inflation is lacking, a measure of core inflation should - to some degree - be less volatile than headline inflation, and it should - to some degree - forecast headline inflation at some "near" or "medium" term horizon (Bryan and Cecchetti, 1994; also see Blinder 1997). In practice this near-to-medium-term horizon equates to anything between one to five years (see, e.g., Bryan and Cecchetti, 1994; Cogley, 2002; Smith, 2004).

Core inflation measures are typically constructed using disaggregate price data on the items underlying the consumption basket.[1] The measures are essentially cross sectional filters, whereby price changes in some items are excluded before taking an average. For example, limited influence estimators (such as the median and trimmed mean) exclude items with price changes in the tails of the distribution, so that the excluded items are permitted to change month to month. Examples of limited influence estimators can be found in Bryan and Pike (1991), Bryan and Cecchetti (1994), Bryan, Cecchetti and Wiggins (1997) and Dolmas (2005), amongst others. In contrast, a second set of core inflation measures simply exclude items that have historically exhibited high volatility. These include PCE and CPI inflation excluding energy and/or food items, and the measures proposed by Clark (2001). We refer to this second group as "excluded item" measures of core inflation.

Although the BLS first began to report a CPI excluding food and energy in 1978 (Rich and Steindel, 2007), Bryan and Cecchetti (1994) first provided the justification for the cross sectional filter as a measure of core inflation. In order to explain how cross-sectional filters can isolate the persistent component of headline inflation, Bryan and Cecchetti (1994) refer to the Ball and Mankiw (1995) menu-cost model of firm-level pricing behavior. This model yields the following equation for observed price changes in the cross section of items (cf. eq.(3) in Bryan and Cecchetti, 1994):

$$\pi_{it} = \pi_t^{core} + \pi_{it}^o, \; i = 1, \ldots, N; \; t = 1, \ldots, T, \tag{1}$$

where $\pi_{it}$ is inflation in the $i$th item over some short-term interval (e.g., 1 month) at time $t$, $\pi_t^{core}$ is a common component across all items $i$, and $\pi_{it}^o$ is an idiosyncratic inflation term for item

[1] A simple high frequency filter would of course yield a less volatile measure of headline inflation. Such measures are untimely and therefore undesirable from a policy-making perspective (see the discussion in Bryan and Cecchetti, 1994).

$i$.[2] Under the Ball-Mankiw model, for any given time period $t$, price changes in the tails of the distribution of $\{\pi_{it}\}_{i=1}^{N}$ are transitory, because only firms that receive sufficiently large shocks to their desired price level will pay the menu-cost and reset prices ahead of schedule.[3] If these shocks are asymmetrically distributed, then a conventional measure of the mean will be temporarily pulled in the direction of the skewness. Thus periodic and short-lived asymmetry in the distribution of $\{\pi_{it}^{o}\}_{i=1}^{N}$ can lead to volatility in headline inflation measures. Limited influence estimators are, by construction, less sensitive to skewness, and therefore exhibit lower variance than headline inflation measures. Limited influence estimators are also more efficient than arithmetic averages under leptokurtoticity, and therefore will exhibit less volatility than headline inflation measures if the distribution of price changes is symmetric but fat-tailed (see, e.g., Bryan, Cecchetti and Wiggins, 1997). Meanwhile excluded-item measures are less volatile than headline inflation because those items that are repeatedly present in the tails of the distribution (i.e., the more volatile items) are removed from the aggregation. Although the different measures will yield different estimates of core inflation in any given month (since the different measures have different sensitivity to skewness), each measure should yield an estimate that is closer to the less-volatile common component $\pi_t^{core}$ than the simple arithmetic mean.

An important feature of (1) is that it implies that prices in levels, rather than first-differences, have a single common stochastic trend. Moreover, long-term movements in the cross section of prices are dominated by this common stochastic trend. This is because the effect of past realizations of transient inflation $\pi_{it}^{o}$ on the price level of the $i$th item is dominated by past realizations of the more persistent common component $\pi_t^{core}$. However the specification of a single stochastic trend in prices is unappealing from a theoretical perspective because it precludes any long-term price divergence. For example, Baumol's cost disease (Baumol, 1967) implies that prices will rise faster in those sectors of the economy with relatively low labor productivity growth, while Ricardian trade theory predicts that prices in the tradeable sector should decrease as barriers to trade fall.[4]

We formally test for a single common stochastic trend in a large cross-section of disaggregate PCE prices by employing the convergence test recently developed by Phillips and Sul (2007). This test is explicitly designed for panel data sets with a large cross section of trending time-series, thereby circumventing the "curse of dimensionality" problem that afflicts conventional methods in

[2] Throughout, all inflation rates will be calculated as the log-difference in prices and, where necessary, appropriately scaled to be annualized.

[3] In fact the Ball-Mankiw model implies this transitory component is negatively autocorrelated. See the discussion in section III C in Ball and Mankiw (1994).

[4] Moreover, a cursory glance at the data supports long-term price divergence. Over the 1980-2009 period the 12-month inflation rate in PCE services was 2.26% higher on average than inflation in PCE goods, and the inflation rate in goods was higher than that of services in only 24 of the 324 months between January 1983 and December 2009.

the large $N$ framework. The test easily rejects the null hypothesis of a single stochastic trend. We then go on to consider whether there is a finite set of stochastic trends driving the cross section of prices by combining the convergence tests with clustering algorithms. We find strong evidence suggesting that the cross section of PCE items can be grouped into four clubs, whereby items within each club follow the same trend.

When there is a finite set of common stochastic trends underlying disaggregate price levels, prices changes are better characterized by a multi-component model, in which items from different clubs have distinct persistent components. One considerable limitation of the conventional core inflation measures is that they are not robust in the presence of more than one persistent component underlying headline inflation. Specifically, the measures become biased over longer time intervals: They will systematically over- or under- estimate the part of inflation that will persist over the near-to-medium-term. To illustrate, consider the case where a subset of inflation rates share a persistent component $\pi_{1t}^{core}$, while the remainder share a different persistent component $\pi_{2t}^{core}$. That is

$$\pi_{it} = \begin{cases} \pi_{1t}^{core} + \pi_{it}^{o} \text{ for } i \in G_1 \\ \pi_{2t}^{core} + \pi_{it}^{o} \text{ for } i \in G_2 \end{cases}. \quad (2)$$

The price levels of the two groups will diverge when $(\pi_{1t}^{core} - \pi_{2t}^{core})$ is non-zero on average. Let $N_1$ and $N_2$ denote the number of items within each group, and, without loss of generality, assume that each item $i$ has the budget share in the consumption basket. As in the Bryan-Cecchetti framework (1), the $\pi_{it}^{o}$ are volatile and transitory, while the common components $\pi_{1t}^{core}$ and $\pi_{2t}^{core}$ are slow-moving and persistent. Thus an ideal definition of core inflation in (2) will be some linear combination of $\pi_{1t}^{core}$ and $\pi_{2t}^{core}$. Indeed,

$$\pi_{t}^{core} := \frac{N_1}{N}\pi_{1t}^{core} + \frac{N_2}{N}\pi_{2t}^{core},$$

provides an unbiased estimate of future headline inflation, $N^{-1}\sum_{i=1}^{N}\pi_{i,t}$.[5] Now consider the median estimator. Suppose that $N_1 > N_2$ and $\pi_{1t}^{core} > \pi_{2t}^{core}$, and the distribution of $\pi_{it}^{o}$ is symmetric and homogenous across $i$. Then the median (and in general limited influence estimators with symmetric truncations) will be closer to $\pi_{1t}^{core}$ than to $\pi_{t}^{core}$ since $N_1 > N_2$. Thus the median will overstate core inflation, simply because more expenditure falls into the high inflation group. Symmetric trimmed mean inflation measures will likewise overstate core inflation. Intuitively the limited influence estimators are biased because they yield an estimate closer to the central tendency of the distribution, just as they do in the single component model (1). However, the difference in this particular example is that the skewness in the distribution is due to the multi-component structure ($\pi_{1t}^{core}$ and $\pi_{2t}^{core}$), not due to asymmetry in the distribution of idiosyncratic inflation

[5] More formally, for $h$ satisfying $E_t(\pi_{i,t+h}^{o}) = 0$, so-defined $\pi_t^c$ uniquely satsifies $E_t(N^{-1}\sum_{i=1}^{N}\pi_{i,t+h}) = \pi_t^c$.

($\pi_{it}^{o}$). Therefore, an estimate closer to the central tendency of the distribution is undesirable, and the arithmetic mean provides a less-biased estimate.[6] Excluded-item measures will also likewise be biased in this example, unless the excluded items are drawn from the two groups in proportion to the population shares $N_1/N$ and $N_2/N$. If, for example, all excluded items are members of the second group, then the excluded-item measure will be biased upwards.

In the more general Bryan-Cecchetti framework the distribution of the idiosyncratic inflation $\pi_{it}^{o}$ exhibits transitory asymmetry, making it impossible to determine the direction of the bias of the limited influence estimator in any given month. However, because the multi-component structure ($\pi_{1t}^{core}$ and $\pi_{2t}^{core}$) creates a persistent form of asymmetry, the bias will become manifest as the core inflation measure is taken over longer intervals. In other words, rolling (time-series) averages of the monthly inflation measures will consistently overstate (or understate) core inflation provided the time-interval of the average is of sufficient length.[7] But it also implies that measures with different sensitivity to the degree of asymmetry in a distribution should be systematically different. (For example, a 10% trimmed mean is less sensitive to skewness in the distribution than the median.) In fact, this is precisely what we find empirically. Even at time intervals as low as 12-months we observe marked divergence in the conventional core inflation measures. This finding is unsettling in itself, since it suggests that different core inflation measures consistently give different indications of the direction of consumer prices. But it is also further evidence against the single component model (1), because the single component model implies that longer term measures should converge as the time interval of the average grows longer: Intuitively, transitory price changes are averaged out while the average of the single persistent component remains. We will illustrate and discuss this divergence in core inflation measures in more detail in the following section.

How can we gain a better estimate of core inflation in a multi-component model such as (2)? Provided that club membership is known, a limited influence estimator applied to items within each club yields a more accurate estimate of the persistent component within each club. We can then gain an better estimate of the persistent component in all price changes by taking a weighted average of each limited influence estimator.

Thus the contribution of this paper is two-fold. First, we present empirical evidence suggesting that there is more than one persistent component underlying the cross section of consumer item inflation rates. We instead demonstrate that item prices can be grouped into four clubs, whereby

[6] The greater the truncation density in the trimmed mean, the further the estimate will be from the arithmetic mean. That is, the median, with a 50% truncation density, will be further from the arithmetic mean than a trimmed mean with a 49% truncation density.

[7] An additional source of bias could arise if consumers subsitute out of the high inflation goods into the low inflation goods. This bias could be addressed by appropriately re-weighting the consumption basket to account for changes in budget shares.

price changes for the items within each club share the same persistent component. Second, we suggest a new method for measuring core inflation that is robust to the price divergence implied by our empirical findings. Having identified the different groups using the Phillips and Sul (2007) algorithm, we construct a core inflation estimate based on a weighted average of an limited influence estimator for each club. The new core inflation measure exhibits low volatility and is able to forecast headline inflation more accurately than extant measures of core inflation.

The remainder of the paper consists of seven sections. The following section demonstrates that different core inflation measures do not converge over the long-run. In section three we generalize the single component model of Bryan and Cecchetti (1994) to permit a finite set of persistent components in the cross section of disaggregate inflation rates. Through some simple examples we show that both the conventional limited influence estimators and the excluded-item measures become biased in the multi-component framework. We also argue that the single component model is irreconcilable with the stylized facts outlined in section 2. In section four we use empirical evidence to argue that disaggregate consumer item price changes are better characterized by a multi-component model rather than the single model component model (1). Based on these findings, in section 5 we propose our new measure of core inflation. In section 6 we compare the new measure to extant measures of core inflation, focusing in particular on the ability of the various measures to forecast headline PCE inflation. Section 7 concludes.

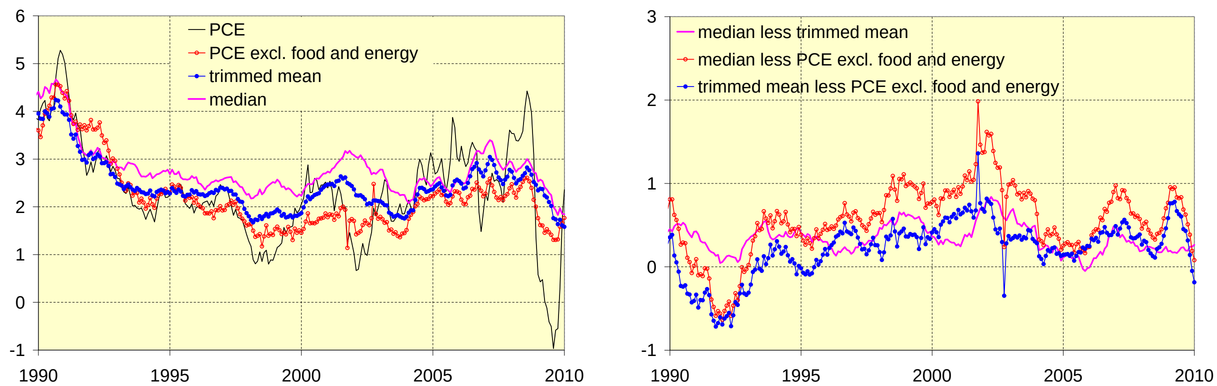
## 2 Stylized Facts of Extant Core Inflation Measures

In this section we establish a couple of stylized facts regarding extant core inflation measures. In order to make the analysis clear, from the many candidate core inflation measures available we choose to focus on three measures proposed by Federal agencies: (i) PCE excluding food and energy ($\hat{\pi}_t^{excl}$), published by the BEA; (ii) trimmed mean PCE published by the FRB Dallas ($\hat{\pi}_t^{trim}$);, and (iii) an expenditure-weighted median PCE inflation rate ($\hat{\pi}_t^{med}$). PCE excluding food and energy is available from the BEA website, and is notable because it is the Federal Reserve Board of Governors' preferred inflation measure. The Dallas FRB trimmed mean (obtainable at http://www.dallasfed.org/data/pce/index.html) uses an asymmetric trimmed mean (cutting out 21% and 25% of the probability mass from the lower and upper tails of PCE expenditure weights, respectively) that has been calibrated to isolate the persistent component of headline inflation based on historical data (Dolmas, 2005). The FRB Cleveland reports a weighted median CPI-U. However, as the other measures are constructed based on PCE prices and expenditures, and not the BLS CPI-U prices and weights, we construct the weighted median PCE ourselves.[8] We construct

[8] Another reason to include the weighted median is that Smith (2004) argues that a weighted median is the "best" core inflation measure in the sense that it provides the best forecast of CPI-U inflation.

the median measure using monthly data for 182 items as obtained from NIPA Table 2.4.4.U at the BEA website.[9]

Panel A in Figure 1 shows the 12 month PCE headline inflation rate and the three 12 month core inflation measures over January 1990 to December 2009.[10] Panel B exhibits the difference between the core inflation measures over the same period.The stylized facts we establish are as follows.




Panel A: 12 month PCE and Core Inflation Rates    Panel B: Differences in 12 month Core Inflation Rates

Figure 1: 12 month PCE and Core Inflation Rates, and Differences between them.

1. Extant inflation measures exhibit lower volatility than headline inflation.

2. Differences between the various inflation measures exhibit high persistence, and are in fact diverging over time.

We now provide some summary statistics to back up observations.

## 2.1 Low Volatility in Core Inflation Measures

Table 1 consists of summary statistics for the various core inflation measures for the 1990-2010 period. We consider both 1-month and 12-month inflation rates, for four different 5-year subsamples.

In accordance with the defining concept of core inflation - a measure that captures low frequency movements in prices - all three core inflation measures exhibit less variation than the headline measure. Of the three measures, the FRB Dallas trimmed mean exhibits the lowest standard deviation in each of the sub-samples considered. It is also interesting that the means of the core inflation measures over the any given five year period can be very different. For example, over the 2005-2009 period, the average 12-month inflation rate in PCE excluding food and energy was

[9] For a detailed expoisition of the construction of the weighted median, see subsection 5.1 below.

[10] The 12 month core inflation rates are backwards 12-month moving average of the month-on-month core inflation rates.

2.13%, whereas the corresponding figure for weighted median PCE inflation was 2.68%. Long-term differences in the level of the various core inflation measures is the issue we turn to in the next subsection.

Table 1: Time Series Properties of Core Inflation Rates

| | 1 Month | | | | 12 Month | | | |
|---|---|---|---|---|---|---|---|---|
| Subsample: | 90-94 | 95-99 | 00-04 | 05-09 | 90-94 | 95-99 | 00-04 | 05-09 |
| Panel A: Time Series Mean | | | | | | | | |
| $\pi_t^{pce}$ | 2.938 | 1.748 | 2.109 | 2.341 | 3.027 | 1.748 | 2.059 | 2.360 |
| $\hat{\pi}_t^{excl}$ | 3.054 | 1.745 | 1.802 | 2.113 | 3.166 | 1.803 | 1.759 | 2.129 |
| $\hat{\pi}_t^{med}$ | 3.136 | 2.441 | 2.645 | 2.617 | 3.300 | 2.461 | 2.641 | 2.676 |
| $\hat{\pi}_t^{trim}$ | 2.825 | 2.058 | 2.222 | 2.393 | 2.972 | 2.078 | 2.201 | 2.470 |
| Panel B: Time Series Standard Deviation | | | | | | | | |
| $\pi_t^{pce}$ | 1.834 | 1.300 | 2.050 | 3.773 | 0.992 | 0.516 | 0.593 | 1.355 |
| $\hat{\pi}_t^{excl}$ | 1.442 | 0.995 | 1.646 | 0.942 | 0.836 | 0.338 | 0.256 | 0.356 |
| $\hat{\pi}_t^{med}$ | 0.927 | 0.490 | 0.759 | 0.816 | 0.685 | 0.149 | 0.292 | 0.386 |
| $\hat{\pi}_t^{trim}$ | 0.916 | 0.475 | 0.533 | 0.798 | 0.662 | 0.246 | 0.242 | 0.342 |

## 2.2 Divergence Between Core Inflation Measures

Evidently, as shown in panel B of Figure 1, differences between the various core inflation measures do not converge to zero; rather they appear to diverge as time passes. Table 2 exhibits statistical evidence suggesting the core inflation rates are indeed diverging over time. In panel A we report the time-series mean of the differences between the three core inflation rates over adjacent 5 year sub-samples. We formally test whether the difference between any pair of core inflation rates is converging to zero by running the following regression

$$\hat{\pi}_t^c - \hat{\pi}_t^d = a_2 + b_2 t + v_t \text{ for } c, d = med, trim, excl. \tag{3}$$

When $a_2 = b_2 = 0$ the pair of core inflation measures are convergent. In other words, we have $\lim_{k\to\infty} E_t\left(\hat{\pi}_{t+k}^c - \hat{\pi}_{t+k}^d\right) = 0$. If $b_2 \neq 0$, then the two series are diverging over time, i.e., $\lim_{k\to\infty} E_t\left(\hat{\pi}_{t+k}^c - \hat{\pi}_{t+k}^d\right) = \pm\infty$. If $a \neq 0$ but $b = 0$, then the two series are neither converging or diverging, but have different long-run levels.

Table 2: Divergence in Core Inflation Measures

| Panel A: Subsample Time Series Mean | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1-month | | | | 12-month | | | |
| Subsample: | 1990-94 | 1995-99 | 2000-04 | 2005-09 | 1990-94 | 1995-99 | 2000-04 | 2005-09 |
| $\hat{\pi}_t^{med} - \hat{\pi}_t^{excl}$ | 0.082 | 0.383 | 0.842 | 0.504 | 0.134 | 0.384 | 0.882 | 0.546 |
| $\hat{\pi}_t^{med} - \hat{\pi}_t^{trim}$ | 0.311 | 0.696 | 0.423 | 0.224 | 0.328 | 0.659 | 0.440 | 0.206 |
| $\hat{\pi}_t^{trim} - \hat{\pi}_t^{excl}$ | -0.229 | 0.314 | 0.419 | 0.280 | -0.194 | 0.275 | 0.442 | 0.340 |

| Panel B: Trend Regressions: 1990-2009 | | |
|---|---|---|
| | 1-month | 12-month |
| $\hat{\pi}_t^{med} - \hat{\pi}_t^{excl}$ | $= 0.082 + 0.0023t +$ residuals | $= 0.075 + 0.0025t +$ residuals |
| | (0.24) (0.0012), Wald=48 | (0.16) (0.0008), Wald=130 |
| $\hat{\pi}_t^{med} - \hat{\pi}_t^{trim}$ | $= 0.389 - 0.0002t +$ residuals | $= 0.411 - 0.0004t +$ residuals |
| | (0.08) (0.0004), Wald=110 | (0.06) (0.0003), Wald=50 |
| $\hat{\pi}_t^{trim} - \hat{\pi}_t^{excl}$ | $= -0.263 + 0.0024t +$ residuals | $= -0.336 + 0.0029t +$ residuals |
| | (0.11) (0.0006), Wald=19 | (0.12) (0.0005), Wald=81 |

Panel B in Table 2 exhibits the point estimates and standard errors for $a_2$, $b_2$, and Wald statistics for the null $a_2 = b_2 = 0$. There is no empirical evidence that all three core inflations are converging. Moreover, except for the difference between the FRB Dallas trimmed mean and the weighted median PCE, the point estimates of the trend coefficients are significantly different from zero, indicating divergence.

## 3 Modelling Disaggregate Inflation

In this section we generalize the single component model of Bryan and Cecchetti (1994) to a multi-component model in which a set of components underlying disaggregate price changes are permitted to be persistent. We discuss the properties of the limited influence estimators and the excluded-item measures in both the single and multi-component framework, and argue that longer term averages of these measures are only consistent estimates of core inflation in the single component model. We will also focus on whether the single-component model can generate the stylized facts discussed in the previous section.

We begin with the Bryan and Cecchetti (1994) single component model.

### 3.1 Single Component Model

Disaggregate inflation rates are assumed to be comprised of two independent series,

$$\pi_{it} = \pi_t^{core} + \pi_{it}^o, \tag{4}$$

where $\pi_t^{core}$ is the common, persistent component and $\pi_{it}^o$ is the idiosyncratic, transitory component. Suppose that each item receives an equal share of expenditure, so that headline inflation is a simple arithmetic average, namely $N^{-1} \sum_{i=1}^N \pi_{it}$. (Arguments presented hereafter can be generalized to the case in which headline inflation is a general weighted arithmetic average.)[11] The Ball-Mankiw framework permits the mean of $\{\pi_{it}^o\}_{i=1}^N$ to deviate from zero in any given period, but it does not permit these deviations to persist over time. (Indeed, it is precisely these short-term deviations between $\pi_t^{core}$ and $N^{-1} \sum_{i=1}^N \pi_{it}$ that lead to volatility in headline measures of inflation in the Ball-Mankiw model.) Thus we may state that $\pi_{it}^o$ in (4) satisfies $E(\pi_{it}^o) = 0$ without precluding $E_{t-1}(\pi_{it}^o) \neq 0$. This condition prevents time series averages $k^{-1} \sum_{t=1}^k \pi_{it}^o$ from being non-zero as $k$ grows large, and implies that $E_t\left(\pi_{it+h}^o\right) = 0$ for sufficiently large $h$. Under this condition we permit a deviation between $\pi_t^{core}$ and $N^{-1} \sum_{i=1}^N \pi_{it}$, but we do not permit this deviation to persist over time.

The limited influence estimators and excluded-item measures can be expressed as

$$\hat{\pi}_t^{core} = \sum_{i=1}^N \omega_i \pi_{it}.$$

where the weights $\omega_i \in [0,1]$ are specific to each cross sectional filter and satisfy $N^{-1} \sum_{i=1}^N \omega_i = 1$. If $\pi_{it}$ exhibit cross-sectional heteroskedasticity, the weights for the median and trimmed mean inflation are inversely related to the individual variance of $\pi_{it}^o$. That is, items with higher variance are assigned a lower weight: see Appendix A.1 for a detailed discussion. For excluded-item measures the weights simply become zero in all periods for those items with large variance.

Consider the following general estimate of core inflation

$$\hat{\pi}_t^{core} = \sum_{i=1}^N \omega_i \left(\pi_t^{core} + \pi_{it}^o\right) = \pi_t^{core} + \sum_{i=1}^N \omega_i \pi_{it}^o.$$

The second equality holds since $\sum_{i=1}^N \omega_i = 1$. Under the single component model (4), the relative weighting of the various items become increasingly irrelevant as we average the measure across time. Consider

$$k^{-1} \sum_{t=1}^k \hat{\pi}_t^{core} = k^{-1} \sum_{t=1}^k \pi_t^{core} + k^{-1} \sum_{t=1}^k \sum_{i=1}^N \omega_{it} \pi_{it}^o = \bar{\pi}_{t,k}^{core} + R_k, \tag{5}$$

[11] Regardless, there is in fact little difference between PCE inflation and a simple arithmetic average of PCE item level inflation.

where $R_k$ is a small order term.[12] This implies the core inflation measure is a consistent estimate of core inflation over longer-time periods, $\bar{\pi}_{t,k}^{core}$, as the time interval of the average. It also implies that the time series mean of the difference between any pair of core inflation rates must converge to zero as $k$ grows large. For example, if we consider the difference between $\hat{\pi}_t^{excl}$ and $\hat{\pi}_t^{med}$ under (4) then

$$\text{plim}_{k\to\infty} k^{-1} \sum_{t=1}^{k} \left( \hat{\pi}_t^{excl} - \hat{\pi}_t^{med} \right) = 0.$$

Thus longer-term measures of core inflation should converge as the time-interval of the average grows larger. Intuitively, if there is a single stochastic component dominating the long-term trends in disaggregate inflation, the relative weighting of the various items in the consumption basket becomes irrelevant when taking long time-series averages of the core-inflation measure. Yet the fact that we do observe any convergence in the 12-month core inflation rates indicates that the single common factor model (4) is not a good model of disaggregate inflation.

### 3.2 Multi-Component Model

Individual inflation rates can be grouped into a few sub-groups such that

$$\pi_{it} = \pi_{rt}^{core} + \pi_{it}^{o}, \text{ for } r = 1, \ldots, m; \quad m \leq N, \tag{6}$$

where $\pi_{it}^{o}$ continues to have zero unconditional expectation, as in (4). Let the size of each sub-group be $N_r$ and without loss of generality, $N_1 > \cdots > N_m$, and $N_1 + \cdots + N_m = N$. For instructive purposes we consider $m = 2$. That is,

$$\pi_{it} = \begin{cases} \pi_{1t}^{core} + \pi_{it}^{o} \text{ for } i \in G_1;\ i = 1, ..., N_1 \\ \pi_{2t}^{core} + \pi_{it}^{o} \text{ for } i \in G_2;\ i = 1, ..., N_2 \end{cases},$$

As above, we assume that each item receives the same weight in the consumption basket. In this case the natural definition of core inflation is a weighted average of two common factors;

$$\pi_t^{core} := \frac{N_1}{N} \pi_{1t}^{core} + \frac{N_2}{N} \pi_{2t}^{core}. \tag{7}$$

So-defined $\pi_t^{core}$ provides an unbiased forecast of future headline inflation, $N^{-1} \sum_{i=1}^{N} \pi_{i,t+h}$, since $E_t \left( \pi_{it+h}^{o} \right) = 0$ for sufficiently large $h$. That is $\pi_t^{core}$ uniquely satisfies $\pi_t^{core} = E_t(N^{-1} \sum_{i=1}^{N} \pi_{i,t+h})$ for some integer $h > 0$.

Consider excluded-item measures of core inflation. Let $N_r^{excl}$ be the number of items left in $G_r$ after excluded items have been removed, and let $N_{excl} = N_1^{excl} + N_2^{excl}$. Analogously let $G_{excl}$

[12] More precisely, $R_k = O_p \left( k^{-1/2} \right)$ for large $k$. In other words, the $R_k$ term is small enough to ignore for sufficiently large $k$.

denote the set of items remaining after excluded items have been removed. Then we have

$$\pi_t^{excl} = \frac{1}{N_{excl}} \sum_{i \in G_{excl}} \pi_{it} = \frac{N_1^{excl}}{N_{excl}} \pi_{1t}^{core} + \frac{N_2^{excl}}{N_{excl}} \pi_{2t}^{core} + \frac{1}{N_{excl}} \sum_{i \in G_{excl}} \pi_{it}^{o},$$

Then take a time series average over $k$ periods;

$$\frac{1}{k} \sum_{t=1}^{k} \left( \pi_t^{excl} - \pi_t^{core} \right) = \left( \alpha_1^{excl} - \alpha_1 \right) \frac{1}{k} \sum_{t=1}^{k} \pi_{1t}^{core} + \left( \alpha_2^{excl} - \alpha_2 \right) \frac{1}{k} \sum_{t=1}^{k} \pi_{2t}^{core} + R_k,$$

where $\alpha_r^{excl} := N_r^{excl}/N_{excl}$ and $\alpha_r := N_r/N$, $r = 1, 2$, are bounded between zero and one. The final term, $R_k$, is very small; see footnote 12 for a detailed discussion. It is apparent then that $\frac{1}{k} \sum_{t=1}^{k} \pi_t^{excl}$ is only a consistent estimator of $\frac{1}{k} \sum_{t=1}^{k} \pi_t^{core}$ as $k$ grows large if $\alpha_1 = \alpha_1^{excl}$.

Next consider the trimmed mean inflation rate. Under (6) the trimmed mean estimator for a given $t$ becomes

$$\pi_t^{trim} = \sum_{i=1}^{N} \omega_i \pi_{it} = \sum_{i \in G_1} \omega_i \pi_{it} + \sum_{i \in G_2} \omega_i \pi_{it} = \alpha \pi_{1t}^{core} + (1 - \alpha) \pi_{2t}^{core} + \sum_{i=1}^{N} \omega_i \pi_{it}^{o}.$$

where $\alpha := \sum_{i \in G_1} \omega_i$. The weight in any given $t$ becomes a function of $\pi_{1t}^{core}$, $\pi_{2t}^{core}$, which are time-varying, and $\sigma_i^2$. To simplify things assume that $\sigma_i^2 = \sigma^2$ and that $\pi_{rt}^{core} = \pi_r^{core}$ for $r = 1, 2$ and all $t$. Then, as shown in Appendix A.2, for symmetric truncations it follows that $\omega_i > \omega_j$ for $i \in G_1$ and $j \in G_2$. This in turn means that $\alpha > N_1/N$ so that

$$\frac{1}{k} \sum_{t=1}^{k} \left( \pi_t^{trim} - \pi_t^{core} \right) = \left( \alpha - \frac{N_1}{N} \right) \frac{1}{k} \sum_{t=1}^{k} \pi_{1t}^{core} + \left( 1 - \frac{N_2}{N} + \alpha \right) \frac{1}{k} \sum_{t=1}^{k} \pi_{2t}^{core} + R_k$$

and hence $\frac{1}{k} \sum_{t=1}^{k} \pi_t^{trim} > \pi_t^{core}$ for large $k$. Thus the trimmed mean $\frac{1}{k} \sum_{t=1}^{k} \pi_t^{trim}$ over (under) estimates the true core inflation $\frac{1}{k} \sum_{t=1}^{k} \pi_t^{core}$ if $N_1 > N_2$ $(N_1 < N_2)$. Appendix A.2 also shows that another trimmed mean $\pi_t^{trim*}$ with a larger truncation has weights $\omega_i^*$ satisfying

$$\omega_i^* > \omega_i,\ i \in G_1;\ \omega_i < \omega_i^*,\ i \in G_2,$$

so that $\alpha < \alpha^*$ and

$$\frac{1}{k} \sum_{t=1}^{k} \left( \pi_t^{trim*} - \pi_t^{trim} \right) = (\alpha^* - \alpha) \left( \pi_1^{core} - \pi_2^{core} \right) + R_k.$$

or $\frac{1}{k} \sum_{t=1}^{k} \pi_t^{trim*} > \frac{1}{k} \sum_{t=1}^{k} \pi_t^{trim}$ for large $k$. Thus symmetric trimmed means with different truncations will be different from each other over the long term. Similarly, since $\alpha \neq N_1^{excl}/N_{excl}$ in general, the trimmed mean and excluded items measures will be persistently different from each other. This is precisely what we observe empirically, as outlined in the previous section.

# 4 Determining the Number of Trends in Disaggregate Prices

Prices indices are, by definition, the cumulated sum of inflation rates (subject to some base-year normalization). Hence if there are $m$ different persistent components underlying the cross section of inflation rates as specified in (6), then there are $m$ stochastic trends in the cross section of price indices. Therefore we can evaluate which model of disaggregate inflation, (4) or (6), better describes disaggregate inflation by estimating the number of stochastic trends underlying disaggregate price levels. If there is more than one trend (i.e., $m > 1$), then this implies that prices are diverging over time.

To estimate the number of trends we use the time-varying factor model employed by Phillips and Sul (2007). They provide convergence tests that can be used both to estimate how many stochastic trends are present, and to cluster the different price indices according to their common trend component. The Phillips and Sul (2007) model is

$$p_{it} = b_{1,it}\theta_{1t} + \cdots + b_{m,it}\theta_{mt} + \varepsilon_{it} = \sum_{r=1}^{m} b_{r,it}\theta_{rt} = b_{it}\theta_t, \tag{8}$$

where $b_{it} := \left(\theta_t^{-1}\sum_{r=1}^{m} b_{r,it}\theta_{rt} + \theta_t^{-1}\varepsilon_{it}\right)$. Here $p_{it}$ is the logged price index of component $i$, $\theta_t$ is the common deterministic or stochastic trend component, and $m$ is the factor number. Notably $b_{it}$ is permitted to be time varying and can be interpreted as relative transitional effects from a common price change $\theta_t$. If $p_{it}$ is dependent across $i$, then $b_{it}$ can be interpreted as the time varying economic distance between $p_{it}$ and $\theta_t$ (see Phillips and Sul, 2007).

The inflation rate of the $i$th component can be then be written as

$$\pi_{it} = \Delta p_{it} = b_{it-1}\Delta\theta_t + \Delta b_{it}\theta_{t-1}.$$

Of particular interest to us in the present application is whether there is systematic behavior in $b_{it}$. For example, if $b_{it}$ converges to 1,[13] $p_{it}$ becomes identical to $\theta_t$ in the long run. In this case $\theta_t$ becomes the dominant trend in all price components, and in this event the common factor to inflation rates becomes $\Delta\theta_t$. Such time varying behavior is modelled by Phillips and Sul (2007) as

$$b_{it} = b_i + \xi_{it} L(t)^{-1} t^{-\alpha_i}, \text{ for } \xi_{it} \sim iid\left(0, \sigma_i^2\right) \tag{9}$$

where $L(t)$ is a slowly time varying function. When $\alpha_i \geq 0$ we have $b_{it} \to b_i$ as $t \to \infty$. When $b_i = 1$, and $\alpha_i \geq 0$ for all $i$, $p_{it} \to \theta_t$ as $t \to \infty$. The dynamics of individual prices are dependent on the time varying factor loadings, $b_{it}$. Individual prices, $p_{it}$, converge to $p_t = \theta_t$ if $b_i = b = 1$ and $\alpha_i \geq 0$ for all $i$. Meanwhile if either $b_i \neq b$ or $\alpha_i < 0$ for any $i$, $p_{it}$ diverge over time.

For instructive purposes it is useful to consider a simple example of this time-varying factor loading model.

[13] Since $b_{it}$ and $\theta_t$ are not separately identifiable, $b_{it}$ can be normalized to be unity when $b_{it}$ converges to a constant over time.

**Example 1** *Let $p_{it}$ follow a quasi-trend (non) stationary process given by*

$$p_{it} = c_i t^{\delta_i} + p^o_{it}, \quad p^o_{it} = \rho p^o_{it-1} + \varepsilon_{it},$$

*where $c_i \sim iid\left(0, \sigma_c^2\right)$, $\varepsilon_{it} \sim iid\left(0, \sigma_i^2\right)$, $\rho < 1$, $\delta_i > 1/2$ for all $i$. Let $\delta^* = \max\{\delta_i\}$.*[14] *Then we can re-express $p_{it}$ as*

$$p_{it} = \left(c_i t^{\delta_i - \delta^*} + \frac{p^o_{it}}{t^{\delta^*}}\right) t^{\delta^*} = b_{it}\theta_t, \text{ for } b_{it} = c_i t^{\delta_i - \delta^*} + p^o_{it} t^{-\delta^*}, \quad \theta_t = t^{\delta^*}. \tag{10}$$

*Hence when $\delta_i \neq \delta^*$, $b_i = 0$ for $\delta_i < \delta^*$ and $b_i = c_i$ for $\delta_i = \delta^*$. Meanwhile when $\delta_i = \delta = \delta^*$, $b_i = c_i$.*

### 4.1 Case of Convergence

Convergence occurs when $b_{it} \to b$. That is, $b_i = b$ and $\alpha_i \geq 0$ (or $\alpha_i^* > 0$) for all $i$ in (9). We rewrite (9) under convergence as

$$b_{it} = b + \xi_{it} t^{-\alpha_i^*} \text{ for any given time } t.$$

Return to the example (10). In this case, $\delta_i = \delta$, $\xi_{it} = p^o_{it}$ and $\alpha_i^* = \delta$ for all $i$. Then under convergence, all prices share the same trend so that we have

$$\pi_{it} = b_{it-1}\Delta\theta_t + \Delta b_{it}\theta_{t-1} = \pi_t^{core} + \Delta p^o_{it} + O_p\left(t^{-1}\right) \tag{11}$$

See Appendix B for detailed derivation of (11). Hence this case asymptotically becomes equivalent to homogeneous factor loading case in (4).

### 4.2 Case of Sub-Convergence

Now we consider the case where there is more than one sub-convergent club. Let

$$p_{it} = \begin{cases} b_{1,it}\theta_{1t} \text{ for } b_{1,it} \to b_1, \ i \in G_1 \\ \vdots \\ b_{m,it}\theta_{mt} \text{ for } b_{m,it} \to b_m, \ i \in G_m \end{cases}. \tag{12}$$

Note that we can obtain the limit result in (12) from (8) by setting $b_{r,it} \to 0$ if $i \notin G_r$. It is important to note that this model does not necessarily imply that the number of sub-clubs should

[14] We can also permit $\rho = 1$. When $\rho = 1$, the model can be rewritten as $p_{it} = c_i t^{\delta_i} + O_p\left(t^{1/2}\right) + \epsilon_{it}$ where we decompose $p^o_{it}$ as the random walk and stationary part (for an example via Beveridge Nelson Decompostion), and assume that the random walk part can be rewritten as $O_p\left(t^{1/2}\right)$. Since the first term, $t^{\delta_i}$, dominates the second term, the example may nest the unit root case of $p^o_{it}$.

be equal to the number of common factors.[15] Let $N_r$ be the total number of members in each club $G_r$. As $T$ grows large the $N$ individual price indices converge to $m$ different trends, $\theta^*_{rt} = b_r\theta_{rt}$ where $b_r \sim \left(b, \sigma^2_b\right)$ and $\theta_{rt} \sim \left(\theta_t, \sigma^2_{\theta r}\right)$.

We now apply the time-varying factor model to the disaggregate price data.

## 4.3 Empirical Application

We now estimate the number of stochastic trends underlying PCE disaggregate prices using the techniques and methods outlined in the previous subsections.

### 4.3.1 Data

Our data set is a panel of 70 annual PCE price indices spanning 1933 through to 2008. Finer levels of disaggregation are possible, but these only begin in 1959, and a longer time series is required to overcome the "base year problem" discussed below.[16] This also permits us to avoid computational problems that arise when a new price index is introduced to PCE, since all 70 indices span the entire 1933 to 2008 time period. The selected PCE components used in our analysis are comprised of 17 durables, 13 nondurables, and 40 services.[17] See the appendix for a full description of the 70 PCE items. Data was obtained from NIPA Table 2.4.4 at the BEA website (www.bea.gov).

### 4.3.2 Convergence Test and Clustering Analysis

For instructive purposes we review Phillips and Sul's (2007) convergence test and clustering algorithm. The so-called "log $t$ - regression" relies on the assumption that the time varying factor loadings can be decomposed as in (9). Then the null hypothesis of convergence is formulated as

$$H_0 : b_i = b \;\text{ and }\; \alpha_i \geq 0, \tag{13}$$

against the alternative hypothesis of

$$H_A : \{b_i = b \;\text{ for all } i \;\text{ with } \alpha_i < 0 \;\} \text{ or } \;\{b_i \neq b \;\text{ for some } i \;\text{ with } \alpha_i \geq 0 \;\text{ or } \alpha_i < 0\}. \tag{14}$$

[15] Suppose that $m = 2$. Then there are possibly three convergent subclubs. To see this, let $b_{1it} \to b_1$ but $b_{2,it} \to 0$ if $i \in G_1$, $b_{1,it} \to 0$ but $b_{2,it} \to b_2$ if $i \in G_2$, and $b_{1,it} \to b_1$ and $b_{2,it} \to b_2$ if $i \in G_3$. Hence there are three convergent clubs but there are only two common factors.

[16] While the majority of PCE items contained in our 70 item set are the most detailed type of product available in the NIPA tables, some higher-level components are included due to this data availability constraint. For example, our data includes "garments" because data on further detail on this category, such as "women's and girls' clothing," "men's and boy's clothing," and "children's and infants' clothing," are not available before 1959.

[17] For alternative levels of disaggregation of the PCE price index data, see Clark (2006). Note that PCE components change periodically, and hence the number of PCE components at the highest level of disaggregation differ across studies.

Phillips and Sul (2007) use the following to test the null hypothesis of convergence:

$$H_t := N^{-1} \sum_{i=1}^{N} (h_{it} - 1)^2, \tag{15}$$

where

$$h_{it} := \frac{p_{it}}{N^{-1} \sum_{i=1}^{N} p_{it}} = \frac{b_{it}}{N^{-1} \sum_{i=1}^{N} b_{it}}, \tag{16}$$

which measures the transition element for individual good $i$ relative to the cross section average. Under the null, the transition distance $H_t$ has the limiting form of $H_t \sim A \left[L(t)^2 t^{2\alpha_i}\right]^{-1}$ as $t \to \infty$. By setting $L(t) = \log t$ in equation (9), we can obtain the following regression

$$\log \frac{H_1}{H_t} - 2\log(\log t) = a + \gamma \log t + u_t, \quad \text{for} \quad t = rT, rT+1, \cdots, T, \tag{17}$$

where the null and alternative hypotheses in (13) and (14) can be transformed into

$$H_0 : \gamma \geq 0, \text{ against } H_A : \gamma < 0.$$

Several points regarding the properties of the $\log t$ regression model (17) are worth pointing out. First, the true value of $\gamma$ under $H_0$ is $2\alpha$. Second, the second term in the right hand side in (17), which is $-2\log(\log t)$, acts as a penalty function. Third, a HAC estimator for the covariance should be used since the regression errors are serially correlated. Finally, only $(1 - r)$ fraction of the sample should be used for the regression. Phillips and Sul (2007) recommend setting $r = 0.3$.

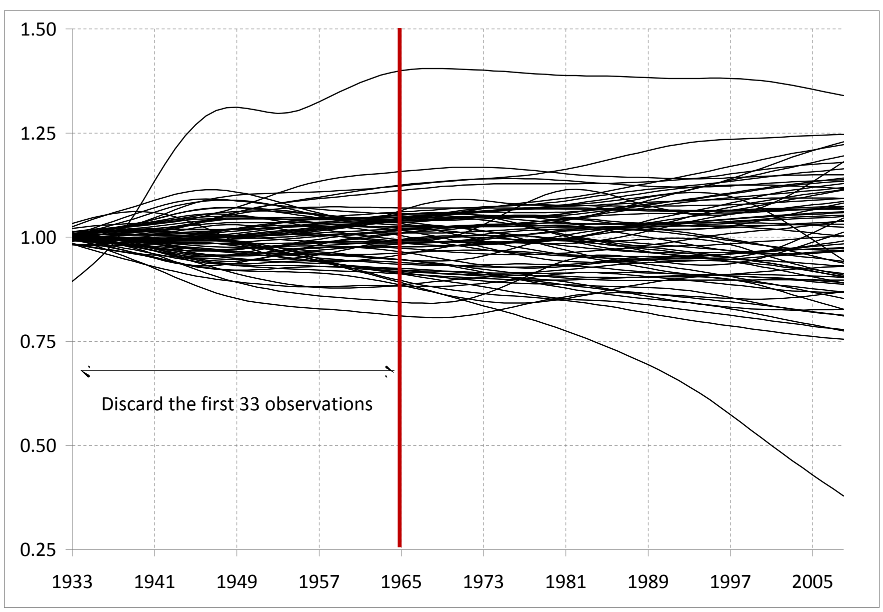



Figure 2: Base Year Effect[18]

[18] This figure plots relative transition curves for $h_{it} = p_{it} \left(N^{-1} \sum_{i=1}^{N} p_{it}\right)^{-1}$ where $p_{it}$ is the Hodrick-Prescott trend of log price index of PCE component $i$ at time $t$. The first 33 observations are discarded to control the base year

Price indices do not typically reflect price disparities between cross sections, but rather reflect price disparities for a given item between different time periods. Hence the Phillips and Sul (2007) convergence tests are subject to a base year problem: If the base year is 2008, then all price series converge to 100 by construction. The problem can be overcome by taking the initial year 1933 as the base year so that all series initially diverge. Figure 2 shows that relative transition curves, $h_{it}$, after filtering out the business cycle components using the Hodrick-Prescott filter. Evidently the base year effect is negligible after 1965. To avoid the base year effect we therefore discard the first 33 observations.

Next, as a robustness check, we further discard the initial $r$ fraction of the sample and test for overall convergence. Table 3 presents the $\log t$ test results using a range of $r \in (0.15, 0.35)$. For $r = 0.15$, the sample begins at 1972 whereas for $r = 0.35$ the sample begins at 1980.

Table 3: The effect of $r$ on overall convergence test

| | 1972 ($r = 0.15$) | 1973 ($r = 0.18$) | 1974 ($r = 0.20$) | 1976 ($r = 0.25$) | 1978 ($r = 0.30$) | 1979 ($r = 0.32$) | 1980 ($r = 0.35$) |
|---|---|---|---|---|---|---|---|
| $\hat{\gamma}$ | -1.25 | -1.28 | -1.31 | -1.39 | -1.46 | -1.51 | -1.59 |
| $t_{\hat{\gamma}}$ | -42.78 | -56.68 | -64.39 | -55.97 | -42.50 | -38.03 | -32.93 |

Evidently, the convergence is strongly rejected regardless of the value of $r$. This is strong evidence against the single component model for inflation.

The empirical evidence in Table 3 shows that the cross sectional variance of price indices are increasing over time. Figure 3 confirms this finding; both the cross-sectional variance of inflation (left scale) and relative convergence parameter $h_{it}$ (right scale) are increasing over time. However, a more important finding is that the rate of divergence was much faster after 1990 than that before 1990. In fact, during the high inflation period between 1970s and 1980s, both measures of cross sectional variance did not significantly increase. Thus, although aggregate variance was high over the 1970-80 period, the cross sectional variance in disaggregate inflation was low. In the past 20

effect. There is little evidence of overall convergence as relative transition curves do not display a marked reduction in their dispersion. Two distinctive paths are "net purchases of used motor vehicles" and "video, audio, photographic, and information processing equipment and media." The former initially deviates from other relative transition paths, but tends to move toward to the others, while the latter appears to be consistently diverging over time.

years the cross sectional variance has more than doubled for both measures.

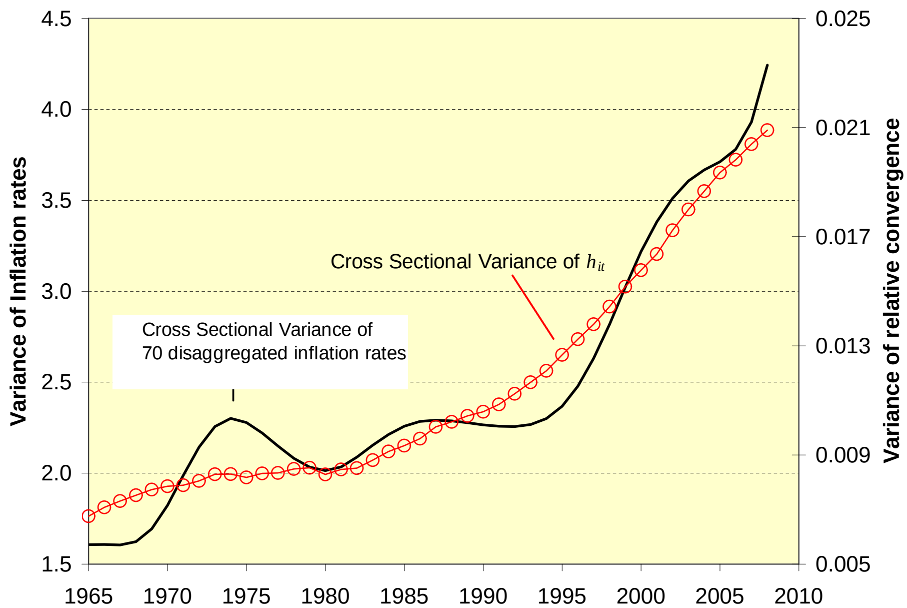



Figure 3: Cross Sectional Variances of Inflation Rates and $h_{it}$

In a recent paper, Stock and Watson (2007) decomposed headline inflation into permanent and transitory components, and demonstrated that by 2004 the variance of the permanent component had declined by about 80% from its peak in the inflationary 1970s. Our documented price divergence in Figure 3 is reconcilable with their findings. Consider the multi-component model for inflation, $\pi_{it} = \pi_{rt}^{core} + \pi_{it}^{o}$. In earlier periods, all items shared the same persistent component in prices, that is $\pi_{rt}^{core} = \pi_{t}^{core}$ for all $r$. Equivalently $\pi_{rt}^{core}$ are perfectly correlated. However, beginning in the early 1980s we observe price divergence caused by the $\pi_{rt}^{core}$ behaving independently from each other. Now when we take a mean across the items, such as $\sum_{i=1}^{N} w_{it}\pi_{it}$, the persistent component of the mean becomes $\sum_{r=1}^{m}\sum_{i\in G_r} w_{it}\pi_{rt}^{core}$. As $\pi_{rt}^{core}$ become less correlated, the low frequency variance of $\sum_{i=1}^{N} w_{it}\pi_{it}$ decreases (all else being equal), which would give rise to the findings of Stock and Watson (2007).

### 4.3.3 Clustering Analysis

Since the log $t$ convergence test will reject the null of convergence in the presence of only one divergent series, if we find in favor of the alternative there could exist subgroups that may converge within each group. Phillips and Sul (2007) propose a clustering procedure that involves the stepwise application of log $t$ regression tests. The clustering procedure differs from previous studies on clustering methods, such as Durlauf and Johnson (1995) and Hobijn and Franses (2000), in the sense that their algorithm focuses on how idiosyncratic transitions behave over time in relation to the common factor component. By analyzing subgroup-convergent behavior among the idiosyncratic transition coefficients $b_{it}$, one may locate the sources of divergence in the entire panel.

Table 4: Convergence Club Classifications

| Initial classification | | Tests of club merging $\hat{\gamma}$ $(t_{\hat{\gamma}})$ | | | | Final classification | |
|---|---|---|---|---|---|---|---|
| Club 1 [25] | 0.06 | | | | | Club 1 [25] | 0.06 |
| | (4.04) | Club 1+2 | | | | | (4.04) |
| Club 2 [15] | 0.32 | -0.38 | | | | Club 2 [15] | 0.32 |
| | (9.09) | (-62.33)* | Club 2+3 | | | | (9.09) |
| Club 3 [10] | 0.76 | | -0.45 | | | Club 3 [19] | 0.23 |
| | (8.65) | | (-5.77)* | Club 3+4 | | | (5.95) |
| Club 4 [9] | 1.37 | | | 0.23 | | | |
| | (3.75) | | | (5.95) | Club 4+5 | | |
| Club 5 [7] | 0.08 | | | | -0.42 | Club 4 [7] | 0.08 |
| | (3.91) | | | | (-13.44)* | | (3.91) |
| Group 6 [4] | -3.25 | | | | | Group 5 [4] | -3.25 |
| | (-86.65) | | | | | | (-86.65) |

Using the clustering algorithm we find that, as the initial classification presented in Table 4, there are five convergent clubs and one divergent group. The automatic clustering method is based on relative convergence among price indices, that is $\alpha^* > 0$. If $\delta > 1$ in (10), it is possible that the cross sectional variance of $b_{it}$ is decreasing but that of individual inflation rates within each club can be increasing. Hence we first examine whether the cross sectional variance of individual inflation rates for each subgroup is increasing or not. For all clubs except for Club 3, the cross sectional variances of individual inflation rates are not increasing over time. For Club 3, the cross sectional variance of individual inflation rates is increasing due to the price index of "telephone and facsimile equipment". Hence we move this item to the divergent group 6.

The Phillips and Sul (2007) clustering algorithm is a very conservative classification for club membership (see, e.g., Phillips and Sul, 2009), we apply tests of club merging used in Phillips and Sul (2009) to determine whether any of the initial subgroups can be merged to form a larger size of convergence club. We consider adjacent subgroups in the original classification and report estimated slope coefficients of $\log t$ regression and corresponding $t-$statistics in Table 4. There appears to be one merger of the original subgroups; the merger of club 3 and 4. Based on our initial club classification and club merging test results, we report the final club classification in Table 4. There are four convergence clubs and one divergent group. For all convergence clubs the point estimates of $\gamma$ are significantly less than zero.

Table 5 displays club membership by major product category. The first convergence club encompasses 25 PCE components. The vast majority (20) are services; including all 5 "health care" sub-categories, 5 out of 6 "financial services and insurance" sub-categories, and all 3 sub-categories of "educational services". Two members of the first club are durables (net purchase of used motor vehicles and educational books), while the remaining 3 members are nondurables (fuel oil and other fuels, pharmaceutical and other medical products, and tobacco). Thus the first club is dominated by health care goods and services, financial services and insurance, and educational good and services. Notably it also includes fuel oil.

Table 5: Club Membership by Major Product Category

| | all | club 1 | club 2 | club 3 | club 4 | group 5 |
|---|---|---|---|---|---|---|
| Durable goods | 17 | 2 | 0 | 7 | 4 | 4 |
| Nondurable goods | 13 | 3 | 3 | 6 | 1 | 0 |
| Services | 40 | 20 | 12 | 6 | 2 | 0 |
| total | 70 | 25 | 15 | 19 | 7 | 4 |

The second convergence club consists of 15 components; 12 service products and 3 nondurables (food and nonalcoholic beverages, purchased for off-premises consumption; motor vehicle fuels, lubricants, and fluids; magazines, newspapers, and stationery). All four "housing" sub-categories, four of the five "recreational services" sub-categories, and both "food services" subcategories fall into club two. Thus the major service components of Club 2 include "housing," "recreational services," and "food services."

About a half of all durables (7 out of 17 "durable" sub-categories) and nondurables (6 out of 13 "nondurable" sub-categories) fall into the third convergence club, along with six service components. The third convergence club thus differs with the two initial clubs in that it includes more imported goods as well as goods and services regulated by government (e.g., electricity and air transportation).

Club 4 includes 7 components; 4 durable, 1 nondurable, and 2 service components. In terms of PCE price index weight, "garments" is the most important member in this club. It also includes "water transportation" and "telecommunication services."

Finally, Group 5 includes three technology items: (i) "telephone and facsimile equipment," (ii) "household appliances," and (iii) "video, audio, photographic, and information processing equipment." These items are known to suffer from significant price measurement problems attributable to frequent and large changes in product quality (see, for example, Triplett, 2006).[19]

[19] The remaining item, "sporting equipment, supplies, guns, and ammunition," is not conventionally thought of as being subject to the same measurement issues. It may be somewhat of an anomaly.

# 5 Constructing a New Measure of Core Inflation

In this section we discuss how to construct a new measure of core inflation based on the convergence clubs created in the previous section. We use a limited influence estimator for each of the four convergence clubs in order to estimate the persistent component for each club. However, there are several limited influence estimators available, such as the median, symmetric trimmed mean and asymmetric trimmed mean, and in some cases, such as the general trimmed means, estimator parameters must be specified. At this stage it is unclear what kind of limited influence estimator should be used for each of the clubs. In order to address this question it is instructive to first consider in more detail how the limited influence estimators are used in practice.

## 5.1 Extant Core Inflation Measures

Motivated by the observation that the distribution of price changes was often skewed, Bryan and Pike (1991) and Bryan and Cecchetti (1994) proposed the median as an estimator of core inflation. While the median gives a less-biased estimate of the mode of a unimodal distribution than the arithmetic average, it does not necessarily have the smallest variance amongst the family of limited influence estimators. Bryan, Cecchetti and Wiggins (1997) propose a more general trimmed mean estimator which is less affected by both skewness and leptokurtosis than the arithmetic average, and thus this estimator may be more efficient than the median. In this sub-section we give a brief but detailed overview of the weighted asymmetric trimmed mean, whereby the weights are given by the proportion of total expenditure allocated to the item. The asymmetric trimmed mean nests both the median and symmetric trimmed mean as special cases.

Let the inflation rates $\pi_{it}$ for $i = 1, \ldots, N$ be ordered such that $\pi_{1t} < \pi_{2t} < \ldots < \pi_{Nt}$ for a given $t$. The index $i = 1$ thus corresponds to the item with the smallest inflation rate in period $t$, $i = 2$ corresponds to the item with the second smallest inflation rate in period $t$, and so-on. (Note that the ordering of the items in the index $i$ thus changes every period.) Let $\hat{\imath}_t(\alpha) := \min\left\{I : \sum_{i=1}^{I} w_{i,t} \geq \alpha\right\}$, where $\alpha \in [0, 1/2]$ is a truncation point for the the tail in the left hand side of a probability density. That is, $\hat{\imath}_t(\alpha)$ is the smallest index $I$ such that the sum of the weights $w_{it}$ over $i = 1$ to $i = I$ is greater than $\alpha$. (Note we have implicitly assumed that $w_{it} \in [0, 1]$ for all $i$ and $t$, and that $\sum_{i=1}^{N} w_{i,t} = 1$.) We then similarly define $\hat{\imath}_t(\beta) := \max\left\{I : \sum_{i=0}^{I-1} w_{N-i,t} \leq \beta\right\}$, where $\beta \in [0, 1/2]$ is a truncation point for the right-hand tail of the density. The weighted asymmetric trimmed mean is then

$$\pi_t(\alpha, \beta) := \frac{1}{1 - \alpha - \beta} \sum_{i=\hat{\imath}_t(\alpha)}^{i=\hat{\imath}_t(\beta)} w_{it}\pi_{it}.$$

The weights $w_{it}$ reflect the share of the $i$th item in the consumption basket in period $t$. For example, these could be fixed weights (as in Bryan and Cecchetti, 1994), or they could be expenditure weights.

For example, in Dolmas (2005) $w_{it}$ are approximations to the Fisher ideal weights, namely

$$w_{it+1} = \frac{1}{2}\frac{Q_{i,t}P_{i,t}}{\sum_i Q_{i,t}P_{i,t}} + \frac{1}{2}\frac{Q_{i,t+1}P_{i,t}}{\sum_i Q_{i,t+1}P_{i,t}}$$

Here $P_{i,t}$ is the price index of the $i$th item, and $Q_{i,t}$ is the quantity of the $i$th item, at time $t$. ($Q_{i,t}P_{i,t}$ is thus the final demand for the $i$th good at time $t$.) The symmetric trimmed mean is nested by the above by setting $\alpha = \beta$, while the weighted median is obtained by setting $\alpha = \beta = 0.5$. A PCE asymmetric trimmed mean is reported by the Dallas FRB as a measure of core inflation, while the FRB Cleveland reports a weighted median CPI-U. Bryan, Cecchetti and Wiggins (1997) study both a weighted symmetric trimmed mean and a weighted median.

For the asymmetric trimmed mean the cut-offs $\alpha$ and $\beta$ must be specified. In practice, this is achieved by choosing $(\alpha, \beta)$ to minimize the mean square error (MSE) between the trimmed mean and an ex-post measure of core inflation (see, e.g., Bryan, Cecchetti and Wiggins, 1997; Dolmas, 2005; Smith, 2004). As the prefix "ex-post" suggests, the measure of core inflation is only available long after the reference month. For example, Bryan, Cecchetti and Wiggins (1997) use a 36 month centered moving average of headline inflation to chose the truncation points. Thus, $\alpha$ and $\beta$ are defined as

$$\left(\hat{\alpha}, \hat{\beta}\right) = \arg\min_{\alpha,\beta} \textstyle\sum_{t=1}^{T} \left(\pi_t\left(\alpha, \beta\right) - \bar{\pi}_t^{core\ ex-post}\right)^2,$$

where $\bar{\pi}_t^{core\ ex-post}$ is the ex-post measure of core inflation. Bryan, Cecchetti and Wiggins (1997) find that a 9% symmetric trimmed mean has the smallest MSE for 36 CPI-U CPI categories. Other ex-post measures of core inflation have been used in practice in addition to the 36 month centered moving average. For example, Dolmas (2005) considers various measures of ex-post core inflation, including a 24 month forward moving average of headline PCE inflation and a band-pass filter of headline PCE inflation.

Unweighted trimmed means are also used in practice, such as the FRB Cleveland 16% symmetric trimmed mean CPI-U. In this case the truncation frequencies $(\alpha, \beta)$ are truncation points in the density of the price changes $\{\pi_{it}\}_{i=1}^{N}$ themselves, and thus are independent of the budget share weights. In what follows we largely ignore these unweighted trimmed means because they tend to perform worse than weighted trimmed means across a range of performance measures.

## 5.2 A New Measure

We follow Bryan, Cecchetti and Wiggins (1997), Dolmas (2005), and Smith (2004) in that we will construct a core inflation measure by choosing truncation points $(\alpha, \beta)$ that minimize the MSE of the measure relative to an ex-post measure of core inflation. For our new core inflation measure we must pick both the truncation points for the trimmed mean of each club as well as the weighting received by each club. Let

$$\pi_t\left(\{\alpha_r\}_{r=1}^m, \{\beta_r\}_{r=1}^m, \{\eta_r\}_{r=1}^m\right) = \sum_{r=1}^m \eta_r \sum_{i=\hat{\imath}_t(\alpha_r)}^{i=\hat{\imath}_t(\beta_r)} w_{it}(r)\, \pi_{it}(r)$$

denote the general new core inflation measure, where $m$ denotes the number of clubs (in our case, we have four clubs), and $(\alpha_r, \beta_r)$ denote the truncation points for the lower and upper tail for a weighted trimmed mean for the $r$th club. The club-weights $\{\eta_r\}_{r=1}^m$ are constrained to add to one and $\eta_r \in (0,1)$ for each $r$. Meanwhile the expenditure-weights for each club are defined as

$$w_{it+1}(r) := \frac{1}{2}\frac{Q_{i,t}P_{i,t}}{\sum_{i\in G_r} Q_{i,t}P_{i,t}} + \frac{1}{2}\frac{Q_{i,t+1}P_{i,t}}{\sum_{i\in G_r} Q_{i,t+1}P_{i,t}},\ i \in G_r$$

such that the weights sum to one within each club. Last, within each club, the truncation point is defined as $\hat{\imath}_t(\alpha_r) := \min\left\{I_r : \sum_{i=1}^{I_r} w_{i,t}(r) \geq \alpha\right\}$, where $w_{i,t}(r)$ denotes the expenditure weights ordered by inflation within each club, i.e. $\pi_{1t}(r) \leq \pi_{2t}(r) \leq \ldots \leq \pi_{N_r t}(r)$. Note that within each club, $i$ indexes the inflation rates from smallest to highest values.

In order to attain club memberships we used annual data from 70 disaggregate PCE items spanning 1933-2008. As discussed earlier, we use the relatively low level of disaggregation (70 items) because it is only at this level of disaggregation that we can attain a panel with a sufficiently long time series dimension with which to test for convergence over time. However, we need not constrain ourselves to this level of disaggregation when constructing a core inflation measure for the 1984-2009 period. We use the most disaggregate level of PCE available, comprised of 182 separate items. Each item is allocated to the club from which the higher level aggregation item has been allocated. For example, the 70-item level item "Food and nonalcoholic beverages purchased for off-premises consumption," which is a club 2 member, is split into "cereals," "bakery products," "beef and veal," etc., and all these sub-items are allocated to club 2.

We use monthly data spanning January 1984 to December 2009. We focus on the 1984-2009 period because Stock and Watson (SW, 2007) demonstrate that there has been a marked reduction in the volatility of the transitory component in headline inflation since 1984. SW argue that this explains the documented failure of multivariate models to accurately forecast headline inflation. (In the 1984 to 2004 period a simple random walk forecast of headline inflation was the most accurate at horizons of one year or more.) We use the monthly sampling frequency as all extant core inflation measures are available at a monthly frequency in order to provide policy-makers in a timely manner.

Table 6: Calibrated parameters of new core inflation

| | club 1 (55 items) | club 2 (59 items) | club 3 (47 items) | club 4 (12 items) |
|---|---|---|---|---|
| upper tail truncation ($\alpha_r$) | 0.175 | 0.275 | 0.275 | 0.200 |
| lower tail truncation ($\beta_r$) | 0.325 | 0.500 | 0.300 | 0.150 |
| club weight ($\eta_r$) | 0.378 | 0.471 | 0.137 | 0.014 |

Calibration to 36 month centered MA of PCE inflation over 1984-2009 using 182 PCE items

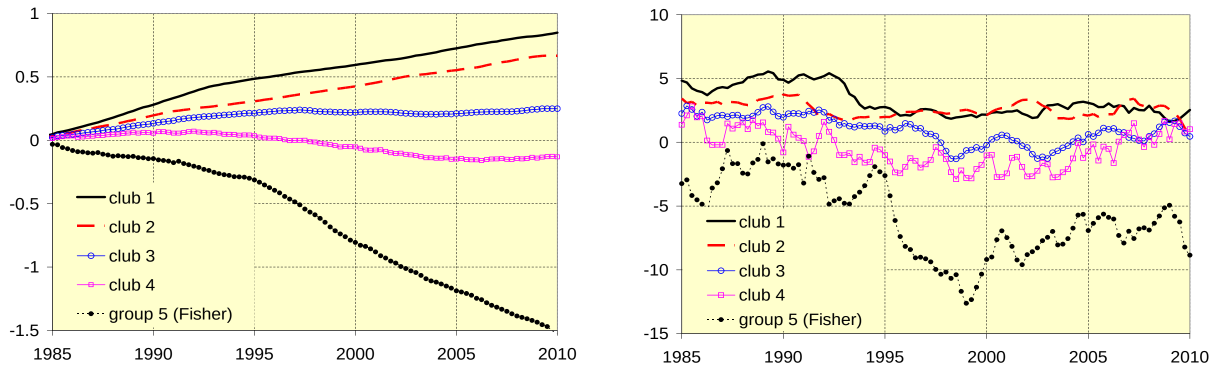
Non-convergent group 5 contains 9 items

Table 6 gives the truncation points and club-weightings of our core. Of course, as elsewhere in the literature we must choose a candidate ex-post measure of core inflation in order to choose these parameters. For the measures reported herein, we will use a 36 month centered moving average for this purpose. Bryan, Cecchetti and Wiggins (1997) and Dolmas (2005) use the same measure.

In Figure 4 below we exhibit the trimmed means for each of the four convergent clubs, as well as an ideal Fisher index for group 5. Panel B shows the 12 month inflation rates within each club, while panel A shows the log price level within each club as implied by the trimmed means. We have normalized each log-price level to be zero in December 1984. Clubs three, four and group five in fact go through varying amounts of deflation in some part of the period after 1990. Group 5, which consists of largely of technology goods, experience a large decrease in the price level beginning in the early 90s. Notably club 4 has the most volatile within-club core inflation rate, yet this may be due to the fact that there are few items within this club (12) compared to the other three clubs. Club 1 and 2 have the core inflation measure with the least volatility.Table 7 exhibits the sample correlation of monthly inflation rates across the five groups before and after 1994. The cross sectional dependence between the each of the convergence club inflation rates has decreased significantly, corroborating the divergence in prices between groups exhibited in Figure 4. Each pairwise correlation between the 4 groups is smaller in the more recent 1995-2009 period. Meanwhile the correlations between group 5 and clubs 1 and 2 have increased in the more recent period, while the correlations between group 5 and clubs 3 and 4 have decreased.

To obtain our new core inflation measure we take a weighted average of the four trimmed means exhibited in panel B of Figure 4 above, where the weights are given in Table 6. The figure below exhibits the 12 month new core inflation measure alongside the extant core inflation measures discussed in section 2 above.

The new core inflation measure generally lies above PCE inflation excluding food and energy, but below the limited influence estimators. In the following section we will evaluate the accuracy of these and other core inflation measures.




Panel A: normalized log price-levels (Dec 1984 = 0) Panel B: 12mth within club core inflation

Figure 4: Prices and Inflation Rates within Convergence Clubs

Table 7: Correlation between club inflation rates

| | Club 2 | Club 3 | Club 4 | Group 5 |
|---|---|---|---|---|
| | | 1984-1994 | | |
| Club 1 | 0.317 | 0.306 | 0.101 | 0.064 |
| Club 2 | | 0.151 | 0.128 | 0.067 |
| Club 3 | | | 0.255 | 0.228 |
| Club 4 | | | | 0.157 |
| | | 1995-2009 | | |
| Club 1 | -0.036 | 0.024 | 0.090 | 0.143 |
| Club 2 | | -0.070 | 0.053 | 0.112 |
| Club 3 | | | 0.116 | 0.095 |
| Club 4 | | | | 0.074 |

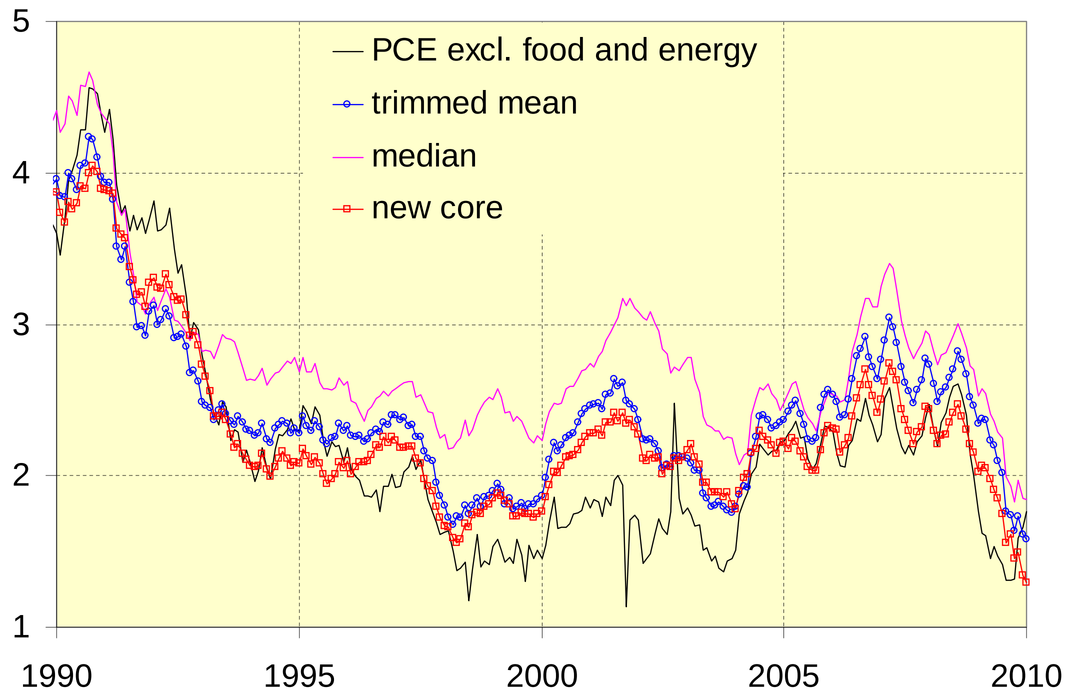



Figure 5: 12 month core inflation rates

## 6 Evaluating Core Inflation Measures

By definition a successful measure of core inflation must be a good proxy for the underlying rate of inflation in prices that is expected to persist over a medium-term horizon (Bryan and Cecchetti,1994; Blinder, 1997). Thus a commonly adopted method for evaluating different core inflation measures is to assess which measure can best predict low frequency movements in headline inflation in practice (see, e.g., Bryan, Cecchetti and Wiggins, 1997; Smith, 2004; Dolmas, 2005). We provide two sets of evaluations. First, we compare the core inflation measures to a 36 month centered average of headline inflation, which is the ex-post measure of core inflation used by Bryan, Cecchetti and Wiggins (1997). Second, we investigate which of the core inflation measures can produce the best out-of-sample forecast of headline inflation over long horizons (between 6 and 60 months). The latter exercise is used by Atkeson and Ohanian (2001), Stock and Watson (2007), and Smith (2004) in order to evaluate various headline inflation forecasts.

We consider the following set of core inflation measures: PCE excluding food and energy, weighted median PCE, the FRB Dallas trimmed mean, and our new core inflation measure. We also construct a weighted symmetric and an asymmetric trimmed mean PCE, with the truncation points calibrated to minimize the MSE to the 36 month centered MA of headline PCE inflation over 1984-2009. As a benchmark we also consider all items PCE inflation as a predictor.

## 6.1 In-sample fit

In the table below we give the mean, variance and MSE of the considered monthly core inflation measures over the 1984-2009 period.

Table 8: In-sample summary of core inflation measures, 1984-2009

| | | | | | | published measures | |
|---|---|---|---|---|---|---|---|
| | PCE | expenditure-weighted | | | | FRB Dallas | BEA |
| | inflation | new[1] | ATrim[1] | STrim[1] | Med | ATrim | PCE ex. |
| mean | 2.57 | **2.57** | 2.58 | 2.99 | 3.03 | 2.69 | 2.58 |
| variance | 5.63 | 0.91 | **0.85** | 0.93 | 0.97 | 0.93 | 2.34 |
| MSE | 3.70 | **0.39** | 0.42 | 0.59 | 0.76 | 0.47 | 1.50 |

[1] measures calibrated to 36 month centered moving average of PCE headline inflation
"ATrim" = asymmetric trim-mean, "STrim" = symmetric trim-mean, "Med" = Median
"PCE ex." = PCE excluding food and energy, "new" is our new measure of core inflation

Amongst the measures calibrated to the 36 centered moving average, the new core inflation measure exhibits the lowest MSE. (It also exhibits the lowest MSE when compared to the other core inflation measures, but this is not surprising given that the other measures are not calibrated to minimize the MSE.) This is despite the fact that it has higher variance than another measure, specifically the asymmetric trimmed mean.[20] The asymmetric trimmed mean has a lower MSE than the symmetric trimmed mean; by construction, its MSE cannot be larger since it nests the symmetric mean. Similarly, the symmetric trimmed mean has a lower MSE than the weighted median.

[20] There is evidence that there is a structrual break in the level of PCE inflation at some point in the early 1990s, and if we split the sample at the break the new core inflation measure has the lowest variance within each of the sub-periods. The Bai and Perron (1998) test for a structural break in the mean of PCE suggests there was a single break in the 1983-2009 series of monthly PCE inflation. The point estimate of the break is October 1990, with a 90% confidence interval of July 1987 to September 1992. To gain a more accurate estimate of the break date, we run the test on PCE excluding food and energy inflation, and find evdeince for a single break at April 1992, with a 90% confidence interval of January 1992 to August 1992. We therefore chose to split the sample at 1993.

### 6.2 Pseudo out-of-sample forecasting

We now turn to the issue of which core inflation measure performs best in terms of out-of-sample forecasting of headline PCE inflation. As discussed in Stock and Watson (2007), the random walk forecast of Atkeson and Ohanian (2001) beats various multivariate and univariate forecasts at horizons in excess of one year. Hence we too will base our forecast on the Atkeson-Ohanian Stock-Watson (AO-SW) specification, given by

$$\hat{\pi}_{t+h|t} = \frac{1}{12}\sum_{s=1}^{12} \hat{\pi}_{t-s+1}^{core}.$$

Thus the forecast of headline inflation over the next $h$ months, $\hat{\pi}_{t+h|t}$, is a 12-month backwards moving average of the given core inflation measure $\hat{\pi}_t^{core}$.[21]

Table 9: Out-of-sample MSFE of core inflation measures 1994-2009

| | | expenditure weighted | | | | published measures | |
|---|---|---|---|---|---|---|---|
| | PCE | | | | | FRB Dallas | BEA |
| $h$ | inflation | new[1] | ATrim[1] | STrim[1] | Med. | ATrim | PCE ex. |
| 6 | 2.74 | **1.91** | 1.92 | 2.19 | 2.20 | 1.93 | 1.92 |
| 12 | 1.68 | **0.99** | 1.02 | 1.29 | 1.29 | 1.03 | 1.01 |
| 18 | 0.97 | **0.53** | 0.56 | 0.76 | 0.81 | 0.58 | 0.57 |
| 24 | 0.64 | **0.40** | 0.42 | 0.56 | 0.67 | 0.45 | 0.44 |
| 36 | 0.55 | **0.31** | 0.32 | 0.43 | 0.54 | 0.34 | 0.38 |
| 48 | 0.53 | **0.24** | 0.26 | 0.36 | 0.43 | 0.27 | 0.39 |
| 60 | 0.49 | **0.21** | 0.24 | 0.33 | 0.39 | 0.24 | 0.41 |

[1] measures calibrated to 36 month centered moving average of PCE headline inflation

"ATrim" = asymmetric trimmed mean, "STrim" = symmetric trimmed mean, "Med" = Median,

"PCE ex." = PCE excluding food and energy, "new" is our new measure of core inflation

The optimal truncations are determined using observable data at the time the forecast is made, and forecasts begin in December 1993 (so that the initial calibration is based on 10 years of data spanning 1984-1993). To maintain computational efficiency, the calibration parameters are re-calculated every December (feasibly the weights can be recomputed every month).

The Table 9 exhibits the mean square forecasting error (MSFE) of the AO-SW RW forecasts for horizons between 6 and 60 months. Across all considered horizons the new core inflation measure exhibits the lowest MSFE. Notably the expenditure-weighted asymmetric trimmed mean (that we

[21] Stock and Watson (2007) use quarterly data, so their definition is slightly different, based on a 4-period backward moving average.

compute) performs second best, while the asymmetric trimmed mean of the FRB Dallas performs third best.[22]

## 7 Conclusion

A measure of core inflation that is simple and contains a great deal of information about persistent movements in inflation is extremely beneficial to the conduct of monetary policy. By focussing on core inflation instead of headline measures of inflation that are inherently noisy, policy-makers may prevent themselves from responding too strongly to transitory movements in inflation. In addition, as the popularity of inflation targeting has spread across economies, it becomes increasingly important to have an accurate measure of core inflation.

Conventional measures of core inflation, such as the weighted median, trimmed mean, and PCE inflation excluding food and energy, remove changes in particular prices to distinguish the persistent inflation signal from transient noise. The underlying assumption behind these conventional measures is that price changes that are relatively large in magnitude are likely to be transitory. Conversely price changes that located towards center of the distribution are more likely to be permanent. Thus an estimate of the central tendency of the distribution should be a better estimate of the persistent component in price changes.

Using the time-varying common factor model developed by Phillips and Sul (2007) we demonstrate that disaggregate prices levels have been diverging over time, and that this divergence can be attributed to a small set of sub-convergent clubs, whereby items within each club follow the same stochastic trend. This finding has stark implications for how we think about persistence and the distribution of disaggregate price changes in any given month. First, persistence in price changes will be associated with several locations in the support of the cross sectional distribution. Hence a measure of the central tendency of the distribution will be inaccurate. Second, divergence implies that different measures of core inflation will not converge over long time intervals, a phenomenon we in fact observe empirically. Based on the results of the clustering analysis, we propose a new measure of core inflation by taking a weighted average of an asymmetric trimmed mean for each convergence club. We find that this new measure of core inflation performs better than extant measures at forecasting future headline inflation, indicating that the new measure is a better estimate of core inflation than the extant estimates available.

[22] We also considered the univariate forecasting method used by Bryan and Cecchetti (1994), in which out-of-sample forecasts are obtained from a fitted long-horizon regression of $h-$period headline inflation $\pi_{t+h}$ on the lagged 12mth core inflation measure $\frac{1}{12}\sum_{s=1}^{12}\hat{\pi}_{t-s+1}^{core}$. For each core inflation measure these forecasts performed much worse than the SW-AO RW forecasts and hence we elected to omit them.

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# 8 Technical Appendix

## 8.1 Appendix A: Median and Trimmed Estimators

### 8.1.1 Single Component

Consider a single-component model of the form

$$\pi_{it} = \pi_t^{core} + \pi_{it}^o,\ \pi_{it}^o \sim i.d.\left(0, \sigma_i^2\right)$$

The trimmed mean can be expressed as a weighted mean as follows.

$$\pi^{trim} = \sum_{i=1}^N \omega_i\left(a, b\right)\pi_i^o + \pi_t^c$$

where

$$\omega_i\left(a, b\right) = \frac{f\left(a < \pi_i < b\right)}{\sum_{i=1}^N f\left(a < \pi_i < b\right)}$$

Here $a$ and $b$ are threshold values for lower and upper bounds of the tail, respectively. Then $\pi_i^o$ with higher variance has smaller probability so that we have

$$\omega_1\left(a, b\right) > \cdots > \omega_N\left(a, b\right) \Longleftrightarrow \sigma_1^2 < \cdots < \sigma_N^2.$$

Now consider symmetric trimed means so that $a = b$. Then it also follows that $\omega_1\left(a, a\right) < \omega_1\left(a+\varepsilon, a+\varepsilon\right)$ and $\omega_N\left(a, a\right) > \omega_N\left(a+\varepsilon, a+\varepsilon\right)$ for $\varepsilon > 0$. Then if $\sum_{i=1}^N \omega_i\left(a, b\right)\pi_i^o = 0$ we have

$$\pi^{a,b} := \sum_{i=1}^N \omega_i\left(a, b\right)\pi_i = \sum_{i=1}^N \omega_i\left(c, d\right)\pi_i =: \pi^{c,d}$$

for all permissible $a, b, c, d$.

### 8.1.2 Double Component

Assume that

$$\pi_{it} = \begin{cases} \pi_1^{core} + \pi_{it}^o \text{ if } i \in G_1 \\ \pi_2^{core} + \pi_{it}^o \text{ if } i \in G_2 \end{cases}$$

where we further assume that

$$\pi_{it}^o \sim iid\left(0, \sigma^2\right).$$

where $\pi_{it}^o$ has a symmetric distribution. Later we relax the homogenous variance to heterogeneous one. Suppose that $\pi_1^{core} \geq \pi_2^{core}$ and $N_1 \geq N_2$. For symmetric density truncations (i.e. $b = a$) the weights satisfy

$$\omega_i\left(a, a\right) \geq \frac{1}{N} \geq \omega_j\left(a, a\right),\ i \in G_1,\ j \in G_2$$

This means that $\pi^a := \sum_{i=1}^{N} \omega_i(a,a)\,\pi_i.$ satsifies $\pi^a \geq N^{-1}\sum_{i=1}^{N} \omega_i(a,a)\,\pi_i$. That is the trimmed mean is greater than the arithmetic mean. We also have

$$\omega_i(a,a) \leq \omega_i(a+\varepsilon, a+\varepsilon)\,,\ i \in G_2;\ \ \omega_i(a,a) \geq \omega_i(a+\varepsilon, a+\varepsilon)\,,\ i \in G_2,\ \varepsilon > 0$$

From the above it is clear that $\pi^{a+\varepsilon} \geq \pi^a$. This means that $\pi^{med} \geq \pi^a \geq \pi^{mean}$ for all $a = 0, ..., 0.5$.

### 8.2 Appendix B: Derivation of (5).

The following lemma will be used for the derivation.

**Lemma 1:** $t^\alpha - (t-1)^\alpha = O\left(t^{\alpha-1}\right)$

**Proof:** when $\alpha \geq 0$,

$$t^\alpha \geq (t-1)^\alpha$$

$$\frac{t^\alpha - (t-1)^\alpha}{t^{\alpha-1}} = \frac{t^\alpha}{t^{\alpha-1}} - \frac{(t-1)^\alpha}{t^{\alpha-1}} = t - \frac{(t-1)^\alpha}{t^{\alpha-1}} \leq t - \frac{t^\alpha}{t^{\alpha-1}} = t - t = 0$$

When $\alpha \leq 0$,

$$t^\alpha \leq (t-1)^\alpha$$

$$\frac{t^\alpha - (t-1)^\alpha}{(t-1)^{\alpha-1}} = \frac{t^\alpha}{(t-1)^{\alpha-1}} - \frac{(t-1)^\alpha}{(t-1)^{\alpha-1}} = \frac{t^\alpha}{(t-1)^{\alpha-1}} - (t-1) \leq (t-1) - (t-1) = 0.$$

Q.E.D.

Note that

$$\pi_{it} = b_{it-1}\Delta\theta_t + \Delta b_{it}\theta_{t-1}.$$

Now let $\theta_t = t^\delta$ and $b_{it} = b + p_{it}^o t^{-\delta}$. Then we have

$$b_{it-1}\Delta\theta_t = b\Delta\theta_t + p_{it-1}^o (t-1)^\delta \Delta\theta_t = \pi_t^c + p_{it-1}^o (t-1)^\delta \Delta\theta_t,$$

but from Lemma 1, we have

$$p_{it-1}^o (t-1)^\delta \Delta\theta_t = O_p(1)\, O\left(t^\delta\right) O\left(t^{\delta-1}\right) = O_p\left(t^{-1}\right)$$

Next,

$$\begin{aligned} \Delta b_{it}\theta_{t-1} &= \left[p_{it}^o t^{-\delta} - p_{it-1}^o (t-1)^{-\delta}\right](t-1)^\delta \\ &= \left\{p_{it}^o \left[t^{-\delta} - (t-1)^{-\delta}\right] + \Delta p_{it}^o (t-1)^{-\delta}\right\}(t-1)^\delta \\ &= p_{it}^o O\left(t^{-\delta-1}\right) O\left(t^\delta\right) + \Delta p_{it}^o \\ &= \Delta p_{it}^o + O_p\left(t^{-1}\right) \end{aligned}$$

Hence finally we have

$$\pi_{it} = \pi_t^{core} + \Delta p_{it}^o + O_p\left(t^{-1}\right).$$

### 8.3 PCE components and club memberships

Club 1: Net purchases of used motor vehicles; Educational books; Fuel oil and other fuels; Pharmaceutical and other medical products; Tobacco; Water supply and sanitation; Natural gas; Physician services; Dental services; Paramedical services; Hospitals; Nursing homes; Ground transportation; Accommodations; Financial services furnished without payment; Life insurance; Net household insurance; Net health insurance; Net motor vehicle and other transportation insurance; Postal and delivery services; Higher education; Nursery, elementary, and secondary schools; Commercial and vocational schools; Professional and other services; Household maintenance.

Club 2: Food and nonalcoholic beverages; Purchased for off-premises consumption; Motor vehicle fuels, lubricants, and fluids; Magazines, newspapers, and stationery; Rental of tenant-occupied nonfarm housing; Imputed rental of owner-occupied nonfarm housing; Rental value of farm dwellings; Group housing; Motor vehicle maintenance and repair; Membership clubs, sports centers, parks, theaters, and museums; Audio-video, photographic, and information processing equipment services; Gambling; Purchased meals and beverages; Food furnished to employees (including military); Personal care and clothing services; Social services and religious activities.

Club 3 (sub group A): Glassware, tableware, and household utensils; Recreational books; Therapeutic appliances and equipment; Food produced and consumed on farms; Other clothing materials and footwear; Household supplies; Other motor vehicle services; Other recreational services; Financial service charges, fees, and commissions; Foreign travel by US residents.

Club 3 (sub group A): New motor vehicles; Furniture and furnishings; Sports and recreational vehicles; Luggage and similar personal items; Alcoholic beverages purchased for off-premises consumption; Recreational items; Personal care products; Electricity; Air transportation.

Club 4: Motor vehicle parts and accessories; Tools and equipment for house and garden; Musical instruments; Jewelry and watches; Garments; Water transportation; Telecommunication services.

Group 5 (No Convergence): Household appliances; Video, audio, photographic, and information processing equipment; Sporting equipment, supplies, guns, and ammunition; Telephone and facsimile equipment.