Credit Cycles and Tightness of the Collateral Constraint

Min-Kyu Song
University of Texas at Austin

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Abstract

This paper investigates the way how collateral constraint can play as a persistence and amplification mechanism even when capital accumulation is not allowed. In particular, I modify Kiyotaki and Moore (1997) to have a look of standard real business cycle models, introducing debt-collateral ratio which is exogenous. Around stable steady states, I show monotonic convergence, hump-shaped relation between persistence and debt-collateral ratio and positive relation between amplification and the ratio; implying trade-off relation between persistence and amplification when debt-collateral ratio gets near to unity. This model can generate relatively flexible persistence to other related literatures, suggesting to focus on persistence measure rather than amplification, when we study actual data involved with collateral constraints. For an application, I study the residential land usage in Korea. Assuming that there is no serial correlation of exogenous shock, I find debt-collateral ratio involved with land usage in Korea; which would be larger than 0.8.

1 Introduction

Typical business cycle models rely on the capital accumulation for exogenous shocks to have amplified and persistent effects. It is not a good idea to apply such a mechanism to movements associated with some types of capital which take very long time to be build up or stay at a relatively constant level over time. Kiyotaki and Moore (1997) suggested a plausible clue of an alternative mechanism not depending on capital accumulation. Their mechanism can generate amplification and persistence via credit-market frictions; they call the cycles by the mechanism "credit cycles," telling from "business cycles" by capital accumulation. Specifically, when debts are fully secured by collateral and the collateral is also a production factor, a small shock to the economy can have amplified and persistent effects. The drastic results of Kiyotaki and Moore (1997) inspired a lot of theoretical researches; Kiyotaki (1998), Kocherlakota (2000), Caballero and Krishnamurthy (2001), Paasche (2001) and Cordoba and Ripoll (2004). However, the theoretical models use some extreme assumptions to get enough amplification and persistence; for instance, Kiyotaki and Moore (1997) use linear technology and preferences.

Cordoba and Ripoll (2004) try to make it closer to the standard real business cycle model improving upon the linearities and show that a credit cycle model needs a drastic parameter specification to have stability around the steady states and persistence and amplification similar to the typical real business cycle models. Setting aside persistence and amplification, their model still has room for improvement. First, the financial transactions are only between agents with production technologies (producers); lenders as well as borrowers are producers but it is hard to see any big portion of firms that are net lenders in a macroeconomy. Second, in equilibrium, low productive firms survive because they don't have a credit constraint; it is hard to believe low productive firms have financial advantages over high firms in reality. Third, to generate amplification, the shock should show relative difference in the productions between the two types; the shock is not such an aggregate shock as in the standard real business cycle models. The model of this

project is compared (in equilibrium) with Kiyotaki and Moore (1997) and Cordoba and Ripoll (2004) in Table 1.

Table 1: Literatures

	This paper	Kiyotaki and Moore(1997)	Cordoba and Ripoll(2004)
Borrower	Producer	Producer and consumer	
			(Superior marginal productivity)
Lender	Consumer	Producer and consumer	
			(Inferior marginal productivity)
Preferences	Concave (lender) (Linear (borrower's life time profit))	Linear	Concave
Technology	Concave	Linear, Concave	Concave
Debt-collateral ratio, η	(0,1]	(0,1)	
Exog. Shock	Aggregate	Heterogenous between borrower and lender	

An objective of this paper is to devise a way to apply the credit cycles to the standard real business cycle models considering the problems found in Cordoba and Ripoll (2004). Second, I introduce a variable which can measure strength of collateral constraints, debt-collateral ratio, which is the amount of debt attainable with amount of collateral given, therefore, when the ratio is higher, he can borrow more implying less financial friction in the economy. And I try to see how amplification and persistence depend on the ratio. Other papers such as Kiyotaki and Moore (1997) and Cordoba and Ripoll (2004) consider only the existence of the constraints; that is, the case of debt-collateral ratio is unity in Table 1. Finally, I take data of land usage in Korea for an example. Then, the residential land usage in Korea shows that the signs of coefficients from AR(2) estimation are positive

in one-lag coefficient and negative in the two-lag coefficient. I check whether the signs of the coefficients are compatible with the theoretical model of the credit cycles and try to find (range of) debt-collateral ratio backing up the coefficients; the real ratio in the country is currently hard to see because collateralization is implicit and habitual.

The model is a closed economy of a representative consumer and a firm with standard preferences and technologies in the sprit of the typical real business cycle models. In particular, a representative agent has concave preferences and is required to collateralize her debts, and a firm maximizes his profit under perfect competition. Collateral is used in production like Kiyotaki and Moore (1997) and is considered in the utility function as well. This does not seem to be a drastic assumption; because if we admit the consumers are a huge part of lenders, the collateral should have intrinsic values to them, for example, when the collateral is literally real estates, the lenders may enjoy them through housing.

First, I study properties of steady state equilibrium, a local dynamics around the steady state and size of persistence and amplification from an unexpected shock in a stochastic model. Second, I execute simulation exercises to see impulse responses from an unexpected shock and fluctuations of output when a shock is autocorrelated. And finally, I apply this model to land usage in Korea.

2 Model

2.1 Economic environment

I describe a simple competitive equilibrium model with a representative household and a representative firm that meets a credit constraint. The model is essentially a version of the neoclassical growth model.

There are two goods in this economy; a durable asset (capital or land, Q and H) and a non-durable commodity (output, C). To hold durable asset is assumed to create utility as well as to consume a non-durable commodity. It is helpful

to think of the durable asset as land, which serves housing to a household. The household maximizes her expected lifetime utility as given by

$$E\sum_{t=0}^{\infty}\beta^t u\left(C_t^L, H_t\right) \tag{1}$$

where $1 > \beta > 0$, C_t^L and H_t are consumption of non-durable commodity and holding of durable commodity at time t, respectively. The utility $u(\cdot)$ is assumed to be additively separable in non-durable consumption and durable holding ($u_{CH} = u_{HC} = 0$), and is concave, strictly increasing, $u_C(0) = u_H(0) = \infty$ and twice continuously differentiable in C and H. The household faces a budget constraint given by

$$C_t^L + p_t H_{t+1} + B_{t+1} \leq R_t B_t + p_t H_t \tag{2}$$

where p is the price of durable good, B is the amount of lending, R is the return rate on the lending. The household behave competitively taking prices as given.

The firm maximizes his expected lifetime consumption, C^B, which is actually profit to the firm, as given by

$$E\sum_{t=0}^{\infty}\tilde{\beta}C_t^B \tag{3}$$

where

$$C_t^B + p_t Q_{t+1} + R_t D_t \leq f(Q_t) + D_{t+1} + p_t Q_t \tag{4}$$

where Q is the durable good which is an input of production, D is the amount of borrowing and $f(Q)$ is a technology of the consumption good production. The technology requires at least $Q_{\min}$ of durable good to produce final goods; it cannot produce anything when the durable input is less than $Q_{\min}$. This is a sunk cost to be sustained; it makes possible a steady state equilibrium analyzed in this paper. And the technology shows the concavity over $Q_{\min}$. That is, when $Q < Q_{\min}$, $f(Q) = 0$ and otherwise, $f(Q) \geq 0$, $f'(Q) > 0$, and $f''(Q) < 0$, furthermore $\lim_{Q\downarrow Q_{\min}} f'(Q) = \infty$. I assume that $\tilde{\beta} < \beta$. This environment makes an agent simply a borrower just like Kiyotaki and Moore (1997) and Cordoba

and Ripoll (2004), and it allows the existence of unfulfilled demand for credit (i.e. credit rationing) around the steady state equilibrium; credit rationing is clearly a widespread phenomenon in developing countries(McKinnon (1973)) and there is substantial evidence of significant rationing of credit even in the United States (Japelli (1990)).

And I assume that the firm can escape without repaying his loans without any other penalty than losing their capital or durable goods. As a result, loans need to be secured (excessively, fully or partially) by the value of the capital, that is,

$$R_{t+1}D_{t+1} \leq \eta p_{t+1}Q_{t+1} \tag{5}$$

where η stands for how much of the loans is collateralized. I call η to be debt-collateral ratio considering an equilibrium which is exogenous in the economy; the loans are collateralized excessively when $0 < \eta < 1$, partially when $1 < \eta$, just or fully collateralized when $\eta = 1$. Almost all the literatures consider only the case of $\eta = 1$ (for instance, Kiyotaki and Moore (1997), Cordoba and Ripoll (2004) and Kocherlakota (2000)). One reason to take the ratio exogenous is that debt-securing depends heavily on legal environments and government policies, which are out of the model; for example, when the government policies tend to support entrepreneurs explicitly or implicitly in their financing, it would turn out to be relatively high debt-collateral ratio.

I confine the ratio up to unity, that is, $\eta \in (0, 1]$ with a few reasons. First, it is to prevent the borrower from intentional default; the net cost of default to the borrower is non-negative. In particular, when the constraint is binding, the cost of default is measured by $p_tQ_t - R_tD_t = (1 - \eta)\, p_tQ_t$, which is non-negative only when $\eta \leq 1$. Second, the liquidation value of collateral, ηp_{t+1}, is assumed to be lower than the prevailing price, p_{t+1}, when the lender secures her lending (Hart and Moore (1998)), implying $\eta \leq 1$. Third, it sufficiently guarantees the user cost (or price) around a steady state to be positive. If the firm borrows up to the maximum of the borrowing constraint, the equation (4) turns out to be

$$C_t^B = f(Q_t) - R_tD_t + p_tQ_t - \left(p_t - \frac{\eta p_{t+1}}{R_{t+1}}\right) Q_{t+1}$$

And $(p_t - \eta p_{t+1}/R_{t+1})$ is the user cost or price, which should be positive, otherwise, borrower's demand of durable goods explodes because the actual price is negative. This means η should be less than $1/\beta$, which is satisfied sufficiently if $\eta \leq 1$.

The firm cannot purchase the durable goods more than his total sources in a period which are composed of his production, net borrowing and holding of the durable goods, that is,

$$p_t Q_{t+1} \leq f(Q_t) + D_{t+1} - R_t D_t + p_t Q_t \tag{6}$$

for all t. Equivalently, this means,

$$C_t^B \geq 0 \tag{7}$$

From now on, I denote the durable good to be capital for simplicity. The capital is available in a fixed aggregate amount which is normalized by unity. This assumption can be interpreted as either investment taking a very long time to build, or as the adjustment costs of investment being very high; simply land is taken as an example[1]. And finally, as in Kiyotaki and Moore (1997) and Cordoba and Ripoll (2004), I exclude the possibility of renting capital[2].

Following the equilibrium considerations, I often call the household a lender and the firm a borrower. For the time being, there is no uncertainty in the model.

2.2 Competitive equilibrium in a steady state equilibrium

Market equilibrium is defined as a sequence of capital prices, interest rates and allocations of capital, debt, consumption of the lender and profit of the borrower, $\{p_t, R_t, Q_t, H_t, B_t, C_t^L, C_t^B\}$ such that the lender chooses $\{H_t, B_t, C_t^L\}$ to maximize the expected discounted utility (1) subject to equation (2); and the borrower chooses $\{Q_t, D_t, C_t^B\}$ to maximize equation (3) subject to equation (4), (7) (or (6)) and 5; and the markets for capital, debt and consumption clear.

[1]Cordoba and Ripoll (2004) argue that fixed amount of capital can help amplification by collateral constraint, but it is not clear.

[2]Adding this possibility would not change the perfect-foresight equilibrium path but would affect how the economy responds to an unanticipated shock (Cordoba and Ripoll (2004))

To characterize equilibrium, I first describe the lender's optimization behavior by the following first-order-conditions.

$$-u_C\left(C_t^L, H_t\right) + \beta R_{t+1} u_C\left(C_{t+1}^L, H_{t+1}\right) = 0 \tag{8}$$

$$-p_t u_C\left(C_t^L, H_t\right) + \beta\left(p_{t+1} u_C\left(C_{t+1}^L, H_{t+1}\right) + u_H\left(C_{t+1}^L, H_{t+1}\right)\right) = 0 \tag{9}$$

Next I examine the borrower's optimization behavior. The Lagrangian of the borrower's optimization problem becomes,

$$\begin{aligned} L = \sum_{t=0}^{\infty} \tilde{\beta}^t \{ & C_t^B - \lambda_{1t} C_t^B \\ & + \lambda_{2t}\left[f\left(Q_t\right) + D_{t+1} - R_t D_t + p_t Q_t - p_t Q_{t+1} - C_t^B\right] \\ & + \lambda_{3t}\left[\eta p_{t+1} Q_{t+1} - R_{t+1} D_{t+1}\right]\} \end{aligned} \tag{10}$$

where all the Lagrangian multipliers are non-negative, that is, $\lambda_{it} \geq 0$ for all t and $i = 1, 2, 3$. The first order conditions are,

$$C_t^B : 1 - \lambda_{1t} - \lambda_{2t} = 0 \tag{11}$$

$$D_{t+1} : \lambda_{2t} - R_{t+1}\lambda_{3t} - \tilde{\beta} R_{t+1} \lambda_{2t+1} = 0 \tag{12}$$

$$Q_{t+1} : -p_t \lambda_{2t} + \eta p_{t+1} \lambda_{3t} + \tilde{\beta}\left[f'\left(Q_{t+1}\right) + p_{t+1}\right]\lambda_{2t+1} = 0 \tag{13}$$

I claim that, in the neighborhood of a steady state, the borrower prefers to borrow up to the maximum and invest in capital, consuming nothing, exactly like Kiyotaki and Moore (1997). And I specify conditions of the collateral-debt ratio, η, which supports the steady state equilibrium. That is, I focus on the case where all the constraints are binding making all the λ_i's strictly positive. From equation (12), we get $\lambda_{3t} = \frac{1}{R_{t+1}}\lambda_{2t} - \beta\lambda_{2t+1}$, which is substituted into equation (13), then we have,

$$\left(-p_t + \frac{\eta p_{t+1}}{R_{t+1}}\right) + \tilde{\beta}\left[f'\left(Q_{t+1}\right) + (1-\eta)\, p_{t+1}\right]\frac{\lambda_{2t+1}}{\lambda_{2t}} = 0 \tag{14}$$

Equation (12) also implies that $\frac{1}{R_{t+1}} - \tilde{\beta}\frac{\lambda_{2t+1}}{\lambda_{2t}} = \frac{\lambda_{3t}}{\lambda_{2t}} > 0$, so it comes to be $\frac{1}{R_{t+1}} > \tilde{\beta}\frac{\lambda_{2t+1}}{\lambda_{2t}}$. Therefore, we have the following inequality,

$$\left(-p_t + \frac{\eta p_{t+1}}{R_{t+1}}\right) + \frac{1}{R_{t+1}}\left[f'\left(Q_{t+1}\right) + (1-\eta)\, p_{t+1}\right] > 0 \tag{15}$$

And since $1 - R_{t+1}\tilde{\beta}\frac{\lambda_{2t+1}}{\lambda_{2t}} = R_{t+1}\frac{\lambda_{3t}}{\lambda_{2t}}$ from equation (12), we have

$$1 - R_{t+1}\tilde{\beta}\frac{\lambda_{2t+1}}{\lambda_{2t}} > 0 \tag{16}$$

Therefore, borrower's optimization behavior of our concern is summarized by the above inequality (15) and (16) and the following binding constraints when η satisfies some conditions.

$$R_{t+1}D_{t+1} = \eta p_{t+1}Q_{t+1} \tag{17}$$

$$f(Q_t) + D_{t+1} - R_t D_t + p_t Q_t - p_t Q_{t+1} = C_t^B = 0 \tag{18}$$

Inequality (16) means the borrower wants to borrow up to the maximum, resulting in equation (17) and inequality (15).means he invests in capital as much as possible ,which implies he consumes nothing, leading to equation (18).

Market clearing conditions are,

$$C_t^L + C_t^B = f(Q_t) \tag{19}$$

$$Q_t + H_t = 1 \tag{20}$$

$$B = D \tag{21}$$

I assume that $\lim_{s\uparrow\infty} E_t(R_{t+s}p_{t+s}) = 0$, which rule out exploding bubbles in the capital price. Then a natural candidate of the steady state equilibrium associated with the equation (8) through (20) shows up dropping all the time subscripts in the equations and inequalities. I denote variables with asterisks to be the variables in the steady state.

Proposition 1 *When η and Q^* satisfy the followings*

$$\frac{1}{\eta}\frac{f(Q^*)}{Q^*} = \beta\frac{u_H(1-Q^*)}{u_C(f(Q^*))} \tag{22}$$

$$\frac{1}{\eta}\frac{f(Q^*)}{Q^*} < \beta f'(Q^*) \tag{23}$$

, Q^* constructs a steady state as,

$$H^* = 1 - Q^* \tag{24}$$

$$R^* = 1/\beta \tag{25}$$

$$p^* = \frac{f(Q^*)}{Q^*}\frac{1}{\eta(1-\beta)} \tag{26}$$

$$C^{L*} = f(Q^*) \tag{27}$$

$$C^{B*} = 0 \tag{28}$$

$$B^* = D^* = \frac{\beta}{1-\beta} f(Q^*) \tag{29}$$

Proof. First, I show that equation (22) and (24) through (29) are solved by dropping time subscripts in the equation (8), (9), (17) and (18). And I show that inequality (16) and (15) are satisfied under the steady state and inequality (23), which confirm equation (17) and (18) back.

This is the first part of the proof. I drop the time subscripts. Equation (25) is solved from equation (8). Equation (24) and (28) are directly from equation (20) and (18) respectively. By equation (19) and (28), equation (27) is true. Boiling out B terms by equation (17) and using $R^* = 1/\beta$, equation (18) is reduced to be the equation (26). Equation (9) turns out to be

$$p^* = \frac{\beta}{1-\beta}\frac{u_H}{u_C} \tag{30}$$

Equating (26) and (30), I get the equation (22). Finally, substitution of equation (26) and $R^* = 1/\beta$ into equation (17) verifies equation (29).

This is the second part of the proof. Since $\beta > \tilde{\beta}$ and $R^* = 1/\beta$, the inequality (16) is verified. By the equation (26), the inequality (23) is,

$$p^*(1-\beta) < \beta f'(Q^*)$$

, which is,

$$-p^*(1-\eta\beta) + \beta\left(f'(Q^*) + (1-\eta)p^*\right) > 0$$

implying the inequality (15) is true. Alternatively, I can show the second part of the proof using the principle of unimprovability exactly like Kiyotaki and Moore

(1997); I show that the firm borrows and invests up to the maximum in appendix A, resulting in equation (17) and (18). ■

The dynamic path in this model follows equation (17) and (18) because the constraints (5) and (6) or (7) are binding around the steady state. The literatures with this kind of collateral constraints usually have the collateral constraints binding, however, they vary in the non-negative conditions of borrower's consumption. I examine the model with the non-negative consumption constraint of the borrower binding because it makes the collateral constraint play more important role in amplification and persistence. For example, Kiyotaki and Moore (1997) have both of the constraints corresponding to equation (5) and (6) binding and generate meaningful credit cycles, however, Cordoba and Ripoll (2004) have the non-negative borrower's consumption constraint not binding. The intuition is that when there occurs a shock, if the non-negative consumption constraint is not binding, it is melted partly by consumption smoothing over time, but if the constraint is binding, only the capital stock (debt simply accompanies the capital stock) reacts to the shock because the capital stock is not attained at the desired level as the equation (15) implies. Other than this, there is a consideration of simplicity of equilibrium. Note that the borrower's consumption in this model is exactly firm's profit. So, when the non-negative consumption constraint is not binding, that is, firm's profit is positive, we need an extra endogenous mechanism to redistribute borrower's profit, which is not necessarily wanted in this paper. I summarize comparing this model, Cordoba and Ripoll (2004) and Kiyotaki and Moore (1997) in Table 2 where the user cost of capital is $s_t \equiv p_t - \eta p_{t+1}/R_{t+1}$[3].

The following lemma tells the reason that this environment needs a positive sunk cost ($Q_{\min} > 0$) in the production function.

Lemma 1 *If $Q_{\min} = 0$, then there is no steady state equilibrium suggested in Proposition 1.*

Proof. Suppose not, then since $Q_{\min} = 0$, $f(Q)/Q > f'(Q)$ for any $Q > 0$. Since

[3]The variable s_t is denoted as a user cost or downpayment in Kiyotaki and Moore (1997)

Table 2: Comparison of models

	Borrower's F.O.C in Q	Borrower's consumption in a steady state
Song (2004) $(0 < \eta \leq 1)$	$-s + \tilde{\beta}\left(f'(Q) + (1-\eta)p\right) > 0$	consumption $= f(Q) + (1-\eta)pQ - sQ$ $= 0$
Cordoba and Ripoll (2004) $(\eta = 1)$	$-s + \tilde{\beta} f'(Q) = 0$	consumption $= f(Q) - sQ$ $= f(Q) - \tilde{\beta} f'(Q) Q$ > 0
Kiyotaki and Moore (1997) $(\eta = 1)$	$-s + \tilde{\beta} f'(Q) > 0$ which is from an assumption of parameters	consumption of tradables $= f(Q) - sQ -$nontradables $= (a+c)Q - aQ - cQ$ $= 0$

$0 < \beta < 1$ and $0 < \eta \leq 1$, we have $f(Q)/Q > \eta\beta f'(Q)$, which clearly contradicts the inequality (23) in Proposition 1. ■

2.3 Parametric Formulation

To understand concretely, I consider an additively separable utility function of

$$u(C, H) = \frac{C^{1-\rho}}{1-\rho} + g\frac{H^{1-\theta}}{1-\theta} \tag{31}$$

and a production of

$$f(Q) = \max\left\{0, (Q - Q_{\min})^{\alpha}\right\} \tag{32}$$

where ρ, θ and g are positive constants and $0 < \alpha \leq 1$. With the particular utility function(31) and production function(32), equation (22) turns out to be

$$\frac{1}{\eta}\frac{(Q^* - Q_{\min})^{\alpha}}{Q^*} = \beta\frac{g(1-Q^*)^{-\theta}}{Q^{*-\alpha\rho}} \tag{33}$$

It will clearly be necessary to establish some properties of Q and η which are stated in the following lemma.

Lemma 2 *Suppose the utility and the production functions are equation (31) and (32) respectively satisfying parameter conditions in the above context. Then when parameters satisfy the following inequality*

$$\eta\alpha^{\alpha}\left(1-\alpha\right)^{\theta-\alpha(1-\rho)} \geq \beta g\frac{Q_{\min}^{1-\alpha(1-\rho)}}{\left(1-\alpha-Q_{\min}\right)^{\theta}} \tag{34}$$

(i) at least one $Q^{*} \in (Q_{\min}, 1]$ *satisfying equation (33) does exist, in particular,* $Q^{*} \equiv Q^{*R} \in \left[\hat{Q}, 1\right]$ *always does and* $Q^{*} \equiv Q^{*L} \in \left(Q_{\min}, \hat{Q}\right]$ *may or may not, where* $\hat{Q} = Q_{\min}/\left(1-\alpha\right)$*,*
(ii) and as η *increases,* Q^{*R} *satisfying equation (33) decreases and* Q^{*L} *increases.*

Proof. The left hand side of equation (33) is continuous in $Q \in (Q_{\min}, 1]$ and has a maximum value when $Q = Q_{\min}/\left(1-\alpha\right) \equiv \hat{Q}$. Then condition (34) is equivalent with

$$\frac{1}{\eta}\frac{\left(\hat{Q}-Q_{\min}\right)^{\alpha}}{\hat{Q}} \geq \beta\frac{g\left(1-\hat{Q}\right)^{-\theta}}{\hat{Q}^{-\alpha\rho}}$$

The right hand side of it is continuous and increasing in $Q \in (Q_{\min}, 1]$ and

$$\lim_{Q\downarrow Q_{\min}} \beta\frac{g\left(1-Q\right)^{-\theta}}{Q^{-\alpha\rho}} = \beta\frac{g\left(1-Q_{\min}\right)^{-\theta}}{Q_{\min}^{-\alpha\rho}} < \infty \quad \text{and} \quad \lim_{Q\uparrow 1} \beta\frac{g\left(1-Q\right)^{-\theta}}{Q^{-\alpha\rho}} = \infty$$

Therefore, (i) is true. Particularly, And (ii) is easily verified by total differentiation in η and Q. ■

Lemma 2 tells that equation (33) may or may not have multiple solutions; the bigger solution is Q^{*R} and the possible smaller one is Q^{*L} whose existence is not guaranteed. Figure 1 shows an example how Q^{*} is set. Even though we have two solutions to equation (33), we focus only on the bigger solution is Q^{*R} because it is usually associated with stability; this is discussed in Proposition 2 (iii) later.

3 Local dynamics

3.1 Linearization of a stochastic model

In order to simplify the analysis and enhance the amplification effects I focus on the case in which the collateral constraint (5) and the non-negative consumption constraint (6) of the borrower are binding, which is the case in the neighborhood of the steady state. To see all of the elasticities concerned, I set up a standard stochastic model[4]. I simply take the expected value of the collateral for the collateral constraint; which makes a stochastic model similar to a deterministic version. Key variables are price variables of p_t, R_t and capital held by the firm Q_t. Considering the productivity shock, I set the production function as,

$$F(Q_t) = Z_t f(Q_t) \tag{35}$$

Z_t follows an AR(1) process given by,

$$\log Z_{t+1} = (1-\Psi)\log \bar{Z} + \Psi \log Z_t + \epsilon_t \tag{36}$$

where $\bar{Z} = 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon)$.

In a stochastic model, the equation (8) is transformed as,

$$u_C(F(Q_t)) = E_t\left[R_{t+1}\beta u_C(F(Q_{t+1}))\right] \tag{37}$$

Likewise the equation (9) implies,

$$-p_t u_C(F(Q_t)) + E_t\left[\beta p_{t+1} u_C(F(Q_{t+1})) + \beta u_H(1-Q_{t+1})\right] = 0 \tag{38}$$

And from the equation (7) we have,

$$F(Q_t) + E_t\left[\frac{\eta p_{t+1}}{R_{t+1}} Q_{t+1}\right] - \eta p_t Q_t + E_t\left[p_t(Q_t - Q_{t+1})\right] = 0 \tag{39}$$

These three equations summarizes the equilibrium path of the model with the productivity shock process. In order to analyze the dynamics of the model, I log-linearize equations (37), (38) and (39) around the steady state.

[4]There are issues of sequences associated with debt repudiation and collateral constraint and contingent claims (Cordoba and Ripoll (2004)).

And note that interest rate determined in the previous period is applied to the debt redemption in the current period and capital holding in the current period is set in the previous period. That is, the current shock affects p_t, R_{t+1} and Q_{t+1}. Therefore, we want the following linearized system finally from the log-linearized equations,

$$\hat{Q}_{t+1} = \varepsilon_{QQ}\hat{Q}_t + \varepsilon_{QZ}\hat{Z}_t \tag{40}$$

$$\hat{R}_{t+1} = \varepsilon_{RQ}\hat{Q}_t + \varepsilon_{RZ}\hat{Z}_t \tag{41}$$

$$\hat{p}_t = \varepsilon_{pQ}\hat{Q}_t + \varepsilon_{pZ}\hat{Z}_t \tag{42}$$

where a hat over a variable denote percentage deviation from its steady state and elasticity ε_{XY} is defined in a normal way. This system is solved by method of undetermined coefficients. The elasticities are determined as below.

Lemma 3 *The elasticities in equation (40) to (26) around the steady state equilibrium are,*

$$\varepsilon_{QQ} = \frac{-\frac{f'}{\eta p} - \frac{1}{\eta} + 1 + (1-\beta)\left(-\frac{u_{CC}}{u_C}f'Q^*\right)}{\left(\beta - \frac{1}{\eta}\right) + (1-\beta)\frac{u_{HH}}{u_H}Q^*} \tag{43}$$

$$\varepsilon_{RQ} = (1-\varepsilon_{QQ})\frac{u_{CC}}{u_C}f'Q^* \tag{44}$$

$$\varepsilon_{pQ} = \frac{\left(\beta - \frac{1}{\eta} + \beta\frac{u_{CC}}{u_C}f'Q\right)\varepsilon_{QQ} + \frac{f}{\eta p} + \frac{1}{\eta} - 1 - \beta\frac{u_{CC}}{u_C}f'Q}{1-\beta\varepsilon_{QQ}} \tag{45}$$

$$\varepsilon_{QZ} = \frac{-(1-\beta) - (1-\beta)\frac{u_{CC}}{u_C}f}{\left(\beta - \frac{1}{\eta}\right) + (1-\beta)\frac{u_{HH}}{u_H}Q^*} \tag{46}$$

$$\varepsilon_{RZ} = \left(-f'Q^*\varepsilon_{QZ} + f(1-\Psi)\right)\frac{u_{CC}}{u_C} \tag{47}$$

$$\varepsilon_{pZ} = \frac{\left(\beta - \frac{1}{\eta} + \beta\frac{u_{CC}}{u_C}f'Q + \beta\varepsilon_{pQ}\right)\varepsilon_{QZ} + \left(1 - \beta - \beta\frac{u_{CC}}{u_C}f(1-\Psi)\right)}{1-\beta\Psi} \tag{48}$$

Proof. See appendix B. ■

3.2 Stability condition and persistence

To see the stability of this system, we have only to check $|\varepsilon_{QQ}| < 1$. Definitely, the stability highly relies on the debt-collateral ratio, η. I see the case where the utility function is equation (31) and the production function is (32); when the shock is considered, the production function is modified as,

$$F(Q) = Zf(Q) = Z \cdot \max\{0, (Q - Q_{\min})^{\alpha}\} \tag{49}$$

And to see the stability condition, I find Q_{st} so that

$$\alpha(1-\rho)\frac{Q_{st}}{Q_{st} - Q_{\min}} = \frac{1-(1-\theta)Q_{st}}{1-Q_{st}} \tag{50}$$

The following lemma tells Q_{st} exists uniquely under above parameter conditions.

Lemma 4 *A unique $Q_{st} \in (Q_{\min}, 1]$ exists when ρ and $\theta \in (0,1)$ and other parameter conditions are in the above context.*

Proof. The left hand side of equation (50) is continuous and decreasing in $Q \in (Q_{\min}, 1]$

$$\lim_{Q \downarrow Q_{\min}} \alpha(1-\rho)\frac{Q}{Q - Q_{\min}} = \infty \quad \text{and} \quad 0 < \lim_{Q \uparrow 1} \alpha(1-\rho)\frac{Q}{Q - Q_{\min}} < \infty$$

, and the right hand side of it is continuous and increasing in $Q \in (Q_{\min}, 1]$

$$0 < \lim_{Q \downarrow Q_{\min}} \frac{1-(1-\theta)Q_{st}}{1-Q_{st}} < \infty \quad \text{and} \quad \lim_{Q \uparrow 1} \frac{1-(1-\theta)Q_{st}}{1-Q_{st}} = \infty$$

Therefore it is proved. ■

Now, when the utility and the production functions are equation (31) and (32) respectively, equation (43) to be

$$\varepsilon_{QQ} = \frac{\frac{1}{\eta} - 1 + (1-\beta)(1-\rho)\alpha\frac{Q^*}{Q^* - Q_{\min}}}{\frac{1}{\eta} - \beta + \theta(1-\beta)\frac{Q^*}{1-Q^*}} \tag{51}$$

The stability condition is described by Q^* and Q_{st}. Further more, since Lemma 2 implies Q^* is a function of η, when denoting $Q^* = Q(\eta)$, it can be stated by η and $Q^{-1}(Q_{st})$, which will be clear below.

Proposition 2 ***(Stability and persistence)*** *When ρ and $\theta \in (0,1)$ and other parameter conditions are in the above context, around the steady states,*
(i) $\varepsilon_{QQ} > 0$
(ii) and $\varepsilon_{QQ} < 1$ *if and only if* $Q^* > Q_{st}$*, equivalently,* $\eta < Q^{-1}(Q_{st})$
(iii) in particular, Q^{*R} *in Lemma 2 is greater than* Q_{st}*, that is, it is stable at the solution* Q^{*R}*.*

Proof. Part (i) and (ii) are clear. Here, I prove the part (iii). Since $Q^{*R} \geq \hat{Q}$ in Lemma 2, it is enough to show that $\hat{Q} > Q_{st}$. Substitution of $\hat{Q} = Q_{\min}/(1-\alpha)$ into Q_{st} shows that the left hand side of equation (50) is $1-\rho$ and the right hand side is $1 + \theta\hat{Q}/\left(1-\hat{Q}\right)$; that is, $LHS < RHS$. Therefore $\hat{Q} > Q_{st}$. ■

Noting that ε_{QQ} implies persistence from a shock as well as the stability condition, Proposition 2 (i) is understandable; seeing that the high current holding of capital produces large amount of output in the next period, which results in high level of capital holding in the next period. The intuition is exactly the credit cycle by Kiyotaki and Moore (1997). Figure 2 shows how ε_{QQ} changes along with η and α when $g = 1$, $\rho = \theta = 1/3$, $\beta = .98$, $Q_{\min} = .1$. As the proposition says, ε_{QQ} is positive as long as available, and the trace of ε_{QQ} along with right(left) solution Q^{*R} $\left(Q^{*L}\right)$ is below (above) unity implying stability(instability) around steady states. Figure 3 magnifies the trace of stable ε_{QQ} only, which is hump-shaped. The hump-shape of the trace can be explained roughly by competing two forces. The debt-collateral ratio increase implies weaker financial constraint to the borrower, so that the credit cycle effect by collateral constraint gets loosened; definitely, this first force drags down the trace. On the contrary, equation (26) shows that higher debt-collateral ratio lowers capital price, which strengthens borrower's purchasing power leading to more capital holding by borrower[5]; this second force pulls up the trace. And the first force overwhelms the second as it gets closer to unity. Of course, positivity and stability in the proposition mean monotonic convergence to a steady state.

[5]Higher ratio allows the firm to borrow more debt which means less demand of capital by the firm, which ends into lower price.

With this fact, we can guess a reason that we don't see much persistence in Kiyotaki and Moore (1997) and Cordoba and Ripoll (2004); the persistence gets around zero after 4 periods[6]. That is because the debt-collateral constraint is unity.

3.3 Amplification

To measure amplification, I measure deviation of output from the steady state in the period $t+1$ when an unexpected shock occurs in the period t (Kocherlakota (2000) and Cordoba and Ripoll (2004)). That is, the amplification is the elasticity of output in period $t+1$ with respect to a productivity shock in period t, which is denoted ε_{YZ}. Then the elasticity is expressed as the product of two components: elasticity of output at time $t+1$ with respect to Q, ε_{YQ}, times the elasticity of Q with respect to Z in the period t, ε_{QZ}.

$$\begin{aligned} \varepsilon_{YZ} &= \varepsilon_{YQ} \cdot \varepsilon_{QZ} \\ &= \alpha \cdot \varepsilon_{QZ} \end{aligned} \tag{52}$$

This equation tells a big difference from Cordoba and Ripoll (2004) and Kiyotaki and Moore (1997) where ε_{YZ} $/\varepsilon_{QZ}$ is very small or negative. Since Cordoba and Ripoll (2004) and Kiyotaki and Moore (1997) are about the relation between two types of production sectors, the shock is amplified only by the productivity gap between them. However, in this model, since only the firm produces, the shock can be amplified by full capital share; α is less than unity in the usual RBC models[7].

Now we examine the effect of capital redistribution, ε_{QZ}. Here since the shock is unexpected, I set $\psi = 0$; then $\varepsilon_{QZ} = \left\{ -(1-\beta) - (1-\beta)\frac{u_{CC}}{u_C} f \right\} / \left\{ \left(\beta - \frac{1}{\eta}\right) + (1-\beta)\frac{u_{HH}}{u_H} Q^* \right\}$. In particular, when the production function and

[6]Kiyotaki and Moore (1997) suggests to consider "cultivation cost" in order to increase persistence.

[7]Kiyotaki and Moore (1997) focuses mainly on capital redistribution effect ε_{QZ}, which is greater than unity. They discuss ε_{YZ} little.

the utility function are equation (35) and (31), then equation (46) comes to be

$$\varepsilon_{QZ} = \frac{(1-\beta)(1-\rho)}{\theta(1-\beta)Q^*/(1-Q^*) + 1/\eta - \beta} \tag{53}$$

Proposition 3 ***(Redistribution)*** *Suppose that the production function and the utility function are equation (35) and (31), and ρ and $\theta \in (0,1)$ and other parameter conditions are in the above context. Then, around the steady state, the followings are satisfied*
(i) $\varepsilon_{QZ} > 0$
(ii). $\partial\varepsilon_{QZ}/\partial\eta > 0$

The redistribution effect ε_{QZ} should be positive in equilibrium around a steady state intuitively. Since borrower's discount factor is smaller than lender's discount factor, he wants as much loan as possible around the steady state equilibrium and since his marginal productivity is positive around the steady state from equation (15), he wants maximum level of capital, too. Therefore, when there happens a positive shock, the borrower always buy capital with the extra revenue rather than lessen his debt. A major reason why the amplification gets larger as the debt-collateral ratio gets larger is that when the ratio is high, the capital price is low from equation (26), which strengthens borrower's financial ability to buy extra capital when output is given.

Figure 4 shows how the amplification changes according to the debt-collateral ratio η and the capital share in the production function α when $g = 1$, $\rho = \theta = 1/3$, $\beta = .98$, $Q_{\min} = .1$. It shows the redistribution effect is not so big as Kiyotaki and Moore (1997) explain (which is greater than unity). However, the result gets closer to theirs as η and ρ get closer to 1 and 0 each; in Kiyotaki and Moore (1997), $\eta = 1$ because they consider only just collateralization and $\rho = 0$ since the utility functions are linear.

Summing up, the amplification is composed of capital-output effect, ε_{YQ}, and capital redistribution effect, ε_{QZ}. The capital-output effect, ε_{YQ}, is greater than Kiyotaki and Moore (1997) and Cordoba and Ripoll (2004) because there exists

only one production sector here. And ε_{QZ} is dependent on the debt-collateral ratio and smaller than unity in the usual parameter specification. Even though the size of amplification is small, this can supply extra amplification other than usual RBC amplification; for example, when the production function has a variable of usual producible capital as well as collateralizable capital, such as $f(K, Q) = K^{1-\alpha}(Q - Q_{\min})^{\alpha}$, the extra amplification is purely additive to the standard RBC amplification around a steady state.

Under a circumstance where borrower's production depends totally on outside debt financing, persistence and amplification by the collateral constraints have trade-off relation in the neighborhood of unity debt-collateral ratio; relatively high requirement of collateral, that is, a small debt-collateral ratio(η) makes persistence strong but amplification weak. The implication is not tiny; a proper selection of the collateralization parameter could help explain the data through the collateral constraints. For example, the debt-collateral ratio of an economy close to unity can help explaining high amplification and low persistence with the same exogenous shock process as other economies, and it could allow us to employ smaller exogenous shocks than the standard real business cycle models. Further more, we can imagine effects by some government policies on persistence and amplification through the collateral constraints; for example, when the government policies tend to support entrepreneurs explicitly or implicitly in their financing, such as developing countries, it would turn out to be low collateralization, of course, resulting in less persistence and bigger amplification in its economy.

4 Exercises

4.1 Impulse response analysis

So far I showed some basic properties of effects of debt-collateral ratio on persistence and amplification. Now I execute an exercise with parameters believed to be standard and see how big amplification and persistence solely by collateral constraints are. I set $\rho = \theta = 1/3$, $\beta = 0.98$, $\alpha = 0.6$, $g = 1.5$, $\eta = 0.97$ and

$Q_{\min} = 0.1$ and see impulse responses of borrower's capital holding (Q), output (Y), interest rate (R) and capital price (p) to 1% of productivity shock. The results are suggested in Figure 5. I find some observations from the figure. (i) It has longer persistence than Kiyotaki and Moore (1997) and Cordoba and Ripoll (2004); persistence of both of the models ends in around period four but this model has longer tail. And (ii) its amplification is not so big because ε_{QZ} is only 0.25, which could be expected mainly from nonlinearity of utility function. This small amplification is mainly by the collateral constraint; which could be an additive amplification when the producible capital is introduced.

Through a lot of simulations with plausible parameter values, Kiyotaki and Moore (1997) can generate the largest amplification among Kiyotaki and Moore (1997), Cordoba and Ripoll (2004) and this model, but this model can outrun both of them in persistence. The exercises show that this model can generate relatively flexible persistence to other related literatures. This suggests to focus on persistence measure rather than amplification, when we study actual data involved with collateral constraints.

4.2 Moments of a stochastic model

Until now, the shock is one-time without any autocorrelation. In a stochastic model, it is possible to compute moments through simulations and use them as a measure of amplification. I suppose $\epsilon_t \sim N(0, \sigma_\epsilon)$ which is i.i.d. in equation (36). Cordoba and Ripoll (2004) use the ratio of standard deviation of output to autocorrelation of the shock, σ_Y/Ψ,as a measure of amplification. They shows $\sigma_Y/\Psi = 1.467$ with $\sigma_\epsilon = 1$. Here, when $\rho = \theta = 1/3$, $\beta = 0.98$, $\alpha = 0.6$, $g = 1.5$, $\eta = 0.97$ and $Q_{\min} = 0.1$, this model have smaller σ_Y/Ψ, which is 0.94, with the same standard deviation of ϵ_t; it seems to be natural because debt-collateral ratio is smaller than Cordoba and Ripoll (2004).

To compare this model with the standard RBC models. I get simulated moments setting $\Psi = 0.95$ and $\sigma_\epsilon = 0.0076$; they are usual in studying U.S. data. Then additive amplification by the collateral constraints is $\sigma_Y = 0.0068$; which is

very small noting that output standard deviations by models of U.S. are $1.3 \sim 1.8$ depending on literatures. This model is still a skeleton to calibrate an economy; at least standard, reproducible and depreciable capital should be included. But, to have a feeling about this model, I suggest cross-correlation table in Table 3; generated data is HP-filtered. The correlations with capital and output are very high over time periods implying high persistence of collateralized capital.

Table 3: Cross-correlation of $(corr\,(v\,(t+j)\,, output\,(t)))$

j	-4	-3	-2	-1	0	1	2	3	4
R	.41	.36	.29	.20	.11	$-.08$	$-.23$	$-.37$	$-.47$
Q	.65	.73	.81	.88	.95	.98	.99	.99	.96
P	.23	.31	.41	.51	.68	.73	.82	.89	.93

4.3 Land usage in Korea

Some policy makers of Japan and Korea worry about the possibility of vicious cycles, which are credit cycles caused by negative shocks, because a lot of lending in those countries are collateralized implicitly and habitually by real-estates (e.g. Park (1998), Park (1999) and Ueda (1999)). How can this simple model explain the movement of the cycles? The question leads to measuring persistence and debt-collateral ratio; it is hard to detect amplification by this model because it has very small effects. And the information of persistence and the ratio is summarized in equation 40; it seems to be less meaningful to use other equations because such variables as usual producible capital are omitted here. The reduced form is transformed as

$$H_{t+2} = c + (\varepsilon_{QQ} + \Psi)\, H_{t+1} - \varepsilon_{QQ}\Psi H_t + e_t \tag{54}$$

where

$$\begin{aligned} c &= 1 - (1-\Psi)\,(1-\varepsilon_{QQ})\, Q^* - (\varepsilon_{QQ} + \Psi) + \varepsilon_{QQ}\Psi \\ e_t &= Q^* \varepsilon_{QZ} \epsilon_t \end{aligned}$$

The derivation of equation (54) is suggested in Appendix C. Now I assume that the persistence is caused totally by collateral constraint without any exogenous shock correlation. It is the case when Ψ is zero in with equation (54); so equation to be estimated is AR(1), which is $H_{t+1} = c + b_1 H_t + e_t$.

I study Korean land usage data taking residential land usage per capita for H_t. Graphs in Figure 6 show original series of residential land usage per capita in Korea, its trend by the Hodrick-Prescott filter and its detrended series from top; the original series is annual from 1972 through 2002 and rescaled for visual purpose. Then the coefficient is estimated as $b_1 = \varepsilon_{QQ} = 0.7$. To find corresponding debt-collateral ratio, I set parameters other than the ratio as $g = 0.5$, $\rho = \theta = 0.33$, $\beta = 0.98$ and $Q_{\min} = 0.1$. Seeing Figure 7, we can find debt-collateral ratio which is associated with $\varepsilon_{QQ} = 0.7$. In particular, the ratio can be 0.44 or 0.80 when $\alpha = 0.54$ which capital share is from the usual capital in Korea (Park (1999)). Since Korean people believe the amplification in the cycles matters, I pick larger η which is 0.80; which can generate more amplification. Note that only collateral constraints explain the persistence in this case. So if other persistent effects such as production shock correlation and usual reproducible capital were considered in the model, actual ε_{QQ} would be smaller than 0.8, implying debt-collateral ratio would be higher than 0.8

5 Conclusion

This paper aims to investigate the way how collateral constraint can play as a persistence and amplification mechanism even when capital accumulation is not allowed. In particular, I modify Kiyotaki and Moore (1997) to have a look of standard real business cycle models, introducing debt-collateral ratio which is exogenous. I interpret a representative producer to be a borrower and a representative consumer to be a lender like standards models. A production and utility functions considered are concave in a standard way, improving on Kiyotaki and Moore (1997). Such efforts as to get closer to standard real business cycle models from Kiyotaki and Moore (1997) are found in other literatures, for example,

Cordoba and Ripoll (2004). Their work still leaves room for improvement. (i) It needs drastic parameter specification to have stability and either of persistence or amplification. (ii) The financial transactions are only between agents with production technologies (producers); lenders as well as borrowers are producers but it is hard to see any big portion of firms that are net lenders. (iii) In equilibrium, low productive firms survive because they don't have a credit constraint; it is hard to believe low productive firms have financial advantages over high firms in reality. And (iv) to generate amplification, the shock should show relative difference in the productions between the two types; the shock is not such an aggregate shock as in the standard real business cycle models.

I construct a closed economy of a representative consumer and a firm with standard preferences and technologies in the sprit of the typical real business cycle models. In particular, a representative agent has concave preferences and is required to collateralize her debts, and a firm maximizes his profit under perfect competition. Collateral is used in production like Kiyotaki and Moore (1997) and is considered in the utility function as well. The collateral holding in utility implies that collateral should have intrinsic values to lenders, for example, when the collateral is literally real estates, the lenders may enjoy them through housing.

I focus on debt-collateral ratio which is exogenous in the model; a reason for exogeneity of the ratio is that debt-securing depends heavily on legal environments and government policies, which are out of the model, for example, when the government policies tend to support entrepreneurs explicitly or implicitly in their financing, it would turn out to be relatively high debt-collateral ratio. Majority of literatures study only the just or exact collateralization but I allow more strict collateral requirement; because (i) net cost of default to the borrower should be non-negative, (ii) the liquidation value of collateral is to be lower than the prevailing price of collateral and (iii) it sufficiently guarantees the user cost (or price) of capital around a steady state to be positive. An unorthodox feature of the model is that the production function has a kind of sunk cost, without which any steady state equilibrium does not exist.

In steady state equilibrium, there exists possibility of multiple equilibrium but

stability condition cuts them down to leave only one solution. Around stable steady states, I show monotonic convergence, hump-shaped relation between persistence and debt-collateral ratio and positive relation between amplification and the ratio. Therefore, we can expect trade-off relation between persistence and amplification when debt-collateral ratio gets near to unity. This characteristic can be used for estimation and have policy implication, for example, when the government policies tend to support entrepreneurs explicitly or implicitly in their financing, such as developing countries, it would turn out to be low collateralization, resulting in less persistence and bigger amplification in its economy.

Impulse response exercises show that this model can generate relatively flexible persistence to other related literatures. This suggests to focus on persistence measure rather than amplification, when we study actual data involved with collateral constraints.

For an application, I study the residential land usage in Korea because it is widely believed that collateralization of land is very habitual in Korea and the land itself is fixed at a certain level or at least hard to produce in short time. For a simple case, I assume that the persistence is caused totally by collateral constraint without any exogenous shock correlation and find debt-collateral ratio to be 0.8; therefore, the ratio would be larger than 0.8 in the real world.

Policy advisors worry about effects of this type of credit cycles on the whole economy. To answer it concretely, a more-extended model is needed; most of all, usual producible capital should be included in the model.

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A Alternative proof of the second part of Proposition 1

Kiyotaki and Moore (1997) prove the second part of the proof by the principle of unimprovability relying on Kreps (1990). I can prove my proposition in the same way. Suppose the present value of borrower's net wealth is $W_{t+1}^{B} \equiv \frac{1}{R_{t+1}}\left(\eta p_{t+1}Q_{t+1} - R_{t+1}B_{t+1}\right)$ and the user cost is $s_t \equiv p_t - \eta\frac{p_{t+1}}{R_{t+1}}$. Then the borrower's problem is,

$$V^{B}\left(X^{B}\right) = \underset{Q',W^{B\prime}\geq 0}{Max}\left\{\left(X^{B} - W^{B\prime} - sQ'\right)\right.$$
$$\left. +\tilde{\beta}V^{B}\left(f\left(Q'\right) + R'W^{B\prime} + \left(1-\eta\right)p'Q'\right)\right\}$$

where $V^{B}\left(\cdot\right)$ is borrower's value function, Q' is restricted by non-negative consumption path and $X^{B} \equiv f\left(Q\right) + RW^{B} + \left(1-\eta\right)pQ$. The collateral constraint is $W^{B\prime} \geq 0$. Then there are three alternatives open to the borrower at t when he has any marginal extra; he can just consume it, reduce his current borrowing or buy more capital by the marginal unit. Contributions of the three options to the life-time borrower's utility around a steady state are measured by $\partial V^{B}/\partial C^{B}$, $\partial V^{B}/\partial\left(-B\right)$ and $\partial V^{B}/\partial\left(sQ\right)$; which turn out to be,

$$\partial V^{B}/\partial C^{B} = 1 \tag{a}$$

$$\partial V^{B}/\partial\left(-B\right) = \frac{1}{1+R^{*}\tilde{\beta}} \tag{b}$$

$$\partial V^{B}/\partial\left(sQ\right) = \frac{1}{1-\tilde{\beta}\left(f'\left(Q^{*}\right) + \left(1-\eta\right)p^{*}\right)} \tag{c}$$

From equation (a) and (b), $\partial V^{B}/\partial C^{B} > \partial V^{B}/\partial\left(-B\right)$, that is, the borrower never lessens his debt. And it can be shown that $f'\left(Q^{*}\right) + \left(1-\eta\right)p^{*} > 0$ from equation (23), therefore, $\partial V^{B}/\partial\left(sQ\right) > \partial V^{B}/\partial C^{B}$. The results mean that the borrower never consumes, neither lessens his debt and wants capital as much as possible around the steady state, verifying that the constraints (5) and (7) are binding which results in equation (17) and (18)

B Proof of Lemma 3

Log-linearize equation (37), (38) and (39),

$$u_{CC}f'Q\hat{Q}_t + u_{CC}f'\hat{Z}_t = u_C\hat{R}_{t+1} + u_{CC}f'Q\hat{Q}_{t+1} + u_{CC}f\hat{Z}_{t+1} \tag{55}$$

$$\begin{aligned}&\beta\left(pu_{CC}f'Q - u_{HH}Q\right)\hat{Q}_{t+1} + \beta pu_C\hat{p}_{t+1} + \beta pu_{CC}f\hat{Z}_{t+1}\\ &= pu_{CC}f'Q\hat{Q}_t + pu_C\hat{p}_t + pu_{CC}f\hat{Z}_t\end{aligned} \tag{56}$$

$$\begin{aligned}&pQ\left(1-\frac{1}{\eta R}\right)\hat{Q}_{t+1} - \frac{pQ}{\eta R}\hat{p}_{t+1} + \frac{pQ}{\eta R}\hat{R}_{t+1}\\ &= \left(f'Q + \left(1-\frac{1}{\eta}\right)pQ\right)\hat{Q}_t - \frac{1}{\eta}pQ\hat{p}_t + f\hat{Z}_t\end{aligned} \tag{57}$$

From equation (55),

$$R_{t+1} = -\frac{u_{CC}}{u_C}f'Q\hat{Q}_{t+1} + \frac{u_{CC}}{u_C}f'Q\hat{Q}_t + \frac{u_{CC}}{u_C}f\left(1-\Psi\right)\hat{Z}_t \tag{58}$$

from equation (56),

$$\begin{aligned}\hat{p}_t - \beta\hat{p}_{t+1} =&\beta\left(\frac{u_{CC}}{u_C}f'Q - \frac{u_{HH}}{pu_C}\right)\hat{Q}_{t+1} - \frac{u_{CC}}{u_C}f'Q\hat{Q}_t\\ &+ \beta\frac{u_{CC}}{u_C}f\Psi\hat{Z}_t - \frac{u_{CC}}{u_C}f\hat{Z}_t\end{aligned} \tag{59}$$

and by substitution of equation (58) into equation (57) using equation (26),

$$\begin{aligned}\hat{p}_t - \beta\hat{p}_{t+1} =& \left((-\eta+\beta) + \beta\frac{u_{CC}}{u_C}f'Q\right)\hat{Q}_{t+1}\\ &+ \left(\eta\frac{f}{p} + (\eta-1) - \beta\frac{u_{CC}}{u_C}f'Q\right)\hat{Q}_t + \left(1-\beta-\beta\frac{u_{CC}}{u_C}f\left(1-\Psi\right)\right)\hat{Z}_t\end{aligned} \tag{60}$$

Equating equation (59) and (60), we get

$$\begin{aligned}\hat{Q}_{t+1} =& \frac{-\eta f'/p - \eta + 1 + (1-\beta)\left(-\frac{u_{CC}}{u_C}f'Q\right)}{(\beta-\eta) + \beta\frac{u_{HH}}{u_C}Q}\hat{Q}_t + \frac{(-1+\beta) + (-1+\beta)\frac{u_{CC}}{u_C}f}{(\beta-\eta) + \beta\frac{u_{HH}}{u_C}\frac{Q^*}{p^*}}\hat{Z}_t\\ =& \varepsilon_{QQ}\hat{Q}_t + \varepsilon_{QZ}\hat{Z}_t\end{aligned} \tag{61}$$

which shows equation (43) and (46).

Substitution equation (61) into equation (58) gets,

$$\begin{aligned}\hat{R}_{t+1} =& (\varepsilon_{QQ} - 1)\left(-\frac{u_{CC}}{u_C} f'Q\right)\hat{Q}_t \\ &+ \left(-\frac{u_{CC}}{u_C} f'Q^* \varepsilon_{QZ} + \frac{u_{CC}}{u_C} f(1-\Psi)\right)\hat{Z}_t \\ =& \varepsilon_{RQ}\hat{Q}_t + \varepsilon_{RZ}\hat{Z}_t\end{aligned}$$

which shows equation (44) and (47).

Since $\hat{p}_t = \varepsilon_{pQ}\hat{Q}_t + \varepsilon_{pZ}\hat{Z}_t$, $\hat{p}_{t+1} = \varepsilon_{pQ}\hat{Q}_{t+1} + \varepsilon_{pZ}\hat{Z}_{t+1} = \varepsilon_{pQ}\varepsilon_{QQ}\hat{Q}_t + \varepsilon_{pZ}\Psi\hat{Z}_t$ and $\hat{Q}_{t+1} = \varepsilon_{QQ}\hat{Q}_t + \varepsilon_{QZ}\hat{Z}_t$, substituting these expressions into equation (60) turns out to be

$$\begin{aligned}&\varepsilon_{pQ}(1-\beta\varepsilon_{QQ})\hat{Q}_t + (\varepsilon_{pZ}(1-\beta\Psi) - \beta\varepsilon_{pQ}\varepsilon_{QZ})\hat{Z}_t \\ &= A\left(\varepsilon_{QQ}\hat{Q}_t + \varepsilon_{QZ}\hat{Z}_t\right) + B\hat{Q}_t + \left(1-\beta-\beta\frac{u_{CC}}{u_C} f(1-\Psi)\right)\hat{Z}_t\end{aligned}$$

where $A = (-\eta+\beta) + \beta\frac{u_{CC}}{u_C}f'Q$ and $B = \eta\frac{f}{p} + (\eta-1) - \beta\frac{u_{CC}}{u_C}f'Q$. Then by the method of undetermined coefficient,

$$\varepsilon_{pQ} = \frac{A\varepsilon_{QQ} + B}{1-\beta\varepsilon_{QQ}} = \frac{\left((-\eta+\beta) + \beta\frac{u_{CC}}{u_C}f'Q\right)\varepsilon_{QQ} + \eta\frac{f}{p} + (\eta-1) - \beta\frac{u_{CC}}{u_C}f'Q}{1-\beta\varepsilon_{QQ}}$$

$$\begin{aligned}\varepsilon_{pZ} &= \frac{(A+\beta\varepsilon_{pQ})\varepsilon_{QZ} + \left(1-\beta-\beta\frac{u_{CC}}{u_C}f(1-\Psi)\right)}{1-\beta\Psi} \\ &= \frac{\left((-\eta+\beta) + \beta\frac{u_{CC}}{u_C}f'Q + \beta\varepsilon_{pQ}\right)\varepsilon_{QZ} + \left(1-\beta-\beta\frac{u_{CC}}{u_C}f(1-\Psi)\right)}{1-\beta\Psi}\end{aligned}$$

C Derivation of equation (54)

From the equation (40),

$$\begin{aligned}\frac{Q_{t+1}-Q^*}{Q^*} &= \varepsilon_{QQ}\frac{Q_t - Q^*}{Q^*} + \varepsilon_{QZ}\hat{Z}_t \\ Q_{t+1} &= \varepsilon_{QQ}Q_t + (1-\varepsilon_{QQ})Q^* + Q^*\varepsilon_{QZ}\hat{Z}_t\end{aligned}$$

So, we have

$$Q^* \varepsilon_{QZ} \hat{Z}_t = Q_{t+1} - \varepsilon_{QQ} Q_t - (1 - \varepsilon_{QQ}) Q^* \tag{62}$$

Also, we know that from the exogenous shock (36)

$$Q^* \varepsilon_{QZ} \hat{Z}_{t+1} = \Psi Q^* \varepsilon_{QZ} \log \hat{Z}_t + Q^* \varepsilon_{QZ} \epsilon_t \tag{63}$$

Substituting equation (62) into (63), we have

$$Q_{t+2} = (1 - \Psi)(1 - \varepsilon_{QQ}) Q^* + (\varepsilon_{QQ} + \Psi) Q_{t+1} - \Psi \varepsilon_{QQ} Q_t + Q^* \varepsilon_{QZ} \epsilon_t$$

Since $Q_t = 1 - H_t$, the above expression is translated into

$$H_{t+2} = c + (\varepsilon_{QQ} + \Psi) H_{t+1} - \varepsilon_{QQ} \Psi H_t + e_t$$

where

$$\begin{aligned} c &= 1 - (1 - \Psi)(1 - \varepsilon_{QQ}) Q^* - (\varepsilon_{QQ} + \Psi) + \varepsilon_{QQ} \Psi \\ e_t &= Q^* \varepsilon_{QZ} \epsilon_t \end{aligned}$$

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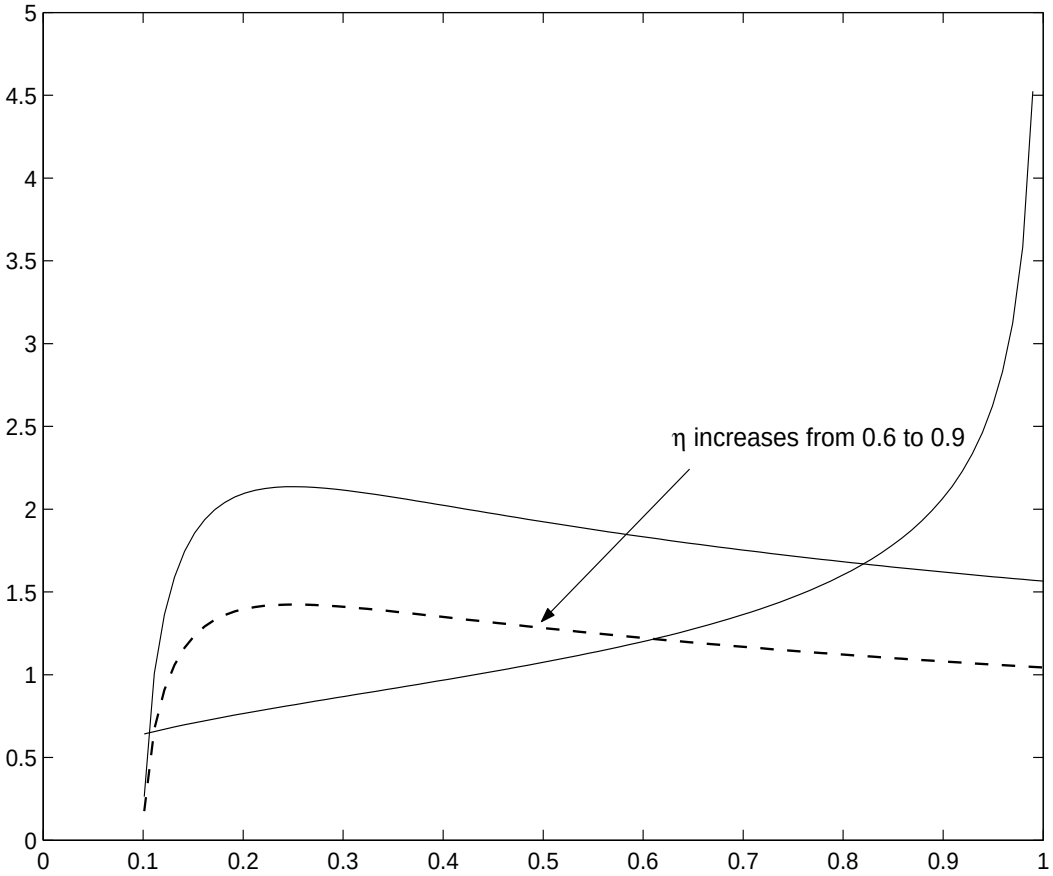

Figure 1: Equilibrium capital distribution

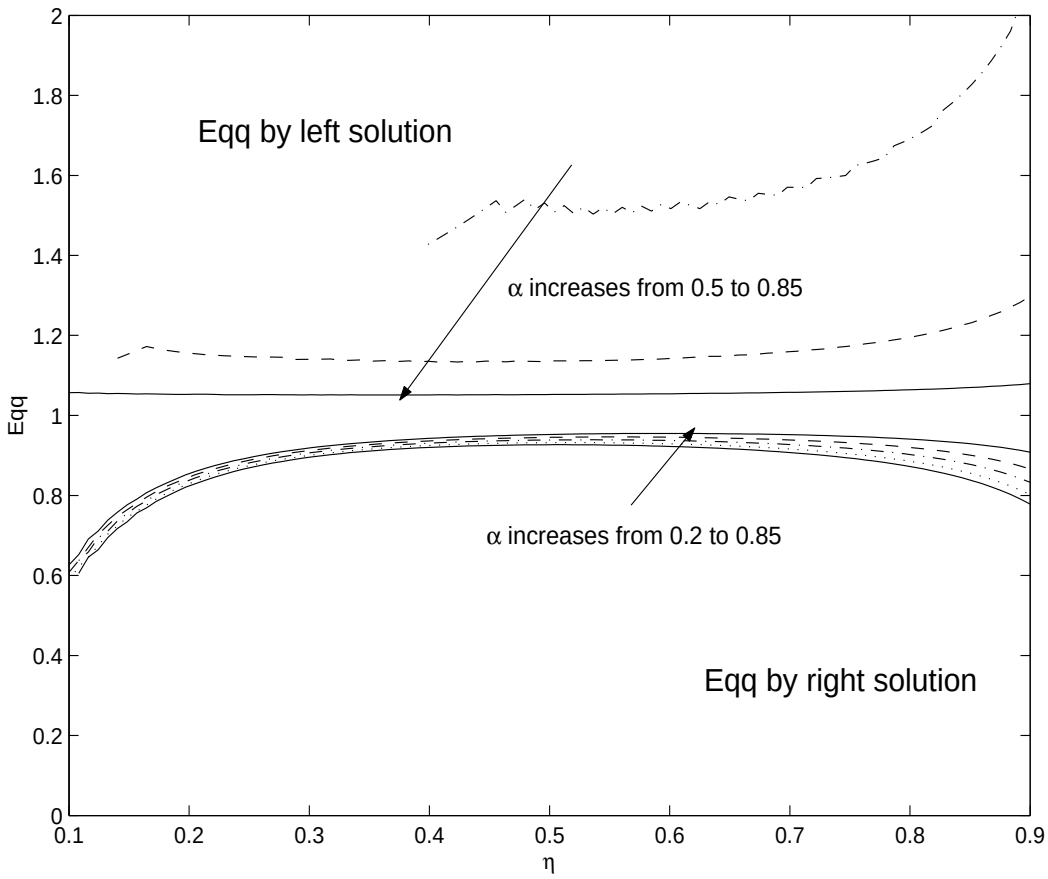

Figure 2: Changes of ε_{QQ} (both stable and unstable) in η and α

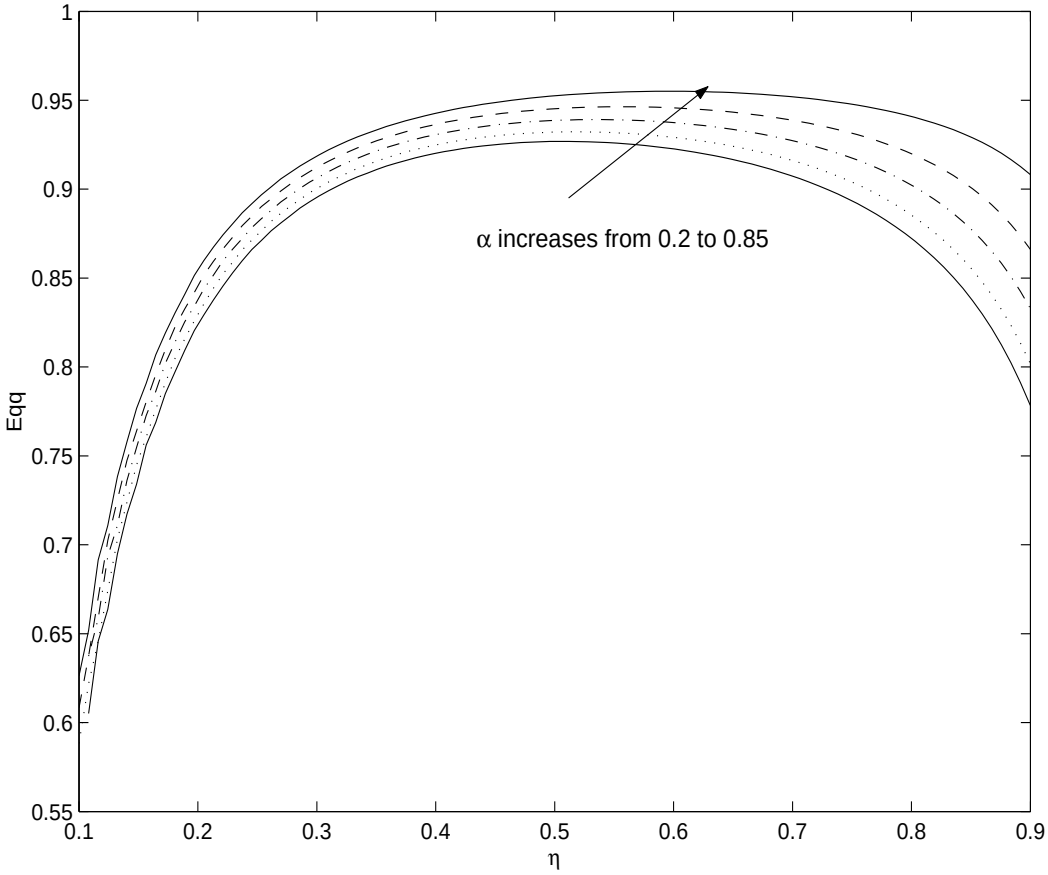

Figure 3: Changes of stable ε_{QQ} in η and α

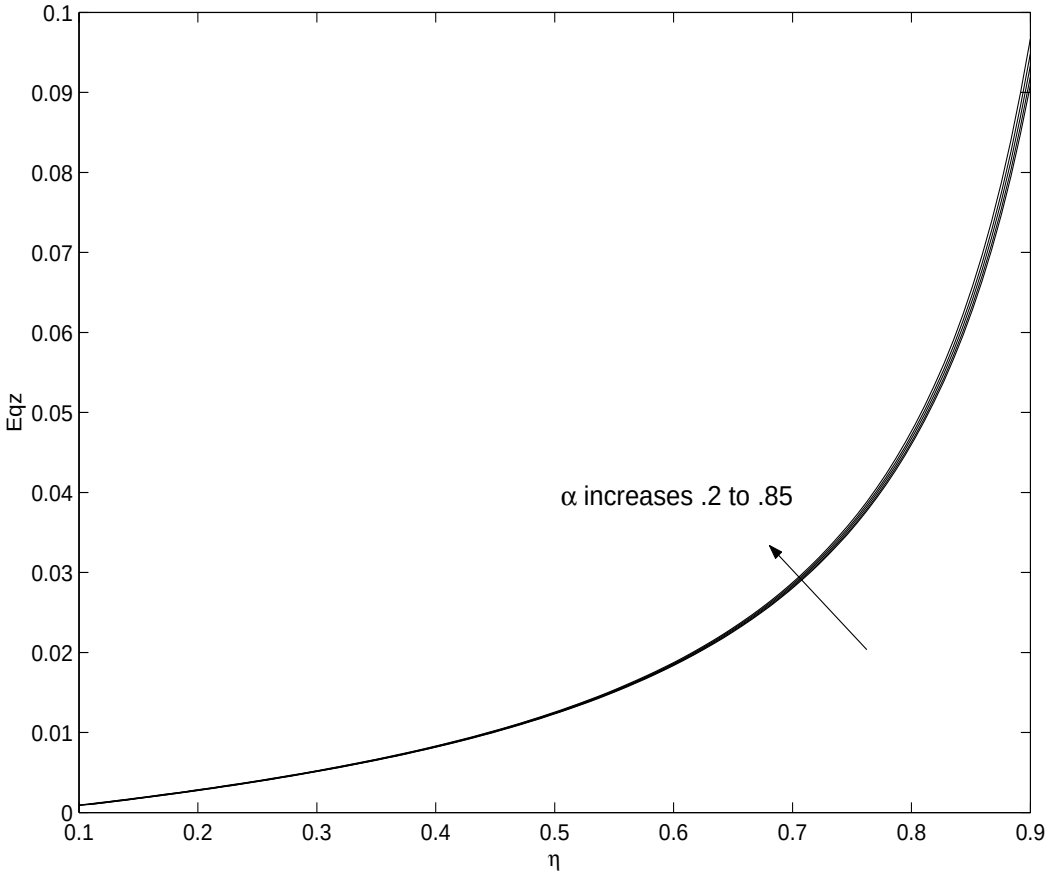

Figure 4: Redistribution of capital, ε_{QZ} in η and α

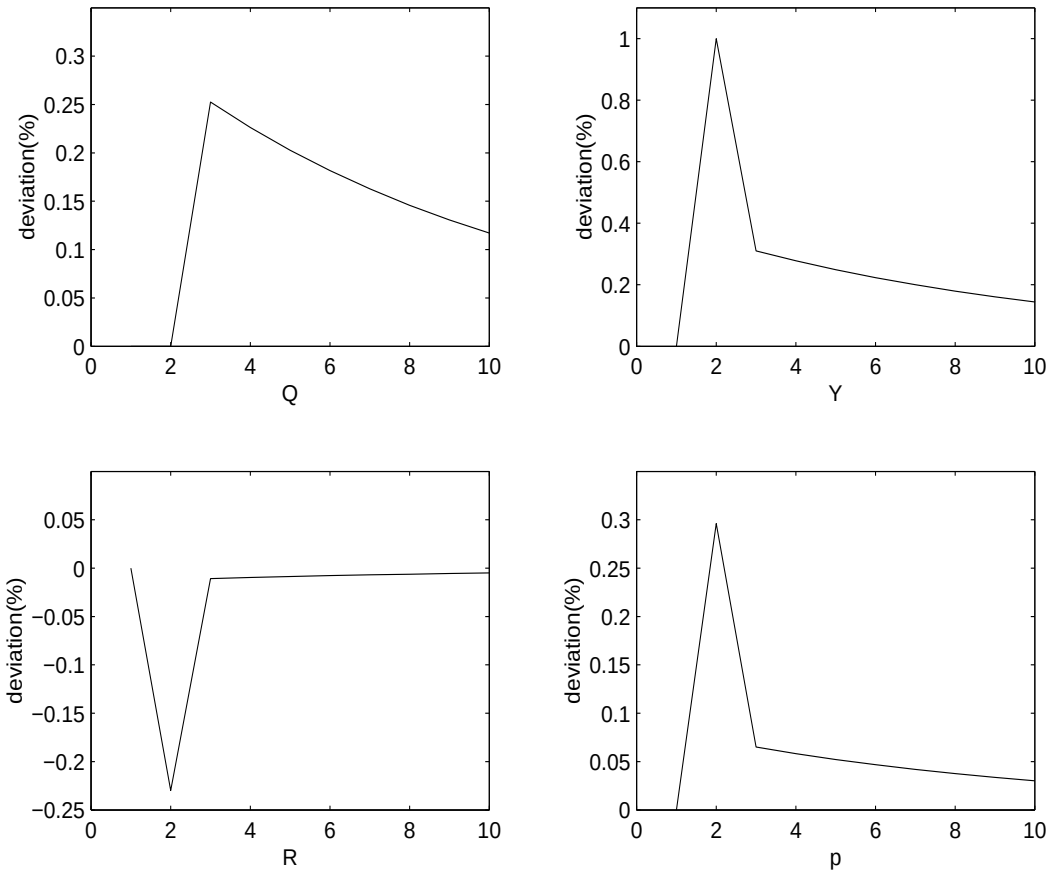

Figure 5: Impulse response functions to a 1% productivity shock

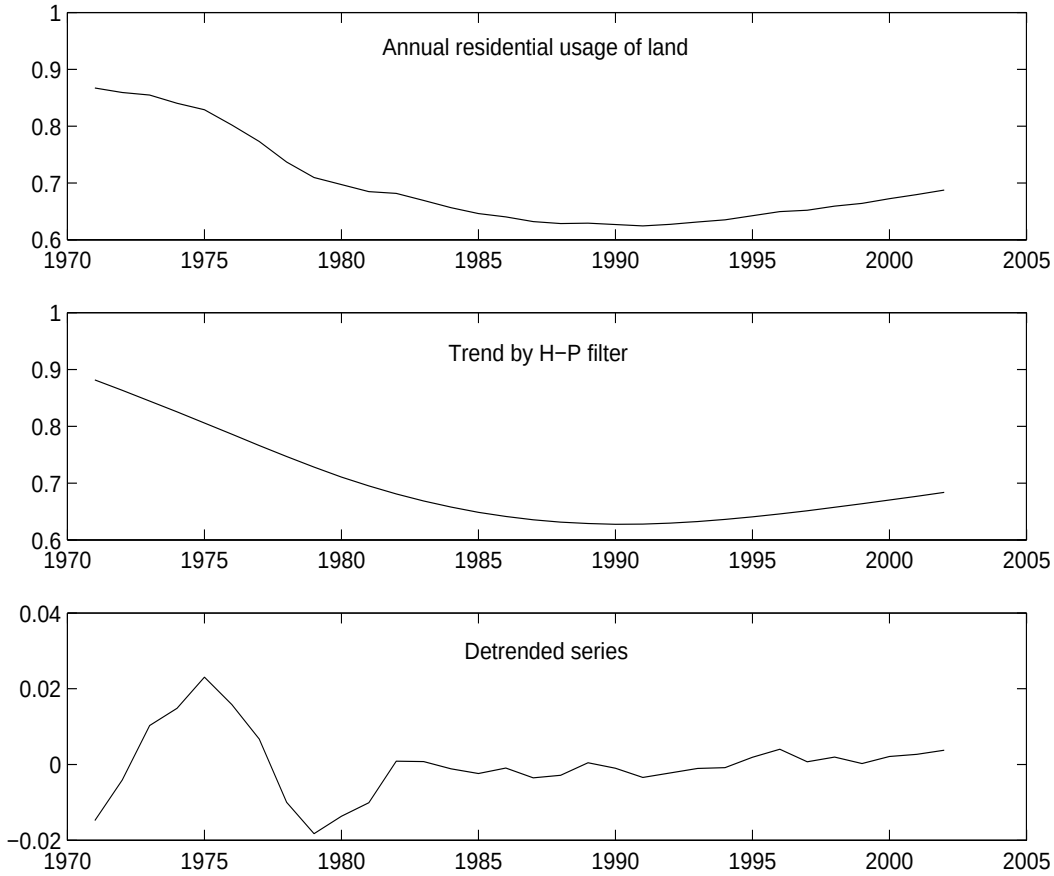

Figure 6: Residential land usage per capita in Korea

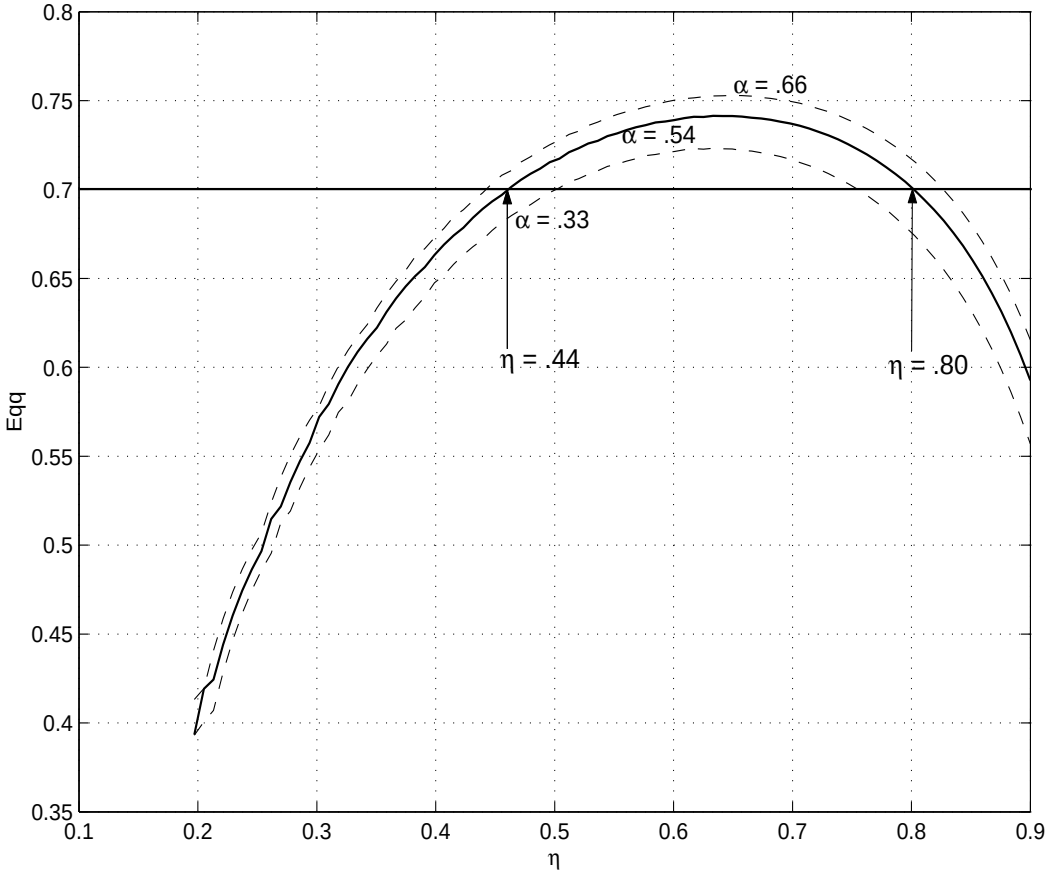

Figure 7: Estimation of debt-collateral ratio in Korea