

# Agency Costs, Net Worth, and Asset Prices

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## Abstract

For the purpose of explaining both business cycles and asset returns, we examine a real business cycle (RBC) model with habit-augmented preferences and *exogenous* costs of adjusting the capital stock. Following the agency-cost models of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), capital adjustment costs are affected by the level of entrepreneur's net worth such that an increase in net worth (following a positive productivity shock) lowers agency costs associated with external financing, and hence makes it easier to expand the capital stock. Along with the restricted labor supply, the model resolves the asset pricing puzzles of the consumption-based model in the sense that the implied stochastic discount factor (or pricing kernel) reaches the Hansen-Jagannathan (1991) volatility bound. Further, this improvement in the asset pricing dimension is achieved without reducing its business cycle performance such as output and consumption volatility. This is in a sharp contrast to the standard RBC model with the reduced-form adjustment cost technology where sufficiently low supply elasticity of capital (or persistently high capital adjustment costs) is required to generate the equity premium at the expense of low output volatility. Here, the capital supply is highly elastic under the plausible calibrations of the structural parameters affecting endogenous capital adjustment costs. The sluggish behavior of net worth, as a shifter of the capital supply curve, is the key mechanism by which capital adjustment is delayed, hampering consumption smoothing desired by households with habit persistence preferences. The agency-costs model reveals that a small curvature in the capital adjustment cost function, viewed as crucial for understanding the fluctuations in Tobin's  $q$ , can be also consistent with both the historical equity premium and the key business cycle facts.

# 1 Introduction

Recent studies in macroeconomics have directed their efforts to explain asset returns in *production* economies where output and consumption choices are determined as agents' optimal response to shocks. They include Lettau and Uhlig (1995, 2000), Jermann (1998), Boldrin, Christiano, and Fisher (2001). A rich set of empirical findings on asset returns is used to discriminate various candidate models of macroeconomic fluctuations, or often called real-business-cycle (RBC) models.

The key question is: what version of the RBC model can explain both business cycles and asset returns? Boldrin, Christiano, and Fisher (2001) suggests three modifications of the standard RBC model (e.g. Hansen 1985) to improve the asset pricing implications without reducing its business cycle performance: (i) habit persistence in preferences; (ii) features of technology that hamper the ability to use variations in leisure or labor supply for consumption smoothing; and (iii) features that lower the elasticity of capital supply in a transient manner. The inclusion of habit persistence in preferences alone turns out to be very difficult to account for substantial risk premium in production economies.<sup>1</sup> This is because endogenous consumption choices become much smoother with habit formation. The literature has found that, in addition to habit, features (ii) and (iii) of technology are required to explain the equity premium by reducing household's ability to smooth consumption.

This paper modifies the agency-cost model of Carlstrom and Fuerst (1997) to include habit persistence preferences, as a candidate to explain both business cycles and the asset returns.<sup>2</sup> Here, the capital adjustment costs are modeled in a less

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<sup>1</sup>Constantinides (1990) has shown that once a habit is added to the standard model with power utility and lognormal distribution, the equity premium puzzle of Mehra and Prescott (1985) is resolved. More recently, Campbell and Cochrane (1999) consider a different habit formation model which avoids a high and volatile risk-free rate. However, these models consider an endowment economy where in equilibrium consumption is equal to an exogenously given dividend process.

<sup>2</sup>Bernanke and Gertler (1989) first developed a general equilibrium model with incomplete asset markets in which entrepreneurs' investment must partially rely on external finance (in addition to

restrictive fashion than the standard reduced-form approach in Jermann (1988) and Boldrin, Christiano, and Fisher (2001). Capital adjustment costs (in the form of agency costs) are endogenized such that an increase in entrepreneurial net worth, following a positive technology shock, reduces the agency costs associated with external financing and hence makes it easier to expand the capital stock. Endogenous agency costs are crucial for the model to generate a close match with business cycle facts, including strongly autocorrelated output growth. For example, changes in agency costs give rise to a significant propagation mechanism: productivity shocks are magnified in their effect on output by inducing households to delay their investment decisions until agency costs are at their lowest several periods after the initial shock.<sup>3</sup>

We first examine the model with flexible labor supply. As elaborated in Lettau and Uhlig (2000) and Boldrin, Christiano, and Fisher (2001), households with habit-augmented preferences fulfill consumption smoothing by adjusting labor-leisure hours. This then implies a negligible volatility of the implied stochastic discount factor, reaching far below the Hansen-Jagannathan (1991) volatility bound (or the HJ bound in short). The model would further require some form of restricted labor supply to explain the equity premium puzzle.

Along with the restricted labor supply, the model resolves the asset pricing puzzles of the consumption-based model in the sense that the implied stochastic discount 

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internal funds or net worth) which is limited because of the agency costs involved in the financial contract. Calstrom and Fuerst (1997) builds on Bernanke and Gertler (1989) by constructing a calibrated, computable general equilibrium model in which endogenous agency costs enhance propagation of productivity shocks by affecting entrepreneurial net worth.

<sup>3</sup>As documented by Cogley and Nason (1993, 1995) and Rotemberg and Woodford (1996), most quantitative business cycle models in the real-business-cycle (RBC) tradition share the feature that model-generated output data exhibit dynamic properties nearly identical to those of the underlying exogenous shocks, so that economic mechanisms play a minimal role in propagating shocks. Among the other models of endogenous propagation mechanisms are the factor-hoarding model of Burnside, Eichenbaum, and Rebelo (1993) and Burnside and Eichenbaum (1996), labor market search models of den Haan, Ramey, and Watson (1997), Andolfatto (1996), and Merz (1995).

factor (or pricing kernel) reaches the HJ volatility bound. This improvement in the asset pricing dimension is achieved without reducing its business cycle performance such as output and consumption volatility. More importantly, the results do *not* depend on a considerably inelastic supply of capital with respect to Tobin's  $q$ : instead, the implied capital supply is highly responsive to a change in the price of capital. This is in a striking contrast to the findings based on the reduced-form capital adjustment costs technology such as Jermann (1988) which has to rely on the lower bound in the range of empirically plausible estimates of the supply elasticity of capital (or upper bound in the capital adjustment costs). In order to hamper consumption smoothing by restricting the investment variations, the model requires substantially inelastic capital supply curve.

How can the model generate the equity premium without relying on a highly inelastic supply of capital? The key is the role of net worth as a “sluggish shifter” of the relatively flat capital supply curve. The immediate impact of a positive technology shock is to increase net worth slightly as the shock raises the entrepreneur's wage and rental income, which then shifts out the capital supply curve. However, net worth's increase is limited by the initially fixed level of entrepreneurial capital. Subsequently, as the increased demand for capital raises the price of capital, pushing up the return to internal fund and hence causing net worth to rise, the capital supply curve shifts further out. This delayed shift of the capital supply curve due to a sluggish adjustment of net worth is analogous to a delayed response of labor-leisure hours in the labor hoarding model by Burnside, Eichenbaum, and Rebelo (1993) or the labor market search model by Merz (1995). Essentially, the delayed response of investment disturbs consumption smoothing desired by the households with habit persistence preferences, so that the implied stochastic discount factor reaches the HJ volatility bound. Therefore, a more general model of capital adjustment costs is shown to be consistent with both the asset returns and the key business cycle facts.

The paper is organized as follows. Section 2 describes the agency-cost model

of Carlstrom and Fuerst (1997) modified to include habit formation in preferences. Section 3 calibrates the model economy following Carlstrom and Fuerst (1997) and Campbell and Cochrane (1999). The quantitative implications for the business cycle and asset pricing are presented in section 4 by working out the log-linearized system of equations characterizing competitive equilibrium. Section 5 summarizes the paper with a few remarks and the derivation of supply elasticity of capital follows in the Appendix.

## 2 Model

We modify the agency-cost model of Carlstrom and Fuerst (1997) to include habit formation as in Campbell and Cochrane (1999). For those readers unfamiliar with the aforementioned models, we briefly describe the key features and their calibrations. There is a continuum of agents with unit mass in the model economy. The fraction  $\eta$  of population are entrepreneurs, and  $1 - \eta$  are households. The entrepreneurs engage in producing the investment good. If a household wishes to purchase capital, it must fund entrepreneurial projects which are subject to agency problems. Entrepreneurs receive their external financing in the form of financial contract with households via intermediaries, called capital mutual funds (CMFs). The economy is also populated with numerous firms producing the single consumption good. These firms are owned by households and they are not subject to any agency problems.

### 2.1 The financial contract

In any given period  $t$ , the financial contract consists of an entrepreneur (as a borrower) with net worth,  $n_t > 0$ , and a household (as a lender) which may wish to lend resources to the entrepreneur. The contract is assumed to be static in the sense that it lasts only one period. The entrepreneur has access to a stochastic technology that contemporaneously transforms  $i_t$  consumption goods into  $\omega_t i_t$  units of capital

where  $\omega_t$  is an i.i.d. random variable (across time and across entrepreneurs) with distribution  $\Phi$ , density  $\phi$ , a nonnegative support, and a mean of unity. Assume that  $\omega_t$  is private information of the entrepreneur: others can observe  $\omega_t$  only at a monitoring cost of  $\mu \in [0, 1]$  per unit of consumption goods invested.

In order to make a moral hazard problem due to the asymmetric information relevant, assume that an entrepreneur's net worth (for internal financing) is sufficiently small that the entrepreneur should get some external financing from households and hence bear some agency costs. An entrepreneur who borrows  $(i_t - n_t)$  consumption goods agrees to repay  $(1 + r_t^k)(i_t - n_t)$  capital goods to the lender. The entrepreneur will default if the realization of  $\omega_t$  is low: that is, if  $\omega_t < (1 + r_t^k)(i_t - n_t)/i_t \equiv \bar{\omega}_t$ . Assuming the pure strategy contract with nonstochastic monitoring as in Williamson (1987), the lender will monitor the project outcome only if the entrepreneur defaults, in which case it will confiscate all the returns from the project. Note that the contract is completely defined by the pair  $(i_t, \bar{\omega}_t)$ , which then implies the lending interest rate,  $(1 + r_t^k) \equiv \bar{\omega}_t i_t / (i_t - n_t)$ .

The optimal contract is characterized by the following first-order conditions:

$$q_t \left[ 1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \right] = 1 \quad (1)$$

$$i_t = \left[ \frac{1}{1 - q_t g(\bar{\omega}_t)} \right] n_t \quad (2)$$

where  $q_t$  denotes the price of capital at the end of  $t$  and  $f(\bar{\omega}_t)$  and  $g(\bar{\omega}_t)$  are respectively the fraction of the expected net capital output received by the entrepreneur and the lender (or household):

$$f(\bar{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\infty} \omega_t \Phi(d\omega_t) - [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t$$

$$g(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} \omega_t \Phi(d\omega_t) - \Phi(\bar{\omega}_t)\mu + [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t$$

Equation (1) implicitly defines  $\bar{\omega}(q_t)$ , the critical value of the entrepreneur's capital output productivity (as an increasing function of  $q_t$ ) below which the entrepreneurial investment projects default. Substituting this into (2) defines the implicit

function  $i(q_t, n_t)$ , total investment of consumption goods into the entrepreneurial capital technology. The new-capital supply function is then given by  $I^S(q_t, n_t) \equiv i(q_t, n_t)\{1 - \mu\Phi[\bar{\omega}(q_t)]\}$  which increases with both  $q_t$  and  $n_t$ . In particular, an increase in net worth is to decrease the need for external financing and hence lower agency costs, boosting the level of capital production for a given price of capital. The level of net worth ( $n_t$ ) and the price of capital ( $q_t$ ) will be determined as a part of competitive general equilibrium which consists of (i) optimizing behaviors of households, firms, and entrepreneurs; and (ii) market-clearing conditions.

## 2.2 Households

Households are infinitely lived and, following Campbell and Cochrane (1999), households' preferences are featured with habit formation:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - L_t; X_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t - X_t)^{1-\psi} - 1}{1 - \psi} + \nu(1 - L_t) \right]$$

where  $E_0$  is the expectations operator conditional on information available at  $t = 0$ ,  $\beta \in (0, 1)$  is the discount factor,  $c_t$  is household consumption, and  $L_t$  is labor supply at  $t$  with the time endowment normalized to unity. Further,  $\{X_t\}_{t=0}^{\infty}$  denotes a (stochastic) sequence of habits which is regarded as exogenous by the individual households and tied to the stochastic sequence of household consumption  $\{c_t\}_{t=0}^{\infty}$ . Let  $s_t$  denote the surplus consumption ratio given by:

$$s_t = \frac{c_t - X_t}{c_t}$$

which is assumed to follow the linear process in the logarithmic form:

$$\ln s_t = (1 - \lambda) \ln s + \lambda \ln s_{t-1} + \left( \frac{1}{s} - 1 \right) (\Delta \ln c_t - g) \quad (3)$$

Here,  $\Delta \ln c_t = \ln c_t - \ln c_{t-1}$ ,  $\lambda \in (0, 1)$ ,  $g$  is the average consumption growth rate,  $g = E(\Delta \ln c_{t+1})$ , and

$$s = \sigma_c \sqrt{\frac{\psi}{1 - \lambda}}$$

where  $\sigma_c^2$  denotes the conditional variance of consumption growth:  $\sigma_c^2 = \text{Var}_t(\Delta \ln c_{t+1})$ .

In any given period  $t$ , households sell their labor to firms at a wage rate of  $w_t$ , rent their previously accumulated capital to the firms at a rental rate of  $r_t$ , purchase consumption at the price of unity (i.e., consumption is the numeraire), and purchase new capital goods at a price of  $q_t$  at the end of period  $t$  via CMFs. Households' optimal choices are summarized by the following first-order conditions:

$$-\frac{U_L(t)}{U_c(t)} = w_t \quad \text{or} \quad \nu(s_t c_t)^\psi = w_t \quad (4)$$

$$q_t U_c(t) = \beta E_t U_c(t+1) [q_{t+1}(1-\delta) + r_{t+1}] \quad \text{or}$$

$$q_t = \beta E_t \left( \frac{s_{t+1} c_{t+1}}{s_t c_t} \right)^{-\psi} [q_{t+1}(1-\delta) + r_{t+1}] \quad (5)$$

Households supply labor according to (4), while their purchase of capital goods is determined by the equation (5) where  $\delta \in (0, 1)$  is the rate of capital depreciation. Each household is assumed to own an equal equity share in each of the firms.

## 2.3 Firms

The firms produce the consumption good using a constant-returns-to-scale production function given by:

$$Y_t = \theta_t F(K_t, H_t, H_t^e) = \theta_t K_t^{\alpha_1} H_t^{\alpha_2} (H_t^e)^{1-\alpha_1-\alpha_2}$$

where  $Y_t$  is aggregate output of the consumption good,  $K_t$  denotes the aggregate capital stock (including entrepreneurial capital),  $H_t$  and  $H_t^e$  denote respectively aggregate labor supply of households and entrepreneurs. Further,  $\theta_t$  denotes the technology (or productivity) shock which is assumed to follow the stochastic process given by:

$$\theta_t = (1 - \rho) + \rho \theta_{t-1} + \epsilon_t \quad (6)$$

where  $\epsilon_t$  is a serially uncorrelated random variable with standard deviation  $\sigma_\epsilon$ ,  $\rho \in (0, 1)$  is the autocorrelation coefficient, and the nonstochastic steady state of  $\theta$  is unity.

Competition in the markets for production inputs implies that wage and rental rates are equal to their respective marginal products:

$$r_t = \theta_t F_1(K_t, H_t, H_t^e), \quad w_t = \theta_t F_2(K_t, H_t, H_t^e), \quad \text{and} \quad x_t = \theta_t F_3(K_t, H_t, H_t^e) \quad (7)$$

where  $x_t$  is the wage rate for entrepreneurial labor.

## 2.4 Entrepreneur

Agency costs associated with external financing by entrepreneurs imply that the return to internal funds is greater than the return to external funds. In order to prevent entrepreneurs from postponing consumption to quickly accumulate sufficient capital for a complete self-financing ( $i = n$ ) and bearing no agency costs, it is assumed that entrepreneurs discount the future more heavily than households do. That is, entrepreneurs' preferences are given by

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_t^e$$

where  $\gamma \in (0, 1)$  denotes additional rate of discounting and  $c_t^e$  denotes an entrepreneur's consumption at  $t$ .

The entrepreneur raises internal funds (or net worth) by inelastically supplying unit endowment of labor to firms, selling undepreciated capital to a CMF for consumption goods, and renting capital to firms. The net worth of the entrepreneur (in consumption units) is given by

$$n_t = x_t + z_t[q_t(1 - \delta) + r_t] \quad (8)$$

where  $z_t$  denotes the capital holdings of the entrepreneur at the *beginning* of period  $t$ . Risk neutrality and the higher return from internal funds imply that the entrepreneur will always choose to put his entire net worth into the investment project.

At the end of the period, those entrepreneurs who are still solvent make their consumption decision based on the following first-order condition:

$$q_t = E_t \beta \gamma [q_{t+1}(1 - \delta) + r_{t+1}] \left\{ \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right\} \quad (9)$$

This equates the benefit of current consumption with the future return on internal funds: the term in braces, which is greater than one, is the gross expected return on internal funds. It is this additional return that encourages entrepreneurs to accumulate capital even though they discount the future more heavily than households. Further, the law of motion of aggregate entrepreneurial capital stock,  $Z_t$ , is obtained by aggregating the entrepreneurs' budget constraints:

$$Z_{t+1} = \{\eta x_t + Z_t[q_t(1 - \delta) + r_t]\} \left[ \frac{f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \right] - \frac{\eta c_t^e}{q_t} \quad (10)$$

Finally, the CMF intermediates capital purchases between households (as buyers) and entrepreneurs (as sellers). On behalf of households, it lends resources to an infinite number of entrepreneurs, so that it can exploit the law of large numbers to ensure a certain return to the household regardless of the entrepreneurs' investment outcomes. That is, an expenditure of  $q_t$  consumption goods guarantees one unit of capital. The households' demand-for-capital schedule is given by the equation (5), whereas the new-capital supply schedule is given by  $I^S(q, n) \equiv i(q, n)\{1 - \mu\Phi[\bar{\omega}(q)]\}$  as an outcome of the financial contract between households and entrepreneurs. Recall that the supply schedule is an increasing function of net worth ( $n$ ).

## 2.5 Competitive equilibrium

Given the distribution  $\Phi$  on idiosyncratic productivity across entrepreneurs and the technology process (6), a (recursive) competitive equilibrium is defined by decision rules for  $K_{t+1}, Z_{t+1}, Y_t, L_t, H_t, H_t^e, q_t, n_t, i_t, \bar{\omega}_t, c_t^e, c_t$ , and  $s_t$  as stationary functions of  $(K_t, Z_t, s_{t-1}, c_{t-1}, \theta_t)$  which satisfy the first-order conditions for optimal financial contract, (1) and (2), the first-order conditions for households' choices, (4) and (5), along with (3) describing the law of motion of habit, the entrepreneur's net-worth (8), the first-order condition for entrepreneur's consumption decision (9), and the law of motion of aggregate entrepreneurial capital stock (10). Further, they should satisfy the following market-clearing conditions in the two labor markets, a consumption-goods

market, and a capital goods market, respectively:

$$H_t = (1 - \eta)L_t \quad H_t^e = \eta \quad (11)$$

$$(1 - \eta)c_t + \eta c_t^e + \eta i_t = Y_t \quad (12)$$

$$K_{t+1} = (1 - \delta)K_t + \eta i_t [1 - \Phi(\bar{\omega}_t)\mu] \quad (13)$$

Finally, let the period- $t$  aggregate consumption be denoted  $C_t$ . Then,

$$C_t = (1 - \eta)c_t + \eta c_t^e.$$

### 3 Calibration

First of all, standard choices are made for the following parameters:  $\beta = 0.99$ ,  $\delta = 0.02$ ,  $\rho = 0.99$ , and  $\sigma_\epsilon = 0.015$ . These parameter values are along the line with Boldrin, Christiano, and Fisher (2001) and Jermann (1998). We also try  $\rho = 0.995$  to examine the effect of a highly persistent technology shock. A household's capital share is set as  $\alpha_1 = 0.36$  and labor share as  $\alpha_2 = 0.6399$ : the latter implies an entrepreneurial labor share of 0.0001. The entrepreneur's labor share needs to be positive to guarantee that each entrepreneur always has at least some labor income as a part of her net worth.

Table 1: Parameter Calibration

Preferences		Technology (Firm)		Technology (Entre.)	
$\beta$	0.99	$\alpha_1$	0.36	$\sigma_\omega$	0.207
$\psi$	2.372	$\alpha_2$	0.6399	$\Phi(\bar{\omega})$	0.00974
$\lambda$	0.97	$\delta$	0.02	$\mu$	0.25
$\sigma_c$	0.0056	$\rho$	0.99		
$\nu$	$\nu(L = 0.3)$	$\sigma_\epsilon$	0.015		
$\gamma$	0.947				

As for the monitoring technology, Altman (1984) estimates the sum of direct and indirect bankruptcy costs at about 20 percent of total firm assets. Since bankruptcy can be viewed as the entrepreneur being closed and his assets being liquidated, another measure of bankruptcy costs could be liquidation costs. Alderson and Betker (1995) use data from Chapter 11 proceedings to calculate liquidation costs equal to approximately 36 percent of firm assets. Carlstrom and Fuerst (1997) set the monitoring cost at the low end of the  $[0.2, 0.36]$  range:  $\mu = 0.25$ . Further, they assume that the distribution  $\Phi$  is lognormal with a mean of unity and a standard deviation of  $\sigma_\omega$ . The two parameters,  $\sigma_\omega$  and  $\gamma$  (additional discount factor for entrepreneur), are calibrated using the two measures of default risk: (i) a quarterly bankruptcy rate of 0.974 percent (Fisher 1994), and (ii) an annual risk premium of 187 basis points measured by the average spread between the prime rate and the 3-month commercial paper rate. Matching these two empirical measures with the model's bankruptcy rate,  $\Phi(\bar{\omega}_t)$ , and a risk premium associated with a loan of one consumption good,  $[q_t(1 + r_t^k) - 1]$ , implies  $\sigma_\omega = 0.207$  and  $\gamma = 0.947$ .

Campbell and Cochrane (1999) use the postwar quarterly data in the U.S. to calibrate  $\sigma_c$  and  $\lambda$  in the habit process. First,  $\sigma_c = 0.56\%$  is chosen to match the moments of postwar quarterly consumption growth. Second, the serial correlation parameter  $\lambda$  is chosen to match the serial correlation of log price-dividend ratios. Since there is a strong seasonality in quarterly price-dividend ratios (and hence deseasonalized data are artificially smooth),  $\lambda$  is calibrated to match the quarterly serial correlation coefficient implied by annual postwar data:  $0.88^{1/4} = 0.97$ . The risk-aversion parameter  $\psi$  is set as 2.372, which is the value chosen by Campbell and Cochrane (1999). Finally, the utility-weight-on-leisure parameter  $\nu$  is chosen so that in the steady state each household devotes 30% of their time to market activities (i.e.  $L = 0.3$ ).

## 4 Results

We analyze the business cycle dynamics with the log-linearized system of equations characterizing competitive equilibrium with  $g = 0$  for the habit equation (3) since we examine a stationary economy. These equations can be solved using the method of undetermined coefficients with the techniques in Uhlig (1999). In order to save notations, let the letters (defined earlier as the level of variables) now denote log-deviations from steady state. The state of the economy in any given period  $t$  is given by the vector  $[K_t, Z_t, s_{t-1}, c_{t-1}, \theta_t]$ . The solution for this dynamic system is a linear vector function:

$$[K_{t+1}, Z_{t+1}, H_t, q_t, n_t, i_t, \bar{\omega}_t, c_t^e, c_t, s_t] = f(K_t, Z_t, s_{t-1}, c_{t-1}, \theta_t)$$

### 4.1 Flexible labor supply

In Table 2 we present the results on the business cycle and stochastic discount factors defined as intertemporal marginal rates of substitution (*IMRS*) along with the corresponding data in the U.S. where  $\sigma_X$  and  $\mu_X$  denote respectively standard deviation in percentage terms and mean of variable  $X$ . The model-implied business cycle statistics and the first two moments of the implied stochastic discount factors are the average of the respective moments calculated respectively from the Hodrick-Prescott filtered data and raw (or "unfiltered") data over 100 simulations each 200 quarters long.

Table 2: Stochastic Discount Factors and the Business Cycle  
Flexible Labor

	Data	$\rho = 0.99$		$\rho = 0.995$	
		Habit	No Habit	Habit	No Habit
$\sigma_{IMRS}$	0.25	0.0216 (0.0008)	0.0178 (0.0007)	0.0223 (0.0008)	0.0182 (0.0007)
$\mu_{IMRS}$	0.99	0.9904 (0.0017)	0.9904 (0.0015)	0.9905 (0.0016)	0.9902 (0.0014)
$\sigma_Y$	1.89 (0.21)	0.720 (0.120)	1.820 (0.290)	0.438 (0.071)	1.61 (0.250)
$\sigma_C$	0.75	0.142 (0.010)	0.805 (0.130)	0.0902 (0.010)	0.851 (0.130)
$\sigma_c$		0.055 (0.007)	0.869 (0.100)	0.058 (0.008)	0.909 (0.110)
$\sigma_{c^e}$		18.76 (0.970)	34.68 (1.800)	11.01 (0.570)	27.97 (1.400)
$\sigma_C/\sigma_Y$	0.40 (0.04)	0.2003 (0.019)	0.4433 (0.007)	0.2076 (0.011)	0.5277 (0.0066)
$\sigma_i/\sigma_Y$	2.39 (0.06)	4.0524 (0.035)	2.8461 (0.018)	3.8824 (0.032)	2.5670 (0.0184)
$\sigma_H/\sigma_Y$	0.80 (0.05)	2.8378 (0.1784)	0.3993 (0.063)	5.5285 (0.2601)	0.4894 (0.0665)

The business cycle data in the second column are replicated from Boldrin, Christiano, and Fisher (2001) covering the sample period from 1964:1 to 1988:2. The volatility figure ( $\sigma_{IMRS}$ ) of stochastic discount factors is the minimum standard deviation of a discount factor in the Hansen-Jagannathan (1991) volatility bound (we call the “HJ bound” from now on). As elaborated in Cochrane and Hansen (1992), quarterly asset returns data (e.g. real value-weighted NYSE return and Treasury Bill

return) imply

$$\frac{\sigma_{IMRS}}{\mu_{IMRS}} \geq 0.25$$

in order for any consumption-based intertemporal asset pricing model to generate the historical equity premium. Noting that the average risk-free rate of about 1 percent implies the mean discount factor close to unity ( $\mu_{IMRS} \approx 0.99$ ), the HJ bound implies  $\sigma_{IMRS} \geq 0.25$ . Finally, columns denoted with “Habit” and “No Habit” display respectively statistics calculated from the model with and without habit formation in preferences. The “No Habit” model corresponds to Carlstrom and Fuerst (1997).

First, Table 2 shows that the Carlstrom-Fuerst (1997) model (“No Habit”) captures basic stylized facts of business cycles reasonably well, whereas the implied volatility of stochastic discount factors is very small. Hence, the model cannot explain substantial equity premium in the data.

It is now well known that models with households maximizing time separable utility function cannot generate substantial equity premium unless the coefficient of relative risk aversion is increased to unrealistically high level. This motivated several researchers to introduce habit formation into household preference specification. The inclusion of a habit in the agency-cost model produces some interesting and novel implications for the two related issues in business cycle and asset pricing. In particular, it is notable in Table 2 that, with habit formation, household consumption becomes extremely smooth: that is, volatility of household consumption ( $\sigma_c$ ) is not even a tenth of the corresponding volatility in the model without habit formation. The fall in volatility of *aggregate* consumption with habit is less dramatic due to the presence of risk-neutral entrepreneurs whose consumption is hardly affected by the inclusion of a habit.

With habit formation,  $\sigma_{IMRS}$  increases, although the improvement is too small to explain the historical equity premium. Intuitively, introducing a habit formation is very similar to increasing the value of coefficient of relative risk aversion to an appropriately high level. As households are (locally) very risk averse with habit, they want

to have very smooth consumption. This consumption smoothing significantly cancels out the direct effect of higher risk aversion on the variability of stochastic discount factors or *IMRS*. Further, as shown in Table 2, an increase in the persistence of productivity shock ( $\rho = 0.995$ ) appears to make hardly any difference in reaching the HJ bound.

In short, the agency-cost model with habit formation and flexible labor supply cannot explain the existence of substantial equity premium in the sense that the implied volatility of consumption-based stochastic discount factor is far below the HJ volatility bound. With habit formation, (locally) very risk averse households desire consumption smoothing by adjusting labor supply. For example, in response to a temporary positive productivity shock, households would increase leisure instead of consumption.

## 4.2 Fixed labor supply

In order to investigate the consumption variability with habit once agents are not able to use labor/leisure to smooth out consumption, we solve the model economy with fixed labor supply. In Table 3, it can be seen that the household consumption variability ( $\sigma_c$ ) increases substantially. This then yields a stark increase in the volatility of stochastic discount factors, approaching close to the HJ volatility bound. As the technology shock becomes more persistent ( $\rho = 0.995$ ), the implied discount factor finally reaches the HJ bound.

The question is whether the model resolves the equity premium puzzles at the expense of the business cycle facts. It is well known in the RBC literature (e.g. Campbell 1994) that a highly variable labor supply is needed to match the variability of output with respect to technology shocks. However, this is not necessarily the case in the model economy where technology shocks affect output through the capital investment channel (rather than the labor supply) by altering agency costs associated with financing the capital investment.

Table 3: Stochastic Discount Factors and Business Cycles

Fixed Labor					
	Data	$\rho = 0.99$		$\rho = 0.995$	
		Habit	No Habit	Habit	No Habit
$\sigma_{IMRS}$	0.25	0.1994 (0.0093)	0.0268 (0.0011)	0.2510 (0.0130)	0.0298 (0.0011)
$\mu_{IMRS}$	0.99	1.0072 (0.015)	0.9903 (0.0021)	1.0205 (0.0180)	0.9910 (0.0021)
$\sigma_Y$	1.89 (0.21)	1.93 (0.260)	1.95 (0.26)	1.93 (0.260)	1.94 (0.26)
$\sigma_C$	0.75	0.488 (0.081)	1.28 (0.170)	0.602 (0.098)	1.45 (0.200)
$\sigma_c$		0.51 (0.067)	1.33 (0.160)	0.637 (0.084)	1.51 (0.190)
$\sigma_{c^e}$		27.69 (1.500)	19.15 (1.000)	26.14 (1.400)	16.60 (0.87)
$\sigma_C/\sigma_Y$	0.40 (0.04)	0.2503 (0.0092)	0.6562 (0.003)	0.3094 (0.0098)	0.7477 (0.002)
$\sigma_i/\sigma_Y$	2.39 (0.06)	3.5509 (0.0449)	2.1298 (0.008)	3.3324 (0.0424)	1.8281 (0.006)

For example, as elaborated in Carlstrom and Fuerst (1997), a positive technology shock increases the return to entrepreneurs' internal funds, which causes entrepreneurial capital and hence net worth to rise. As net worth continues to grow with a somewhat persistent technology shock, entrepreneurs will continue to increase the supply of new capital (or investment) by exploiting lower agency costs associated with less use of external financing. As for the demand side, the positive shock increases marginal product of capital, which shifts out the demand-for-capital schedule. This is the agency-cost driven propagation mechanism by which the investment response

to the technology shock leads to a magnified change in output. Notice in Table 3 that the output variability is kept at a reasonably high level along with the other moments which remain relatively close to the data.

The favorable results of the agency-cost model with fixed labor supply are in a sharp contrast to Lettau and Uhlig (2000) where the fixed labor supply yields consumption only a third as variable as the U.S. consumption and output half as variable as that in the U.S. This lack of consumption variability in Lettau and Uhlig (2000) can be attributed to the absence of the propagation mechanism driven by the supply and demand for investment. Further, in Lettau and Uhlig (2000), the difference between the Habit and No Habit models are much smaller with fixed labor supply. In the agency-cost model, however, habit formation still plays a significant role in the business cycle as well as the asset pricing. For instance, the model with no habit yields too high a volatility in household consumption, whereas the habit model implies a much more reasonable one.

### **4.3 Supply elasticity of capital**

Jermann (1998) also shows that a standard RBC model with habit formation preferences and capital adjustment costs can explain the historical equity premium as well as the key business cycle properties, although labor supply is assumed to be fixed. However, the results depend crucially on the lower bound in the range of empirically plausible estimates of the supply elasticity of capital with respect to its price  $q$  (or upper bound in the curvature of the capital adjustment costs technology). More specifically, the supply elasticity of capital is specified 0.23 to match a set of business-cycle and asset-returns moments. In order to hamper consumption smoothing by inhibiting the variation in capital, the model requires the substantially low elasticity of capital supply (or close to vertical capital supply curve).

Noting that the agency-cost model as considered here can be regarded as an endogenous formulation of capital adjustment costs, we can compute the corresponding

supply elasticity of capital from the log-linearization of the capital supply function  $I^S(q, n) \equiv i(q, n)\{1 - \mu\Phi[\bar{\omega}(q)]\}$  as discussed in section 2.1:

$$I^S(\hat{q}_t, \hat{n}_t) = \hat{n}_t + \eta_q^S \hat{q}_t$$

where  $\eta_q^S$  denotes the supply elasticity of new capital with respect to  $q$ , evaluated at a given set of parameter values in the steady state (see the Appendix for its derivation). It can be shown that the supply elasticity  $\eta_q^S$  is a decreasing function of  $(\mu, \sigma_\omega)$ , the monitoring cost and the degree of idiosyncratic uncertainty in entrepreneurial investment projects, respectively. This property of the supply elasticity of capital is consistent with its counterpart in the standard capital adjustment cost model in the sense that these two parameters determine the model's capital adjustment costs in the form of the agency costs. In a striking contrast to Jermann (1998), the structural parameters in the model economy (e.g.  $\mu = 0.25$  and  $\sigma_\omega = 0.207$ ) implies highly elastic capital supply:  $\eta_q^S = 8.94!$

How can the model generate the equity premium without relying on a considerably inelastic supply of capital? The key is the role of net worth,  $n$ , as a “sluggish shifter” of the capital supply curve: recall that  $I_n^S(q, n) > 0$  as well as  $I_q^S(q, n) > 0$ . In the model economy, the immediate impact of a positive technology shock is to increase net worth slightly as the shock raises the entrepreneur's wage ( $w_t$ ) and rental income ( $r_t$ ): see equation (8). However, net worth's increase is limited by the initially fixed level of entrepreneurial capital ( $z_t$ ). Subsequently, as the increased demand for capital pushes up the price of capital ( $q_t$ ), the return to internal funds  $qf/[1 - qg]$  goes up causing net worth to rise: see equation (10) which is the law of motion of entrepreneurial capital stock. This sluggish behavior of net worth (or accumulated capital) leads to the delayed shift of the capital supply curve following the shock, which is similar to a delayed response of labor supply due to the labor hoarding behavior or the search frictions in the labor market.<sup>4</sup> Essentially, the delayed response of investment puts the “burden” of cyclical variations onto consumption. Despite habit-augmented

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<sup>4</sup>As properly noted by Carlstrom and Fuerst (1997), when the adjustment costs technology de-

preferences, the resulting variations in consumption enable the implied stochastic discount factor to reach the HJ volatility bound.

## 5 Concluding Remarks

We have attempted to explain both business cycle facts and asset returns using an RBC model which incorporates habit-augmented preferences and endogenous costs of adjusting the capital stock. Following the agency-cost model of Carlstrom and Fuerst (1997), capital adjustment costs are affected by the level of entrepreneur’s net worth such that an increase in net worth (following a positive productivity shock) lowers agency costs associated with external financing, and hence makes it easier to expand the capital stock. In the presence of the restricted labor supply, the agency-cost model of capital adjustment costs is capable of resolving the key asset pricing puzzles without reducing its business cycle performance.

In contrast to the standard reduced-form approach to the capital adjustment costs technology, the results do not depend on the lowest possible value in the empirically plausible estimates of the supply elasticity of capital with respect to Tobin’s  $q$  (or maximum value in the capital adjustment costs). Instead, the capital supply is highly elastic under the plausible calibrations of the “deep” structural parameters. The sluggish behavior of net worth, as a shifter of the now relatively flat capital supply curve, is the key mechanism by which capital adjustment is delayed, hindering consumption smoothing desired by households with habit persistence preferences. Therefore, a small curvature in the capital adjustment cost function, viewed as crucial for understanding the fluctuations in Tobin’s  $q$ , can be also consistent with the historical equity premium and the key business cycle facts.

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depends on the investment-capital ratio (not investment only) as in Jermann (1998), the capital supply curve also shifts out as capital begins to grow. However, there is no delayed response of investment because households internalize the effect and increase their initial investment in anticipation.

## 6 Appendix: supply elasticity of capital

As discussed in section 2.1 Financial contract, the period- $t$  investment (or new-capital) supply function is defined as  $I^S(q_t, n_t) \equiv i(q_t, n_t)\{1 - \mu\Phi[\bar{\omega}(q_t)]\}$  where  $I_q^S(q_t, n_t) > 0$  and  $I_n^S(q_t, n_t) > 0$ . Log-linearization around steady state yields:

$$I^S(\hat{q}_t, \hat{n}_t) = \hat{i}_t - \left[ \frac{\phi(\bar{\omega})\mu\bar{\omega}}{1 - \Phi(\bar{\omega})\mu} \right] \hat{\bar{\omega}}_t \quad (\text{A.1})$$

In order to write  $\hat{\bar{\omega}}_t$  as a function of  $\hat{q}_t$ , we can log-linearize (1) to obtain:

$$\hat{\bar{\omega}}_t = \frac{1}{X} \left( \frac{1}{q} \right) \hat{q}_t \quad (\text{A.2})$$

where

$$X = \mu\phi(\bar{\omega})\bar{\omega} \frac{f(\bar{\omega})}{f'(\bar{\omega})} \left[ \frac{f''(\bar{\omega})}{f'(\bar{\omega})} - \frac{\phi'(\bar{\omega})}{\phi(\bar{\omega})} \right].$$

Similarly, we can log-linearize (2) to obtain:

$$\hat{i}_t = \left[ \frac{g(\bar{\omega})q}{1 - qg(\bar{\omega})} \right] \hat{q}_t + \left[ \frac{qg'(\bar{\omega})\bar{\omega}}{1 - qg(\bar{\omega})} \right] \hat{\bar{\omega}}_t + \hat{n}_t \quad (\text{A.3})$$

Now, substituting (A.3) and (A.2) into (A.1) to obtain the log-linearized version of capital supply function:

$$I^S(\hat{q}_t, \hat{n}_t) = \hat{n}_t + \eta_q^S \hat{q}_t$$

where

$$\eta_q^S \equiv \frac{g(\bar{\omega})q}{1 - qg(\bar{\omega})} + \left[ \frac{qg'(\bar{\omega})\bar{\omega}}{1 - qg(\bar{\omega})} - \frac{\phi(\bar{\omega})\mu\bar{\omega}}{1 - \Phi(\bar{\omega})\mu} \right] \frac{1}{X} \left( \frac{1}{q} \right).$$

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