Abstract:
The paper investigates a spatial competition model of Salop in a bank loan and deposit market. Asymmetric information between bank and borrower exists about the refinancing probability of the investment project. The banks set up a menu of loan contracts under which the borrower makes a bank choice. We study the short-run and long-run loan and deposit market equilibrium.

JEL Classification: D43; D82; G21

Key words: Banking, Spatial Competition, Entry, Market Structure; Agency Problem

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1. Introduction

The vinculum of industrial organization with corporate finance has been scant in the literature, even if each has progressed rapidly as distinct fields of inquiry\(^1\). This paper intends to add on how capital markets influence competition in banking industry\(^2\). Specifically, the Bolton and Scharfstein (1990) threat of termination model is applied to bank loan contract and then to its results the Salop’s location model (1979) is applied to look into banking industry.

Many models depicting price competition in the banking industry consider Bertrand approach that lacks product differentiation. Another approach that allows product differentiation in the banking industry is found in a few papers that include Chiappori et al. (1995), Dell’Ariccia (2001), and Hyytinen (2003). All these papers use Salop’s model of a circular economy of banks and economic agents. Economic agents can be depositors and/or borrowers. Chiappori et al. consider both deposit and loan market in a banking industry and study the effects of regulation on the equilibrium deposit and loan rates. They find that regulation causes bundling of the two rates as a way to avoid the regulation. Dell’Ariccia’s paper, unlike Chiappori et al., introduces the loan market with informational asymmetry.

Banks are differentiated because of differences in ATM facilities, availability in various regions, internet banking services. Location model fits naturally with the intuition of bank branching and regional location. Recent theoretical papers highlight the importance of distance in explaining the availability and pricing of bank loans and deposits. Lending and borrowing conditions may depend on the distance between the borrower and the closest competing bank and the distance between the lender and the closest competing bank.

Our paper considers the bank loan market in a spatial economy and studies

\(^1\) Michael Riordan (2004) claims that the industrial organization effects of capital structure is one of four important topics for the theory of capital structure identified recently by Milton Harris and Artur Raviv (1991), in which “…it appears that models relating to product and input markets are underexplored, while the asymmetric information approach has reached a point of diminishing returns…”

\(^2\) There are a few important additions to the theoretical literature.
the effects of informational asymmetry on the equilibrium loan size and deposit rate in a model of spatial competition. The outline of the model is as follows. There are \( n \) banks located symmetrically on a unit circle. A continuum of borrowers and lenders with a unit mass are independently distributed uniformly along the circle. Borrower’s investment project gives uncertain return; only borrower knows how good the project is. The risk characteristics may be classified as a parameter, \( \Theta \). Each borrower and a nearby bank make a loan contract. The bank charges different interest rates according to the reported value of \( \Theta \). Given the menu of the interests, each borrower signs the loan contract with the bank, and then chooses and travels to a bank offering a varying loan size depending on the expected profit size between two adjacent banks. Revelation Principle determines the interest rate in this adverse selection problem. In the literature there are two different well-established loan contracts. The one is the one-period costly state verification model of Townsend (1979), Gale and Hellwig (1985), and further developed by Williamson (1987), where the bank cannot observe the result of the investment made by the borrower, unless costly audit is performed. The second loan contract, which our model explores, is the two-period repeated borrower-lender relationship model of Bolton and Scharfstein (1990) in which the threat of termination by the bank at the end of the first period provides incentives for the borrower to repay the loan. The informational asymmetry lies between each borrower and each bank. In the deposit market, depositors travel to the bank to deposit their savings. Both traveling costs make bank possess market power through their product differentiation in loan and deposit markets.

The model has three stages. At Stage I, each depositor travels to his favorite bank and in turn the bank allocates capital from depositors between loan assets and other assets. This choice determines not only the quantity of

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3 For an application of the threat of termination to banks, see Bond and Krishnamurthy (2004).
4 In the Dell’Aricea model (2001), the asymmetry is between banks.
5 There is a model of asymmetry between bank and borrower, See Hyttinen (2003). But his informational asymmetry stems from an exogenous belief asymmetry on the project’s return. So there is no issue of adverse selection in such a model.
production by the borrower/entrepreneur, but also the banking industry equilibrium. A bank, which diverts loan assets to other assets, can lose market share to the nearby bank and leave the industry in the long run. Banks have access to a fixed amount of equity at a cost of $\rho$. The fixed supply of bank equity represents the notion that bank equity is not only costly but also that banks must compete among them to raise it, at least on short notice. This is consistent with Stein (1998), Almazan (2002), and Dell’Ariccia and Robert Marquez (2003), among others, who argue that, in the short run, banks may face a nearly fixed supply of equity capital.

At Stage II, entrepreneurs travel to a nearby bank incurring travel cost to borrow loan from the bank. Each entrepreneurial firm and bank makes a loan contract.

At Stage III, each firm invests each over two periods by using the borrowed capital from bank, provided that it obtains refinancing from the bank in the second period based on the first period’s performance.

Industry equilibrium satisfies two conditions that link industrial organization and corporate finance perspectives. First, at Stage I, banks’ total deposits are determined via their competition in the deposit market that is monopolistically competitive. Second, banks’ loan size choices at Stage II form a Nash equilibrium given loan contract constraints. Loan contracts satisfy capital market incentive compatibility, limited liability, and individual rationality constraints. If these constraints are satisfied, then entrepreneurs have no incentive to renegotiate with banks.

A loan contract is specified for a borrower located at a given location. We use the Salop’s location model to describe the banking industry equilibrium, where we specify for each borrower both the participation constraint and location indifference constraint to derive the demand function for each bank and solve the bank’s expected profit maximization problem. Then we find the market clearing Nash equilibrium both in the short run and the long-run equilibrium.

The next section presents the model. It discusses moral hazard and adverse
selection; the two period adverse selection and short-run equilibrium. The bank entry and long-run equilibrium are discussed in subsequent sections. We also consider purely competitive deposit market structure. The final section concludes.
2. Financial Contract and Competition for Bank Loans

For our purpose we modify Salop (1979) model in which banks as lenders compete for loans to entrepreneurial firms as borrowers, and for deposits. We assume that there are $n$ banks located symmetrically around a circle of measure 1, and firm (depositor) population is uniformly (independently) distributed around the circle. When contracting a loan, firms consider the interest rate they have to pay, and the per length ‘transportation cost’, $\tau$, they travel to the chosen bank. $\tau$ can be viewed as a measure of the degree of product differentiation. When depositing, depositors consider the interest rate they receive from banks, and the per length ‘transportation cost’, $t$, they travel to the chosen bank. The transportation cost, $t$ is similarly viewed as a measure of the degree of product differentiation. Banks could be differentiated because of differences in ATM facilities, availability in various regions, internet banking services.

In order to solve for the equilibrium, we apply backward induction. First we describe the investment projects of the borrowers at Stage III. Second, we describe and solve the loan contract between the banks and borrowers at Stage II. Then, at Stage I, we describe the deposit market and solve for a sub-game perfect equilibrium.

2.1 Banks and Borrowers

We consider agency problem between the bank and borrower in the location model. Both bank and firm are risk neutral. Assume that the type of a firm has a probability measure 1 at $\theta$ in $(0, 1)$. We study a repeated borrower-lender relationship of Bolton and Scharfstein (1990) in which the threat of termination by the bank provides incentives for the borrower to repay the loan.
The timing of events is as follows: At the initial date 0, a firm must borrow a fixed size of loan to finance an investment project. Bank makes a take-it-or-leave-it contract offer to firm, which the firm accepts if the contract provides nonnegative expected value. If contract is accepted, borrower travels one time to the bank to retrieve the loan. Then there are two more periods unfolded, date 1 and 2 of production and staged financing. Please see Figure 1 below for the time-line.

Firm’s gross profit from financing the amount $F$ (before financing cost, or the size of the loan, of $F$) in each period is either $\pi_1$ with probability $\theta$ or $\pi_2$ with probability $1- \theta$ ($b> \rho >a >0$). Profits are independently distributed across periods. We assume that gross profits are positively dependent on the loan size $F$ that bank provides. For simplicity we assume that this dependence is linear; $\pi_1(F) = a F$ and $\pi_2(F) = b F$ where $b> \rho >a >0$, where $\rho$ is the deposit rate. We make $F$ endogenous that bank chooses in competition with the other
banks nearby. F is a function of the deposit rate over which banks compete. The expected net present value of the investment is assumed to be positive:

\[ \bar{\pi} \equiv \theta \pi_1 + (1- \theta) \pi_2 > \rho F, \text{ or equivalently,} \]

\[ \theta a +(1- \theta)b- \rho > 0 \]

Solving the agency problem, given F, between a typical bank and borrower yields the Bolton and Scharfstein optimal contract. Let \( \{R_1^*, \beta_1^*, R_2^*, \beta_2^*, R_1^*, R_2^*\} \) denote the optimal contract, where \( R_i \) is the transfer at date 1 if the firm reports profits of \( \pi_i \) in date 1, \( \beta_i \in [0, 1] \) is the probability that the bank gives the firm F dollars at date 1 to fund the second-period production if the firm reports profits of \( \pi_i \) in date 1, and \( R_i^* \) is the second-period transfer if the first-period reported profit is \( \pi_i \). The symbol * denotes optimality. (The dependence of profits on F is omitted as long as it is clear).

Then \( \{R_1^*, \beta_1^*, R_2^*, \beta_2^*, R_1^*, R_2^*\} = \{\pi_1, 0, \bar{\pi}, 1, \pi_1, \pi_1\}. \) (For proof, see Bolton and Scharfstein. (1990)) The main point of this optimal contract is that for a given one borrower, it maximizes the expected profits of the bank subject to the borrower’s incentive compatibility condition for dates 1 and 2, and the borrower’s limited liability and individual rationality conditions. The bank cuts off funding if the firm reports low profits. The firm’s performance affects its financing costs and its access to capital.

Then the maximum expected profit of each bank given the contract is

\[ E \pi \equiv \pi_1 - \rho F + (1- \theta)(\bar{\pi} - \rho F). \]

The maximum expected profit of each borrower given the contract is

\[ G \equiv (1- \theta)(\pi_2 - \pi_1). \]

Now that agency problem was discussed between each bank and each

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6 We make this assumption to distinguish and extend our results from the model of Bolton and Scharfstein where gross profits are constant, independent of F.

7 Bank also chooses how much to lend in Dell’Ariccia and Robert Marquez’s paper (2003).
borrower, we move to competition among banks.

At date 0, the bank is able to attract the borrower located at distance x from it only if the borrower’s participation constraint is satisfied and the bank’s offer is more lucrative than the offers of the rival banks.

The distance between bank i and the borrower is x. Then firm \( \theta \) will demand a unit of loan if

i) \( G^i - \tau x \geq 0 \)

and

ii) \( G^i - \tau x \geq G - \tau(1/n - x) \)

where \( G^i \) is the expected profit of borrower from bank i and \( G \) is its rival bank’s lending offer of the borrower’s expected profit.

Condition i) is the participation constraint and Condition ii), the incentive compatibility condition, says that bank i’s loan offer is more lucrative than the offers of rival banks.

Then, the total demand and profits now become as follows:

The total demand of bank i, under full-scale competition, is

(a) \( D^i = 1/n + (1/\tau) (G - G^i) \)

and the total expected profit of bank i from all the borrowers attracted to the bank is

(b) \( \Pi^i = D^i E \pi^i \).

### 2.2 Banks and Depositors

A typical bank collects funds from depositors by offering interest rate \( \rho \), and invests the proceeds in the project described in the previous subsection. Funds are supplied to the banks by a continuum of depositors, uniformly distributed along the circumference. Each depositor is endowed with a unit of cash that she can invest with in bank deposits, incurring a transportation
cost of $t$ per unit of distance, or in a risk-free asset with normalized net return of zero for simplicity.

Consider a depositor located between banks $i$ and $i+1$, at a distance of $y \in [0,1/n]$ of bank $i$. Let $\rho^i$ denote the deposit rate charged by bank $i$. A depositor located at a distance $y$ of bank $i$ is indifferent between depositing in bank $i$ and a nearby bank, if $\rho^i - t \ y = \rho - t \ (1/n - y)$, from which we obtain the following deposit supply function for bank $i$:

$$S^i = 2y = 1/n + (\rho^i - \rho)/t$$

where $\rho$ denotes the nearby bank deposit rate. Banks have no equity in this model. Therefore, in order to meet the demand for loan, $D^iF^i$, the following balance sheet identity condition holds true:

(c) $S^i = D^iF^i + (1-\theta)D^iF^i$.

Formally, a typical bank $i$’s problem in the spatial market is to choose the loan size $F^i$ in the following:

(P) Maximize $\Pi^i = D^i E \pi^i$
subject to
(1) $D^i = 1/n + (1/\tau) (G - G^i)$;
(2) $E \pi^i = \pi_1 - \rho^i F^i + (1-\theta)(\bar{\pi} - \rho^i F^i)$
   $$= [(a - \rho^i) + (1-\theta) \ (\theta a + (1-\theta) b - \rho^i)] F^i;$$
(3) $G^i = (1-\theta)(\pi_2 - \pi_1) = (1-\theta)(b-a)F^i$
(4) $S^i = D^iF^i + (1-\theta)D^iF^i$

where superscript $i$ denote bank $i$ or its contracting borrower or its depositor.

Previous participation constraint and incentive compatibility condition of each bank is altered and reflected in the total demand (1) that bank $i$’s offer of the borrower’s profits is nonnegative and more lucrative than the offers of rival banks. As in equation (2), each bank supplies capital if and only if
\[ A \equiv (a - \rho) + (1 - \theta) \{ \theta a + (1 - \theta)b - \rho \} > 0 \]  \(8\)

where \( A \) is the expected profit of the bank per unit of the loan in the optimal loan contract.

All the projects are funded, as long as they produce positive net present value over the loan cost, i.e., \( A > 0 \). Since \( \pi > \rho F \) the term, \( \{ \theta a + (1 - \theta)b - \rho \} \) in \( A \), is strictly positive, but \( a < \rho \). The first term \( a - \rho \) in \( A \) represents the per unit incentive-compatible expected sum of the first-period transfer if low profits are reported in the first period and a portion of the first-period transfer if high profits are reported in the first period, subtracting the per unit loan cost. The portion is the first period transfer by the firm who tells a lie. Therefore, it is the bank’s incentive compatible loss in the first period. The second term in \( A \), \( (1 - \theta) \{ \theta a + (1 - \theta)b - \rho \} \), is the bank’s discounted gain in the second period, when the contract induces the firm to report profits truthfully. When the low profits are reported, the second period funding is terminated and the firm is liquidated. The contract honors first-period payment of \( \pi \) for \( F \) units of funding when high profits are reported in the first period. And the second period funding continues. So \( (1 - \theta) \{ \theta a + (1 - \theta)b - \rho \} \) represents the bank’s portion of the expected surplus of the firm in the second period operation. It is not obvious that capital is always supplied, even though the second term \( (1 - \theta) \{ \theta a + (1 - \theta)b - \rho \} \) in \( A \), the expected net present value of the investment from the second period, is strictly positive and exceeds the per unit deposit cost, \( \rho \). The first term in \( A \), the first period loss \( a - \rho \), may outweigh the second period gain. There must be an upper bound on \( \theta \) above which capital is not supplied. We assume that \( \theta \) is below that upper bound so that capital is supplied\(9\).

It is noteworthy that the expected profit of the bank per unit of the loan in the optimal loan contract, \( A \), increases with \( a \) or \( b \), but decreases with \( \theta \). An increase in \( a \) will make larger the expected profit of the bank; by enlarging

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\(8\) The above \( A > 0 \) is equivalent to the Bolton Scharfstein nonnegative profit condition \( F^1 < \frac{\pi}{\pi_2} - (\frac{\pi}{\pi_2} - \pi_1)/(2 - \theta) \).

\(9\) In order for \( \pi = \theta \pi_1 + (1 - \theta) \pi_2 > \rho F \), it is necessary that \( \theta \) must be below \( (b - \rho)/(b - a) \). Now \( \theta \) must be smaller than this value to have \( A \) to be positive.
the per unit expected net present value of investment from the second period and, at the same time, lowering the incentive compatible loss in the first period. An increase in \( b \) (high profits) or a decrease in \( \theta \) (the probability of low profits) raises the expected profit of the bank by increasing the per unit expected net present value of investment.

3. **Equilibrium**

We will be solving for a sub-game perfect equilibrium for this game. Solving the bank’s problem in the previous section yields;

**Proposition 1:** Suppose \( \pi_1(F) = a F \) and \( \pi_2(F) = b F \) where \( b > \rho > a > 0 \) and \( \theta a + (1-\theta)b - \rho > 0 \)

The bank supplies capital if and only if \( A \equiv (a - \rho) + (1-\theta)(\theta a + (1-\theta)b - \rho) \) > 0

*In this case, at the sub-game perfect Nash equilibrium, the optimal deposit rate \( \rho^* \) and optimal loan size \( F^* \) are, respectively:*

\[
\rho^* = \left[ a + (1-\theta)(\theta a + (1-\theta)b) \right] / (2-\theta) - t/n
\]

and

\[
F^* = 1 / (2 - \theta).
\]

**Proof:** See the Appendix.

**Corollary 1:** \( \rho^* \) decreases with \( \theta \) or \( t \), but increases with \( n \) or \( a \) or \( b \).

\( F^* \) increases in \( \theta \)

The first part of Corollary 1 demonstrates that an increase in the average return on a bank’s portfolio (with a decrease in \( \theta \) or an increase in \( a \) or \( b \)) will increase the deposit rates in equilibrium. (See for the same result with a different differentiated banking industry model Shy, O., and R. Stenbacka, 2004). The second part of Corollary 1 demonstrates how the degree of competition affects the deposit rates. When competition is intense (\( t \) is low or \( n \) is high), banks must pay higher deposit rates (For similar interpretation, see Shy, O., and R. Stenbacka, 2004 for \( t \) value and Besanko and Thakor, 1992 for \( n \) value). The third part of Corollary 1 shows that the investment
project becomes smaller with an increase in the average return on the bank’s portfolio.

**Proposition 2:** For a typical bank, the subgame perfect equilibrium expected bank profit from each loan contract, $E \pi$ and total loan contracts, $\Pi$, are, respectively,

$$E \pi = \frac{t}{n}$$
and
$$\Pi = \frac{t}{n^2}.$$  

For a typical borrower, the subgame perfect equilibrium expected borrower profit $G$ is

$$G = \frac{(1- \theta)(b-a)}{2\theta}.$$  

**Corollary 2:** $G$ decreases with $\theta$, but increases with $b-a$.

$\Pi$ decreases with $n$, but increases with $t$.

Now, we ask how entry by banks is influenced by adverse selection and what the effect of adverse selection in a loan contract on loan market structure.

Dewatripont and Maskin (1995) in a model of no market structure showed that when a loan market becomes decentralized (or competitive), the negative effect of agency problem is being reduced because the lender’s incentive to monitor the borrower becomes less effective as competition in the market is intensified. We want to obtain the converse of this result; the severe agency problem makes, in the long run equilibrium, the loan market less competitive. The market structure studied here is one of the varieties of monopolistic competition, Salop’s spatial economy.

In order to make $n$ endogenous, we assume that, at the entry stage of a bank, there is a cost of $S$ of setting up a bank. (Chiappori et. Al., 1995 and Hyytinen, 2003). The number of banks in the monopolistically competitive
market in the long-run equilibrium is determined by the Chamberlinian free entry condition that the expected profits net of entry costs become zero, i.e., \( \Pi(n^*) - S = 0 \), where \( n^* \) denotes the long-run equilibrium number of banks. From Proposition 2, solving for \( n^* \) yields the following:

**Proposition 3:** The long run equilibrium number of banks, \( n^* \) is

\[ n^* = \left\{ \frac{t}{S} \right\}^{1/2}. \]

The interpretation is straightforward.

### 4. Other Deposit Market

Proposition 1 demonstrated that when depositors have a choice of banks according to product differentiation offered by banks competition induces banks to offer higher deposit rates to attract the fund to be loaned out to the borrowers, and to secure the loanable funds based on the success probability of the investment projects. The effect of market structure on the characteristics of the loanable fund might depend upon on what kind of deposit markets the banks face. We will therefore concentrate in this section our analysis on one type of deposit market structure; the perfectly elastic deposit supply.

#### 4.1. Perfectly Elastic Deposit Supply

Here we assume that banks can secure as much funding as possible at a given market rate. The deposit supply function thus is perfectly elastic with respect to the deposit rate, \( \rho \). The situation may arise because of either competitive deposit market or the role of the central bank. In the latter case, the central bank decides on the policy deposit rate to stabilize output, say, according to Taylor rule (for example, see Toolsema, L.A. (2004)). A typical bank’s problem P now becomes:
(PC) Maximize $\Pi^i = D^i E \pi^i$
subject to
\begin{align*}
(1) & \quad D^i = 1/n + (1/\tau) (G - G^i); \\
(2a) & \quad E \pi^i = \pi_1 - \rho^i F^i + (1- \theta)(\bar{\pi} - \rho^i F^i) \\
& \quad = [(a - \rho) + (1- \theta) \{\theta a + (1- \theta) b - \rho\}] F^i; \\
(3) & \quad G^i = (1- \theta) (\pi_2 - \pi_1) = (1- \theta) (b - a) F^i
\end{align*}

Solving the bank’s problem yields the following:

**Proposition 4:** At subgame perfect Nash equilibrium,
\begin{enumerate}
\item The optimal optimal loan size, $F$ is $\tau / n(1- \theta)(b-a)$ so that $F$ decreases with $b-a$ or $n$, but increases with $\theta$ or $\tau$,
\item The expected bank profit, $\Pi$ is $A\tau/[ n^2 (1- \theta)(b-a)]$ so that $\Pi$ decreases with $\theta$ or $n$, but increases with $a$ or $b$ or $\tau$,
\item The expected borrower profit, $G$ is $\tau/n$ so that $G$ increases with $\tau$, but decreases with $n$,
\item The long-run equilibrium number of banks, $n$ is $\{A\tau/[ S (1- \theta)(b-a)]\}^{1/2}$ so that the larger the negative effect of adverse selection, $(b-a)$, is, the less decentralized the banking industry is,
\item The free entry loan size is $F^* = \{S \tau/[A(1- \theta)(b-a)]\}^{1/2}$ so that $F^*$ increases with $S$, or $\tau$ or $\theta$ or $a$.
\end{enumerate}

The greater the level of product differentiation is the larger $\tau$ is and the size of the loan increases.\(^{10}\) Ceteris paribus, when the gap between $b$ and $a$ increases, the agency problem is aggravated and thus the optimal loan size decreases. Paul Povel and Michael Raith (2004) show that more asymmetric information leads to lower investment and more sensitive to changes in internal funds. The connection between the nature of market competition and the lending relationship is in Jonathan Conning et.

\(^{10}\) Reployability of an asset, the asset’s value in alternative uses (see Oliver Williamson (1988), is a form of product differentiation. Efraim Benmelech, Mark J. Garmaise and Tobias J. Moskowitz (2004) shows that more redeployable assets receive more loan of larger size.
al. (2003). The risk factor $\theta$ also affects the optimal loan size. The riskier the project is the larger the loan size becomes. Finally as the number of banks increases with more competition, each bank’s loan size decreases.
5. Concluding Remarks

The main objective of this paper is to investigate the effects of adverse selection in a loan contract on loan and deposit market structure. Dewatripont and Maskin (1995) showed that when a loan market becomes decentralized (or competitive), the negative effect of agency problem is being reduced because the lender’s incentive to monitor the borrower becomes less effective as competition in the market is intensified. Their model lacks a market structure. Our model intends to fill the gap. The most appropriate model is one of monopolistic competition. We have chosen one of its varieties, Salop’s spatial economy. Therefore in our paper we have attempted to bridge the loan contract between two parties to the loan market and further explore how the capital market affects the product market in the banking industry. Also we have integrated deposit market with loan market.

We conclude this paper with some possible directions for future research. First, the basic model in this paper may be used in evaluating various policies in the banking industry. Implication of deposit insurance may be analyzed. For instance, how does a risk based deposit insurance premium affect the equilibrium deposit rates? Does a full risk-based insurance improve welfare? Second, we may introduce different market structures from the current monopolistically competitive one in order to see how banks deposit rates, loan size, and welfare would respond to the changes in market structure.
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Appendix

The proofs are given. Here, subscripts denote partial differentiation.

**Fact:** First note that $A_a = 1 + (1 - \theta) \theta > 0,$ $A_b = (1 - \theta)^2 > 0,$ and $A_\theta = 2(b-1) - (b-a) + 1-b < a -1 < 0.$

The second to the last inequality is due to the assumption that $\theta$ is bounded below $(b-\rho)/(b-a)$ (see footnote 7).

**Proposition 1:** Substituting (1) through (4) into the objective function, $\Pi^i$ and differentiating $\Pi^i$ with respect to $\rho^i$ yields the first order condition. Then imposing the equilibrium condition that $\rho^i = \rho$ and $G^i = G$ yields $\rho^*$ and $F^*$ value.

**Corollary 1:** All are straightforward, except $\rho^*_{\theta}.$ Differentiation yields: $\rho^*_{\theta} = -(b-a)(3 - \theta) (1 - \theta)/(2 - \theta)^2 < 0.$

**Proposition 4:** From Proposition 4.1, $\Pi = A\tau/[ n^2(1- \theta)(b-a)].$ Differentiation of $\Pi$ and $G$ with respect to their parameters yields:

\[
\begin{align*}
\Pi_a &= [\tau/ n^2(1- \theta)(b-a)^2] [A_a (b-a) + A] > 0 \\
\Pi_b &= [\tau/ n^2(1- \theta)(b-a)^2] [(1-a) (2- \theta)] > 0 \\
\Pi_n &= -[2A\tau/ n^3(1- \theta)(b-a)] < 0 \\
\Pi_\theta &= [\tau/ n^2(1- \theta)^2(b-a)] [(1- \theta)^2(a-b) + (a- 1)] < 0 \\
\Pi_\tau &= A/ n^2(1- \theta)(b-a) > 0.
\end{align*}
\]