Spatial Competition for Bank Loans and Adverse Selection

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Abstract:

The paper investigates a spatial competition model of Salop in a bank loan market. Asymmetric information between bank and borrower exists about the success probability of the investment project. The banks set up a menu of loan rates under which the borrower makes a bank choice. We study the short-run and long-run loan market equilibrium.

1. Introduction

Many models depicting price competition in the banking industry consider Bertrand approach that lacks product differentiation. Another approach that allows product differentiation in the banking industry is found in a few papers that include Chiappori et al. (1995), Dell'Ariccia (2001), and Hyytinen (2003). All these papers use Salop's model of a circular economy of banks and economic agents. Economic agents can be depositors and/or borrowers. Chiappori et al. consider both deposit and loan market in a banking industry and study the effects of regulation on the equilibrium deposit and loan rates. They find that regulation causes bundling of the two rates as a way to avoid the regulation. Dell'Ariccia's paper, unlike Chiappori et al., introduces the loan market with informational asymmetry.

Our paper considers the bank loan market in a spatial economy and studies the effects of informational asymmetry on the equilibrium loan rate in a model of spatial competition. The outline of the model is as follows. There are n banks located symmetrically on a unit circle. A continuum of borrowers with a unit mass is distributed uniformly along the circle. Borrower's investment project gives uncertain return, only borrower knows how good the project is. The risk characteristics may be classified as a parameter, θ. Each borrower and a nearby bank make a loan contract. The bank charges different interest rates according to the reported value of θ. Given the menu of the interests, each borrower chooses a bank depending on the expected profit size between two adjacent banks. Revelation Principle determines the interest rate in this adverse selection problem. We will consider two different loan contracts. The one is the one-period costly state verification model of Townsend (1979), Gale and Hellwig (1985), and further developed by Williamson (1987), where the bank cannot observe the result of the investment made by the borrower, unless costly audit is performed. The second loan contract is the two-period repeated borrower-lender relationship model of Bolton and Scharfstein (1990) in which the threat of termination by the bank at the end of the first period provides incentives for the borrower to repay the loan. The informational asymmetry lies between banks[1] in the Dell'Ariccia model (2001). In our model, the asymmetry is between each borrower and each bank.

Once the loan contract is specified for a borrower located at a given location, we marriage it to the Salop's location model. Specifically, we specify for each borrower both the participation constraint and location indifference constraint to derive the demand function for each bank and to solve its expected profit maximization problem. Then we find the market clearing Nash equilibrium both in the short run and the long-run equilibrium.

The next section presents the model. The benchmark case of no enforcement is presented in section 3, in which the characteristics of the loan contract are determined only by risk of investment technology. Section 4 discusses adverse selection; the two period adverse selections in section 4.1, and the one-period costly-verification in section 4.2. The final section concludes.

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[1] Hyytinen (2003)'s informational asymmetry stems from an *exogenous* belief asymmetry on the project's return. So there is no issue of adverse selection in such a model.

2. Model

2.1 Model Framework

For our purpose we modify Salop (1979) model in which banks as lenders compete over interest rates for loans to entrepreneurial firms as borrowers. We assume that there are n banks located symmetrically around a circle of measure 1 and firm population is uniformly distributed around the circle. When contracting a loan, firms consider the interest rate they have to pay, and the per length 'transportation cost', τ, they travel to the chosen bank. τ can be viewed as a measure of the degree of product differentiation.

Borrowers are heterogeneous in their investment technology. Each firm θ has an investment technology which transforms 1 unit of investment borrowed from a bank into a random output X characterized by a probability distribution function F(X) with density function f(X) on R_+.

The project size is normalized to unity, financed entirely by a bank loan. The banks compete for borrowers by lending rate offers; let R_i denote the gross lending rate (one plus the interest rate) of bank i. The bank can raise funds at an interest ρ. ρ is the bank's cost of raising one unit of the fund.

The timing of events is as follows: At a date 0, a borrower must borrow a unit of investment to finance an investment project. Prior to production, both bank and borrower share a common prior belief f(X). After production, the borrower knows X but the bank does not. A contract is written. If contract is accepted after borrowers' travel to the bank from which they would like to borrow, then one unit of the good is transferred to the borrower and production takes place. Nature then chooses X. At date 1, the borrower, but not the banker, observes X. The borrower reports its output to the bank. Then the bank decides whether to seize or not the assets of the borrower. When the assets are not seized, the continuation value for the borrower is V. When they are seized the assets generate S value to the bank. The financial contract is written with a pair of repayment interest and probability of seizure.

3. No enforcement

In this section we consider the benchmark case where the bank does not have any enforcement mechanism such as costly verification or termination of future loan. So the contract is completely characterized by only repayment interest. We will analyze the case with two examples.

3.1. Example 1.

The investment technology F(X) is described as follows. Each firm θ transforms 1 unit of investment borrowed from a bank into X_θ units of output with probability 1- θ and 0 with probability θ. We assume that θ is distributed uniformly on [0, 1]. Therefore, the higher θ the riskier the investment project is. We also assume that $(1-\theta) X_\theta = X$ so that all the projects have the same expected return (mean-preserving). The technology of borrowers is of Mankiw's (1986) in that each borrower knows his own expected return and repayment probability, but not observable by the banks. 1- θ is the repayment probability of firm θ. It is a special case in which the expected return is constant across borrowers.

The project size is normalized to unity, financed entirely by a bank loan. The banks compete for borrowers by lending rate offers; let R_i denote the gross lending rate (one plus the interest rate) of bank i. The bank can raise funds at an interest ρ. ρ is the bank's cost of raising one unit of the fund.

The timing of events is as follows: In stage 1, the banks compete for borrowers by announcing simultaneously R_i, to which they commit. In stage 2, the borrowers observe the offers and travel to the bank from which they would like to borrow. And the borrowers invest in the projects, the return results and the loan payment, if possible, is made to the banks.

We solve backwards. In stage 2, the borrower θ's decision is simple. On the one hand, if X_θ is larger than or equal to R_i plus the travel cost, then the loan is paid off. On the other hand, if X_θ is smaller than R_i plus the travel cost, then the loan is defaulted and the bank will receive nothing.[2]

The distance between bank i and the borrower is x. Then Firm θ will demand a unit of loan if

i) $X_\theta - R_i - \tau x \geq 0$

and

ii) $X_\theta - R_i - \tau x \geq X_\theta - R - \tau(1/n - x)$

where *R* is its rival bank's lending rate offer..

Condition i) is the participation constraint and Condition ii), the incentive compatibility condition, says that bank i's loan offer is more lucrative than the offers of rival banks. *R* denotes the offer of a typical rival bank.

Then under full-scale competition, the total demand of bank i is

(1) $D_i = 1/n + (1/\tau)(R - R_i)$

and the profit of bank i from a borrower who pays off the loan is

(2) $\pi_i = D_i (R_i - \rho)$.

In stage 1, given the other banks' choices, bank i maximizes its expected profits. To find the expected profits, first note that, from the participation constraint i), only the risky firms with θ values in the interval $[1 - X/(R_i + \tau x), 1]$ ask for the loan. And the total amount lent equals $X/(R_i + \tau x)$. When $X > R_i + \tau x$, then all the firms obtain a loan. Note that as x increases, the interval becomes smaller attracting more risky firms. Therefore, the expected profits for bank i is:

[2] The firm's behavior is not of risk neutrality. In fact, the firm is risk-loving in this example.

$$(3)\ E\,\pi_i = \int_{1-X/(R_i+\tau x)}^{1} (1-\theta)\pi_i\, d\theta$$

Maximizing (3) with respect to the bank lending rate R_i yields the optimal lending rate, R_i^* that bank chooses. The optimal lending rate satisfies:

(4) $\partial\ E\ \pi_i/\partial\ R_i = 0$ or, equivalently, ε, the elasticity of π_i with respect to $R_i + \tau x$ is equal to 2,

where $\varepsilon \equiv (\partial\ \pi_i/\partial\ R_i)((R_i + \tau x)/\ \pi_i)$.

Now it is straightforward to establish the following result:

Proposition 1. *In a symmetric Nash equilibrium with fixed number of banks on the loan market,*
i) the lending rate, $R^* = R_i^*$ *at a Nash equilibrium is a solution to the following quadratic equation in R:*

(5) $a\, r^2 + (2 + a\,\rho)\, r - (\rho + a^{-1}) = 0,$

where $a \equiv n/\tau$, $r \equiv R - \rho$,

ii) the lending rate is increasing in ρ*, the bank's cost of raising one unit of the fund.*
iii) the net spread between the lending rate and cost of capital for the bank, r is decreasing in the ratio of the number of the banks to the unit transportation cost, a, and, therefore,
iv) the threshold θ *below which the loans is denied is an increasing function of* ρ *and a decreasing function of a*

This result demonstrates that as competition intensifies in the banking industry, i.e., a increases, the pool of borrowers takes more low values of θ and the net spread between the lending rate and cost of capital decreases. As the threshold θ decreases, there will be more risky projects, but, if successful, with much higher return. This result is similar to Hyytinen (2003)[3] where the more optimistic the borrowers are, the smaller the spread becomes. However, his treatment of belief is exogenous while ours is endogenous.

3.2. Example 2. (Alternative Specification of Technology):

The example in the previous section specifies the project's return to be mean-preserving, but not variance-preserving. In order to track more, let us change the investment technology of the entrepreneurial firm. We assume that θ is fixed.

Assumption A: At date 0, a firm has an investment technology which transforms 1 unit of investment borrowed from a bank into X (constant) units of output at date 1 with probability θ and 0 with probability 1- θ.

The lower θ is the riskier the investment project is. As before, we let τ be the transportation cost per length or interpreted as a measure of the degree of product differentiation. The distance between bank i and the borrower is x. Let R_i and *R* be, respectively, Bank i's and its rival bank's lending rate offers. Firm will demand a unit of loan if

i) $\theta X - R_i - \tau x \geq 0$,

and

ii) $\theta X - R_i - \tau x \geq \theta X - R - \tau(1/n - x)$[4].

[3] The firm in his model is risk neutral.

[4] Now the firm is risk neutral.

Then the total demand of bank i is $D_i = 1/n + (1/\tau)\ (R - R_i)$ thus the expected profit is $\pi_i = D_i\ (\theta\ R_i - \rho)$, where ρ is the bank's cost of raising one unit of the fund[5]. The bank chooses R_i^*. The optimal lending rate satisfies:

$$\partial\ \pi_i / \partial\ R_i = 0$$

$R^* = R_i^*$ at a Nash equilibrium becomes:

$$R^* = \tau/n + \rho/\ \theta.$$

We summarize the result in the following:

Proposition 2: *In a symmetric Nash equilibrium with a fixed number of banks on the loan market with all firms having identical technology specified in Assumption A,*

i) the lending rate, $R^* = R_i^*$ at a Nash equilibrium is $R^* = \tau/n + \rho/\theta$

ii) $dR^*/d\tau > 0$, $dR^*/dn < 0$, $dR^*/d\rho > 0$, and $dR^*/d\theta < 0$.

We obtain, therefore, that on the one hand, the more costly it is to travel to the bank or for the bank to raise its capital, the higher the equilibrium loan rate is. On the other hand, the higher the probability of success the project has or the more the number of the banks is the lower the equilibrium interest rate becomes.

4. Adverse Selection

4.1 Threat of Termination

Now we would like to consider adverse selection problem between the

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5 Θ is common prior for the bank and borrower.

bank and borrower in the location model in section 3.2. Both are risk neutral. Assume that type θ is uniformly distributed on [0, 1] as in section 3.1. We study a repeated borrower-lender relationship of Bolton and Scharfstein (1990) in which the threat of termination by the bank provides incentives for the borrower to repay the loan. At the end of date 1, the assets of the firm can be seized by the bank, generating a value S(θ). If they are not seized, the continuation value for the firm is V(θ). Then a financial contract is composed of a repayment R(.) and a probability of seizing the assets, 1 - β (.), both contingent on the firm's report about its type.

Assume that E (X) = θ X > 1 + τx , thus, in a world of perfect information, the investment would be undertaken. However, the realized output is not observable by the bank. This may lead the bank to refuse to grant the loan. In a one-shot relationship, there would be no lending, because the borrower can always pretend that his output is zero, which is therefore the maximum repayment that the bank can enforce. However, in a two-period relationship, the bank can commit to renew the initial loan by threatening to "cut off funding in the second period if the firm defaults in the first." (Bolton and Scharfstein (1990, p 96). The probability 1 - β (.) captures the threat in the model.

Now the firm's utility at date 1 is:

$$(6) \qquad U(\theta, \hat{\theta}) = \theta X - R(\hat{\theta}) - \tau x + \beta(\hat{\theta})V(\theta),$$

where θ is the true value of the firm and $\hat{\theta}$ the value reported by the borrower. The Spence-Mirrlees single crossing condition requires that V(θ) be monotonic in θ, which we assume. Under risk-neutrality, the expected profit of the bank at date 0 is:

$$(7) \qquad \int_0^1 [R(\theta) + (1 - \beta(\theta)S(\theta)]dF(\theta) - 1$$

In order to solve for the adverse selection problem we make the following additional assumptions on V and S.

Assumption. $V(\theta) = \max \{ L, \theta X - \tau x \}$ and $S(\theta) = L$.

From the argument presented above, at the end of the second period, the firm will be in the same situation as in a one-shot relationship and will always repay the minimum possible amount, 0. Thus firm θ can carry the second period profit $\theta X - \tau x$. The continuation value for the firm, however, is the larger one between L and $\theta X - \tau x$. So $V(\theta)$ and $S(\theta)$ capture the standard debt contract[6]; *for V(θ) being vertically translated by the amount of L*. So the punishment for default is not at maximum.

The optimal financial contract {R*(.), β*(.)} maximizes the banker's profit subject to the incentive compatibility constraints and the limited liability constraint, i.e., it solves

(P) max $\int_0^1 [R(\theta) + (1 - \beta(\theta)L] dF(\theta) - 1$

{R*(.), β*(.)}

subject to $U(\theta, \theta) \geq U(\theta, \hat{\theta}) \quad \forall (\theta, \hat{\theta}) \in [0,1]^2$ (IC)

$\theta X - R(\theta) - \tau x \geq 0$ (LL)

Let $u(\theta) \equiv U(\theta, \theta)$ and rewrite (P) as:

(P*) max $\int_0^1 [\theta X - \tau x + \beta(\theta)(V(\theta) - L) - u(\theta) + L] dF(\theta) - 1$

{R*(.), β*(.)}

subject to $du/d\theta = X + \beta(\theta)\, dV/d\theta$ for a.e. $\theta \in [0, 1]$

and β (.) increasing (IC)

[6] The set-up of the contract follows the line of Faure-Grimaud et.al. (1999).

$$u(\theta) \geq \beta(\theta)\, V(\theta) \qquad \forall\ \theta \in [0,1] \qquad \text{(LL)}$$

Letting u as a state variable and β as a control variable, one can solve the optimal control problem (P*) by applying the maximum principle. Then assuming the standard hazard rate $h(\theta) \equiv f(\theta)/(1-F(\theta))$ increasing in θ, the optimal contract is characterized as: Given x,

$$\text{(S)} \qquad \beta^*(\theta) = 0 \text{ and } R^* = -\tau x, \qquad \text{for } \theta \in [0, \theta^*],$$

$$\beta^*(\theta) = 1 \text{ and } R^* = L, \qquad \text{for } \theta \in (\theta^*, 1],$$

where $\theta^* \equiv (L + \tau x)/X$.

So there are two types of firms, 'bad' firms (low θs) are subsidized with τx while 'good' firms (high θs) pay L. Now the question is "What is the optimal L for the bank?" To find it , we now let x be varied. As discussed earlier, the bank is able to attract the borrower located at distance x from it only if the borrower's participation constraint and the bank's offer is more lucrative than the offers of the rival banks. Applying (S) to Section 3.1 equations (1) and (2) (the total demand and profits)[7], the expected profit for the bank, Eπ is:

$$\text{(8)} \qquad E\pi = \theta^* D_i (R^*=-\tau x)(-\tau x-\rho) + (1-\theta^*) D_i(R^*=L)(L-\rho).$$

Maximizing Eπ with respect to L yields;

Proposition 3: *The optimal contract is a pair of {R*(θ), β*(θ)}, where R*(θ) and β*(θ) are given by (S) and L = L**

$$\text{(9)} \qquad L^* = \tau/n + \rho$$

[7] Note that equations (1) and (2) do not depend on the investment technology.

(Proof in the appendix)

4.2. Costly Verification

In this section, we assume that the firm's utility is U(θ, $\hat{\theta}$) = θ X – R ($\hat{\theta}$) – τx - β ($\hat{\theta}$)V(θ, $\hat{\theta}$), where β ($\hat{\theta}$) is interpreted as the probability of verifying the exact value of output when the firm reports $\hat{\theta}$ and V(θ, $\hat{\theta}$) is the penalty imposed on a borrower who reports $\hat{\theta}$ when the bank verifies that the true value is θ. We assume that V is monotonic in θ so that Spence-Mirrlees single crossing condition is satisfied. The rest is the same as section 4.1.

5. Concluding Remarks

The main objective of this paper is, not yet fully addressed in the beginning, to investigate the effects of adverse selection in a loan contract on loan market structure. Dewatripont and Maskin (1995) showed that when a loan market becomes decentralized (or competitive), the negative effect of adverse selection problem is being reduced because the lender's incentive to monitor the borrower becomes less effective as competition in the market is intensified. Their model lacks a market structure. Our model intends to fill the gap. The most appropriate model is one of monopolistic competition. We have chosen one of its varieties, Salop's spatial economy. Therefore in our paper we have attempted to bridge the loan contract between two parties to the loan market.

Appendix

Lemma: $d\,\theta^*/dL = 0$

Proof: Note that $\theta^* \equiv (L + \tau x)/X$ and, from section 3.2, $D_i = 1/n + (1/\tau)(R - L)$. Since θ^* is a function of L and x and x is equal to D_i, differentiating θ^* with respect to L yields zero.

Proof of Proposition 3: It follows immediately from both Lemma and the fact that $D_i(R^*=-\tau x)(-\tau x-\rho)$ in (8) is constant.

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