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The Cross Section of Temporally Aggregated Continuous Time Expected Returns

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Abstract

This paper discusses how continuous time CAPMs (ICAPM, Merton (1973); CCAPM, Breeden (1979)) can be fitted into the long horizon data, and also shows how the discrete version of the Continuous Time CAPM (DICAPM) can be a three factor model, where the risk premiums for the three factors are the cost of the market risk, the rebalancing cost, and the hedging cost, such as Fama and French (FF, 1996). Furthermore, we test and show that 1) SMB and HML be linearly spanned by both the cost to rebalance continuously and the cost to hedge against the state variable with regards to the covariance structure, and 2) that FF stand on the ground of ICAPM. The results have clear implications for asset pricing test, performance measurement, and more.

Key Words: Fama-French Model, Continuous Time Capital Asset Pricing Model, Temporal Aggregation, Discrete version of the Continuous Time CAPM (DICAPM)

JEL classification: C60, G10, G12

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1. Introduction

We examine explanatory power of continuous time capital asset pricing models (CAPMs) such as ICAPM of Merton (1973) and CCAPM of Breeden (1979) on long-term empirical regularities. The regularities are those static CAPMs of Sharpe (1964), Lintner (1965), Mossin (1966), and Black (1972) explain weakly, and those that have no "…a coherent story that (1) relates the cross-section properties of expected returns to the variation of expected returns though time, and (2) relates the behavior of expected returns to the real economy in a rather detailed way (Fama, 1991, pp 1610)…" The financial researchers wonder whether other models except the static CAPMs are more explanatory ((Banz (1981), Fama-French (FF, 1992), and others). Researchers believe that size and B/M variables are the components of multi factor models (Chan-Chen (1988 & 1991), Fama-French (1993, 1995, 1996, 1998, & 1999), Ferson-Harvey (1999), Ferson-Siegel (2001), and more); on the contrary, some doubt that the explanatory powers of size and B/M variables are from the data problems (Kothari-Shanken-Sloan (1995), Kim (1995), MacKinlay (1995), and others), or the empirical design (Berk, 1995). Believing that the empirical regularities are evident, some build up new theories based on the behavioral patterns (Brav-Heaton, 2002) or the option structure (Berk-Green-Naik, 1999). Others believe that varieties of the current CAPMs explain well the regularities so that the new will be dispensable (Constantinides (2002), FF (1993, 1995, 1996, 1998, & 1999), and more). FF and others guess that ICAPM or APT (Ross, 1976) show a light on the regularities; however, some empirical results of APT were weak (Chan-Chen-Hsieh (1985), Chen (1983 and 1991), and Chen-Roll-Ross (1986)). Regardless of some results on

ICAPM or APT, there are questions as to whether the continuous time CAPMs explain the cross section of average returns.

Continuous time CAPMs[2] are made to be discrete in order to test the CAPMs with financial data. The horizon of the data can be viewed as two ideas[3] even though we are not sure to know with certainty which length of time period[4]is discrete or cumulative; the horizon of the data is viewed as the time interval during which investors trade their portfolios many times or only once. Assuming that the investors do something only once, researchers interpret the data with the discrete time CAPMs as continuous time CAPMs. Long (1974) and Fama (1970 & 1996a) derived discrete time ICAPM through the intuitive method; Fama (1996a) derived a discrete time ICAPM with respect to mean-variance efficiency (Markowitz, 1959). Both of the discrete time ICAPM and Sharpe's CAPM are indistinguishable in the absence of the state variable since the discrete time ICAPMs disregard rebalancing portfolios continuously for the horizon. Inputting the rebalancing cost into empirical models of ICAPM, we have to assume that investors trade their portfolios continuously in the

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2 We refer Sundaresan (2000) as a general guide of the finance in continuous time.

3 There are two methods to use when applying continuous time CAPMs with the discrete data. The first is to use the discrete continuous time CAPMs on the implicit assumption that the investors trade things only once within the horizon. The second method is to use the temporally aggregated continuous time CAPMs on the assumption that the investors uncountably many times do the things. Two methods are related to two forms in the stochastic calculus; the discrete method is the differential form, and the temporal aggregation is the integral form. The focus in this paper is on how continuous time CAPMs are temporally aggregated to fit into the long-term data, and what DICAPM temporally aggregated explain empirical regularities.

4 All horizons, time intervals, and frequency intervals refer to the time interval in the data. "…the time interval between successive market openings is sufficiently small to make the continuous-time assumption a good approximation(Merton(1973, p869)." The long horizon is defined as the time interval while it takes for investors to rebalance their portfolios continuously, and the data refers to the cumulative results in the paper.

horizon, and to aggregate ICAPM temporarily. Especially, in the market governed by the state variable, we must assume that investors rebalanced their portfolios many times with viewpoints of two kinds of pillars within the horizon: the market portfolio and the state variable, and so that the data reflected the cumulative results. The temporally aggregated method dictates that the instantaneous continuous time CAPMs be cumulated for the horizon. This method is better than the discrete method when taking into account rebalancing over a long horizon.

We shall wander over "… the horizon in all tests of multifactor ICAPM's or APT's…(FF, 1996)" as Fama (1996a) walks in the Merton forest, finance in continuous time (Merton, 1992), but shall step along into the other lane through the forest to catch up with the long-term regularities, the current hottest issue in the finance. This lane reflects how ICAPM can be tested empirically in regards to the temporal aggregation property. Through this lane we shall show that FF is like a tree in the ICAPM forest. Furthermore, we shall show that 1) DICAPM is a multi-factor model like FF 2) SMB and HML of FF factors are regarded as a linear combination with the continuously rebalancing cost in terms of the market portfolio and the hedging cost against the state variable. If we construct a reasonable lane, DICAPM should have many implications upon the asset pricing test, performance measurement and more. We believe that we have found a missing link between theories of asset pricing models, especially continuous time models such as ICAPM of Merton (1973), CCAPM of Breeden (1979), and general equilibrium model of Cox-Ingersoll-Ross (CIR, 1985 a & b), and empirical regularities most notably the empirical models of

Fama-French (1993, 1995, 1996, 1998, & 1999) regardless of many behavioral theories (Fama (1998b), and Brav-Heaton (2002)).

For their temporal aggregated continuous CAPMs, Breeden-Gibbons-Litzenberger (1989) used a cumulative expression of rates of return by testing a CCAPM (Breeden, 1979) without the state variable with quarterly horizon data, and Longstaff (1989) showed cumulative raw expressions of instantaneous rates of return under the ICAPM of a state variable and a three factor model with monthly horizon data; covariance between raw return and market portfolio, variance of the return, and autocovarince of the return.

The Longstaff three factor model is from 'Discrete version of the Continuous Time CAPM (DICAPM)' in this paper at some points. At first, the Longstaff model cannot be taken as cross-sectional since the autocovariance is a time series moment. DICAPM is a three factor cross sectional model; the covariance between excess return and excess return of the market portfolio, the variance of the excess return, and the covariance between the excess return and T-bill rate as the proxy of the state variable. The first factor is from the market, the second from the rebalancing, and the third from the hedging against the state variable. DICAPM is directly linked to capture the rebalancing effect and the hedging effect against the state variable. However, the Longstaff model fails to capture both the rebalancing and the hedging effects. Secondly, the Longstaff model is based on raw returns. Investors in the continuous time trade their portfolios with regards to the rate of interest; therefore, the cumulative returns must be based on the integration of excess instantaneous rates over the rate of interest. DICAPM is based on cumulative expressions of excess returns. DICAPM is so a Sharp's CAPM as to the Longstaff model is a Black's CAPM. Finally, the Longstaff model is an approximation of the their equation. DICAPM is a linear approximation derived from the second equation. Furthermore, DICAPM is more logical than the Longstaff model.

This paper shall also be distinguishable from Berk-Green-Naik (1999). They develop the partial equilibrium model that explains the empirical results of FF (1992), and simulate the partial model to show the empirical results on the time series and cross-sectional relations of the SMB and HML. DICAPM in this paper is the discrete version of the general equilibrium model (CIR, 1985a & b) regarding the temporal aggregation, and based on the one state variable, and test DICAPM with the data of the market (FF, 1996).

The paper shall also be different from Brennan-Wang-Xia (2003). Brennan-Wang-Xia has shown that FF should be based on the ICAPM on the premise that risk premia on SMB and HML are associated with Vasicek-based ICAPM of two state variables with regards to discreteness. In this paper we shall show that covariance structure on SMB and HML are linked to CIR-based ICAPM of one state variable with viewpoints of the temporal aggregation and examined the DICAPM by the iterated Generalized Method of Moment (itGMM) (Ferson-Foester, 1994). The temporal aggregation is easily adopted into Vasicek-based ICAPM even though we do not show explicitly what DICAPM of Vasicek-based ICAPM is built up. Furthermore, we shall show that FF is based on the ICAPM more compactly and effectively than Brennan-Wang-Xia (2003) have shown.

Now we shall introduce some notations and drive DICAPM, and the covariance structure of FF three factors in section 2, describe how FF and ICAPM are empirically compared to each other in section 3, our empirical findings in section 4, and our conclusion in section 5.

2. The Cross Section of Temporally Aggregated Continuous Time Expected Returns and Fama-French (1996)

2.1 The Cross Section of Temporally Aggregated Continuous Time Expected Returns

In this section, we shall build up DICAPM in regards to rebalance portfolios continuously within a long horizon. The continuous time CAPM in this paper is ICAPM based on CIR (1985a & b). The CAPM shall be called CIR-based ICAPM[5], and has the same assumptions as those in Longstaff (1989);

Let the market be an economy akin to those of Cox-Ingersoll- Ross, and investors with utility $U(C,t)=e^{-\rho t}\ln(C)$ want to maximize $E[\int U(C_s,s)ds]$ such that assets have the dynamics of $d\ln P_i=(a_i-\frac{1}{2}\sigma_i^2)Xdt+\sigma_i\sqrt{X}\,dZ_i$ controlled by the state variable X, $dX=k(\mu-X)dt+s\sqrt{X}\,dZ_X$, and instantaneous correlation rates $E[dZ_i dZ_X]=\rho_{iX}dt$ and $E[dZ_i dZ_j]=\rho_{ij}dt$, $i=1,\ldots,n$, respectively. Investors also can rebalance their portfolio continuously in the market.

Next, we may assume that the market will be well defined; Market portfolio P_m

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[5] Without the loss of logic, we shall omit general assumptions of continuous time CAPMs; limited liability, existence of free financing market and so on. Since we have interest in testing empirically the CAPMs for long horizon, we shall assume that the investor rebalances her portfolio infinite many times during the horizon. Furthermore, we accept continuous time CAPM and ICAPM as exchangeable and CCAPM as one of continuous time CAPMs. The ICAPM based on the CIR. The state variable in the Vasicek-based ICAPM is $dX=k(\mu-X)dt+sXdZ_X$. For the consistency of the paper, the over-all of the temporal aggregation does not change even though we do not investigate Vasicek-based ICAPM.

has the dynamic of $d \ln P_m = (a_m - \frac{1}{2}\sigma_m^2)Xdt + \sigma_m \sqrt{X}\, dZ_m$, and ICAPM:

$(a_i - r_0) = \lambda \sigma_i$[6].

Taking $E[dZ_i dZ_m] = \rho_{im} dt$, and $E[dZ_m dZ_X] = \rho_{mX} dt$, define some instantaneous correlation rates $\sigma_{iX} = \sigma_i s \rho_{iX}$, $\sigma_{im} = \sigma_i \sigma_m \rho_{im}$, and $\sigma_{mX} = \sigma_m s \rho_{mX}$ among P_i, P_m, and X. Then we may also assume that σ_{im} indicates an instantaneous rate of correlation between individual assets and the market portfolio. Rewriting ICAPM, we can get the following;

$$(a_i - r_0) = \lambda \sigma_{im} \tag{1}$$

Thus, (1) gives us a linear relation for the instantaneous expected rates of returns over the rate of interest rate; cross sectional relation in instantaneous expected rates of returns can be rewritten as cross-sectional variation in the instantaneous covariance's with the market's return and the state variable. Since (1) is based on the continuous time parameters for instantaneous returns, we have to rewrite (1) with the long horizon data, and also match the parameters with the unconditional moments of returns for the long horizon.

We shall define the cumulative expressions of the instantaneous rates of returns over the rate of interest, and the moments of their cumulative expressions. These moments represent the unconditional means of cumulative expressions of instantaneous rates of returns over the rate of interest, the unconditional variances of

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[6] Consider Merton (1992, pp 489) and Cox-Ingersoll-Ross (1985a & b). The market risk premium, the risk premiums of assets and pricing kernels in this paper are time varying.

cumulative expressions of instantaneous rates of returns over the rate of interest, the unconditional covariances between cumulative expressions of instantaneous rates of returns over the rate of interest and cumulative expressions of instantaneous rates of the market portfolio over the rate of interest, the unconditional covariances between cumulative expressions of instantaneous rates of returns over the rate of interest and the state variable, and the unconditional covariances between cumulative expressions of instantaneous rates of the market portfolio over the rate of interest and the state variable, respectively.

Investors in the ICAPM rebalance their portfolios with regard to the market portfolio and against the state variable. For the expected returns over the unit horizon, we define $R(i)_0^\tau$ as the cumulative expression[7] of the instantaneous rate of return over interest rate $r_0 Xdt$ as the following;

$$R(i)_0^\tau = \int_0^\tau ((a_i - r_0 - \frac{1}{2}\sigma_i^2)Xdt + \sigma_i \sqrt{X}\, dZ_i) \tag{2}$$

Now, letting $X_0 = \mu$ and considering the appendix for the unit long horizon, by the Fubini Theorem (Ikeda and Watanabe, 1989), we can take some unconditional moments: M_i as the means of R_i, V_i as the variances of R_i, C_{im} as the covariance of R_i and R_m, C_{iX} as the covariance of R_i and X, and C_{mX} as the covariance of R_m and X among $R_i = R(i)_0^1$, $R_m = R(m)_0^1$, and $X = X_0^1$, respectively as the followings;

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[7] Longstaff (1989) defined the cumulative of raw instantaneous rate of return considering $a_i = \lambda_0 + \lambda_1 \sigma_{im}$. Since investor consider the excess over the interest rate, the cumulative of this paper is practically better than those of Longstaff .

$$M_i = \mu(a_i - r_0 - \frac{\sigma_i^2}{2}) \tag{3}$$

$$V_i = \mu\sigma_i^2 + \frac{2\mu c_1}{k}\sigma_{iX}(a_i - r_0 - \frac{\sigma_i^2}{2}) + (\frac{\mu s^2 c_2}{k^2} - \mu^2)(a_i - r_0 - \frac{\sigma_i^2}{2})^2 \tag{4}$$

$$C_{im} = \mu\sigma_{im} + \frac{\mu c_1 c_4}{k}\sigma_{iX} + (\frac{\mu s^2 c_2 c_4}{k^2} - \mu^2 c_4 + \frac{\mu c_1}{k}\sigma_{mX})(a_i - r_0 - \frac{\sigma_i^2}{2}) \tag{5}$$

$$C_{iX} = \frac{\mu c_3}{k}\sigma_{iX} + \frac{\mu s^2 c_3^2}{2k^2}(a_i - r_0 - \frac{\sigma_i^2}{2}) \tag{6}$$

$$C_{mX} = \frac{\mu c_3}{k}\sigma_{mX} + \frac{\mu s^2 c_3^2 c_4}{2k^2} \tag{7}$$

where $c_1 = 1 - \frac{1-e^{-k}}{k}$, $c_2 = 1 - 2\frac{1-e^{-k}}{k} + \frac{1-e^{-2k}}{2k}$, $c_3 = 1 - e^{-k}$, and $c_4 = a_m - r_0 - \frac{\sigma_m^2}{2}$

Equations (3), (4), (5), (6), and (7) form a system of links between instantaneous continuous time parameters and discrete data. Using the system, we shall develop the DICAPM. Now, substituting moments M_i, C_{im}, V_i, C_{iX}, and C_{mX} for $a_i - r_0$, σ_{im}, σ_i^2, σ_{iX}, and σ_{mX},

$$a_i - r_0 = \frac{1}{2\mu}V_i^2 + (\frac{1}{\mu} - \frac{c_1}{\mu^2 c_3}C_{iX})M_i + \frac{s^2}{2\mu^2 k^2}(c_1 c_3 - c_2 + \mu k^2)M_i^2 \tag{8}$$

$$\sigma_i^2 = \frac{1}{\mu}V_i - \frac{2c_1}{\mu^2 c_3}C_{iX}M_i + \frac{s^2}{\mu^2 k^2}(c_1 c_3 - c_2 + \mu k^2)M_i^2 \tag{9}$$

$$\sigma_{im} = \frac{1}{\mu}C_{im} - \frac{c_1 c_4}{\mu c_3}C_{iX} + \frac{s^2}{\mu}[\frac{(\mu+1)c_1 c_3 c_4}{2k^2} + \frac{\mu c_4}{s^2} - \frac{\mu c_2 c_4}{k} + \frac{c_1}{s^2 c_3}C_{mX}]M_i \tag{10}$$

$$\sigma_{iX} = \frac{k}{\mu c_3}C_{iX} - \frac{s^2 c_3}{2\mu k}M_i \tag{11}$$

$$\sigma_{mX} = \frac{k}{\mu c_3}C_{mX} - \frac{s^2 c_3 c_4}{2k} \tag{12}$$

Considering (1) and (3),

$$M_i = \mu\lambda\sigma_{im} - \mu 2^{-1}\sigma_i^2 \tag{13}$$

By integrating (9) and (10) with (13), we can get the 2nd order equation for M_i.

Hence, approximating the equation for M_i linearly,

Claim For the unit horizon and $R_f = r_0 X$,

$$M_i = mC_{im} + vV_i + xC_{iX} \tag{14}$$

where $m = d_5 d_3^{-1}$, $v = d_6 d_3^{-1}$, $x = d_4 (r_0 d_3)^{-1}$, and $d_i = f(\lambda, a_m, \sigma_m, k, \mu, s)$, $i = 0...6$, respectively.

Proof) more details in the appendix

It can be called (14) as in the DICAPM below.

Without the state variable,

$$M_i = mC_{im} + vV_i \tag{15}$$

Even though a multi period model is not state-dependent, the model is a static model (Fama, 1970). The assumption that investors rebalance their portfolios continuously makes ICAPM not degenerate static CAPM. Without the state variable, the smaller the variance is, the more future is certain, and the smaller the rebalancing cost is; the farther the horizon is, the less rebalancing cost is. In the market governed by the state variable, the rebalancing cost does not increase as the horizon does since investors trade their portfolios concurrently with regards to the market portfolio and the state variable.

Suppose that the market is like the sea with time-varying wave (state variable), and investors as surfers rebalancing their positions in term of the market portfolio and against the state variable. And suppose that financial economists look at the snapshots of surfers (the data from the market). If the economists disregarded that the surfers rebalanced their positions continuously between snapshots, they would doubt whether the surfers should take exotic positions. The economists would conclude that the positions of the surfers were 'abnormal'; to wit, discrete ICAPM is not suitable for a long horizon. In the continuous time, the investors can trade their portfolios continuously to rebalance and hedge for the long horizon.

Now, DICAPM could be a three factor model: m is from the market factor, v from the rebalancing factor, and x from the hedging factor. DICAPM is a face of ICAPM in the discrete time world.

2.2 The Covariance Structure of FF

In this section, we shall investigate whether FF is based on ICAPM. We can rewrite FF as the following;

$$\begin{aligned} E[R_i] &= b_i E[R_M] + s_i E[SMB] + h_i E[HML] \\ &= cbCov(R_i, R_m) + csCov(R_i, SMB) + chCov(R_i, HML) \\ &= cbC_{im} + csC_{iSMB} + chC_{iHML} \end{aligned} \tag{16}$$

where SMB = Difference between Small and Big stocks, HML = Difference between High-book-to-market stocks and Low-book-to-market stocks, $cb = E[R_m]V_J$, $cs = E[SMB]V_J$, $ch = E[HML]V_J$, and V_J is the inverse of covariance matrix. We shall rewrite FF and call it as modified FF. Both modified FF and DICAPM are three factor models; furthermore, both of the two models are have the common factor C_{im} from the market portfolio. The other two factors of modified FF are C_{iSMB} from SMB and C_{iHML} from HML. It is sufficient that there exist sv, sx, hv, and hx for $sv \cdot hx - sx \cdot hv \neq 0$ such that

$$\begin{aligned} C_{iSMB} &= Cov(R_i, svR_i + sxR_f) \\ C_{iHML} &= Cov(R_i, hvR_i + hxR_f) \end{aligned} \tag{17}$$

Hence, rewriting (17) for V_i and C_{iX},

$$\begin{bmatrix} C_{iSMB} \\ C_{iHML} \end{bmatrix} = \begin{bmatrix} sv & sx \\ hv & hx \end{bmatrix} \begin{bmatrix} V_i \\ C_{iX} \end{bmatrix} \tag{18}$$

Regarding the covariance structure, we aggressively guess that FF is based on the ICAPM (FF, 1996, pp 57) only if there exist sv, sx, hv, and hx for $sv \cdot hx - sx \cdot hv \neq 0$ such that

$$\begin{bmatrix} C_{iSMB} \\ C_{iHML} \end{bmatrix} = \begin{bmatrix} sv & sx \\ hv & hx \end{bmatrix} \begin{bmatrix} V_i \\ C_{iX} \end{bmatrix} \tag{18'}$$

Moreover, we dare that SMB and HML are linear spanned by the rebalancing cost and hedging cost.

3. Empirical Designs

We built up DICAPM implied by ICAPM unconditionally with respect to the information of assets but conditionally with respect to the state variable. In this section, we develop econometric tests for comparisons between FF and ICAPM with the FF 25 portfolios. Tests are iterated Generalized Method of Moment (itGMM) by Ferson-Foester (1994)[8]. itGMM is a upgraded GMM (Hansen-Singleton, 1982) and is the most popular in asset pricing tests by Ferson-Foerster (1994), Ferson-Constantinides (1991), Ferson-Harvey (1992, 1993, & 1997), Longstaff (1989), and others. Harvey (1989) developed the conditional covariance tests of assets pricing models.

C_{im}, V_i and C_{iX} of DICAPM are based on the market factor, the rebalancing factor of portfolio i and the hedging factor of portfolio i, $i = 1...25$, respectively; C_{iSMB} and C_{iHML} on SMB, and HML of FF portfolio i, $i = 1...25$, respectively. Moreover, cm from the market risk premium, cs from $E[SMB]$ and ch from $E[HML]$ are directly linked to FF three factors, respectively and v from the rebalancing and x from the hedging against the state variable. Since C_{im} is the common factor of both the modified FF and DICAPM, we shall investigate how well C_{iSMB} and C_{iHML} are linked to V_i and C_{iX}, concurrently and simultaneously, $i = 1...25$.

First, to show that DICAPM and FF seem to explain the expected returns sufficiently, we shall define a full model nesting DICAPM and modified FF as the following;

$$M_i = cmC_{im} + vV_i + xC_{iX} + csC_{iSMB} + chC_{iHML} \tag{19}$$

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8 Without loss of logic, we shall omit subscript t for time and i for asset.

where $cm = m$ for DICAPM and $cm = cb$ for FF, respectively

Considering Longstaff (1989, pp877), we shall set up itGMMs as following;

For CAPM, we define 26 parameters as cb, $C_{1m} \ldots$ C_{25m} with 50 restrictions. For DICAPM, we define 78 parameters as m, v, x, $C_{1m} \ldots$ C_{25m}, $V_1 \ldots$ V_{25}, $C_{1X} \ldots C_{25X}$ with 100 restrictions. For modified FF, we define 78 parameters as cb, cs, ch, $C_{1m} \ldots$ C_{25m}, $C_{1SMB} \ldots$ C_{25SMB}, $C_{1HML} \ldots$ and C_{25HML} with 100 restrictions. Moreover, for the full model, we define 130 parameters as cm, v, x, cs, ch, $C_{m1} \ldots$ C_{25m}, $V_1 \ldots$ V_{25}, $C_{1X} \ldots$ C_{25X}, $C_{1SMB} \ldots$ C_{25SMB}, $C_{1HML} \ldots$ C_{25HML} with 150 restrictions. The parameter vector is the following;

$$I(b) = \begin{bmatrix} (R_1 - E[R_1])(R_m - E[R_m]) - C_{1m} \\ \vdots \\ (R_{25} - E[R_{25}])(R_m - E[R_m]) - C_{25m} \\ (R_1 - E[R_1])(R_1 - E[R_1]) - V_1 \\ \vdots \\ (R_{25} - E[R_{25}])(R_{25} - E[R_{25}]) - V_{25} \\ (R_1 - E[R_1])(R_f - E[R_f]) - C_{1X} \\ \vdots \\ (R_2 - E[R_{25}])(R_f - E[R_f]) - C_{25X} \\ (R_1 - E[R_1])(SMB - E[SMB]) - C_{1SMB} \\ \vdots \\ (R_{25} - E[R_{25}])(SMB - E[SMB]) - C_{25SMB} \\ (R_1 - E[R_1])(HML - E[HML]) - C_{1HML} \\ \vdots \\ (R_{25} - E[R_{25}])(HML - E[HML]) - C_{25HML} \\ R_1 - (cmC_{1m} + vV_1 + xC_{1X} + csC_{1SMB} + chC_{1HML}) \\ \vdots \\ R_{25} - (cmC_{25m} + vV_{25} + xC_{25X} + csC_{25SMB} + chC_{25HML}) \end{bmatrix} \qquad (20)$$

Under the null hypothesis that CAPM, DICAPM, modified FF and full model are true, $E[I(b)] = 0$, respectively. We shall accept that a model is meaningful as the expected excess returns only if cm, v, x, cs and ch are statistically meaningful regarding Longstaff (1989). Under the null hypotheses: CAPM, DICAPM, modified FF and full model, models statistics are χ^2 (chi-sq) with the degree of freedom (df): 24 (=50-26), 22 (=100-78), 22 (=100-78), and 20 (=150-130), respectively. We shall substitute the other details of test statistics for Longstaff (1989) and Ferson-Foester (1994).

Next, knowing whether or not ICAPM is a background of FF, we shall set up itGMM test for simultaneous comparison between V_i and C_{iX} vs. C_{iSMB} and C_{iHML} as the following;

$$\begin{bmatrix} C_{iSMB} \\ C_{iHML} \end{bmatrix} = \begin{bmatrix} sv & sx \\ hv & hx \end{bmatrix} \begin{bmatrix} V_i \\ C_{iX} \end{bmatrix} \tag{18''}$$

where sv, sx, hv, hx, $V_1 \ldots$ V_{25}, C_{1X} … and C_{25X} are itGMM parameters.

The number of parameters and the number of the restrictions for itGMM are 54 and 100. We shall accept that a pair of C_{iSMB} and C_{iHML} are simultaneously related to a pair of V_i and C_{iX} if each of sv, sx, hv, and hx is statistically meaningful and; especially if SMB and HML are spanned by the rebalancing and hedging factors considering the covariance structure only if $sv \cdot hx - sx \cdot hv \neq 0$. Under the null hypotheses that a pair of V_i and C_{iX}, and a pair of C_{iSMB} and C_{iHML} are simultaneous, the model statistics is chi-sq with 46 (=100-54), with a parameter vector as follows;

$$I(b) = \begin{bmatrix} \begin{bmatrix} (R_1 - E[R_1])(R_1 - E[R_1]) \\ (R_1 - E[R_1])(R_f - E[R_f]) \end{bmatrix} - \begin{bmatrix} V_1 \\ C_{1X} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} (R_{25} - E[R_{25}])(R_{25} - E[R_{25}]) \\ (R_{25} - E[R_{25}])(R_f - E[R_f]) \end{bmatrix} - \begin{bmatrix} V_{25} \\ C_{25X} \end{bmatrix} \\ \begin{bmatrix} (R_1 - E[R_1])(SMB - E[SMB]) \\ (R_1 - E[R_1])(HML - E[HML]) \end{bmatrix} - \begin{bmatrix} sv & sx \\ hv & hx \end{bmatrix} \begin{bmatrix} V_1 \\ C_{1X} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} (R_{25} - E[R_{25}])(SMB - E[SMB]) \\ (R_{25} - E[R_{25}])(HML - E[HML]) \end{bmatrix} - \begin{bmatrix} sv & sx \\ hv & hx \end{bmatrix} \begin{bmatrix} V_{25} \\ C_{25X} \end{bmatrix} \end{bmatrix} \qquad (21)$$

4. Data and Empirical Results

4.1 Data

We obtained monthly returns on U.S. common stocks and a one-month T-bill rate for 1963.07-1993.12. 25 FF portfolios are formed by the method of 5 size and 5 book to market (B/M) classification in FF (1993)[9]. We substitute more details of portfolio formations for FF (1993). Table 1 shows descriptive statistics for the mean and variance of the excess returns of FF 25 portfolios over the one month T-bill rate, and covariances between excess returns of FF 25 portfolios over the one month T-bill rate and excess return of market portfolio over the one month T-bill rate, SMB, HML, and one month T-bill rate, respectively. R_m, SMB, and HML are FF three factors.

[Table 1 maybe here]

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[9] We surely appreciate Kenneth French for FF 25 portfolios and FF factor data available on his homepage: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

Table 1 shows that variances of excess returns of portfolios and covariances between excess returns of portfolios and the T-bill rate have a great spread across the portfolios. The dispersions of the variances strongly depend on the size and B/M, and the covariances between excess returns of portfolios and T-bill rate strongly depend on the size but less on the B/M. Covariances of excess returns of portfolios and SMB of FF and covariances between excess returns of portfolios and HML have a great spread across the portfolios. Both covariances of excess returns of portfolios and SMB and covariances between excess returns of portfolios and HML strongly depend on size and B/M. Interestingly, covariances of excess returns of portfolios and SMB of FF negatively depend on size and B/M, but covariances between excess returns of portfolios and HML positively depend on size and B/M.

4.2 Tests of DICAPM and modified FF

We use itGMM to test CAPMs including DICAPM and modified FF. We get itGMM 26 estimates of parameters and chi-sq test statistic with 24 (=50-26) overidentifying restrictions for CAPM, 78 with 22 (=100-78) for DICAPM, 78 with 22 (=100-78) for modified FF, and 130 with 20 (=150-130) for full model, respectively on excess returns of FF 25 portfolios over a one month T-bill rate for 1963.07-1993.12. Considering the null hypotheses, we need parameters cm, v, x, cs and ch to explain cross-sectional variations of unconditional excess returns.

[Table 2 maybe here]

Table 2 shows that chi-sq test statistics (df) are 92.28 (24), 105.59 (22), 109.37 (22) and 120.90 (20) for CAPM, DICAPM, modified FF and full model, respectively. Thus, itGMMs show that four models are meaningful with regard to linear relationships for the expected excess returns. For itGMM parameter estimates of CAPM, estimates (t-statistics) of cm is 0.02 (2.11); for itGMM estimates of DICAPM, estimates (t-statistics) of cm, v, and x 0.10 (4.78), -0.04 (-4.14), and –10.67 (-7.58); for itGMM parameter estimates of modified FF, estimates (t-statistics) of cm, cs, and ch 0.04 (3.68), 0.05 (3.60), and 0.14 (7.34); for itGMM estimates of full model, estimates (t-statistics) of cm, v, x, cs and ch, 0.03 (3.61), -0.04 (-11.19), 1.23 (3.02), 0.13 (8.34) and 0.18 (8.68), respectively. As shown, we know that four models have sufficient explanatory powers for the expected excess returns of FF 25 portfolios over the one month T-bill rate.

4.3 Comparison between DICAPM Factors vs. FF Factors

We use itGMM to test simultaneous relationship of a pair of V_i and C_{iX} vs. a pair of C_{iSMB} and C_{iHML}. We take itGMM 54 estimates of parameters and χ^2 test statistic with 46 (=100-54) overidentifying restrictions.

[Table 3 maybe here]

Table 3 shows that chi-sq test statistics (df) are 123.69 (46); that is, the simultaneous linear relationship is meaningful. Estimated coefficients sv, sx, hv, and hx are -0.14 (-3.54), -44.38 (-7.54), -0.17 (-4.47), and –36.99 (-7.76), respectively; especially, $sv \cdot hx - sx \cdot hv \neq 0$. As we show in table 3, C_{iSMB} and C_{iHML} are in the set linearly spanned by V_i and C_{iX}; SMB and HML are linearly spanned by the rebalancing and the hedging factors in regards to the covariance structure.

Additionally, we use itGMM to test if $C_{iSMB} = svV_i$, $C_{iSMB} = sxC_{iX}$, $C_{iHML} = hvV_i$, and $C_{iHML} = hxC_{iX}$, respectively.

[Table 4 maybe here]

The number of parameters and the number of the restrictions for each itGMM are 26 and 50. The results show that chi-sq test statistics (24) are 103.92, 70.57, 90.72, and 89.07; coefficients sv, sx, hv, and hx are 0.16 (15.53), -95.84 (-3.65), 0.31 (10.32), and -7.48 (-4.04), respectively.

We want to conclude that both of C_{iSMB} and C_{iHML} are strongly linearly spanned by V_i and C_{iX}; aggressively, SMB and HML be regarded such as the cost to rebalance portfolio continuously in terms of the market portfolio and the cost to hedge against the state variable for a long horizon. FF (1996, pp 57) argued "… is an equilibrium pricing model, a three factor version of Merton's (1973) intertemporal CAPM (ICAPM) or Ross's (1976) arbitrage pricing theory (APT)". In this view, SMB and HML mimic combinations of two underlying risk factors or state variables of special hedging…" On the stand of the temporal aggregation for a horizon and covariance structure, we aggressively suggest that *SMB* and *HML* are a linear combination of the cost to rebalance and the cost to hedge; furthermore, FF is a tree in the Merton forest.

5. Summary and Future Research

We have discussed how continuous time CAPMs can be fitted into the long horizon data, and shown that DICAPM of CIR-based ICAPM can be a three factor model, where the premiums for the factor risks are the cost of the market risk, the rebalancing cost, and the hedging cost, such as FF. Furthermore, considering temporal aggregation and covariance structure, we test and show that FF stands on the ground of ICAPM; furthermore, SMB and HML are linearly spanned by both of the rebalancing cost in terms of the market portfolio and the hedging cost against the state variable.

Appendix

1) The cumulative rate of Excess Return of the asset $\ln P_i$ over $r_0 X$

Consider dynamics of Xe^{kt}. By the Ito's Lemma and the change of integration,

$$X_t = X_0 + (X_0 - \mu)(e^{-kt} - 1) + se^{-kt}\int_0^t \sqrt{X}e^{ku}dZ_X \tag{A1}$$

Define the cumulative rate of interest X_0^τ as $X_0^\tau = \int_0^\tau dX = X_\tau - X_0$.

Then, taking $X_0 = \mu$,

$$X_0^\tau = se^{-k\tau}\int_0^\tau \sqrt{X}e^{ku}dZ_X \tag{A2}$$

Define $R(i)_0^\tau$, cumulative rate of return of the asset i $\ln P_i$ over the instantaneous rate of interest $r_0 X$, from 0 to τ as the following;

$$R(i)_0^\tau = \int_0^\tau (d\ln P_i - r_0 X dt) = \int_0^\tau ((a_i - r_0 - \frac{1}{2}\sigma_i^2)Xdt + \sigma_i \sqrt{X}dZ_i) \tag{A3}$$

Now, substituting the dynamic of X_t, and considering the stochastic version of Fubini's Theorem,

$$R(i)_0^\tau = (a_i - r_0 - \frac{\sigma_i^2}{2})[\mu\tau + \frac{s}{k}\int_0^\tau \sqrt{X}(1 - e^{-k\tau}e^{ku})dZ_X] + \sigma_i \int_0^\tau \sqrt{X}dZ_i \tag{A4}$$

2) The Claim $M_i = mC_{im} + vV_i + xC_{iX}$

Hence, define moments: M_i the mean of cumulative expression of excess return of i over the rate of interest rate, V_i the variance of cumulative expression of excess return assets, C_{im} covariance of cumulative expression of excess return of i over the rate of interest rate and cumulative expression of excess return of market portfolio over the rate of interest rate, C_{iX} covariance of covariance of cumulative expression of excess return of i over the rate of interest rate between the state variable, and C_{mX} covariance between cumulative expression of excess return of market portfolio over the rate of interest rate and the state variable for $R_i = R(i)_0^1$, $R_m = R(m)_0^1$, and $X = X_0^1$ for a unit interval as follows;

$$M_i \equiv E[R_i] = \mu(a_i - r_0 - \frac{\sigma_i^2}{2}) \tag{A5}$$

$$\begin{aligned} V_i &\equiv V(R_i) = E[R_i \cdot R_i] - E[R_i]E[R_i] \\ &= \mu\sigma_i^2 + \frac{2\mu c_1}{k}\sigma_{iX}(a_i - r_0 - \frac{\sigma_i^2}{2}) + (\frac{\mu s^2 c_2}{k^2} - \mu^2)(a_i - r_0 - \frac{\sigma_i^2}{2})^2 \end{aligned} \tag{A6}$$

$$\begin{aligned} C_{im} &\equiv Cov(R_i, R_m) = E[R_i \cdot R_m] - E[R_i]E[R_m] \\ &= \mu\sigma_{im} + \frac{\mu c_1 c_4}{k}\sigma_{iX} + (\frac{\mu s^2 c_2 c_4}{k^2} - \mu^2 c_4 + \frac{\mu c_1}{k}\sigma_{mX})(a_i - r_0 - \frac{\sigma_i^2}{2}) \end{aligned} \tag{A7}$$

$$\begin{aligned} C_{iX} &\equiv Cov(R_i, X) = E[R_i \cdot X] - E[R_i]E[X] \\ &= \frac{\mu c_3}{k}\sigma_{iX} + \frac{\mu s^2 c_3^2}{2k^2}(a_i - r_0 - \frac{\sigma_i^2}{2}) \end{aligned} \tag{A8}$$

$$\begin{aligned} C_{mX} &\equiv Cov(R_m, X) = E[R_m \cdot X] - E[R_m]E[X] \\ &= \frac{\mu c_3}{k}\sigma_{mX} + \frac{\mu s^2 c_3^2 c_4}{2k^2} \end{aligned} \tag{A9}$$

where $c_1 = 1 - \frac{1 - e^{-k}}{k}$, $c_2 = 1 - 2\frac{1 - e^{-k}}{k} + \frac{1 - e^{-2k}}{2k}$, $c_3 = 1 - e^{-k}$, and $c_4 = a_m - r_0 - \frac{\sigma_m^2}{2}$

Now, substituting moments (A5), (A6), (A7), (A8), and (A9) for $a_i - r_0$, σ_{im},

σ_i^2, σ_{iX}, and σ_{mX},

$$a_i - r_0 = \frac{1}{2\mu}V_i + (\frac{1}{\mu} - \frac{c_1}{\mu^2 c_3}C_{iX})M_i + \frac{s^2}{2\mu^2 k^2}(c_1 c_3 - c_2 + \mu k^2)M_i^{\,2} \qquad \text{(A10)}$$

$$\sigma_i^2 = \frac{1}{\mu}V - \frac{2c_1}{\mu^2 c_3}C_{iX}M_i + \frac{s^2}{\mu^2 k^2}(c_1 c_3 - c_2 + \mu k^2)M_i^{\,2} \qquad \text{(A11)}$$

$$\sigma_{im} = \frac{1}{\mu}C_{im} - \frac{c_1 c_4}{\mu c_3}C_{iX} + \frac{s^2}{\mu}[\frac{(\mu+1)c_1 c_3 c_4}{2k^2} + \frac{\mu c_4}{s^2} - \frac{\mu c_2 c_4}{k} + \frac{c_1}{s^2 c_3}C_{mX}]M_i \qquad \text{(A12)}$$

$$\sigma_{iX} = \frac{k}{\mu c_3}C_{iX} - \frac{s^2 c_3}{2\mu k}M_i \qquad \text{(A13)}$$

$$\sigma_{mX} = \frac{k}{\mu c_3}C_{mX} - \frac{s^2 c_3 c_4}{2k} \qquad \text{(A14)}$$

Considering the continuous time CAPM: $a_i - r_0 = \lambda\sigma_{im}$ and (A5),

$$M_i = \mu\lambda\sigma_{im} - \mu 2^{-1}\sigma_i^2 \qquad \text{(A15)}$$

Plugging equations σ_{im} and σ_i^2 into (A15), we obtain the equation for $M_i = M_i(C_{im}, V_i, C_{iX}, C_{mX})$:

$$d_0 M_i^{\,2} - F_i M_i + G_i = 0 \qquad \text{(A16)}$$

where $F_i = d_1 C_{iX} + d_2 C_{mX} + d_3$, $G_i = d_4 C_{iX} + d_5 C_{im} + d_6 V_i$, $d_0 = \frac{s^2}{2\mu k^2}(c_1 c_3 - c_2 + \mu k^2)$,

$d_1 = \frac{2c_1}{\mu c_3}$, $d_2 = \frac{\lambda c_1}{\mu c_3}$ $d_3 = \frac{s^2}{\mu}[\frac{(\mu+1)c_1 c_3 c_4}{2} + \frac{\mu}{s^2}c_4 - \frac{\mu c_2 c_4}{k}] - 1$, $d_4 = \frac{\lambda c_1 c_4}{c_3}$, $d_5 = -\lambda$, and $d_6 = 0.5$

Consider the case $V_i = C_{im} = C_{iX} = C_{mX} = 0$. Since $M_i = \lambda C_{im} - 2^{-1}V_i = 0$ without the state variable,

$$M_i(0,0,0,0) = 0 \qquad \text{(A17)}$$

Thus, we can obtain the closed solution of M as following;

$$M_i(C_{im}, V_i, C_{iX}, C_{mX}) = mC_{im} + vV_i + xC_{iX} + mmC_{im}^2 + mvC_{im}V_i + mxC_{im}C_{iX} + myC_{im}C_{mX} + vvV_i^2 + vxV_iC_{iX} + vyV_iC_{iX} + xxC_{iX}^2 + xyC_{iX}C_{mX} \quad \text{(A18)}$$

where $m = d_5 d_3^{-1}$, $v = d_6 d_3^{-1}$, $x = d_4 d_3^{-1}$, $mm = d_0 d_5^2 d_3^{-3}$, $mv = 2d_0 d_5 d_6 d_3^{-3}$, $mx = (2d_0 d_4 - d_1) d_5 d_3^{-3}$, $my = d_2 d_5 d_3^{-2}$, $vv = d_0 d_6^2 d_3^{-3}$, $vx = (2d_0 d_4 - d_1 d_3) d_6 d_3^{-2}$, $vy = -d_2 d_6 d_3^{-2}$, $xx = (d_0 d_4 - d_1 d_3) d_4 d_3^{-3}$, and $xy = -d_2 d_4 d_3^{-2}$

Hence, approximating the equation for M_i linearly, and taking $R_f = r_0 X$, we can obtain the solution for M called as a time discretized version of ICAPM (DICAPM) as the following:

$$M_i = mC_{im} + vV_i + xC_{iX} \quad \text{(A19)}$$

where $m = d_5 d_3^{-1}$, $v = d_6 d_3^{-1}$, and $x = d_4 (r_0 d_3)^{-1}$ QED

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Table 1: Means and Variances of excess returns of FF 25 Portfolios over a One Month T-Bill Rate, Covariances between excess returns of FF 25 Portfolios over a One Month T-Bill Rate and FF 3 Factors, and Covariances between excess returns of FF 25 Portfolios over a One Month T-Bill Rate and a One Month T-Bill Rate

This table is constructed with FF 25 portfolios, FF 3 factors, and a one month T-bill rate for 1963.07-1993.12. The portfolios and FF 3factors are constructed by FF portfolio formation (FF, 1993). The data represent excess monthly returns of FF portfolios over the one-month T-bill rate for 1963.07-1993.12. R_i is the excess return of FF 25 portfolio i over the one month T-bill rate, R_m excess return of market portfolio over the one month T-bill rate , and R_f one-month T-bill rate. R_m, SMB and HML are FF 3 factors. M_i is the mean of the excess monthly return of portfolio i over the one month T-bill rate, V_i the variance of the excess monthly return of portfolio i over the one month T-bill rate, C_{im} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and excess monthly return of market portfolio over the one month T-bill rate, C_{iSMB} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and SMB, C_{iHML} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and HML, and C_{iX} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and one month T-bill rate, respectively.

	Book-to-Market Equity (BE/ME) Quintiles										
Size	Low	2	3	4	High		Low	2	3	4	High
			M_i						V_i		
Small	0.26	0.69	0.73	0.92	1.08	Small	58.34	44.92	37.65	33.99	38.14
2	0.38	0.63	0.89	0.92	1.05	2	51.36	38.08	31.13	27.83	34.69
3	0.41	0.72	0.68	0.89	0.94	3	42.78	30.95	25.91	23.05	30.45
4	0.47	0.37	0.63	0.79	0.95	4	33.51	28.05	24.28	22.60	31.01
Big	0.32	0.36	0.42	0.52	0.61	Big	23.00	21.09	18.40	17.69	22.54
			C_{im}						C_{iSMB}		
Small	27.98	24.68	22.77	21.20	21.62	Small	16.48	14.78	13.43	12.85	13.58
2	28.24	24.34	21.94	20.56	22.20	2	13.59	12.23	10.93	9.82	11.24
3	26.74	22.78	20.47	19.19	21.06	3	10.97	9.59	8.70	7.63	9.32
4	24.19	22.25	20.36	19.00	21.48	4	7.65	6.98	6.39	5.82	7.67
Big	19.76	19.32	17.23	16.52	17.11	Big	3.06	2.98	2.10	2.45	3.81
			C_{iHML}						C_{iX}		
Small	-7.09	-4.23	-3.06	-1.99	-0.56	Small	-0.24	-0.18	-0.16	-0.16	-0.20
2	-8.35	-4.69	-2.91	-1.35	-0.50	2	-0.18	-0.14	-0.16	-0.12	-0.15
3	-7.91	-4.42	-2.31	-1.10	-0.21	3	-0.15	-0.14	-0.14	-0.14	-0.12
4	-7.54	-4.32	-2.36	-0.83	-0.29	4	-0.13	-0.13	-0.12	-0.13	-0.12
Big	-6.76	-4.03	-2.50	-0.38	0.95	Big	-0.12	-0.09	-0.11	-0.09	-0.10

Table 2: itGMM Parameter Estimates for Modified FF and DICAPM

$$M_i = cmC_{im} + vV_i + xC_{iX} + csC_{iSMB} + chC_{iHML}$$

The parameters are from itGMM of monthly excess returns of FF 25 portfolios over the one month T-bill rate for 1963.07-1993.12. The t-statistics (t-stat) for the estimated parameter are in parentheses. M_i is the mean of the excess monthly return of portfolio i over the one month T-bill rate, V_i the variance of the excess monthly return of portfolio i over the one month T-bill rate, C_{im} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and excess monthly return of market portfolio over the one month T-bill rate, C_{iSMB} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and SMB, C_{iHML} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and HML, and C_{iX} the covariance between the covariance between the excess monthly return of portfolio i over the one month T-bill rate and one month T-bill rate, respectively. The parameter cm is m if the estimated model is DICAPM, cb if modified FF, and $cb+m$ if full model, respectively; v and x from DICAPM; cs and ch from FF, respectively. df represents the degrees of freedom, N-P the number (N) of model restrictions minus the number (P) of model parameters, and χ^2 (chi-sq) model statistics with df, respectively. This table is based on the Table Ⅱ of Longstaff (1989, pp882).

Model	cm	v	x	cs	ch	Df	N-P	chi-sq
CAPM	0.02					24	50-26	92.28
t-stat	**2.11**							
DICAPM	0.10	-0.04	-10.67			22	100-78	105.59
t-stat	**4.78**	**-4.14**	**-7.58**					
Modified FF	0.04			0.05	0.14	22	100-78	109.37
t-stat	**3.68**			**3.60**	**7.34**			
Full model	0.03	-0.04	1.23	0.13	0.18	20	150-130	120.90
t-stat	**3.61**	**-11.19**	**3.02**	**8.34**	**8.68**			

Table 3: itGMM Parameter Estimates for Simultaneous Comparison between DICAPM 3 factors and FF 3 factors

$$\begin{bmatrix} C_{iSMB} \\ C_{iHML} \end{bmatrix} = \begin{bmatrix} sv & sx \\ hv & hx \end{bmatrix} \begin{bmatrix} V_i \\ C_{iX} \end{bmatrix}$$

The parameters are from itGMM for simultaneous comparison between FF factors and DICAPM factors on monthly excess returns of FF 25 portfolios over the one month T-bill rate for 1963.07-1993.12. The t-statistics (t-stat) for the estimated parameter are in parentheses. V_i the variance of the excess monthly return of portfolio i over the one month T-bill rate, C_{im} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and excess monthly return of market portfolio over one month T-bill rate, C_{iSMB} the covariance between the covariance between the excess monthly return of portfolio i over the one month T-bill rate and SMB, C_{iHML} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and HML, and C_{iX} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and one month T-bill rate, respectively. The parameters sv, sx, hv, and hx are for simultaneous linear relationships between modified FF and DICAPM. df is the degree of freedom, N-P the number (N) of model restrictions minus the number (P) of model parameters, and χ^2 (chi-sq) model statistics with df, respectively.

Covariance	*sv*	*sx*	*hv*	*hx*	df	N-P	chi-sq
C_{iSMB}	-0.14	-44.38			46	100-54	123.69
t-stat	**-3.54**	**-7.54**					
C_{iHML}			-0.17	-36.99			
t-stat			**-4.47**	**-7.76**			

Table 4: itGMM Parameter Estimates for relationships among DICAPM 3 factors and FF 3 factors

$$C_{iSMB} = svV_i\ ;\ \ C_{iSMB} = sxC_{iX}\ ;\ \ C_{iHML} = hvV_i\ ;\ \ C_{iHML} = hxC_{iX}$$

The parameters are from itGMM for simultaneous comparison between FF factors and DICAPM factors on monthly excess returns of FF 25 portfolios over the one month T-bill rate for 1963.07-1993.12. The t-statistics (t-stat) for the estimated parameter are in parentheses. V_i the variance of the excess monthly return of portfolio i over the one month T-bill rate, C_{im} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and excess monthly return of market portfolio over one month T-bill rate, C_{iSMB} the covariance between the covariance between the excess monthly return of portfolio i over the one month T-bill rate and SMB, C_{iHML} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and HML, and C_{iX} the covariance between the excess monthly return of portfolio i over the one month T-bill rate and one month T-bill rate, respectively. The parameters sv, sx, hv, and hx are for simultaneous relationships between modified FF and DICAPM. df is the degree of freedom, N-P the number (N) of model restrictions minus the number (P) of model parameters, and χ^2 (chi-sq) model statistics with df, respectively.

Covariance	*sv*	*sx*	*hv*	*hx*	df	N-P	Chi-sq
C_{iSMB}	0.16				24	50-26	103.92
t-stat	**15.53**						
C_{iSMB}		-95.84			24	50-26	70.57
t-stat		**-3.65**					
C_{iHML}			0.31		24	50-26	90.72
t-stat			**10.32**				
C_{iHML}				-7.48	24	50-26	89.07
t-stat				**-4.04**			