Intraday Volatility, Jump Probability and Periodicity Filters in Korean Won–U.S. Dollar Exchange Rates

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I. Introduction

The discontinuous jump in financial assets, such as exchange rates, has become one of the most important issues since the global financial crisis of 2007–2008 and the Euro crisis of 2010–2012. In July 2018, the then U.S. President Donald Trump imposed sweeping tariffs on China for alleged unfair trade practices. The uncertainty in the U.S.–China Trade War may lead to reduced world trade, and ultimately affect the Korean economy and trade and subject the Korean won–U.S. dollar exchange rate to greater volatility. In the 2010s in particular, jumps in the Korean won–U.S. dollar exchange rates occurred frequently: 1152.10 on July 1, 2016; 1070.31 on March 20, 2018; 1217.45 on March 16, 2020; and 1117.15 on April 30, 2021.

As noted, the volatility of Korean won–U.S. dollar exchange rates is connected to the trade between Korea and the U.S. The volatility and jumps in this rate also influence the trade between Korea and the U.S. This makes it crucial to accurately estimate the volatility as well as the frequency and probability of jumps in the volatile Korean won–U.S. dollar exchange rates that occurred in the 2010s using more efficient, robust volatility and jump estimation. Thus far, parametric approaches, particularly the autoregressive conditional heteroskedastic (ARCH) models and stochastic volatility, which were popularly used for such analyses before the 2000s, have not adequately addressed this problem. There is a need to incorporate discontinuous jumps in the volatility process observed in recent years. In this regard, the effectiveness of the volatility estimations has been tested in various ways while considering the volatility and jumps in exchange rates.

The parametric approaches rely on explicit functional forms that cannot be inherently specified in detail. It is nearly impossible to explain the discontinuous jump parts of intraday return volatility using parametric models. Dewachter et al. (2014) show that the Gaussian quasi-maximum likelihood estimates of generalized ARCH (GARCH) models, subject to the presence of additive jumps, tend to overestimate the volatility for the days following the jumps and produce upward-biased estimates of long-term volatility.

To overcome these drawbacks of parametric approaches, scholars have introduced and developed nonparametric approaches that use high-frequency daily and intraday asset returns data (Andersen et al., 2001, 2003; Andersen et al., 2002, 2004; Barndorff-Nielsen and Shephard, 2005a, 2005b, 2006). Notably, it is nearly impossible to analyze discontinuous jumps in the volatility of Korean won–U.S. dollar exchange rates. Therefore, we use a modified version of the nonparametric approach proposed by scholars, particularly Andersen et al. (2004, 2007), Huang and Tauchen (2005), and Lee and Mykland (2008).

Nevertheless, the studies cited above do not account for intraday volatility periodicity. According to Boudt et al. (2011b), disregarding intraday volatility periodicity can influence the accuracy of the estimated jump statistics and jump detection considerably. To overcome this, Boudt et al. (2011a, 2011b) used volatility periodicity filters with the nonparametric method. Yi (2014) also included the periodicity filters of volatility but analyzed the volatility of U.S. SPC500 returns using Z-type jump statistics. Similarly, Yi (2020) used Z-type normal distribution jumps of exchange rates based on outlying weighted quarticity statistics using a Gumbel distribution.

Lee and Hannig (2010) proposed another version of the Lee and Hannig (LH) test to obtain robust estimates for both finite and infinite exchange rate jumps. The detection method for big jumps was the same as that of Lee and Mykland (2008). The jump component captures both finite and infinite activity price jumps. Mancini (2009) and Bollerslev and Todorov (2011) and Yi (2023) suggested using truncated power variation to consistently estimate integrated volatility.

Although the linear drift process falls within the general asset price specification, Laurent and Shi (2020) showed that the infinite sample performance of their Laurent and Shi (LS) test for additive jumps is far from satisfactory. When asset prices deviate locally from the random walk, the test shows a strong size distortion and dramatic power loss. They applied the tests on 21 years of five-minute log-returns of the Nasdaq stock price index and found that, unlike Lee and Mykland's (2008) test by the same name (Lee and Mykland [LM] test), their test allows the detection of jumps when log prices exhibit clear upward or downward trend movements. Specifically, Laurent and Shi (2020) showed that the mean of five-minute log returns generated by a model with or without jumps that deviates from the random walk can be non-negligible, which invalidates the use of bipower variation.

Although most studies (e.g., Barndorff-Nielsen and Shephard, 2005a, 2005b, 2006; Andersen et al., 2004, 2007; Boudt et al., 2011b; Yi 2014) have adopted the standard normal Z-type jump statistics of the standard normal distribution, we adopt the Gumbel distribution to determine whether significant daily and intraday jumps occur. Therefore, maximum outlying (or max outlying) daily jumps and intraday jump probabilities are analyzed using volatility periodicity filters in five-minute returns of the Korean won–U.S. dollar exchange rates.

Essentially, we use a nonparametric realized volatility model with intraday volatility periodicity to explain the discrete jumps and continuous volatility of the Korean won–U.S. dollar exchange rates during 2010–2021. We adopt the realized volatility and jump statistics using the volatility periodicity with LH, LS, and LM jump statistics.

Our approach has several highlights. First, unlike most studies, this study utilizes the newly developed periodicity window factors of volatility of financial assets, or periodicity filters, such as weighted standard deviation (WSD), shortest half scale (ShortH), and median absolute deviation (MAD).

Second, the severe volatility of Korean won–U.S. dollar movements in the 2010s has violated the Gaussian distribution. Thus, a robust estimator for jumps is required for non-Gaussian data. We adopt the

Gumbel distribution to determine whether a significant daily and intraday jump occurs using max outlying daily jumps. Further, we employ the aforementioned jump statistics to determine whether a significant jump occurs.

Third, to estimate the volatility and jumps in Korean won–U.S. dollar exchange rates, efficient and robust estimators of the jumps should be obtained using the periodicity filters of intraday volatility during the volatile 2010s, as in Lee and Mykland (2011a, 2011b). However, while these authors used the periodicity filters of intraday volatility with bipower variations, we use local robust variances to estimate the integrated volatility.

Fourth, besides the periodicity filters of intraday volatility during the 2010s, as in Lee and Mykland (2011a, 2011b), we also use the LH test to obtain robust estimates for both finite and infinite exchange rate jumps. Fifth, Laurent and Shi (2020) showed that the LM test detects upward or downward jumps, which are negligible. This invalidates the use of bipower variation.

Therefore, we use the truncated power variation to estimate the integrated volatility consistently. Besides using the LS test to determine the volatility of Korean won–U.S. dollar movements, we compare the probability of jumps in the 2010s during which events such as the Euro crisis and the U.S.–China Trade War occurred. Therefore, we account for the periodicity window factors of intraday volatility and jumps to derive robust jump statistics and avoid biased estimators.

The remainder of this study is organized as follows. Section II undertakes the literature review. Section III introduces the realized volatility, jump statistics, and jump tests with periodicity filters of volatility. Section IV presents the high-frequency data and empirical results of several jump statistics associated with jump probabilities. Finally, section V summarizes the empirical findings and presents the conclusions of this study.

II. Literature Review

Around the beginning of the 2000s, many studies adopted volatility models that use the parametric approach (Andersen et al., 2003; Chernov et al., 2003; Eraker, 2004; Eraker et al., 2003; Pan, 2002; Johannes, 2004). However, it is considerably difficult to estimate frequent jump diffusions through continuous parametric approaches. Moreover, there are difficulties in incorporating the complexities of micro financial markets appropriately into empirical parametric models. Models using low-frequency data cannot explain why discontinuous jumps occur frequently, even in intraday returns in financial assets.

Therefore, nonparametric approaches using realized volatility with high-frequency intraday returns were developed to capture discontinuous jump parts and the continuous volatility of financial asset returns. Bollerslev and Zhou (2002,

2006) and Fleming et al. (2003) developed realized volatility as a new volatility measure. By incorporating realized high-frequency intraday returns, the nonparametric volatility models in Andersen et al. (2003) provide enhanced empirical results compared with complicated parametric volatility models, such as GARCH and stochastic volatility models. This nonparametric volatility approach uses high-frequency intraday return data, such as one-minute or five-minute returns.

Some authors have analyzed the total variation (Andersen and Bollerslev, 1998a, 1998b; Andersen et al., 2003; Barndorff-Nielsen and Shephard, 2005a, 2005b, 2006; Andersen et al., 2004, 2007; Huang and Tauchen, 2005), which can be separated into continuous variation parts and discontinuous jump parts. These studies have concluded that discontinuous jumps are an important source of non-predictable exchange rate volatility. Among recent studies, Bollerslev and Todorov (2011) estimated jump tails and premia, which cannot be explained using continuous volatility.

Others have examined jumps, co-jumps, macro events, and macro news, and their effects. Lahaye et al. (2011) examined jumps, co-jumps, and macro announcements. Laakkonen and Lanne (2013) analyzed the impact of macroeconomic news on exchange rate volatility. Bibinger et al. (2014) estimated the spot covariation of assets, while Bibinger and Winkelmann (2013) estimated co-jumps in high-frequency data with noise. Dewachter et al. (2014) examined the intraday impact of communication on U.S. dollar-Euro volatility and jumps. Similar to Lahaye et al. (2011), Chatrath et al. (2014) analyzed currency jumps, co-jumps, and the role of macro news. Délèze and Hussain (2014) examined information arrival, jumps, and co-jumps in European financial markets.

Meanwhile, Pukthuanthong and Roll (2015) analyzed internationally correlated jumps in stock returns in 82 countries based on the quarticity of standard deviations proposed by Barndorff-Nielsen and Shephard (2006) and the jump statistic based on bipower variations. Siroos and Narayan (2019) analyzed the intraday effects of the currency market using hourly exchange rates from 2004 to 2014. Arouri et al. (2019) analyzed international asset allocation in the presence of systematic co-jumps using Huang and Tauchen's (2005) jump test statistics and the rescaled intraday returns proposed by Bollerslev et al. (2009).

However, Arouri et al. (2019) did not consider intraday periodicity filters of volatility. According to Lee and Mykland (2008) and Boudt et al. (2011a, 2011b), disregarding this aspect of intraday volatility periodicity may hinder the accuracy of jump detection. Recently, research has started adopting intraday periodicity filters in volatility to estimate discontinuous volatility and jumps in high-frequency financial assets.

Some solutions have been proposed. Lee and Hannig (2010) proposed using the LH test to obtain robust estimates for both finite and infinite exchange rate jumps. Mancini (2009) and Bollerslev and Todorov (2011) suggested using truncated power variation to consistently estimate integrated volatility. Laurent and Shi (2020) showed that the infinite sample performance of their LS test for additive jumps is far from satisfactory, unlike the Lee and Mykland (2008) test. Moreover, Laurent and Shi (2020) showed that the mean of five-minute log returns generated by a model with or without jumps that deviate from the random walk can be non-negligible, which invalidates the use of bipower variation.

In summary, the reasoning and methods used in this study align with those of Andersen et al. (2004, 2007), Lee and Mykland (2008), and Boudt et al. (2011a, 2011b). However, this study progresses differently and differs from earlier models, and makes the following contributions to the literature.

First, we introduce newly developed periodicity window factors of the volatility of financial assets. In this respect, our study differs vastly from Andersen et al. (2004, 2007) and Lee and Mykland (2008), who did not consider these periodicity window factors of volatility. Second, we use both daily and intraday jumps because high volatility and significant jumps within a day can affect daily volatility. This differs from previous studies. For example, Siroos and Narayan (2019) analyzed only the intraday effects of the currency market. Furthermore, unlike previous studies that used standard normal Z statistics, we adopt the Gumbel distribution to determine whether there are significant daily and intraday jumps.

Third, we analyze and compare the volatility and probability of jumps in the Korean won and U.S. dollar exchange rates using LM and LH tests, and the LS jump statistics. We use the truncated power variation to estimate consistently the integrated volatility instead of bipower volatility.

III. Realized Volatility, Jumps, and Periodicity Filters

3.1. Realized Volatility and Jump with Normal Distribution

Following Barndorff-Nielsen and Shephard (2004) and Bollerslev et al. (2009), let us consider that T days of M equally-spaced intraday returns and the j-th intraday return of day t by $r_{t,i}$. M represents the observed intraday sampling frequency Thus, the daily realized volatility or variation of day t (RV_t) is represented by the sum of the intraday realized squared variation in equation (1).

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 , \quad j=1,\ 2,\ \ldots,\ M. \tag{1}$$

The daily realized volatility converges to the increment of the quadratic variation process as the sampling frequency (M) of the underlying returns goes to infinity or ((1/M)≡Δ) approaches zero, as pointed out by Andersen, Bollerslev, and Diebold (2007). In reality, however, jumps in exchange rates occur occasionally and the occurrence of jumps is generally assumed to follow a Poisson, which is a continuous-time discrete process in which the realized volatility inherits the continuous sample path process and the discrete jump process. In the presence of jumps, realized volatility is no longer a consistent estimator of integrated volatility. Thus, for Δ→0, the daily realized volatility on day t converges in probability to the sum of continuous integrated variance and the daily summation of discrete N jumps of size κ_t, as in equation (2).

$$\lim_{M\to\infty} RV_t = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2 . \tag{2}$$

This study adopts the standard realized bipower variation (BV_t) developed by Barndorff-Nielsen and Shephard (2004, 2006). Huang and Tauchen (2005) showed that the realized bipower variation converges in probability to the integrated variation, as $\Delta \to 0$ or M becomes sufficiently large.

$$BV_t = \mu_1^{-2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}|r_{t,j-1}||r_{t,j}| = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}|r_{t,j-1}||r_{t,j}|,$$

$$\lim_{M\to\infty} BV_t = \int_{t-1}^{t} \sigma^2(s)ds, \quad \mu_1 = \sqrt{\frac{2}{\pi}} = E(|Z|),\ Z \sim N(0,1). \tag{3}$$

The bipower variation is robust to jumps because it uses the product between two consecutive returns instead of the squared return. As M approaches infinity and there are no jumps in the volatility of financial assets, the joint distribution of the realized volatility and the realized bipower variation has the property of the normal distribution, as shown by Barndorff-Nielsen and Shephard (2006).

3.2. Volatility Periodicity Filters

As Boudt, Croux, and Laurent (2011a) proposed, the high-frequency return variance $\sigma_{t,i}^2$ has a periodic component $f_{t,i}^2$ due to the weekly cycle of opening, lunch, and closing times at financial centers. However, Barndorff-Nielsen and Shephard (2004) and Andersen, Bollerslev, and Diebold (2007) did not consider the periodicity of volatility. Therefore, this study adopts nonparametric estimators in the presence of jumps using periodicity. To identify the periodicity factor $f_{t,i}^2$ for the average variance of day t, the squared periodicity factor has a mean of one over the local window.

In line with Boudt, Croux, and Laurent (2011b), to estimate the periodicity factor, this study uses a nonparametric estimator of the standardized returns scale. Thus, the estimator will be robust to jumps. The nonparametric estimator is based on a scale estimate of the standardized returns $\overline{r_{t,\iota}} = r_{t,i}/\widehat{s_{t,\iota}}$, where $\widehat{s_{t,\iota}} = \sqrt{\frac{1}{M-1}BV_t}$. These standardized returns share the same periodicity factor as $r_{t,i}$, and are observed at the same time of the day and day of the week as $r_{t,i}$. The periodicity estimator can be assumed to follow the classical estimator, which is based on the standard deviation, and is similar to Taylor and Xu's (1997) periodicity estimate. This, in turn, is based on the averages of squared returns.

$$f_{t,i}^{SD} = \frac{SD_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=0}^{M} SD_{t,j}^2}}, \qquad SD_{t,i} = \sqrt{\frac{1}{n_{t,i}}\sum_{j=1}^{n_{t,i}} \overline{r_{j;t,\iota}^2}} \tag{4}$$

This estimator of standard deviation is efficient only in the absence of jumps. We cannot use this estimator in the presence of jumps because the observation can be affected by jumps, which makes the periodicity estimate arbitrarily large.

First, following various filters proposed by Boudt, Croux, and Laurent (2011a, 2011b), this study adopts the MAD. The MAD of a sequence of observations $y_1, \cdots, y_n$ is defined as $1.486 median_i|y_i - median_i y_i|$, where 1.486 is the correction factor that guarantees that the MAD will be a consistent scale estimator for normal distributions.

The MAD estimator for the periodicity factor is presented in equation (5).

$$f_{t,i}^{\widehat{MAD}} = \frac{MAD_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} MAD_{t,i}^2}} \tag{5}$$

Second, the ShortH estimator, proposed by Rousseeuw and Leroy (1988), is considered as efficient as the MAD under normality because the ShortH estimator is consistent in the presence of infinitesimal contaminations based on jumps in the data. According to these authors, it gives the smallest maximum bias possible; the jump estimator can cause bias among a wide class of scale estimators. Moreover, it is computationally convenient and does not require a location estimation. The ShortH estimator for the periodicity factor is given by equation (6).

$$f_{t,i}^{\widehat{ShortH}} = \frac{ShortH_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} ShortH_{t,j}^2}} \tag{6}$$

$$ShortH_{t.i} = 0.741 \min\left\{\overline{r_{(h_{t,\iota}):t,\iota}} - \overline{r_{(1):t,\iota}}, \cdots, \overline{r_{(n_{t,\iota}):t,\iota}} - r_{(h_{t,i-1}):t,i}\right\}$$

Boudt, Croux, and Laurent (2011a, 2011b) showed that the standard deviation applied to the returns weighted in the function of their outlyingness offers a better trade-off between the efficiency of the standard deviation under normality and the high robustness to jumps of the shortest half dispersion. The estimator for the periodicity factor under the WSD estimate equals the following:

$$f_{t,i}^{\widehat{WSD}} = \frac{WSD_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} WSD_{t,i}^2}}, \qquad WSD_{t,j} = \sqrt{1.081\frac{\sum_{l=1}^{n_{t,j}} w\left[\left(r_{l:t,j}/f_{t,j}^{\widehat{ShortH}}\right)^2\right]\overline{r_{l:t,J}^2}}{\sum_{l=1}^{n_{t,j}} w\left[\left(r_{l:t,j}/f_{t,j}^{\widehat{ShortH}}\right)^2\right]}} \tag{7}$$

where the factor 1.081 ensures consistency of the estimator under normality.

3.3. Intraday Gumbel Distribution Jump Tests with Periodicity Filters

Next, we test whether any intraday return $r_{t,i}$ has a purely continuous diffusion or jump in the price process, following Lee and Mykland (2008) and Boudt, Croux, and Laurent (2011b). If a return contains a jump component, it should be abnormally large. An abnormal return in times of high volatility is larger than that in times of low volatility. Therefore, the aforementioned authors use the ratio of the tested return to a measure of local volatility. The intraday jump statistic $IDJ_{t,i}$ tests whether a jump occurred between intraday periods $i-1$ and i of day t. It is defined as the absolute return divided by an estimate of the local standard deviation $\widehat{\sigma_{t,i}}$:

$$IDJ_{t,i} = \frac{|r_{t,i}|}{\widehat{\sigma_{t,i}}}. \tag{8}$$

Under the null of no jump at the time of testing, the process belongs to the family of Brownian semimartingale jump models. A suitable choice of window size for local volatility asymptotically follows a standard normal distribution. We can replace the local variance $\widehat{\sigma_{t,i}}$ by $\widehat{s_{t,i}} = \sqrt{\frac{1}{M-1}BV_t}$.

$$IDJ_{t,i} = \frac{|r_{t,i}|}{\sqrt{\frac{1}{M-1}BV_t}}. \tag{9}$$

To minimize spurious jump detection, the filtered LM jump statistic $IDJ_{t,i}$, proposed by Broudt, Croux, and Laurent (2011b), follows a Gumbel distribution when $\Delta \rightarrow 0$ under the assumption of no jump in the interval $i-1$ and i of day t. Hence, we reject the null of no jump during day t at the α % critical level if:

$$IDJ_{t,i} > G^{-1}(1-\alpha)S_n + C_n$$
$$C_n = (2\log n)^{0.5} - \frac{\log(\pi) + \log(\log n)}{2(2\log n)^{0.5}}, \;\; S_n = \frac{1}{(2\log n)^{0.5}}. \tag{10}$$

When $n = M$ (i.e., number of observations per day) and $n = MT$ (i.e., the total number of observations), the expected number of detected jumps is equal to αT and α (i.e., ≈ 0), respectively.

3.3.1. Lee and Mykland (LM) Jump Tests with Periodicity Filters

If we ignore periodic volatility patterns, it can lead to spurious jump identifications. Boudt, Croux, and Laurent (2011b) suggested accounting for the strong periodicity in volatility, showing that it provides more appropriate results. In this study, to consider periodic volatility patterns, we can derive the three robust nonparametric estimators $\widehat{s_{t,i}}\widehat{f_{t,i}^{MAD}}$, $\widehat{s_{t,i}}\widehat{f_{t,i}^{ShortH}}$, and $\widehat{s_{t,i}}\widehat{f_{t,i}^{WSD}}$ to estimate $\widehat{\sigma_{t,i}}$ and the Lee and Mykland (LM) jump statistics as follows:

$$LM\ IDJ_{t,i}^{MAD} = \frac{|r_{t,i}|}{\widehat{s_{t,\iota}} f_{t,i}^{\widehat{MAD}}} = \frac{|r_{t,i}| \sqrt{\frac{1}{M-1} \sum_{j=1}^{M} MAD_{t,i}^2}}{\hat{s}_{t,i} MAD_{t,i}} \tag{11}$$

$$LM\ IDJ_{t,i}^{ShortH} = \frac{|r_{t,i}|}{\widehat{s_{t,\iota}} f_{t,i}^{\widehat{ShortH}}} = \frac{|r_{t,i}| \sqrt{\frac{1}{M-1} \sum_{j=1}^{M} ShortH_{t,i}^2}}{\hat{s}_{t,i} Short_{t,i}} \tag{12}$$

$$LM\ \ IDJ_{t,i}^{WSD} = \frac{|r_{t,i}|}{\widehat{s_{t,\iota}} f_{t,i}^{WSD}} = \frac{|r_{t,i}| \sqrt{\frac{1}{M-1} \sum_{j=1}^{M} WSD_{t,i}^2}}{\hat{s}_{t,i} WSD_{t,i}} \tag{13}$$

Under the null of no jump and a consistent estimate $\widehat{\sigma_{t,\iota}}$, $IDJ_{t,i}$ follows a standard normal distribution, which has an absolute value. If the statistic exceeds a plausible maximum, the null of no jump can be rejected. However, Brownlees and Gallo (2006) found that a normal distribution spuriously detects many jumps. If we ignore periodic volatility, it leads to over-detection of jumps in periods of high intraday periodic volatility.

3.3.2. Lee and Hannig Jump Test using the Truncated Power Variation

Lee and Hannig (2010) proposed another version of the LH test. The detection method for big jumps was the same as that of Lee and Mykland (2008). According to Laurent (2018), the only difference is in the way spot volatility is estimated. The nonparametric estimator is based on a scale estimate of the standardized returns $\overline{r_{t,\iota}} = r_{t,i}/\widehat{S_{t,\iota}^{LH}}$, where $\widehat{S_{t,\iota}^{LH}} = \sqrt{\frac{1}{M-1} TV_t}$, where TV_t is the truncated power variation. If we have intraday periodicity in volatility and estimate spot volatility, we have the following:

$$\widehat{S_{t,\iota}^{LH}} = \widehat{f_{t,\iota}} \sqrt{\frac{1}{M-1} TV_t}. \tag{14}$$

Under the Brownian semimartingale with infinite activity jumps (BSMIAJ) model, the diffusion component captures the smooth variation of the price process. The jump component accounts for both rare, large discontinuities, and frequent, small jumps in prices. A BSMIAJ log-price diffusion admits the following representation:

$$dp(t) = \mu\,(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t) + h(t)dL(t),\ \ t \geq 0, \tag{15}$$

where q(t) is a counting process (possibly a Poisson process), L(t) represents either an α-stable process or a Cauchy process, as in Lee and Hannig (2010), and κ(t) and h(t) denote the jump sizes of the corresponding jump processes, respectively.

The jump component captures both finite and infinite activity price jumps. Under the BSMIAJ, Mancini

(2009) and Bollerslev and Todorov (2011) suggest using the truncated power variation TV_t to consistently estimate IV_t. TV_t is defined as follows:

$$TV_t(\Delta) \equiv \sum_{i=1}^{M}(r_{t,i})^2 1\, 1_{|r_{t,i}|\le g(\Delta)^{\tilde{\omega}}} \xrightarrow{p} \int_{t-1}^{t} \sigma^2(s)ds, \tag{16}$$

where g > 0 and $\tilde{\omega} \in (0, 1/2)$ are the thresholds for truncating the returns. TV_t eliminates the large returns and retains those that are lower than the specified thresholds. In the LH test, the typical values for g and $\tilde{\omega}$ are 3 (or 4) and 0.47, respectively. Now we can derive three robust nonparametric estimators $\widehat{S_{t,i}^{LH} f_{t,i}^{MAD}}$, $\widehat{S_{t,i}^{LH}} f_{t,i}^{\widehat{ShortH}}$, and $\widehat{S_{t,i}^{LH}} f_{t,i}^{\widehat{WSD}}$ to estimate $\widehat{\sigma_{t,i}}$ and the Lee and Hannig (LH) jump statistics as follows:

$$LH\ IDJ_{t,i}^{MAD} = \frac{|r_{t,i}|}{\widehat{S_{t,i}^{LH}} f_{t,i}^{\widehat{MAD}}} = \frac{|r_{t,i}|\sqrt{\frac{1}{M-1}\sum_{j=1}^{M} MAD_{t,i}^2}}{\widehat{S_{t,i}^{LH}} MAD_{t,i}} \tag{17}$$

$$LH\ IDJ_{t,i}^{ShotrH} = \frac{|r_{t,i}|}{\widehat{S_{t,i}^{LH}} f_{t,i}^{\widehat{ShortH}}} = \frac{|r_{t,i}|\sqrt{\frac{1}{M-1}\sum_{j=1}^{M} ShortH_{t,i}^2}}{\widehat{S_{t,i}^{LH}} ShortH_{t,i}} \tag{18}$$

$$LH\ IDJ_{t,i}^{WSD} = \frac{|r_{t,i}|}{\widehat{S_{t,i}^{LH}} f_{t,i}^{\widehat{WSD}}} = \frac{|r_{t,i}|\sqrt{\frac{1}{M-1}\sum_{j=1}^{M} WSD_{t,i}^2}}{\widehat{S_{t,i}^{LH}} WSD_{t,i}} \tag{19}$$

3.3.4. Laurent and Shi Test using the Median

However, Laurant and Shi (2020) showed that despite the fact that the linear drift process falls within the general asset price specification of Lee and Mykland (2008), the infinite sample performance of their test for additive jumps under this data-generating process is far from satisfactory.[2] Indeed, when asset prices deviate locally from the random walk, the test shows a strong size distortion and dramatic power loss.

Laurent and Shi (2020) proposed an alternative construction of a test that does not deteriorate its performance in the random walk case but significantly improves the performance. Specifically, the authors showed that their test allows the detection of jumps with or without jumps when log prices exhibit clear upward or downward trend movements, which invalidate the use of bipower variation. Therefore, we use the Laurent and Shi (LS) jump statistics as follows:

$$J_{t,i}^{LS} = \frac{|\, r_{t,i} - \hat{\mu}_{t,i}\,|}{\hat{\sigma}_{t,i}^{LS}}, \tag{20}$$

where $\hat{\mu}_{t,i}$ is an estimate of the empirical mean obtained from the same set of observations as $\hat{\sigma}_{t,i}$.

$$\hat{\sigma}_{t,i}^{LS} = \hat{s}_{t,i}^{LS} = \sqrt{\frac{1}{M-1} BV_t^{LS}}, \tag{21}$$

where BV_t^{LS} corresponds to the bipower variation computed on log-returns centered by $\hat{\mu}_{t,i}$. Laurent and Shi (2020) proposed using the median instead of the empirical mean for $\hat{\mu}_{t,i}$. Indeed, while the empirical mean has a breakdown point of 0%, the median has a breakdown point of 50%, and therefore, is robust to jumps. When this LS statistic is selected, spot volatility is multiplied by an estimate of the periodicity. Similarly, we can derive the three robust nonparametric estimators $\widehat{S_{t,i}^{LS}}\widehat{f_{t,i}^{MAD}}$, $\widehat{S_{t,i}^{LS}}\widehat{f_{t,i}^{ShortH}}$, and $\widehat{S_{t,i}^{LS}}\widehat{f_{t,i}^{WSD}}$ to estimate $\widehat{\sigma_{t,i}}$ and the Laurent and Shi (LS) jump statistics as follows:

$$LS\ IDJ_{t,i}^{MAD} = \frac{|r_{t,i} - \hat{\mu}_{t,i}|}{\widehat{S_{t,i}^{LS}}\widehat{f_{t,i}^{MAD}}} = \frac{|r_{t,i} - \hat{\mu}_{t,i}|\sqrt{\frac{1}{M-1}\sum_{j=1}^{M} MAD_{t,i}^2}}{MAD_{t,i}\sqrt{\frac{1}{M-1}BV_t^{LS}}} \tag{22}$$

$$LS\ IDJ_{t,i}^{ShotrH} = \frac{|r_{t,i} - \hat{\mu}_{t,i}|}{\widehat{S_{t,i}^{LS}}\widehat{f_{t,i}^{ShortH}}} = \frac{|r_{t,i} - \hat{\mu}_{t,i}|\sqrt{\frac{1}{M-1}\sum_{j=1}^{M} ShortH_{t,i}^2}}{ShortH_{t,i}\sqrt{\frac{1}{M-1}BV_t^{LS}}} \tag{23}$$

$$LS\ IDJ_{t,i}^{WSD} = \frac{|r_{t,i}|}{\widehat{S_{t,i}^{LS}}\widehat{f_{t,i}^{WSD}}} = \frac{|r_{t,i} - \hat{\mu}_{t,i}|\sqrt{\frac{1}{M-1}\sum_{j=1}^{M} WSD_{t,i}^2}}{WSD_{t,i}\sqrt{\frac{1}{M-1}BV_t^{LS}}} \tag{24}$$

IV. Empirical Results

4.1 Data

Our empirical analysis is based on data from Olsen and Associates in Zurich, Switzerland. The dataset consists of five-minute observations on the Korean won–U.S. dollar exchange rate from June 1, 2010, to April 30, 2021. All volatility measures are based on the five-minute returns as the first difference of the logarithm of the Korean won–U.S. dollar exchange rate, which results in a total of M($= \frac{1}{\Delta}$) = 288 high-frequency return observations per day, that is, $\Delta = \frac{1}{288}$. This five-minute interval is short enough for the underlying realized volatility measures to work well as well as long enough for market micro frictions to not overwhelm the process.

Next, weekend holidays and several fixed holidays and July 4th, are removed. Moreover, the moving holidays of Good Friday, Easter Monday, Memorial Day, Labor Day, and Thanksgiving are also eliminated, as well as the days immediately after, in order to remove the holiday effects, following Andersen et al. (2001). Finally, we have a total of 3,304 days. The corresponding daily returns of Korean won–U.S. dollar for these days can be represented as $r_{t+1} \equiv r_{t+1,1} \equiv r_{t+\Delta,\Delta} + r_{t+2\Delta,\Delta} + \cdots + r_{t+1,\Delta}$, $t = 1, 2, \cdots, 3{,}304$. Therefore, we have 951,552 (= 3,304 × 288) sample observations.

4.2 Basic Statistics and Density Function Distribution

Table 1 shows the basic statistics of the realized returns (RR), realized variances (RV), realized bipower variation (BV), and realized jumps by BV (RJ_BV) for the Korean won–U.S. dollar exchange rate using realized bipower variation (BV) at α = 0.999.

Table 1. Basic Statistics

Variable	min	mean	max	std.dev
RR	0.029893	-0.00010017	0.037691	0.0053144
RV	7.0392e-11	0.00012082	0.0063475	0.00021004
BV(0.999)	0	0.0001054	0.0037839	0.00017039
RJ_BV(0.999)	0	1.5419e-05	0.0025637	6.246e-05

“Min” represents minimum values, “mean” represents the average, “max” represents maximum values, and “std.dev” represents the standard deviation. The number in parentheses denote the significance levels.

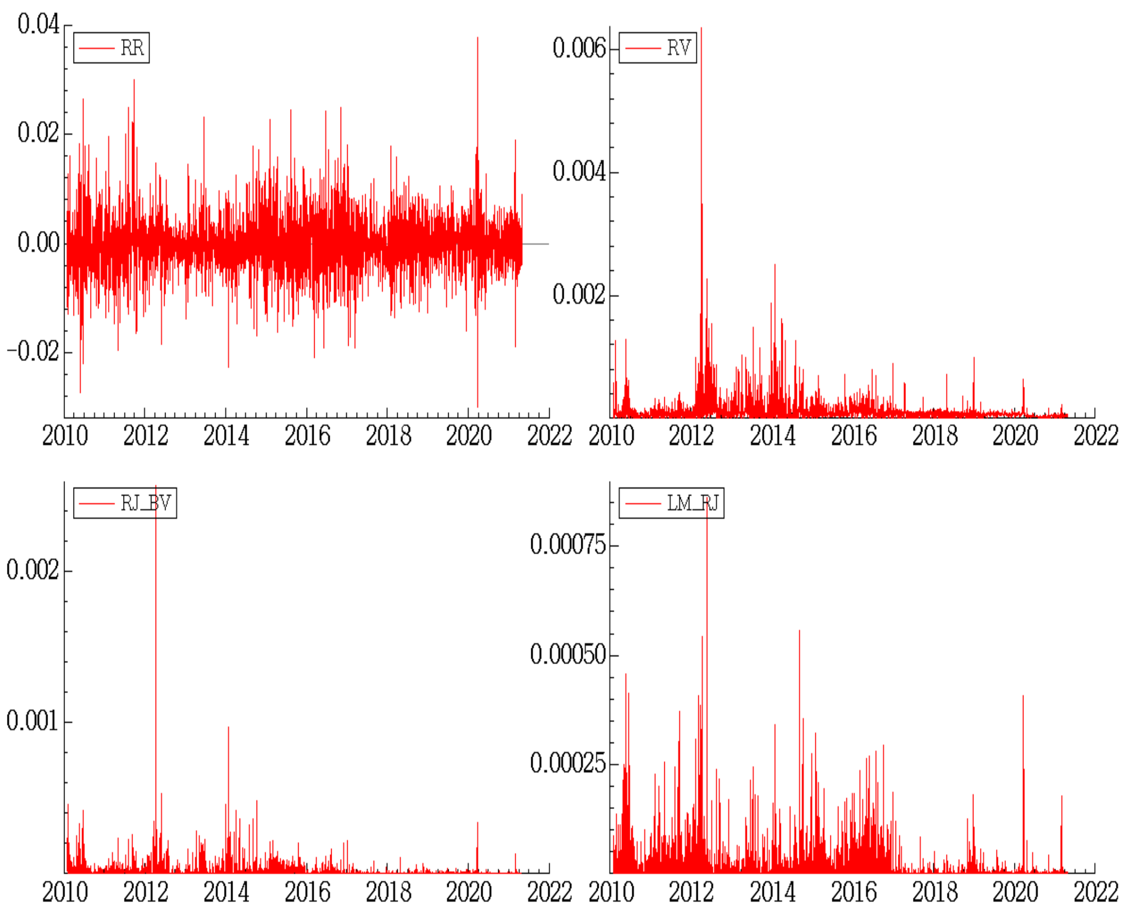
4.3. Volatility and Jump Distribution

Figure 1 presents the graphs of the volatility of RR (top left panel), RV (top right), RJ_BV (bottom left), and the L-M realized jumps (LM_RJ) (bottom right) by outlying weighted quarticity using frequency periodicity at a 0.01% significance level. Figure 1 also shows the stylized volatility clustering effects and dynamic dependence of the returns of the Korean won–U.S. dollar in this period.

Figure 1 presents the graphs of the volatility of RR, RV, the realized jumps by BV (RJ_B BV), and the realized jumps (LM_RJ) by outlying weighted quarticity using frequency periodicity at a 0.01% significant level.In the top left panel, the RR of the Korean won–U.S. dollar exchange rate appeared volatile around zero, and some large jumps occurred during 2010–2011 and 2012, amid the Euro crisis, 2015–2016, and 2020. The RV (top right panel in Figure 1) of the Korean won–U.S. dollar exchange rate also shows extraordinary volatility in the second half of 2012 and 2014. Hence, RJ_BV (bottom left panel in Figure 1) occurred in the second half of 2012 and 2014. Extremely large jumps occurred particularly during 2012. However, LM_RJ (bottom right panel) using frequency periodicity occurred more frequently in the second half of 2010, 2012, 2015, 2017, and 2020.

While the size of RJ_BV for the Korean won–U.S. dollar became very smooth after the second half of 2014, LM_RJ using frequency periodicity occurred frequently even in and after 2014, in particular, in 2016 and 2020. Therefore, the volatility of the Korean won–U.S. dollar exchange rate returns should include not only continuous volatility but also a significant discrete jump volatility process, as stated by Andersen, Bollerslev, and Diebold (2007).

Figure 1. Volatility and Jump Distribution



4.4. Intraday LM jump Test using the Local Robust Variance with Periodicity Filters

This subsection examines the intraday jump probability instead of the daily jump probability, both with and without periodicity filter cases. We use intraday observations ($n = MT = 951{,}552$).

To obtain the intraday LM jump statistics, we use the 951,552 intraday observations of Korean won–U.S.

dollar exchange rates from June 1, 2010, to April 30, 2021. Table 6 shows the results when we use the local robust variance (Average BV) with the jump statistics at the α = 0.990, α = 0.995, and α = 0.999 critical levels. Table 2 reports the intraday jump detection probability with returns with the no periodicity window and filters using intraday observations for the intraday jump statistics.

First, at the critical level α = 0.999 (Φ_α = 6.20451), 3,048 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,254, or 37.95%, days out of a total of 3,304 days.

In the case of jump detection probability with filtered returns with MAD periodicity, 2,792 jumps were detected. At least one significant jump occurred on 1,091, or 33.029%, days out of 3,304 days. In the case of ShortH, 2,372 jumps were detected. At least one significant jump occurred on 867, or 26.24%, days. In the case of WSD, 2,288 jumps were detected. At least one significant jump occurred on 882, or 26.69%, days.

Table 2. Intraday LM Jump Probability using Local Robust Variance: Intraday Observations

Jump and Critical Value	**No Filter**	**Intraday LM Jump with Periodicity Filters**		
a (α = 0.999)	$LM\ IDJ_{t,i}$	$LM\ IDJ_{t,i}^{MAD}$	$LM\ IDJ_{t,i}^{ShortH}$	$LM\ IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	6.20451	6.20451	6.20451	6.20451
Number of detected jumps	3048	2792	2372	2288
probability of jumps	0.00320319	0.00293415	0.00249277	0.00240449
Number of periods (typically days) with at least one significant jump	1254	1091	867	882
Proportion of periods with at least one significant jump	0.37954	0.330206	0.262409	0.266949
Expected number of spurious detected jumps (under H0=no jumps)	0.001	0.001	0.001	0.001
b (α = 0.995)	$LM\ IDJ_{t,i}$	$LM\ IDJ_{t,i}^{MAD}$	$LM\ IDJ_{t,i}^{ShortH}$	$LM\ IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	5.8974	5.8974	5.8974	5.8974
detected number of jumps	3563	3331	2821	2716
Proportion of detected jumps	0.00374441	0.0035006	0.00296463	0.00285428
Number of periods (typically days) with at least one significant jump	1382	1229	980	989
Proportion of periods with at least one significant jump	0.418281	0.371973	0.296610	0.299334
Expected number of spurious detected jumps (under H0=no jumps)	0.005	0.005	0.005	0.005
c (α = 0.990)	$LM\ IDJ_{t,i}$	$LM\ IDJ_{t,i}^{MAD}$	$LM\ IDJ_{t,i}^{ShortH}$	$LM\ IDJ_{t,i}^{WSD}$

Critical value, i.e. G(Beta)*Sn+Cn	4.5113	4.5113	4.5113	4.5113
Number of detected jumps	3830	3572	3031	2930
Proportion of detected jumps	0.004025	0.00375387	0.00318532	0.00307918
Number of periods (typically days) with at least one significant jump	1453	1294	1032	1033
Proportion of periods with at least one significant jump	0.43977	0.391646	0.312349	0.312651
Expected number of spurious detected jumps (under H0=no jumps)	0.01	0.01	0.01	0.01

Second, at α = 0.995 (Φ_α = 4.51132), 3,563 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,382, or 41.82%, days out of 3,304 days. In the case of jump detection probability with filtered returns with MAD, 3,331 jumps were detected. At least one significant jump occurred on 1,229, or 37.20%, days out of 3,304 days. In the case of ShortH, 784 jumps were detected. At least one significant jump occurred on 980, or 29.66%, days. In the case of WSD, at least one significant jump occurred on 989, or 29.93%, days.

Third, at α = 0.990 (Φ_α = 4.5113), 3,830 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,453, or 43.98%, days. In the case of the jump detection probability with filtered returns with MAD, 3,572 jumps were detected. At least one significant jump occurred on 1,294, or 39.16%, days out of 3,304 days. With ShortH, 3,031 jumps were detected. At least one significant jump occurred on 1,032, or 31.23%, days. In the case of WSD, 2,930 jumps were detected. At least one significant jump occurred on 1,033, or 31.27%, days.

Therefore, we can infer that if periodicity filters are not considered, the intraday jump detection probability is significantly higher than when periodicity filters such as MAD, ShortH, and WSD are used.

4.5. Intraday Lee-Hannig Jump Test using Local Robust Variance

To obtain the intraday LH jump statistics, we use the 951,552 intraday observations of Korean won–U.S. dollar exchange rates from June 1, 2010, to April 30, 2021. Table 3 presents the jump statistics at the α = 0.990, α = 0.995, and α = 0.999 critical levels. It reports the intraday jump detection probability with returns with the no periodicity window and filters using intraday observations for the intraday jump statistics.

First, at the critical level α = 0.999 (Φ_α = 6.20451), 1, 987 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,010, or 30.57%, days out of 3,304 days. In the case of jump detection probability with filtered returns with MAD periodicity, 1,750 jumps were detected. At least one significant jump occurred on 868, or 26.27%, days out of 3,304 days. In the case of ShortH, 1,489 jumps were detected. At least one significant jump occurred on 689, or 20.85%, days. In the case of WSD, 1,477 jumps were detected. At least one significant jump occurred on 702, or 21.25%, days.

Second, at α = 0.995 (Φ_α = 4.51132), 2,366 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,136, or 34.38%, days out of 3,304 days. In the case of jump detection probability with filtered returns with MAD, 2,146 jumps were detected. At least one significant jump occurred on 1,021, or 30.9%, days out of 3,304 days. In the case of ShortH, 1,806 jumps were detected. At least one significant jump occurred on 797, or 24.12%, days. In the case of WSD, 1,760 jumps were detected, and at least one significant jump occurred on 798, or 24.15%.

Table 3. Intraday LH Jump Probability using Local Robust Variance: Intraday Observations

Jump and Critical Value	**No Filter**	**Intraday LH Jump with Periodicity Filters**		
a (α = 0.999)	$LH\,IDJ_{t,i}$	$LH\,IDJ_{t,i}^{MAD}$	$LH\,IDJ_{t,i}^{ShortH}$	$LH\,IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	6.20451	6.20451	6.20451	6.20451
Number of detected jumps	1987	1750	1489	1477
Proportion of detected jumps	0.00208817	0.0018391	0.00156481	0.0015522
Number of periods (typically days) with at least one significant jump	1010	868	689	702
Proportion of periods with at least one significant jump	0.30569	0.262712	0.208535	0.21247
Expected number of spurious detected jumps (under H0=no jumps)	0.001	0.001	0.001	0.001
b (α = 0.995)	$LH\,IDJ_{t,i}$	$LH\,IDJ_{t,i}^{MAD}$	$LH\,IDJ_{t,i}^{ShortH}$	$LH\,IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	4.51132	4.51132	4.51132	4.51132
detected number of jumps	2366	2146	1806	1760
Proportion of detected jumps	0.00248646	0.00225526	0.00189795	0.00184961
Number of periods (typically days) with at least one significant jump	1136	1021	797	798
Proportion of periods with at least one significant jump	0.343826	0.309019	0.241223	0.241525
Expected number of spurious detected jumps (under H0=no jumps)	0.005	0.005	0.005	0.005
c (α = 0.990)	$LH\,IDJ_{t,i}$	$LH\,IDJ_{t,i}^{MAD}$	$LH\,IDJ_{t,i}^{ShortH}$	$LH\,IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn		4.3046		
Number of detected jumps	2571	2336	1986	1915
Proportion of detected jumps	0.0027019	0.00245494	0.00208712	0.0020125

Number of periods (typically days) with at least one significant jump	1198	1083	854	855
Proportion of periods with at least one significant jump	0.362591	0.327785	0.258475	0.258777
Expected number of spurious detected jumps (under H0=no jumps)	0.01	0.01	0.01	0.01

Third, at α = 0.990 (Φ_α = 4.3046), 2,571 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,198, or 36.26%, days out of 3,304 days. In the case of jump detection probability with filtered returns with MAD, 2,336 jumps were detected. At least one significant jump occurred on 1,083, or 32.78%, days out of 3,304 days. With ShortH, 1,986 jumps were detected. At least one significant jump occurred on 854, or 25.85%, days. In the case of WSD, 1,915 jumps were detected. At least one significant jump occurred on 855, or 25.88%, days.

Therefore, we can infer that if periodicity filters with the LH test are not considered, the intraday jump detection probability is significantly lower than when periodicity filters with the LH test are considered, such as MAD, ShortH, and WSD.

The use of periodicity filters with the LH test can help obtain increasingly robust and consistent estimators of volatility jumps and jump probabilities of Korean won–U.S. dollar exchange rates; this is in contrast to Barndorff-Nielsen and Shephard (2004a, 2004b, 2005a, 2005b, 2006) and Andersen, Bollerslev, and Diebold (2007), who did not consider the periodicity window factors of volatility nor the increasingly efficient outlying weighted variances.

4.6. Intraday Laurent-Shi Jump Test using Local Robust Variance

To obtain the intraday LS jump statistics, we use the 951,552 intraday observations of Korean won–U.S. dollar exchange rates from June 1, 2010, to April 30, 2021. Table 4presents the jump statistics at the α = 0.990, α = 0.995, and α = 0.999 critical levels. It reports the intraday jump detection probability with returns with the no periodicity window and filters using intraday observations for the intraday jump statistics.

First, at the critical level α = 0.999 (Φ_α = 6.20451), 3,050 jumps were detected with no periodicity window. At least one significant jump occurred on 1,010, or 38.01%, days out of a total of 3,304 days. In the jump detection probability with filtered returns with MAD periodicity, 2,869 jumps were detected. At least one significant jump occurred on 1,116, or 33.78%, days out of 3,304 days. In the case of ShortH, 2,788 jumps were detected. At least one significant jump occurred on 1,054, or 31.90%, days. In the case of WSD, 2,362 jumps were detected. At least one significant jump occurred on 899, or 27.20%, days.

Second, at α = 0.995 (Φ_α = 4.51132), 3,563 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,382, or 41.82%, days out of 3,304 days. In the jump detection probability with filtered returns with MAD, 3,383 jumps were detected. At least one significant jump occurred on 1,248, or 37.77%, days out of 3,304 days. In the case of ShortH, 3,304 jumps were detected. At least one significant jump occurred on 1,177, or 35.62%, days. In the case of WSD, 2,792 jumps were detected, and at least one significant jump occurred on 1,013, or 30.66%, days.

Table 4. Intraday LS Jump Probability using Local Robust Variance: Intraday Observations

Jump and Critical Value	**No Filter**	**Intraday LS Jump with Periodicity Filters**		
a (α = 0.999)	$LS\ IDJ_{t,i}$	$LS\ IDJ_{t,i}^{MAD}$	$LS\ IDJ_{t,i}^{ShortH}$	$LS\ IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	6.20451	6.20451	6.20451	6.20451
Number of detected jumps	3050	2869	2788	2362
probability of jumps	0.00320529	0.00301507	0.00292995	0.00248226
Number of periods (typically days) with at least one significant jump	1256	1116	1054	899
Proportion of periods with at least one significant jump	0.380145	0.337772	0.319007	0.272094
Expected number of spurious detected jumps (under H0=no jumps)	0.001	0.001	0.001	0.001
b (α = 0.995)	$LS\ IDJ_{t,i}$	$LS\ IDJ_{t,i}^{MAD}$	$LS\ IDJ_{t,i}^{ShortH}$	$LS\ IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	4.51132	4.51132	4.51132	4.51132
detected number of jumps	3563	3383	3304	2792
Proportion of detected jumps	0.00374441	0.00355524	0.00347222	0.00293415
Number of periods (typically days) with at least one significant jump	1382	1248	1177	1013
Proportion of periods with at least one significant jump	0.418281	0.377724	0.356235	0.306598
Expected number of spurious detected jumps (under H0=no jumps)	0.005	0.005	0.005	0.005
c (α = 0.990)	$LS\ IDJ_{t,i}$	$LS\ IDJ_{t,i}^{MAD}$	$LS\ IDJ_{t,i}^{ShortH}$	$LS\ IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	4.3046	4.3046	4.3046	4.3046
Number of detected jumps	3830	3634	3552	3022
Proportion of detected jumps	0.004025	0.00381902	0.00373285	0.00317586
Number of periods (typically days) with at least one significant jump	1453	1307	1235	1064

Proportion of periods with at least one significant jump	0.43977	0.395581	0.373789	0.322034
Expected number of spurious detected jumps (under H0=no jumps)	0.01	0.01	0.01	0.01

Third, at α = 0.990 (Φ_α = 4.30461), 3,830 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,453, or 43.98%, days out of 3,304 days. With the jump detection probability with filtered returns with MAD, 3,634 jumps were detected. At least one significant jump occurred on 1,307, or 39.56%, days out of 3,304 days. In the case ShortH, 3,552 jumps were detected. At least one significant jump occurred on 1,235, or 37.38%, days. With WSD, 3,022 jumps were detected. At least one significant jump occurred on 1,064, or 32.20%, days.

Therefore, we can infer that if periodicity filters with the LS jump test are not considered, the intraday jump detection probability is significantly lower than when periodicity filters with the LS test are considered, such as MAD, ShortH, and WSD.

4.7. Combined Lee-Hannig and Laurent-Shi Intraday Jump Test

To obtain the combined intraday LH and LS jump statistics using local robust variance (average truncated power variation) with parameters g = 4 and omega = 0.47, we use the 951,552 intraday observations of Korean won–U.S. dollar exchange rates from June 1, 2010, to April 30, 2021. Table 5 presents the jump statistics at the α = 0.990, α = 0.995, and α = 0.999 critical levels. It reports the intraday jump detection probability with returns with the no periodicity window and filters using intraday observations for the intraday jump statistics.

Table 8. Intraday LH and LS Jump Probability using Local Robust Variance: Intraday Observations

Jump and Critical Value	**No Filter**	**Intraday LH and LS Jump with Periodicity Filters**		
a (α = 0.999)	$IDJ_{t,i}$	$LHLS\ IDJ_{t,i}^{MAD}$	$LHLS\ IDJ_{t,i}^{ShortH}$	$LHLS\ IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	6.20451	6.20451	6.20451	6.20451
Number of detected jumps	1986	1802	1713	1534
Proportion of detected jumps	0.00208712	0.00189375	0.00180022	0.0016121
Number of periods (typically days) with at least one significant jump	1010	890	820	723
Proportion of periods with at least one significant jump	0.30569	0.26937	0.248184	0.218826

Expected number of spurious detected jumps (under H0=no jumps)	0.001	0.001	0.001	0.001
b (α = 0.995)	$IDJ_{t,i}$	$LHLS\ IDJ_{t,i}^{MAD}$	$LHLS\ IDJ_{t,i}^{ShortH}$	$IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	4.51132	4.51132	4.51132	4.51132
detected number of jumps	2367	2201	2097	1833
Proportion of detected jumps	0.00248752	0.00231306	0.00220377	0.00192633
Number of periods (typically days) with at least one significant jump	1137	1043	961	823
Proportion of periods with at least one significant jump	0.344128	0.315678	0.29086	0.249092
Expected number of spurious detected jumps (under H0=no jumps)	0.005	0.005	0.005	0.005
c (α = 0.990)	$IDJ_{t,i}$	$LHLS\ IDJ_{t,i}^{MAD}$	$LHLS\ IDJ_{t,i}^{ShortH}$	$LHLS\ IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	4.30461	4.30461	4.30461	4.30461
Number of detected jumps	2570	2391	2298	1992
Proportion of detected jumps	0.00270085	0.00251274	0.002415	0.00209342
Number of periods (typically days) with at least one significant jump	1198	1099	1026	877
Proportion of periods with at least one significant jump	0.362591	0.332627	0.310533	0.265436
Expected number of spurious detected jumps (under H0=no jumps)	0.01	0.01	0.01	0.01

First, at the critical level α = 0.999 (Φ_α = 6.20451), 1,986 jumps were detected with the no periodicity window. One significant jump occurred on 1,010, or 30.57%, days out of 3,304 days. In the jump detection probability with filtered returns with MAD periodicity, 1,802 jumps were detected. At least one significant jump occurred on 890, or 26.94%, days out of 3,304 days. In the case of ShortH, 1,713 jumps were detected. At least one significant jump occurred on 820, or 24.82%, days. In the case of WSD, 1,534 jumps were detected. At least one significant jump occurred on 723, or 21.88%, days.

Second, at α = 0.995 (Φ_α = 4.51132), 2,367 jumps were detected with the no periodicity window. One significant jump occurred on 1,137, or 34.41%, days out of 3,304 days. In the jump detection probability with filtered returns with MAD, 2,201 jumps were detected. At least one significant jump occurred on 1,043, or 31.57%, days out of 3,304 days. In the case of ShortH, 2,097 jumps were detected. At least one significant jump occurred on 961, or 29.09%, days. In the case of WSD, 1,833 jumps were detected, and at least one significant jump occurred on 823, or 24.91%, days.

Third, at α = 0.990 (Φ_α = 4.30461), 2,570 jumps were detected with the no periodicity window. At least one significant jump occurred on 1,198, or 36.26%, days out of 3,304 days. In the jump detection probability with filtered returns with MAD, 2,391 jumps were detected. At least one significant jump occurred on 1,099, or 33.27%, days out of 3,304 days. In the case of ShortH, 2,298 jumps were detected. At least one significant jump occurred on 1,026, or 31.05%, days. In the case of WSD, 1,992 jumps were detected. At least one significant jump occurred on 877, 26.54%, days.

Therefore, we can infer that if periodicity filters such as MAD, ShortH, and WSD are used, the intraday jump detection probability is significantly lower than when periodicity filters with the combined intraday LH and LS test are considered.

V. Conclusion

This study analyzes the realized volatility and discrete jump volatility of Korean won–U.S. dollar exchange rate returns using high-frequency five-minute returns from June 1, 2010, to April 30, 2021. We consider several periodicity filters, such as MAD, ShortH, and WSD, to obtain increasingly efficient and robust jump estimators. We estimate the volatility and jumps using the maximum outlyingness and local robust variance with truncated power variation. Although most studies adopted the standard normal Z-type jump statistics of the standard normal distribution, we adopt the Gumbel distribution to determine whether a significant daily and intraday jump occurs.

Therefore, overall, when we utilize MAD, ShortH, and WSD, the five-minute returns of Korean won–U.S. dollar exchange rates have considerably lower daily and intraday jump probabilities. If the periodicity filters of volatility are not considered, the jump probabilities may be overestimated.

For a more robust analysis, while expensive, future studies can consider using longer periods and exchange rate data from several countries. Furthermore, considering the influencing factors such as economic events and psychological traits can yield noteworthy results for volatility and jumps in Korean won–U.S. dollar exchange rates during the 2010s.

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