International Reserve Accumulation: Balancing Private Inflows with Public Outflows[*]

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preliminary and incomplete

Abstract

International reserve accumulation is a favored policy tool against excessive capital inflows in many Emerging Market Economies (EMEs). Since it is always unclear whether the country is leaning against the wind or manipulating the exchange rate, serious debates center around reserve accumulation. This paper provides a novel theory of reserve accumulation that suggests previously unformulated motivation. The model views reserve accumulation as public capital outflows that supplement less-than-optimal capital outflows by the private sector. When an EME receives large capital inflows, it is optimal to invest abroad to maintain macroeconomic balance and prepare for a sudden reversal in the capital inflows. Due to financial frictions, however, the private sector does not invest abroad sufficiently. Then, the public sector may come in to increase gross capital outflows in the form of reserves and achieve a constrained efficient equilibrium. We analyze 33 EMEs from 1999 to 2008 and provide evidence supporting our theory. We find that countries with less-open financial markets accumulate larger reserves in response to exogenous gross capital inflows.

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1 Introduction

The massive international reserve[1] accumulation of Emerging Market Economies (EMEs) from the late 1990s caused active discussion on the motivation. One widely accepted explanation is that EMEs have accumulated reserves as a buffer against sudden reversal of capital flows. This is the "Precautionary view." Among many emerging economies, reserve accumulation has been a favored policy response for capital inflows, and the proceeds have been used as a buffer stock when sudden stops come. Theoretically, however, the exact working of the reserve accumulation policy is yet to be fully clarified. Absent a comprehensive theory of reserves, important questions remain unanswered: When and how should a country accumulate reserves? What is the optimal reserve accumulation for EMEs? Is the role of reserves substitutable with capital controls?

In this paper, we suggest a novel theory of reserve accumulation that corroborates the precautionary view. Unlike other existing theories, our model is more focused on the process of accumulation. Specifically, we view reserve accumulation as an optimal response to excessive capital inflows. A sudden surge of gross capital inflows causes imbalances in the current account. If there is no market friction, there should come corresponding gross capital outflows by the private sector. However, the private sector may not be able to invest abroad sufficiently due to the underdevelopment of financial markets, lack of knowledge and skills for foreign investment, and other legal restrictions. Since the country needs to make capital outflows to maintain macroeconomic balance, the overseas investments need to be done by the public sector in such a case. Reserve accumulation makes the job done.

We provide a theoretical model of reserve accumulation in this regard. It is a three-period model of the Fisherian deflation in capital control literature (e.g. Bianchi, 2011, Korinek, 2018, Jeanne and Korinek, 2019). We have two additional ingredients: 1) imperfect capital mobility for debt inflows and outflows, and 2) gross capital flows in direct investments. First, we model imperfect mobility by adopting the idea of limited participation (Fanelli and Straub, 2020) for debt inflows. Then, similarly, for debt outflows, we assume that a country should rely on international financial intermediaries (IFIs) with increasing intermediation costs depending on the level of overseas investments.[2] Due to this cost, larger investments by private sectors lead to lower returns, while the private agents do not take account of it. Second, we model direct investments as gross flows by assuming that a direct investor purchases a claim on a country's capital. This modeling helps us to capture large inflows mainly driven by global push factors.

[1] Throughout this paper, we use the term reserve, international reserve, and foreign reserve interchangeably.

[2] There are different sources of inefficiencies in private sector outflows. We suggest different micro-foundations for it in Appendix F and G.

Our parsimonious model[3] explains why EMEs facing large capital inflows have incentives to accumulate reserves. In the model, receiving direct investments rather than debt inflows makes EMEs less prone to sudden stops. However, when the direct investment inflows are beyond a certain level, EMEs need to save abroad because larger investments lead to higher capital returns to the foreign investors; direct investment is better than external debt, but it is still a different form of external liability. The problem occurs when the private sector is not able to make sufficient outflows (overseas investments). In the absence of reserve accumulation, the large capital inflows cause domestic currency appreciation and consumption booms. Therefore, the social planner is incentivized to accumulate reserves to generate capital outflows as a reaction to the massive capital inflows. In the model, reserve accumulation supplements insufficient capital outflows by the private sector. This specific role of reserves is unique among existing literature because capital controls cannot perfectly replace the reserve management policy.

We provide empirical evidence in support of the proposed mechanism. We analyze quarterly capital flows of 33 EMEs over the massive reserve accumulation period: from 1999 to 2008. Using the data, we investigate reserve accumulation response to gross capital inflows. Our identification strategy is twofold. First, we compare countries with varying degrees of financial market openness. In our theory, the public sector accumulates reserves to supplement less-than-optimal private capital outflows. In countries with less open capital markets, the need for reserve accumulation would be greater than in countries with more open capital markets. We examine this prediction with a difference-in-difference research design. Second, we use two-stage regression. We first estimate exogenous gross capital inflows by projecting each country's capital inflows on global push factors of capital. In the second stage, we use the estimated "exogenous factors-explained" gross capital inflows to examine reserve accumulation responses. The results are in support of our theory. We find that the countries with less-open capital markets accumulate more reserves when they receive exogenous gross capital inflows.

Our theory, combined with the empirical finding, can help us understand the fast increases in reserve holdings of EMEs beginning from the late 1990s. After opening up the financial account in the 1990s, many EMEs had to cope with surges in gross capital inflows, particularly in the form of direct investment. We can also explain why some countries hoard more reserves than others. In countries with less developed financial markets, the official sector has to complement capital outflows to keep the economy in balance. In addition, our model provides important implications for the debate on currency manipulation: Do EMEs depreciate their currencies to boost their exports? We argue that the amount of reserves is not a good litmus for

[3] In our model, there is no complicated structure except for the two new ingredients. Unlike other existing research, we do not need any particular structures in the model, such as longer maturity in the external debt than reserves, the existence of a long-term project, or constantly binding credit constraints.

the test of currency manipulation. As discussed, reserve accumulation may be a passive reaction to exogenous capital inflows. Accordingly, EMEs accumulate reserves because they should cope with large inflows, not because they want to boost their exports.

Related Literature Our paper is related to several strands of literature in international macroeconomics. First and foremost, our paper relates to the literature that studies the reserve accumulation of EMEs. The main objective of the literature is to find why EMEs hold large amounts of costly reserves. While there are a few different approaches, the literature broadly falls into two different views: The mercantilist view and the precautionary view. The mercantilist view argues that reserve accumulation is a byproduct of exchange rate policies to boost exports by depreciating local currencies. Early works in the literature include Dooley et al. (2004), and there have been a few notable recent papers of similar ideas (e.g. Korinek and Serven, 2016, Choi and Taylor, 2017). On the other hand, the precautionary view pays attention to the historical fact that most EMEs began building their stocks of reserves after experiencing financial crises, in particular after the East Asian crisis in 1997. The papers in this view argue that EMEs have accumulated reserves believing that the reserves will protect them against sudden stops. Earlier works on this explanation include Aizenman and Lee (2007), which analyzes the macroprudential role of reserves in the framework of the Diamond-Dybvig model, and Jeanne and Ranciere (2011), which quantify the optimal reserve holding by assuming reserve is a sort of Arrow-Debreu security. More recently, Bianchi et al. (2018) show how reserves can help EMEs with reducing roll-over risks of external sovereign debts[4]. A recent work that shares similar insights with ours is Jeanne and Sandri (2020). The paper introduces a model where EME policy authorities accumulate reserves to complement insufficient capital outflows by private sectors and to generate enough capital outflows (having more liquid foreign assets) that stabilize domestic debt prices from volatile capital flows. We contribute to this literature by proposing a novel theory of reserve accumulation from the precautionary view[5]. We show that in environments where there are frictions on capital outflows, EME policy authorities facing large capital inflows are incentivized to accumulate reserves, and it is not based on any mechanism in preceding papers. Although we share some insights with Jeanne and Sandri (2020), we construct our model using different micro-foundation of frictions on capital outflows, and we link reserve accumulation to the types of direct investment or equity portfolio inflows, which we will discuss more below.

[4]Other papers that studied reserve accumulation in the precautionary view are Durdu et al. (2009), Obstfeld et al. (2010), Aizenman (2011), Hur and Kondo (2016), Shousha (2017), and Bocola and Lorenzoni (2020).

[5]We do not include the mercantilist view-related ingredients in our model. In the formalization of the mercantilism view, it is required to model externalities in export sectors, which cannot be verified easily. It is often documented that the correlation between reserve accumulation and export growth is low for many EMEs, which contradicts the prediction from the mercantilism view (BIS, 2019).

Our work is also related to the papers investigating the positive correlations between FDI external liabilities and official reserve assets. The positive correlation between direct investment inflows and reserve accumulation is documented in Dooley et al. (2004). The paper argues that EMEs depreciate their currencies to attract direct investments so as to utilize otherwise wasteful resources in the economy, such as labor. Matsumoto (2022) and Wang (2019) share similar ideas with Dooley et al. (2004). They introduce a small open economy model where EMEs accumulate reserves to have more FDI to the economy. While these papers interpret the observed correlations with the mercantilist view, we provide another way of looking at the same fact. It is from the point of the precautionary view[6]. Furthermore, our model can explain empirical regularities, which papers listed above might have difficulties explaining: positive correlations between FDI inflows and reserve outflows in a short run[7], and positive correlations between equity portfolio investments inflows and reserve accumulation, which resembles the correlations between direct investments and reserve accumulation. In our model, the reserve accumulation is an almost linear function of capital inflows in the form of direct investment or equity portfolio investment[8], and thus those empirical facts are explained by the model.

This paper also adds to the nascent literature that studies the effectiveness of foreign exchange market interventions under imperfect capital mobility. Gabaix and Maggiori (2015) show how limits to the arbitrage in global asset markets - UIP violation - can explain important puzzles in the exchange rate literature. They also show interventions in the foreign exchange market should be effective and can be welfare-improving. Cavallino (2019) build a continuous-time New Keynesian general equilibrium model that analyzes foreign exchange market intervention, using almost the same specification as Gabaix and Maggiori (2015). From a slightly different micro-foundation, Fanelli and Straub (2020) derives general principles of foreign exchange market interventions.[9] To model limited arbitrage in a foreign exchange market, we mostly follow Fanelli and Straub (2020) but extend their modeling technique to private capital outflows from EMEs. Our new insights are that EME policy authorities might intervene not just

[6] However, we also showed that reserve accumulation attracts more direct investments in an extension of the baseline model.

[7] In Appendix, we show the positive correlation between FDI and international reserve is also clear in the flow data in the frequency in quarter or year. This is problematic in the explanation of the mercantilism view since the policymakers do not necessarily react to capital flows in such short runs; it usually takes several years for direct investors to decide which country to invest.

[8] In addition, the approaches in Matsumoto (2022) and Wang (2019) cannot clearly explain why EMEs rely on reserve accumulation to attract more foreign direct investments rather than use other seemingly more efficient tools. For instance, simple tax cuts on the profits of foreign direct investors probably are more efficient than reserve accumulation. Another difficulty in the approaches of those papers is that some exogenous factors such as geographic location might be dominant in determining FDI. Therefore, with this view alone, we will not see such clear positive correlations between FDI liabilities and reserve assets.

[9] For other papers that analyzed foreign exchange market intervention, see Basu et al. (2016), Chang (2018) and Amador et al. (2020).

to manage spreads on the borrowing rates but also to supplement insufficient overseas investments by private sectors in the EME. We also incorporate the idea of imperfect capital mobility into a sudden stop model and show how policy authorities can use foreign exchange market intervention to prepare for sudden stops in the future.

Layout The rest of this paper is organized as follows. In Section 2, we introduce our baseline model that provides new insights on reserve accumulation. In Section 3, we test the model implication with cross-country panel data. Section 4 concludes and discusses implications.

2 Baseline Model

In this section, we present our model to explain why EMEs accumulate reserves. Since the mechanism in the model is novel, we adopt a "minimum ingredients" strategy so that we can derive a few "pen and paper" analytical results. The minimum ingredients are 1) "exogenous" capital inflows in the form of direct investment, and 2) imperfect capital mobility for both capital inflows and outflows. We plug in these two into the framework of the small economy version of Fisherian deflation model, which was developed to model currency crises in the context of pecuniary externality[10].

2.1 Model Setup

We consider a small open economy that lasts three periods, t = 0, 1, 2. There are two domestic agents in our model: households and the social planner. On the other side, there are two international investors: international financial intermediary who intermediates capital inflows (outflows) to (from) the small open economy and direct investors who purchase the domestic capitals in the economy in period 0. The small open economy faces a credit constraint only in period 1,[11] which may or may not bind depending on the states. Thus, for precautionary purposes, the social planner accumulates reserves in period 0 when there is no concern over the binding credit constraint.[12]

[10] For more details of Fisherian deflation model, see Mendoza (2010) and Korinek and Mendoza (2014).

[11] This is same as Korinek and Sandri (2016). We can put the credit constraint in period 0. But, it makes it harder to solve the model, without providing any extra insight.

[12] Remember the level of reserves of EMEs soared from the late 1990s to the mid 2000s. Considering many EMEs experienced or witnessed the currency crises around the mid 1990s, it is reasonable to think that policy authorities in EMEs during the time had a fear of currency crisis in the future, which is captured by the occasionally binding credit constraint in our model.

Production To model direct investments, while keeping the simplicity, we assume that both tradable and nontradable goods are produced by AK technology. That is, there are two different capitals, namely K^T and K^N. They are capitals to produce tradable goods and nontradable goods, respectively. Two different kinds of capital cannot be converted from one to another and, more importantly, there is no capital accumulation or depreciation. Such a "semi-production" economy is a useful modeling technique to allow direct investments in the model while keeping only the necessary ingredients. We denote output of sector j at time t by y_t^j; therefore $y_t^j = A_t^j K^j$ where A_t^j is the TFP. Throughout this section, we do not allow direct investors to purchase the nontradable goods sector capitals. Then, it is convenient to drop the upperscript j and let $y_t^T = A_t K$[13].

We set $y_0^N = y_1^N = y_2^N$, but $y_0^T < y_1^T < y_2^T$ (hence $A_0 < A_1 < A_2$) so that households need to borrow against higher outputs in the future. It is to investigate how reserve accumulation in our model is linked to the precautionary motivation[14].

Households The overall utility of the representative household is given by

$$U = u(c_0^T, c_0^N) + \mathbb{E}_0\left[\beta u(c_1^T, c_1^N) + \beta^2 u(c_2^T, c_2^N)\right]$$

where the utility function is $u(c_t^T, c_t^N) = ln\left(\left(c_t^T\right)^{\alpha}\left(c_t^N\right)^{1-\alpha}\right)$. α and $1-\alpha$ are the shares of tradable goods and nontradable goods respectively, and β is the discount rate.

The households enter period 0 with some legacy debts d_0, d_1, d_2, which the households have to repay in each period respectively.[15] Given the output streams and the legacy debts, households determine their borrowing or saving of tradable goods. Since there is no saving technology domestically, they must invest abroad to save. Their saving[16] decision will be discussed in detail in the paragraphs of overseas investments. We denote borrowing (saving) of households at time t by b_{t+1}; $b_{t+1} < 0$ ($b_{t+1} > 0$) means borrowing (saving).

Direct investments Direct investment inflows are novel and one of the most important elements in our model. Since we consider direct investments from a different research ques-

[13] In fact, in terms of nontradable goods, the economy is same as an endowment economy.

[14] However, even with a decreasing output stream of $y_0^T > y_1^T > y_2^T$, the model generates positive amounts of reserve accumulation and the amounts of reserve accumulation can potentially be larger.

[15] This is not necessary for us to derive desired results. The seemingly ad-hoc assumption is to match our model to the empirical regularity: although the total outstanding external debt to GDP ratio has declined, EMEs have had substantial external debts. Such drawback arises because our model is a representative agent model. In Appendix, we will examine an extension to heterogeneous agents setup. It turns out that the heterogeneous agent extension matches the empirical findings better and our key insight survives in the new environment, though it is harder to solve.

[16] Throughout this paper, we use households' saving, lending abroad and overseas investments interchangeably.

tion from the international trade literature, we model direct investments in a different structure rather than following the traditions in trade literature. In other words, we take some aspects of foreign direct investments based on our purpose, even if the related environments look too barren or unnatural.

There is a direct investor interested in the tradable goods sector capitals. At the beginning of period 0, the direct investor determines how much capital she will purchase depending on technological features from which we abstract. Here purchasing capitals means to buy claims on the returns to the capital like equities in reality. One can imagine direct investments in our model as a merger and acquisition process.

Regarding the price of the capital, we assume it is determined through a bargaining process, again similarly as like a merger and acquisition process. We assume that direct investors value the capitals higher than domestic households, implying the equilibrium price of capital is higher than the valuation of households. Let θ be the share of capitals sold to the direct investor: thus among the total returns of $A_t K$, $\theta A_t K$ belongs to the direct investors and remaining returns for households.

Also, let us denote the price of capital by Q_0, which is determined through the bargaining process. Then, we have

$$Q_0 > \sum_{t=0}^{2} M_{0,t}^h A_t$$

where $M_{0,t}^h$ is the stochastic discount factor of households; hence, $M_{0,t}^h = \beta^t E \frac{u_t^T}{u_0^T}$ and u_t^T is the marginal utility of tradable goods consumption in period t. Notice that the equilibrium price of capital price Q_0 is higher than the valuation of households as long as all the bargaining power belongs to the direct investors. With the equilibrium price, the direct investment capital inflows in period 0 are $Q_0 \theta K$.

International financial intermediary and household overseas investment We have not explicitly solved the model yet, but can easily envision that large amounts of direct investment inflows induce households to save. Since there is no available saving technology, households must invest abroad to transfer their incomes to the future. In reality, there should be different sources of frictions on overseas investments, which we cannot include all in our model. We mainly focus on frictions in intermediation of global investment banks. Later we will separately discuss how we can derive similar results with other specification and assumptions.

Reflecting on the fact that much of overseas investments are intermediated by international financial intermediaries (IFIs), we assume that any overseas investment by households must be intermediated by IFIs. There is a continuum of IFIs and they have a heterogeneous operation cost to intermediate.[17] Following Fanelli and Straub (2020), let the fixed cost be uniformly distributed on IFIs. That is, if there exists a continuum of IFIs, labeled by $j \in [0,\infty)$, then IFI j pays a cost of j. Further, we assume that IFIs of measure χ can manage the assets by the amount of $\gamma\chi$. If the equilibrium intermediation fee is determined by the marginal cost of the intermediation, then it implies

$$\gamma\chi = b_1 \tag{1}$$

And the return facing households, say r_{t+1}, is

$$r_{t+1} = r^* - \Gamma_s b_{t+1} \tag{2}$$

where $\Gamma_s = \gamma^{-1}$ and r^* is direct return to the overseas investment before paying the intermediation fee[18]. As a result, $r_{t+1} < r^*$ since $b_{t+1} > 0$.

Furthermore, the gross return to the overseas investment b_{t+1} will be decomposed as follows.

$$b_{t+1}\left(1+r^*\right) = \underbrace{b_{t+1}\left(1+r_{t+1}\right)}_{returns\ to\ households} \quad \underbrace{+\frac{1}{2}\Gamma_s b_{t+1}^2}_{returns\ to\ IFIs} \quad \underbrace{+\frac{1}{2}\Gamma_s b_{t+1}^2}_{total\ fixed\ costs} \tag{3}$$

The return to financial experts $\frac{1}{2}\Gamma_s b_{t+1}^2$ is a sort of rent to IFIs and it represents a cost of overseas investment, which is not taken account by households.

While the reliance on foreign financial intermediary due to the lack of expertise of domestic financial institution is a source of undesirable negative externality for many EMEs and certain types of capital outflow, such as portfolio outflows, tax evasion using overseas investments is another important source of negative externality related to capital outflows by residents in EMEs. In the paragraph we discuss potential externality from overseas investment, we illustrate another model setup, in which tax evasion motivation gives similar results with overseas investments intermediation by IFIs.

[17]Global banks in this context indicate that international investment banks operating in multiple countries. In fact, almost all renowned global banks such as JP Morgan, HSBC, and etc. have branches in major cities in most EMEs and do important roles in both capital inflows and outflows. In this context, the cost can be understood to run these branches or offices in EMEs; e.g., costs to hire and train new people in the EMEs. Also, the participation of a new IFI can be both entrance of new IFI (extensive margin) or more operation of an incumbent IFI (intensive margin).

[18]We implicitly make two assumptions. First, the heterogeneous fixed cost is the cost in terms of tradable goods; each expert must pay tradable goods to participate. This is for simplicity and tractability. However, much of such cost following overseas investment in reality is the costs denominated by foreign currencies; for example, costs to manage branches in foreign countries. Second, the only available option to invest abroad is a fixed income security that pays a net return at the rate of r^*. This is surely counterfactual, but incorporating portfolio decision problem into our model will be overly complicated.

International financial intermediation and household borrowing The friction on capital outflows described above is a version of imperfect capital mobility, which has been popularized since the influential work by Gabaix and Maggiori (2015)[19]. The feature in the paper is the break-down of interest parities so that exchange rates are determined by capital flows and accordingly foreign exchange market intervention becomes effective by changing the spreads. For the borrowing of households and the borrowing rates determination, we adopt results in Fanelli and Straub (2020). The key idea is that limited participation arises due to participation costs. If an EME borrows more, the EME attracts more IFIs into the bond market by paying higher rates; IFIs with higher participation costs need to join the market. That is,

$$-b_{t+1} = \frac{1}{\Gamma_b}\left(r_{t+1} - r^*\right) \tag{4}$$

Hence, the amount of capital inflow in the debt form linearly increases in the spread $r_{t+1} - r^*$. Rearranging the equation (4) yields a similar form with (2)

$$r_{t+1} = r^* - \Gamma_b b_{t+1} \tag{5}$$

Credit constraint Households face a credit constraint in period 1. That is,

$$-b_2 \leq \phi\left(y_1^T(1-\theta) + p_1 y_1^N\right) \tag{6}$$

The credit constraint is just the same as the collateral constraint in the recent capital control literature (e.g. Bianchi, 2011, Korinek, 2018) where the collateral is GDP[20]. The idea of the standard specification of credit constraint in EMEs is that the borrowers in an EME may default on their external borrowing, but in such cases, international investors can take some properties in the country, which prevents a default of the EME. Since the international investors cannot fully utilize the properties in the EME, the international investors should discount the values of the collateral.

We add one property to this standard form. In our model, ϕ is stochastic[21]. More formally,

[19]Other papers sharing a similar framework are Basu et al. (2016) and Cavallino (2019)

[20]Some readers might wonder how the credit constraint can coexist with the limited participation. To handle the issue, we can conceptually decompose the borrowing process. In the first step, whether the credit constraint binds or not is determined by the realized states. If it turns out that the constraint binds, the amount of borrowing and the borrowing rates by the equation (5) and the credit constraint in (8).

[21]The exogenous change of the ϕ mostly reflects "global financial shocks - changes in global financial market conditions. Similar to Shin (2012), Bruno and Shin (2015), and Miranda-Agrippino and Rey (2020), the change in global financial market conditions, which may stem from center economics, would cause changes in risk appetites of international investors. For example, when the conditions in global financial market become worsen, the risk appetite of the investors will be lower (risk-off), and therefore the investors will ask EMEs to provide more collaterals.

$\phi(\omega)$ depends on the realized state ω. It is frequently argued that an important driver of sudden stops is a change in the amount of funds that international investors are willing to provide for a given amount of collateral, i.e. changes in the leverage parameter ϕ. A few theoretical works in macro-finance, such as Geanakoplos (2010) document such pro-cyclical leverage ratios as a general feature of financial markets, and more recently Arce et al. (2019) have a similar feature with ours. It isn't necessary, but for convenience and tractability, we assume ϕ has a support of an interval, and further its CDF and PDF are both continuous.

An additional note about the credit constraint in (8) is that it captures the infimum of the cost of sudden stop. Notice that disruption of consumption smoothing by drops of the consumption is the only cost of sudden stop. This is counter-factual; many of sudden stops accompany significant falls in output.[22] Furthermore, Nakamura et al. (2013) document that the negative impacts of a financial crisis on output may last much longer than expected in a standard model. In this regard, the conditions under which the social planner in our model is incentivized to accumulate reserves can be interpreted as "sufficient" conditions in the sense that it is socially desirable to accumulate reserves with the infimum cost of sudden stop.

Social planner The social planner accumulates international reserves in period 0 when there is no concern over the binding credit constraint. To finance the accumulation of reserves, the planner imposes lump-sum taxes by the amount of T units of tradable goods. With the revenue from the tax, the planner purchases foreign bonds in period 0, which will earn $1+\overline{r}$ units of tradable goods in period 1 per one unit of the bond. Accordingly, the dynamics of international reserves holding is given by

$$T\left(1+\overline{r}\right)=R_1$$

2.1.1 Discussion of assumptions

Direct investments The description of FDI in our model is much different from international trade literature. This is because the features we want to look at in this paper are different from the literature. We model direct investment inflows as an almost exogenous variable, as we want to capture large capital inflows driven by "push" factors. The amounts of capital inflows do not necessarily align with the optimality conditions of the households.

Although we posit exogenous capital inflows according to our analysis purpose, we can endogenize the direct investment along with the capital price that direct investors pay. In Appendix, we introduce an extension where the amounts of direct investments and the price of the capital are endogenous. We found that the social planner will accumulate more reserves

[22]See Basu et al. (2016).

in the new environment. Intuitively, more reserve accumulation derives down the capital price through lower marginal utilities in the future.

In our model, the direct investors never undo their investment during sudden stops. This reflects the idea that the direct investments are more stable than other capital flows. However, few papers such as Ostry et al. (2011) document that certain FDI can outflow during sudden stops. Similarly, it is also argued that irreversible 'greenfield' FDI is actually small parts of the total. Considering all these points, it is more realistic to think that some FDI is partly reversible. But, throughout this paper, we maintain the common view that FDIs are more stable than others. However, letting FDI outflow during sudden stops will give us a similar results with our baseline model.

Overseas investment To the best of our knowledge, the only preceding paper in the international reserve literature that includes capital outflows of private sectors is Jeanne and Sandri (2020). In fact, almost all papers in the emerging market economy literature focus on capital inflows (or net inflows while assuming that EMEs have negative net foreign asset positions).[23] However, thanks to the few influential empirical papers such as Forbes and Warnock (2012) and Broner et al. (2013), now the importance of analysis of gross flows is widely recognized. We also believe capital outflows from EMEs are really important, but unfortunately we still have a scant theory about private capital outflows from EMEs. Since we nearly have no giant on which we can stand, we borrow some modeling tools from recent papers assuming imperfect capital mobility and describe frictions of private overseas investment in the simplest way that gives us a clean result.

To model frictions on capital outflows of private sectors, we assume that all the overseas investments are intermediated by global investment banks. Of course, it is extreme, but also reflects reality. It is true that domestic banks in EMEs often need assistance from global investment banks when they invest abroad, probably due to the lack of expertise. For instance, sovereign wealth funds or national pension funds are often advised by global investment banks for overseas investments. In Appendix, we documents that the profit growth of branches of global banks in Korea is positively correlated with the growth of overseas investment in Korea, which suggests that much of the overseas investments are directly or indirectly intermediated by the global banks.

However, we do not want to limit the frictions on capital outflows to rent extractions to foreign banks. Actually, another important cost about capital outflows might be capital "migrations" for the purpose of tax evasion or improper concealment of assets, especially for EMEs

[23] An exception is a strand of papers that interpreted the global imbalance as a result heterogeneous financial development between developed and developing countries. See Caballero et al. (2008), Mendoza et al. (2009), Maggiori (2017)

with low quality institutions. The fear of such capital migration would have been a reason behind strict capital control measures on outflows, as documented in Fernández et al. (2016). To reflect such features in EMEs, we can model social costs of overseas investment, which arise due to tax evasions using the investments. Following Aizenman and Marion (2004), we posit there is a cost of collecting tax, but we assume the cost only exists for overseas investments. Let C_t be the cost of collecting tax, Then, we assume

$$\mathrm{C}_t = C_0 + \frac{\xi}{2} b_{t+1}^2 \tag{7}$$

Hence, the tax collection cost is a quadratic function of overseas investments. There is a cost of auditing incomes and assets of entities and training officials to have them equipped with adequate expertise. This is reflected in the constant C_0. The cost is increasing in the overseas investments as it is increasingly hard to audit incomes and assets abroad. One important note regarding the cost is that decentralized agents do not take account of the costs as they take the amounts of tax as given.[24]

This approach, which is derived from a different story, gives us an almost same setup, in which we can derive similar implications about the relationship between reserve accumulation and large capital inflows. If we add some quadratic costs of overseas investments, which do not necessarily result in rents to foreign parties, then we can end up with an exactly same result with equation 3.

In Appendix, we provide a microfoundation for the cost of overseas investments due to tax evasion. We illustrate that overseas investments are used as opportunities of tax evasion and as a consequence, more overseas investments cause less tax revenue and accordingly less government expenditures. The surveillance capacity of the government decreases in overseas investments since it will be increasingly harder for the government to monitor more overseas investments. If government expenditures become short of the social optimum because of the less tax revenues, overseas investments create social costs other than direct cost of the investments. This illustration corresponds to several papers, which documented pervasive tax evasion using foreign direct investments of large companies in EMEs.[25]

As another suggestive evidence of social costs of overseas investments, we document the relative strict controls on the outflows than inflows, and illustrate the evolution of the regulations in Korea, as an example. Such strict regulations might imply that regulators in EMES were aware of different cost of the overseas investments, discussed above.[26]

[24]Of course, we implicitly assume that all the taxes are lump-sum. Higher tax rates due to more tax collection cost surely result in another distortion. That would be more realistic costs of overseas investments.

[25]See Perez et al. (2012) and Janskỳ and Palanskỳ (2019).

[26]If the controls on capital outflows strictly limit the physical amounts of capital, to reflect such features in the model, we can set an arbitrary limit on b_s or it can be understood as a case of $\Gamma_s = \infty$ when the limit on b_s is zero.

2.2 Solving the Model

Now we solve the model. First, we solve the household problem in decentralized equilibrium, and then solve for the solution of the social planner who accumulate reserves.

2.2.1 Decentralized Equilibrium

We first illustrate the household decisions. Then, we introduce our first analytical result, which proves the existence of pecuniary externality in our model and shows how the externality depends on the frictions on capital outflows. Finally, based on the comparative statics and the analytical result, we illustrate how the benefit of direct investment inflows is greatly reduced.

Utility maximization of households We derive the solution of households via backward induction. In the last period, by construction, there is no dynamic decision of households. Households consume all the available tradable and non-tradable goods after paying back all the debts and receiving remaining reserves from the planner. As we will show later, in our three period model, the social planner depletes all the reserves in period 1 regardless of the realization of ϕ, which corresponds to our intuition. However, the social planner might not deplete all the reserves once we make the model more dynamic; the number of periods is equal to or larger than four. Of course, the main conclusion and insights from the model are not affected by the reserve depletion decision. Back to the household problem, since all the reserves are depleted in period 1, the consumption of the households is as follows.

$$c_2^T = (1-\theta)\, y_2^T + b_2\,(1+r_2) - d_2,\ c_2^N = y_2^N$$

In period 1, the households take the states of the economy as given and solve the utility maximization problem. It is important to note that the states include ϕ. As we emphasized earlier, ϕ is a random variable whose value is determined at the beginning of period 1. A difficult question is "what would be a good distribution of ϕ that resembles the reality?" To materialize an idea of a disaster, it seems a positively skewed distribution would be good, i.e., its pdf has a left-leaning curve. Here, we only assume that the distribution has a support on an interval of positive real numbers. That is, the support of ϕ is $\left[\underline{\phi}, \overline{\phi}\right]$ where $\underline{\phi} > 0$ and $\bar{\phi} < \infty$. Also, we suppose it is nicely well-defined so that we don't have any trouble in using calculus. The utility maximization is formally defined below.

$$\max_{c_t^T, c_t^N, b_t} u\left(c_0^T, c_0^N\right) + \mathbb{E}\left[\beta u\left(c_1^T, c_1^N\right) + \beta^2 u\left(c_2^T, c_2^N\right)\right] \ \textit{subject to}$$

$$\begin{aligned}
c_0^T &= (1-\theta)\, y_0^T + p_0 y_0^N - p_0 c_0^N - b_1 - T + Q_0 \theta k - d_0 \\
c_1^T &= (1-\theta)\, y_1^T + p_1 y_1^N + b_1 (1 + r_1) + R - p_1 c_1^N - b_2 - d_1 \\
c_2^T &= (1-\theta)\, y_2^T + p_2 y_2^N + b_2 (1 + r_2) - p_2 c_2^N - d_2 \\
-b_2 &\le \phi(y_1^T (1-\theta) + p_1 y_1^N)
\end{aligned}$$

where $u(c_t^T, c_t^N) = ln\left(\left(c_t^T\right)^{\alpha}\left(c_t^N\right)^{1-\alpha}\right)$ and $R = T(1+\bar{r})$.

Market clearing conditions will be given by the pricing functions.

$$p_t = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{c_t^T}{c_t^N}\right) \ \text{and} \ r_t = -\Gamma_j b_t + r^* \ \text{where} \ j = b, s$$

If the credit constraint does not bind, i.e., the realized ϕ is high enough, then the household determines its consumption of tradable goods according to her Euler equation. The amount of borrowing in period 1 is determined by

$$-b_2 = \frac{-\beta (1+r_2)\left((1-\theta)\, y_1^T + b_1 (1+r_1) + R - d_1\right) + (1-\theta)\, y_2^T - d_2}{(1+r_2)\left(1+\beta\right)} \tag{8}$$

The interest rate r_2 is accordingly determined by $r_2 = -\Gamma_b b_2 + r_*$. Plugging in the pricing function into (8) yields

$$-b_2 = \frac{-\zeta_1 + \sqrt{\zeta_1^2 - 4\left(1+\beta\right)\Gamma\left(\beta\left(1+r^*\right)\left((1-\theta)\, y_1^T + b_1 (1+r_1) + R - d_1\right) - (1-\theta)\, y_2^T - d_2\right)}}{2\left(1+\beta\right)\Gamma} \tag{9}$$

where $\zeta_1 = (1+r^*)\left(1+\beta\right) + \beta\Gamma\left((1-\theta)\, y_1^T + b_1 (1+r_1) + R - d_1\right)$.

If the credit constraint binds, the consumption of tradable goods will be determined by the credit constraint. Plugging in the budget constraint into the credit constraint equation, we can derive[27][28]

$$-b_2 = \left(\frac{1}{1-\phi\left(\frac{1-\alpha}{\alpha}\right)}\right)\left(\phi\left((1-\theta)\, y_1^T + \frac{1-\alpha}{\alpha}\left((1-\theta)\, y_1^T + b_1 (1+r_1) - d_1 + R\right)\right)\right) \tag{10}$$

[27]To have a unique equilibrium, we need a condition $\phi\left(\frac{1-\alpha}{\alpha}\right) < 1$. Here, it is easily satisfied since the credit constraint only binds for low values of ϕ. The parametric restriction is imposed to make sure households cannot increase the limit in the credit constraint just by consuming more. Such parametric restrictions are usual in a small open economy model with a credit constraint including asset price (Korinek (2018)).

[28]The equation (10) implies that the borrowing rate r_2 is low during sudden stop. This is a little unsatisfactory since the spread often soars during sudden stop. This can be corrected by letting Γ_b as a function of ϕ and $\Gamma_b' < 0$ or similarly assuming r^* is a decreasing function of ϕ. However, since the counterfactual does not seriously affect our key insight and the modifications make the model complicated without providing extra insights, we keep Γ_b and r^* as constants.

Since ϕ has a support of an interval, we can derive a formula of the cut-off of ϕ, below which the credit constraint binds, given other states and the reserve. We can obtain

$$\phi^c = \frac{-b_2}{\frac{1}{\alpha}(1-\theta)\, y_1^T + \frac{1-\alpha}{\alpha}\left(b_1(1+r_1)+R-d_1-b_2\right)} \tag{11}$$

where $-b_2$ is determined by (9).

We now turn to the period 0 optimization problem. Since there is no credit constraint, trivially the solution is the Euler equation. By denoting the marginal utility of good x in period t as u_t^x ,

$$u_0^T(b_1, ;) = \beta(1+r_1)\left(\int_{\underline{\phi}}^{\phi^c} u_1^T\left(b_1, b_{2,c}, ;\right) d\mathscr{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T\left(b_1, b_{2,u}, ;\right) d\mathscr{F}_\phi\right) \tag{12}$$

where $b_{2,c}$ is determined by (10), while $b_{2,u}$ is determined by (9).

Pecuniary externalities Before we move on to the subsection of the planning problem, we introduce our first analytical result. We can easily notice the amount of borrowing and saving in the Euler equation (12) is not necessarily socially optimal. b_1 will impact the interest rates r_1, r_2, and p_1, and households do not take account of it. The externalities arise because agents do not take account of the impact of their action on the prices and such externalities are often named as pecuniary externality.[29] We here note that there are two different sources of pecuniary externalities; one through the real exchange rate in period 1 (p_1), and the other one through interest rates in period 0, 1 (r_1, r_2). Interestingly, when households borrow, both externalities work in the same way, whereas the two different externalities work in the opposite way from each other when households save. In the same way as the models in the capital control literature, households do not consider the effects of their decision on the real exchange rate in the future, which creates overborrowing (Bianchi (2011)). At the same time, more borrowing by the households also creates another source of overborrowing; more borrowing by households pushes up the interest rate on the borrowing. On the contrary, when households save more (higher $b_1 > 0$), it pushes down the yields on the saving, but it also makes the economy better prepared for a possible crisis in the next period. Consequently, higher saving by households creates two different externalities working in the opposite ways in terms of the distance between the saving by households and the socially optimal saving. Whether the saving decision by households will be "undersaving" or "oversaving" in the eye of the social planner is itself indeterministic and depends on the state of the economy and values of important parameters; the distribution of ϕ and Γ_s. We formally state these results in the following lemma. The formal proof is relegated to

[29] For a more detailed discussion of pecuniary externality, see Dávila and Korinek (2018).

the appendix A.

Lemma 1. *Assume $\phi^c > \underline{\phi}$ and $\Gamma_s, \Gamma_b > 0$. b_t^h be the solution of (12), and b_t^{sp} be the solution by a social planner.*

1) If $b_t^h < 0$, then $b_t^h < b_t^{sp}$

2) If $b_t^h > 0$, then there always exists $\gamma_0 \in (0, \infty]$ such that for $\Gamma_s \in (0, \gamma_0)$ $b_t^h < b_t^{sp}$. If $\gamma_0 \in (0, \infty)$, then there exists $\gamma_1 \in (\gamma_0, \infty)$ such that for $\Gamma_s \in (\gamma_0, \gamma_1)$ $b_t^h > b_t^{sp}$

Proof) See the Appendix A.

Lemma 1 illustrates two (or three) different externalities from capital flows and when we are more likely to have undersaving or oversaving by households. First, as long as $prob(\phi < \phi^c) > 0$[30], any borrowing by households has an externality of tightening the credit constraint in period 1. The borrowing of households will lower the real exchange rate in period 1, p_1, which is included in the credit constraint $-b_2 \leq \phi\left((1-\theta)\, y_1^T + p_1 y_1^N\right)$; obviously, more borrowing will reduce the tradable consumption in period 1, and thereby resulting in lower real exchange rates. Second, any borrowing or lending abroad results in extra costs of capital flows, of which households do not take account. Recall that $r_t = -\Gamma_j b_t + r^*$ and one unit of borrowing or lending gives $b_1(1+r_1)$ in period 1. Differentiating $b_1(1+r_1(b_1))$ with respect to b_1 gives

$$\begin{aligned}\frac{d\left(b_1\left(1+r_1\left(b_1\right)\right)\right)}{db_1} &= 1 + r^* - 2\Gamma_j b_1 \\ &= 1 + r_1(b_1) - \Gamma_j b_1\end{aligned}$$

Households are price takers and hence they only consider $1+r_1$ as a cost (return) to their borrowing (lending). Thus, any borrowing or lending by households leaves a term that is not in the calculation of the households: the second externality through borrowing rates or returns to overseas investments. Here it is important to note that the term $-\Gamma_j b_1$ is positive for $b_1 < 0$ while it is negative for $b_1 > 0$. These different signs show reasons why we always have $b_t^h < b_t^{sp}$ for $b_t^h < 0$, but $b_t^h > b_t^{sp}$ for $b_t^h > 0$ only if Γ_s is large enough. If households borrow abroad, additional borrowing raises the borrowing rate. Hence less borrowing, which leaves more resources for period 1, is desirable in terms of both the borrowing rates and the preparation for sudden stops in period 1. In contrast, once the borrowing alters to lending due to direct investment inflows, additional lending increases the marginal cost of overseas investments, equivalently lowers returns to the lending. This makes more lending by households undesirable for the social planner, while it still makes the economy better prepared for possible sudden stops in the next period. Thus the saving creates two externalities that work in the opposite direction from

[30] We assumed that the support of ϕ is $\left(0, \overline{\phi}\right)$ the condition is equivalent to $b_2 < 0$.

each other: the negative externality of lower returns, and the positive externality of less probable and less severe sudden stops. Of course, the negative externality increases in the measure of the frictions on oversea lending. Therefore the socially optimal overseas investments are lower than the investments by households for Γ_s large enough and vice versa.

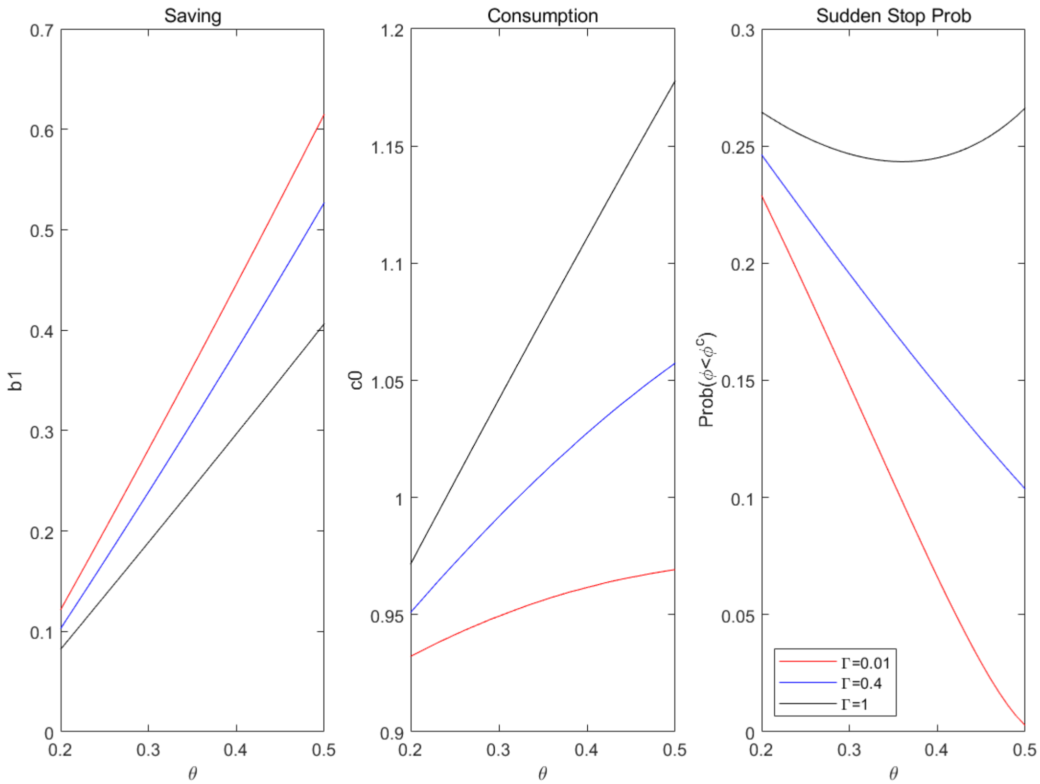
Direct investment inflows and decentralized equilibrium Since we assumed $y_2^T > y_1^T > y_0^T$, the optimum for the households is to borrow against larger outputs in the future. However, borrowing creates two externalities as discussed above. Those externalities hinder households from "transferring" resources from the future to the present.

The direct investment inflows provide a better way of external financing for the households with the problems described above. Capital inflows in the form of direct investment are free from concerns about the externalities, or at least are better in terms of the externalities as we discussed in the last section; for example, the required return is less sensitive to the amount of capital inflows than debt inflows. However, the friction imposed on capital outflows, Γ_s, gives a new challenge for the households. For direct investments inflows above a certain level, households need to save; they need to lend tradable goods abroad. Previously, they needed to bring the resources from the future to the present for the purpose of consumption smoothing, but now they need to reallocate the extra resources in the present to the future, again for the consumption smoothing. However, if Γ_s is non-negligible more saving by households leads to lower returns, which in turn lead to insufficient saving and inefficient consumption boom accordingly.

Figure 1 below shows how the decentralized equilibrium changes along with direct investment inflows, parameter θ in the model. As one can easily envision from the comparative statics, more capital inflows cause higher tradable goods consumption in period 0, so higher real exchange rates. Despite the inefficient consumption booms in period 0, the direct investment inflows make the economy more robust to sudden stop: lower probability of binding credit constraint and less tight constraint for given ϕ. Hence, as it is commonly argued, external financing in the forms of direct investments or equity portfolio investments is better in terms of macro-prudence in our model. However, the gain is strictly diminishing in the friction of capital outflows. Figure vividly indicates that magnitudes of the decline of the sudden stop probability $prob(\phi < \phi^c)$ rapidly decrease in Γ_s. For an extremely large Γ_s, it is observed that the probability of sudden stop against direct investment inflows exhibits "U-shaped" curve: For direct investment flows above a certain level, the economy becomes more vulnerable to sudden stop as more direct investment capitals inflow.[31] Such an inefficiency arises because households cannot save enough by themselves due to the friction underlying overseas investments.

[31]We can see the relationship between sudden stop and direct investment more explicitly through $-\frac{\partial b_2}{\partial \theta} |_{\phi<\phi^c}$.

Figure 1: Decentralized Equilibrium



Note: As a benchmark, the parameter values for our numerical results are as follows: lower bound of borrowing rate(r^*)=0.05, interest rate on reserves($\bar{r}$)=0.01, weight on tradables(α)=0.35, discount factor (β)= 0.94, distribution of credit coefficient=Beta(1.5,5), wedges in UIP($\Gamma_s = 0.2$, Γ_b=0.25), endowment stream(y_0^T=0.8, y_1^T=1, y_2^T=1.5, y^{NT}=1), legacy debts(d_0=0.1, d_1=0.2, d_2=0.1).

To summarize, the economy suffers from the inefficiency of overseas lending by households, which generates extra costs of the lending, $-\Gamma_j b_1$. The low returns significantly dampen the benefit of direct investment flows. It will be introduced and analyzed in details, but the analytics so far already hint what would be the role of the social planner: Since agents in private sectors cannot save enough and it is done inefficiently, the planner invests instead of the private agents.

2.2.2 Equilibrium with Social Planner

Now we solve for the solution of the planning problem. We will derive the solution of a social planner without capital controls. The results of reserve accumulation with capital controls are

$-\frac{\partial b_2}{\partial \theta}\ |_{\phi<\phi^c} = \phi \frac{-\frac{1}{\alpha} y_1^T + (\frac{1-\alpha}{\alpha})(1-2\Gamma_s)\frac{db_1}{d\theta}}{\l 1-\phi(\frac{1-\alpha}{\alpha})}$ The first term in the derivative $-\frac{1}{\alpha} y_1^T$ is less collateral in period 1; since the claim on the capital was sold to foreigners, it cannot be used as pledged collaterals. The second term is effect from more saving from period 0. More tradable goods saved for period 1 raise the real exchange rate so as to ease the credit constraint.

introduced in the next section.

Reserve accumulation without capital control To solve for the reserve accumulation in period 0, we first need to solve for the reserve depletion in period 1. As one easily expects, the social planner, regardless of whether the credit constraint binds or not, depletes all the reserves. It apparently looks natural, but it is actually a little unsatisfactory considering the fact that many EMEs during sudden stops did not deplete much of their reserves: fear of losing reserve (Aizenman and Sun, 2012). Such phenomenon can be captured in a similar environment with this paper, once we extend our model to a more dynamic version, i.e., the number of periods larger than 3. The intuition is straightforward. If the social planner facing an ongoing sudden stop expects that the crisis might be recurrent in the near future, then the social planner leaves a part of the reserves for a possible crisis in the future. We refer interested readers to Han and Park (2022).[32]

Given that the social planner will deplete all the reserves in period 1, we can formulate the planning problem as below.

$$\max_R V = u\left(c_0^T, y_0^N\right) + \mathbb{E}_0\left[\beta u\left(c_1^T, y_1^N\right) + \beta^2 u\left(c_2^T, y_2^N\right)\right] \qquad subject\ to$$

$$\begin{aligned}
-b_1 &= \text{the solution of (12)} \\
-b_2 &= \begin{cases} \dfrac{\phi\left((1-\theta)y_1^T + \frac{1-\alpha}{\alpha}\left((1-\theta)y_1^T + b_1(1+r_1) - d_1 + R\right)\right)}{1-\phi\left(\frac{1-\alpha}{\alpha}\right)} & if \quad \phi \in \left[\underline{\phi}, \phi^c\right] \\ \dfrac{-\zeta_1 + \sqrt{\zeta_1^2 - 4(1+\beta)\Gamma\left(\beta(1+r_*)\left((1-\theta)y_1^T + b_1(1+r_1) + R - d_1\right) - (1-\theta)y_2^T - d_2\right)}}{2(1+\beta)\Gamma} & if \quad \phi \in \left[\phi^c, \overline{\phi}\right] \end{cases} \\
c_0^T &= (1-\theta)\,y_0^T + p_0 y_0^N - p_0 c_0^N - b_1 - T + Q_0\theta k - d_0 \\
c_1^T &= (1-\theta)\,y_1^T + p_1 y_1^N + b_1(1+r_1) + R - p_1 c_1^N - b_2 - d_1 \\
c_2^T &= (1-\theta)\,y_2^T + p_2 y_2^N + b_2(1+r_2) - p_2 c_2^N - d_2 \\
r_t &= -\Gamma b_t + r^*
\end{aligned}$$

where $u(c_t^T, c_t^N) = ln\left(\left(c_t^T\right)^{\alpha}\left(c_t^N\right)^{1-\alpha}\right)$ and $R = T(1+\bar{r})$. Then deriving the first-order condition and rearranging the equation yields the proposition 1.

Proposition 1. *The optimal reserve accumulation at t=0 is characterized by*

[32] Bocola and Lorenzoni (2020) construct a three period model, in which social planner holds international reserves to prevent bank run to withdraw foreign currency deposits.

If $b_1^h < 0$

$$\underbrace{\beta\Gamma_b\mathbb{E}\left[u_1^T\right]b_1\frac{db_1}{dR_1^*}}_{Higher\, r_1}+\underbrace{\left[u_0^T-\beta\left(1+\overline{r}\right)\mathbb{E}\left[u_1^T\right]\right]}_{Consumption\, Wedge\, at\, \overline{r}}=$$

$$\beta\frac{dw_1}{dR_1^*}\left[\underbrace{\int_{\underline{\phi}}^{\phi^c}\frac{d\left(-b_2\right)}{dw_1}\left(u_1^T-\beta\left(1+r_2\right)u_2^T\right)d\mathscr{F}_\phi}_{Marginal\, Value\, of\, Borrowing}-\underbrace{\beta\Gamma_b\mathbb{E}\left[u_2^T b_2\frac{db_2}{dw_1}\right]}_{Lower\, r_2}\right] \tag{13}$$

If $b_1^h > 0$

$$\underbrace{\left[u_0^T-\beta\left(1+\overline{r}\right)\mathbb{E}\left[u_1^T\right]\right]}_{Consumption\, Wedge\, at\, \overline{r}}=$$

$$\beta\frac{dw_1}{dR_1^*}\left[\underbrace{\int_{\underline{\phi}}^{\phi^c}\frac{d\left(-b_2\right)}{dw_1}\left(u_1^T-\beta\left(1+r_2\right)u_2^T\right)d\mathscr{F}_\phi}_{Marginal\, Value\, of\, Borrowing}-\underbrace{\beta\Gamma_b\mathbb{E}\left[u_2^T b_2\frac{db_2}{dw_1}\right]}_{Lower\, r_2}\right]-\underbrace{\beta\Gamma_s\mathbb{E}\left[u_1^T\right]b_1\frac{db_1^s}{dR_1^*}}_{Higher\, r_1} \tag{14}$$

where $w_1 = R_1^* + b_1(1+r_1)$ *and* $\frac{dw_1}{dR_1^*} = \frac{\partial w_1}{\partial R} + \frac{\partial w_1}{\partial b_1}\frac{db}{dR} = 1 + \left(1 + r^* - 2\Gamma_j b_1\right)\frac{db_1}{dR}$

Proof) See the Appendix A.

The proposition above well illustrates what determines the optimal reserve accumulation. The terms in the LHS are the marginal costs of reserve accumulation and the terms in the RHS are the benefits. The term $u_0^T - \beta\left(1+\overline{r}\right)\mathbb{E}\left[u_1^T\right]$ in the LHS indicates the cost of reserve accumulation due to low returns to reserve, in terms of utility[33]. The two terms in the RHS are the benefits from higher wealth in period 1[34]. Because of the imperfect capital mobility in both of capital inflows and outflows, reserve accumulation raises the wealth in the future, which in turn helps with sudden stops and drives down expected borrowing rates in period 1[35].

An interesting term is $\beta\Gamma_j\mathbb{E}\left[u_1^T\right]b_1\frac{db_1}{dR_1^*}$. The term is in LHS in equation (13) whereas it is in RHS in equation (14): the term is a marginal cost when $b_1 < 0$, but it is a marginal benefit when $b_1 > 0$. This is related to the mechanism of how reserve accumulation works. When households

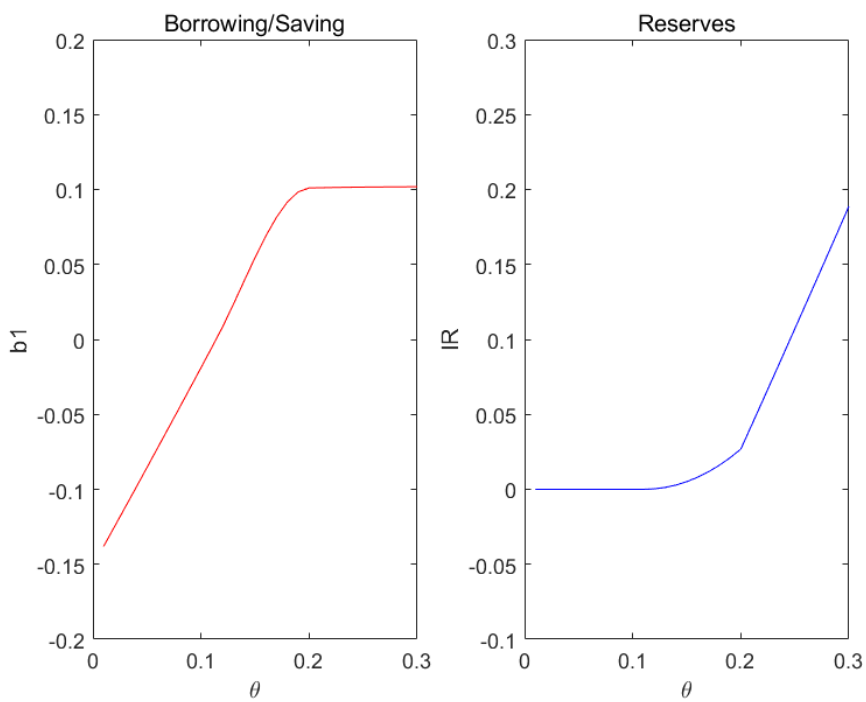
[33] As long as $\overline{r} < r_1$, $u_0^T - \beta\left(1+\overline{r}\right)\mathbb{E}\left[u_1^T\right]$ is positive; hence positive marginal cost.

[34] Please notice that this excludes the direct investments. The term wealth can be understood as net foreign currency liquidity.

[35] Throughout this paper, we implicitly assume $b_2\mid_{\phi>\phi^c} < 0$. That is, households still want to borrow in period 1. However, if direct investment inflows in period 0 is so overwhelming; i.e., θ is large enough, it is possible to have $b_2\mid_{\phi>\phi^c} > 0$ under the reserve accumulation. This might be relevant to some EMEs with large amounts of external assets comparing with the external debts. However, it is a little difficult to interpret the results in such a case. We relegate the analysis of the case of $b_2\mid_{\phi>\phi^c} > 0$ to the appendix C. Also notice that we likely to have $b_2\mid_{\phi>\phi^c} > 0$ when the planner optimally accumulates reserves. Otherwise, very low Γ_s and large θ are required to generate $b_2\mid_{\phi>\phi^c} > 0$.

borrow, the Ricardian equivalence breaks down due to higher borrowing rates; hence $\frac{db_1}{dR} > -1$. On the contrary, the Ricardian equivalence breaks down due to higher yields for households when the households save. Thus the changes in the interest rates driven by reserve accumulation are the costs when households borrow, but are benefits when households save. It naturally implies that we are more likely to have an interior solution, i.e., positive reserve accumulation when households save rather than borrow. Figure 2 below illustrates this point.

Figure 2: Reserve accumulation without capital control



Notes: All the parameter values are the same as the benchmark except for distribution of credit coefficient (=Beta(1.2,5)).

We elaborate more on the different efficacy of reserve accumulation, depending on whether households borrow or save. In Figure 2, for θ such that $b_1 < 0$, the optimal reserve accumulation is not to accumulate reserve. Of course, our model is stylized and thus we should not think of the results quantitatively. However, although the analytical results are from the simple model, the results in Proposition 1 and Figure 2 highlight deficiencies of reserve accumulation as a macroprudential policy tool in an environment where agents borrow from outside of the economy. As noted in lemma 1, when households borrow, the economy suffers from the overborrowing problem. Since reserve accumulation always incentivizes households to borrow more if the households borrow in the absence of reserve accumulation, the reserve accumulation calls a side effect: the economy suffers even more from the overborrowing. This is similar to a few preceding papers that documented the moral hazard from reserve accumulation (Acharya and Krishnamurthy (2018)) capital controls, we illustrate more on the mechanism behind Proposition 1. First, we need to understand the response of borrowing/saving of households to direct investment flows and reserve accumulation. Although we cannot solve for b_1 more explicitly, it is easy to see the key characteristics of the b_1 as a function of θ and R. We can easily show

$$\frac{\partial b_1}{\partial \theta} > 0, \quad \frac{\partial b_1}{\partial R_1} < 0$$

It is straightforward that b_1 increases in θ since more capital purchase by direct investors in period 0 induces more excess tradable goods to save for the future[36]. Similarly, b_1 decreases in the amounts of reserve accumulation because households borrow more or save less as resources are transferred from the current period to future periods through the reserve accumulation.

Next we also need to see the relationship between Γ and $\frac{\partial b_1}{\partial R}$. Higher Γ suppresses the responsiveness of the borrowing (saving) to the reserve accumulation. It is little cumbersome to show it formally, but we can easily see $\mid \frac{db_1}{d\theta} \mid$ decrease in Γ_s in equation (15). To an extreme, as $\Gamma_s \to \infty$, $\frac{db_1}{d\theta} \to 0$. Obviously, for $\Gamma_s \to \infty$, any positive saving derives down the gross yield to zero, and therefore households don't save in any amount. Same logic also applies to the household borrowing.

$$\frac{db_1}{dR_1} = \frac{-u_0^{''} + \beta(1+r_1)\mathbb{E}_0\left[u_1^{''}\left((1+\bar{r}) - \frac{\partial b_2}{\partial R}\right)\right]}{u_0^{''} - \beta(1+r_1)\mathbb{E}_0\left[u_1^{''}\left(1+r^* - 2\Gamma_j b_1 - \frac{\partial b_2}{\partial b_1}\right)\right] - \mathbb{E}_0\left[\beta u_1^{'}\Gamma_j\right]} \tag{15}$$

where $u_t^{''} = \frac{\partial u_t}{\partial c_t^T}$.

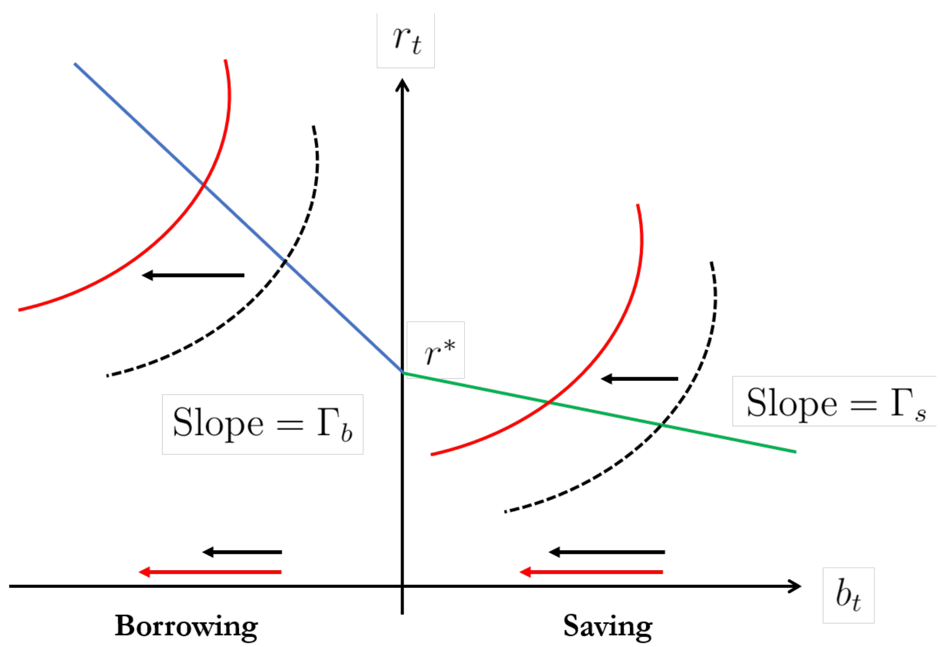
Now, we contrast reserve accumulation under household overseas borrowing and reserve accumulation under household overseas investments. First, when households borrow, the households face higher interest rates as they borrow more responding to reserve accumulation; therefore the households will borrow less. As a result, the borrowing increases less than reserve accumulation: net foreign assets of the economy increase. Higher borrowing rates improve the net foreign asset positions, but at the same time it generates more rents to IFIs. On the contrary, when households save, the less savings due to reserve accumulation drive up the yields facing the households, which results in more savings: falls in the household savings are less than reserve accumulation. Therefore, the reserve accumulation improves the net foreign asset positions, and also reduces the rents to IFIs as opposed to the case of reserve accumulation under household borrowing. This is well illustrated in Figure 3. Reserve accumulation raises the rates facing the households regardless of whether households borrow abroad or lend abroad,

[36] Recall the budget constraint in period 0, $c_0^T + p_0 c_0^N = (1-\theta)\, y_0^T + p_0 y_0^N - b_1 - T + Q_0 \theta k - d_0$

but reserve accumulation under household saving reduces the rents to IFS, whereas reserve accumulation under household borrowing increases the rents.

Keeping in mind the higher efficacy of reserve accumulation when households invest abroad, the illustration of the mechanism behind reserve accumulation in Proporsition 1 is as follows. Massive direct investments in period 0 incentivize households to save so much for the consumption smoothing, but severe friction on capital outflows (high Γ_s) hinder the households from investing abroad to transfer extra resources to the future periods. The friction on capital outflows by households is the key to understanding the motivation of reserve accumulation in our model. Direct investments or equity portfolio investments, which are not necessarily in line with optimal consumption smoothing of the households, give households more resources than needed to consume now and if households cannot save (invest abroad) to reallocate the resources enough or it is done inefficiently, the social planner needs to supplement (or substitute for) the insufficient savings by households.

Figure 3: Comparative statics of the households decision



Note: The shift-left of the curves indicates that changes in household borrowing/saving due to reserve accumulation. Reserve accumulation induce more borrowing/less saving of the households. See that more borrowing raise the borrowing rate facing the households, whereas less saving raises the return rates to the households.

2.3 Reserve accumulation with capital control

Now we analyze the reserve accumulation in an environment where the social planner has control over the capital flows[37]. In this subsection, we only consider the capital controls on debt flows, although there is another type of capital flows—direct investment flows. Analysis of relationship between capital controls on direct investment flows and reserve accumulation is introdued in Appendix.

Suppose the social planner can tax or subsidize (hence negative tax) debt instruments type capital flows, i.e., b_{t+1}. As it is well known in the capital control literature, the optimal tax can achieve the same equilibrium where the social planner directly chooses the capital flow. Because this is well known and analyzed in the literature, we do not discuss it further and introduce the optimal tax on the debt type capital flows. The optimal tax is characterized as below.

$$\tau_b = \frac{1}{u_0^T}\beta[\int_{\underline{\phi}}^{\phi^c} -\frac{db_2}{db_1}\left(u_1^T - \beta(1+r_2)u_2^T\right) d\mathscr{F}_\phi - \int_{\underline{\phi}}^{\overline{\phi}} \Gamma b_1 u_1^T - \beta\Gamma b_2 \frac{db_2}{db_1} u_2^T d\mathscr{F}_\phi] \tag{16}$$

As one can easily see, the optimal tax increases in the externalities from borrowing or saving of households[38].

Now we present the two equations that characterize the equilibrium where the social planner optimally accumulates reserve and tax (or choose borrowing or saving). Assuming an interior solution, which is not always, the equilibrium is characterized by the two equations.

$$-u_0^T + \beta u_1^T\left(1 + r^* - 2\Gamma_s b_1^{sp}\right) + \beta\int_{\underline{\phi}}^{\phi^c}\left(-u_1^T + \beta u_2^T(1+r_2)\right)\frac{db_2}{dw_1}\left(1 + r^* - 2\Gamma_s b_1^{sp}\right) = 0 \tag{17}$$

$$-u_0^T + \beta u_1^T\left(1 + \overline{r}\right) + \beta\int_{\underline{\phi}}^{\phi^c}\left(-u_1^T + \beta u_2^T(1+r_2)\right)\frac{db_2}{dw_1}\left(1 + \overline{r}\right) = 0 \tag{18}$$

where $w_1 = b_1(1+r_1) + R_1$ and b_2 is characterized in equation (9), (10). It is straightforward that we can solve for b_1^{sp} once we assume an interior solution for both of b_1^{sp} and R_1. Equation (17)

[37]Throughout this subsection, we assume that the capital control is perfect; the social planner can compute the optimal tax or subsidy to the different capital flows and can impose it perfectly. This is obviously counterfactual. In reality, the capital control is imperfect due to incomplete information or any other reasons and furthermore cannot be imposed perfectly. To cover those realistic features is obviously beyond the coverage of this paper. For a formal analysis of the leakage of capital control, see Bengui and Bianchi (2018).

[38]Some papers named such taxation 'Pigouvian taxation'. See Jeanne and Korinek (2019). Also, please notice that there are multiple policy instruments that achieve the same equilibrium. For deeper discussions, see Benigno et al. (2016).

and (18) give us

$$b_1^{sp} = \frac{r^* - \overline{r}}{2\Gamma_s} \tag{19}$$

See in equation (19), for $b_1^{sp} > 0$, $b_1^{sp} \to 0$ as $\Gamma_s \to \infty$. On the contrary, $b_1^{sp} \to \infty$ as $\Gamma_s \to 0$, which is a contradiction. The contradiction implies that we cannot have an interior solution for Γ_s small enough. More broadly, the reserve accumulation increases in Γ_s, while overseas investments are chosen by the social planner decrease in Γ_s. This is intuitive and corresponds to our main message throughout this paper. The social planner accumulates reserves since households cannot lend abroad enough by themselves. Γ_s is the measure of the friction on the private capital outflows and therefore the results above are straightforward.

$b_1^{sp} > 0$ in equation (19) since $r^* > \overline{r}$. This implies that $R^* = 0$ if $b_1^{sp} < 0$. The result of no reserve accumulation when the social planner chooses to borrow is also important and interesting analytics. In this paper, we aim at explaining why EMEs choose reserve accumulation as a macroprudential policy tool against sudden stop. For this purpose, suggesting mechanisms of how reserve accumulation works against sudden stop is not enough: It is a necessary condition, but not a sufficient condition. If there are multiple policy options, we need to explain why reserve accumulation is chosen over others and what is the unique role of it. Equation (19) already answers those questions. Although the social planner optimally subsidizes investments by households, which is unrealistic since such taxation or subsidy is never perfect in reality, the social planner would like to accumulate reserves for Γ_s large enough. If Γ_s is large, the marginal benefit of overseas investment rapidly falls, and therefore beyond a certain level, the planner accumulates reserves to have more foreign assets. On the contrary, when the social planner optimally chooses to borrow, the capital control always dominates the reserve accumulation. Recall that the function of reserve accumulation is to raise net foreign currency external assets. In this regard, the taxation on external borrowing is always better than the reserve accumulation since the reserve accumulation works by raising the borrowing rates; it gives a penalty to the borrowers. Then it is obvious to see why taxing external borrowings is better than the reserve accumulation. The capital control lowers the borrowing rates whereas the reserve accumulation raises the rates[39]. We summarize these findings in the second proposition.

Proposition 2. *Suppose the planner optimally taxes or subsidies debt capital flows, then*

1. $b_{t+1}^{sp} = b_{t+1}(\tau_t)$
2. *If* $b_{t+1}^{sp} < 0$ *and* $b_1^{sp} = b_{t+1}$ *then* $R^* = 0$

[39] Few theoretical papers (e.g. Davis et al., 2021, Arce et al., 2019) documented the equivalence between reserve accumulation and capital controls based on the assumption that the credit constraints are always binding. However, these papers also assumed that the returns to reserve are the same as the borrowing rates, which is counterfactual.

3. *If* $b^{sp}_{t+1} > 0$ *and* $b^{sp}_1 = b_{t+1}$ *then we may have* $R^* > 0$

Proof) See the Appendix A.

We can formulate the proposition in a different way, which gives us a corollary. It restates that the capital control on debt flows always dominates reserve accumulation for the borrowing, but reserve accumulation can be more efficient than the saving[40].

Corollary 1. *Let* $V_0(R)$ *is the value function of the planner with the optimal reserve accumulation at t=0. Similarly, define* $V_0(\tau)$*. Then,* $V_0(\tau) > V_0(R)$ if $b^{sp}_1 < 0$*. On the contrary, if* $b^{sp}_1 > 0$*, then we may have* $V_0(R) > V_0(\tau)$ *for* $\overline{r}$ *high enough or* Γ_s *high enough.*

Proof) See the Appendix A.

Overall, the proposition and the corollary above imply that there is a unique role of reserve management policy, which cannot be substituted by another policy tool, capital control. This is in contrast to the preceding reserve accumulation papers, such as Arce et al. (2019) and Davis et al. (2021), in which the role of reserve accumulation as a prudential policy tool can be fully replaced with capital control. In this regard, our theoretical results explain why reserve accumulation is often preferred to capital control in many EMEs, as documented in IMF (2012).

2.4 Reserve accumulation and currency manipulation

In this subsection, we characterize the optimal reserve accumulation more specifically as a function of direct investment capital inflows. It uncovers the nature of the reserve accumulation as a macroprudential policy tool against sudden stop and vividly maps our model to the empirical finding. In addition, the new proposition will provide important implications for the policy debate of currency manipulation.

As a first step, we denote $b_1 = b_1(\theta, R)$. That is, b_1 is a function of θ and R. From Remark 1, we know $\frac{\partial b_1}{\partial \theta} > 0$ and $\frac{\partial b_1}{\partial R} < 0$. And recall the FOC of reserve accumulation without capital

[40] The proposition and corollary also explain another puzzle about international reserve accumulation. One puzzle about international reserve is why central banks in EMEs limit the compositions of their reserves to certain safe assets such as US treasury bonds. Because of the low returns to these assets, sovereign wealth funds, whose portfolios accommodate more risky assets were expected to replace international reserves by central banks to a substantial extent. However, most of the external assets held by the public sector in EMEs are still international reserve. Proposition 2 suggests that the sovereign wealth funds are probably subject to the same frictions with private sectors. If sovereign wealth funds need to rely on foreign banks to make overseas investments, they must be subject to the same restriction (Γ_s). Also, the inefficiencies of domestic financial sectors or low quality institutions may matter in a similar way; for example, corruption in the sovereign wealth funds. In other words, although the social planner chooses how much to save and invest abroad, the social planner still faces the same constraint and thereby being incentivized to accumulate reserves.

control.

$$\left[u_0^T - \beta\left(1+\overline{r}\right)\mathbb{E}\left[u_1^T\right]\right] = \beta\frac{dw_1}{dR_1^*}\left[\int_{\underline{\phi}}^{\phi^c}\frac{d\left(-b_2\right)}{dw_1}\left(u_1^T-\beta\left(1+r_2\right)u_2^T\right)d\mathscr{F}_{\phi}-\beta\Gamma_b\mathbb{E}\left[u_2^T b_2\frac{db_2}{dw_1}\right]\right]-\beta\Gamma_s\mathbb{E}\left[u_1^T\right]b_1\frac{db_1}{dR_1^*} \quad (20)$$

Rearranging the equation (20) and using $u_0^T - \beta\left(1+r_1\right)\mathbb{E}\left[u_1^T\right]=0$ give us

$$\left[r^*-\overline{r}-\Gamma_s b_1\left(\theta\right)\left(1-\frac{db_1\left(\theta\right)}{dR_1}\right)\right]\mathbb{E}\left[u_1^T\right] = \frac{dw_1}{dR_1^*}\left[\int_{\underline{\phi}}^{\phi^c}\frac{d\left(-b_2\right)}{dw_1}\left(u_1^T-\beta\left(1+r_2\right)u_2^T\right)d\mathscr{F}_{\phi}-\beta\Gamma_b\mathbb{E}\left[u_2^T b_2\frac{db_2}{dw_1}\right]\right] \quad (21)$$

See the LHS in the equation (21) decreases in θ. However, the RHS is always positive as long as $b_2 < 0$.[41] We can think of the LHS as adjusted marginal costs and RHS as adjusted marginal benefits accordingly. Now define θ^c such that the LHS in equation (21) is zero. That is

$$r^*-\overline{r}-\Gamma_s b_1\left(\theta^c\right)\left(1-\frac{db_1\left(\theta^c\right)}{dR_1}\right)=0 \quad (22)$$

At $\theta=\theta^c$, the social planner must accumulate reserves because the marginal cost is zero while the marginal benefit is positive. Thus θ^c is a cut-off of direct investment above which the social planner must accumulate reserves. Along with a few other related analytical results, we introduce our third proposition.

Proposition 3. *(Passive Reserve Accumulation) Let $Q_0=\overline{Q}$ and define θ^c such that $r^*-\overline{r}=\Gamma_s b_1\left(\theta^c\right)\left(1-\frac{db_1\left(\theta^c\right)}{dR_1}\right)$. Then we have*

1) There exists δ_0 such that for $\theta>\theta^c-\delta_0$, $R_1^*>0$ and R_1^* increases in θ

2) $R_1^*=\left(Q_0-A_0\right)\left(\theta-\theta^c\right)K+b_1\left(\theta^c,0\right)-b_1\left(\theta,R_1^*\right)+c_0^T\left(\theta^c,0\right)-c_0^T\left(\theta,R_1^*\right)$

3) $b_1\left(\theta^c,0\right)>b_1\left(\theta,R_1^*\right)$

4) There exists δ_1 such that $c_0^T\left(\theta^c,0\right)-c_0^T\left(\theta,R_1^*\right)>0$ for $\theta\in\left(\theta,\theta+\delta_1\right)$. Therefore $R_1^*>\left(Q_0-A_0\right)\left(\theta-\theta^c\right)K$ for $\theta\in\left(\theta^c,\theta^c+\delta_1\right)$.

Proof) See the Appendix A.

Broadly speaking, the proposition 3 illustrates the existence of the cut-off and the amounts of reserve accumulation are bounded below by $\left(Q_0-A_0\right)\left(\theta-\theta^c\right)K$. Now assume that δ is small

[41] If b_2 is negative in any state, then it is negative in all the states. The only stochastic variable is ϕ. Once the decision is to borrow, lower ϕ only yields lower borrowing.

and $b_1(\theta^c, 0) - b_1(\theta, R^*) \simeq 0$ then we have

$$R_1^* \simeq (Q_0 - A_0)\left(\theta - \theta^c\right) K \tag{23}$$

More importantly, the real exchange rate in period 0 becomes almost invariant to θ. More formally, define

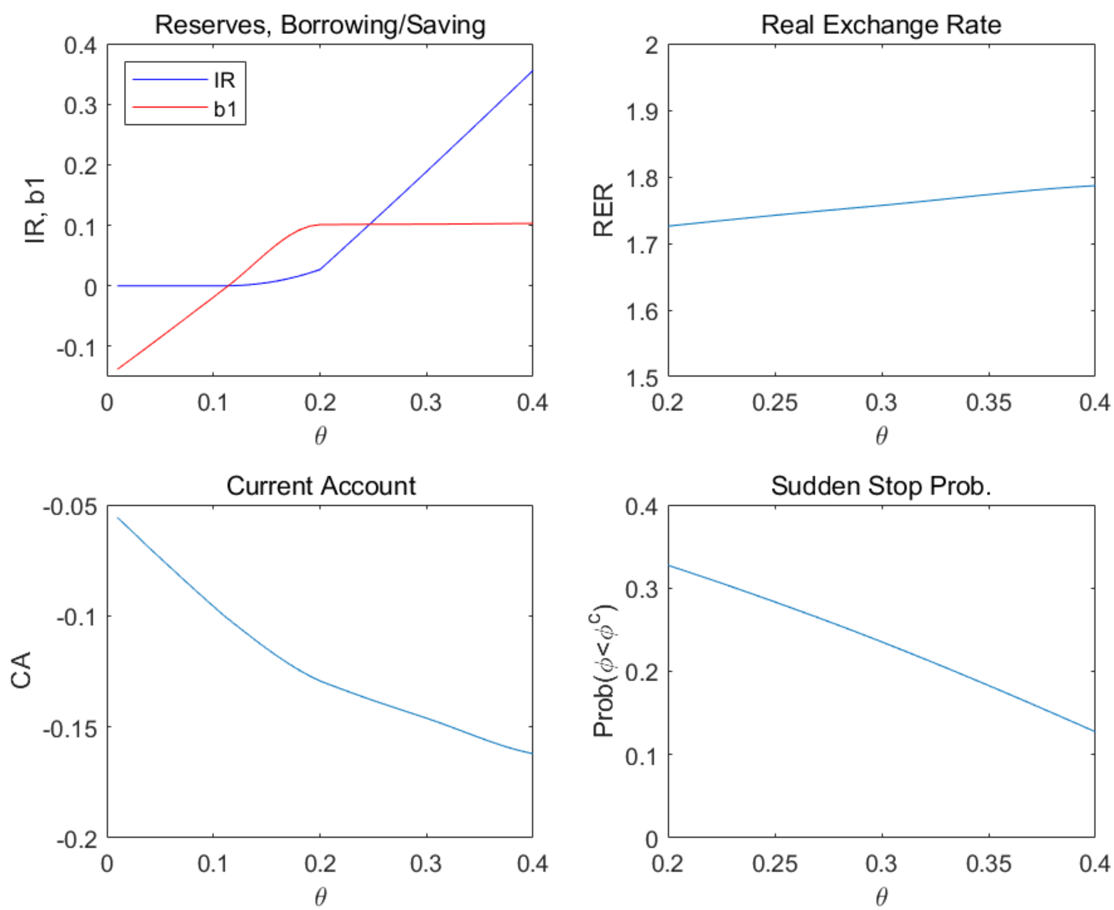
$$\begin{aligned} p_0 &= \left(\frac{1-\alpha}{\alpha}\right) \frac{y_0^T + (Q_0 - A_0)\,\theta K - d_0 + b_1(\theta) - R(\theta)}{y_0^N} \\ &\simeq \left(\frac{1-\alpha}{\alpha}\right) \frac{y_0^T + (Q_0 - A_0)\,\theta^c K - d_0 + b_1(\theta^c)}{y_0^N} \end{aligned} \tag{24}$$

Then p_0 is almost invariant to $\theta \in (\theta^c, 1)$. In other words, because the social planner absorbs the extra liquidity from direct investments by accumulating more reserves, the real exchange rate turns out to be almost "invariant" to direct investment capital inflows. Figure 4 illustrates such "passive" reserve accumulations and invariant real exchange rates under the passive reserve accumulation. To explain more, the social planner facing direct investment flows above a certain level "passively" absorbs the extra inflows beyond the level: The reserve accumulation increases by the almost same amount of the increase in direct investments. We explain more about such a passive reserve accumulation and following invariant real exchange rates in the discussion of currency manipulation below. Before we move on to the discussion, we restate the findings in the remark 1.

Remark 1. For direct investments inflows beyond a certain level, 1) the social planner must accumulate reserves, and 2) for direct investments more than the level, the planner absorbs the extra inflows by accumulating reserves and thereby making the real exchange rates almost invariant to the capital inflows.

Remark 1 and Figure 4 map our model to the empirical regularities. The equation (23) is the theoretical counterpart to the empirical regularity. Furthermore, once we posit the direct investment capital flows and equity portfolio flows as given, we can explain the evolution of international reserve holding of EMEs for the last two decades: as more direct investment and equity portfolio capitals flow into EMEs, many of the EMEs absorb the capital inflows by accumulating reserves. Also, we can explain much of the cross country difference of reserve holdings: The more direct investments or equity portfolio investments EMEs receive, the more reserves the EMEs accumulate. The model predicts that EMEs with more direct investment or equity

Figure 4: Passive Reserve Accumulation



Note: A 0.1 unit of θ approximately corresponds to the inflow in the amount of the 9.5% of GDP. All the parameter values are the same as the benchmark except for distribution of credit coefficient(=Beta(1.2,5)).

portfolio external liabilities accumulate more reserves as we discuss in Appendix[42]. Another important parameter to explain the reserve accumulation is Γ_s. The model predicts lower Γ_s induces more reserve accumulation. Unfortunately, we have no clear idea of what determines Γ_s or the empirical counterpart of the parameter. Despite such difficulty, it seems that the recent divergence between private sector external assets and official reserves in EMEs is a result of less friction on overseas investment by the private sectors: that is, lower Γ_s. For more discussion and related empirical findings, we relegate it to the appendix.

Discussion of currency manipulation Reserve accumulation is often viewed as evidence that some EMEs depreciate their currencies to boost their exports, and such one side interventions to depreciate domestic currencies are often called "currency manipulation." For example, one

[42]The EMEs holding reserves more than 40% of GDP, such as Malaysia, Thailand or Bulgaria also have sizable direct investment and equity portfolio investment external liabilities.

of the criteria that the US treasury examines to judge whether an EME is manipulating their currency value is the amounts of reserve accumulation; detailed information is in table below.

Table 1: The U.S Treasury's Foreign Exchange Report

Each country reports shall contain:
(1) country's bilateral trade balance with the United States
(2) country's current account balance as a percentage of its GDP
...
(4) country's **foreign exchange reserves** as a percentage of its short-term debt
(5) country's **foreign exchange reserves** as a percentage of its GDP
Enhanced analysis shall include:
(1) a significant bilateral trade surplus with the United States
(2) a material current account surplus
(3) engaged in persistent one-sided intervention in the foreign exchange market

Source: Section 701 of the Trade Facilitation and Trade Enforcement Act of 2015

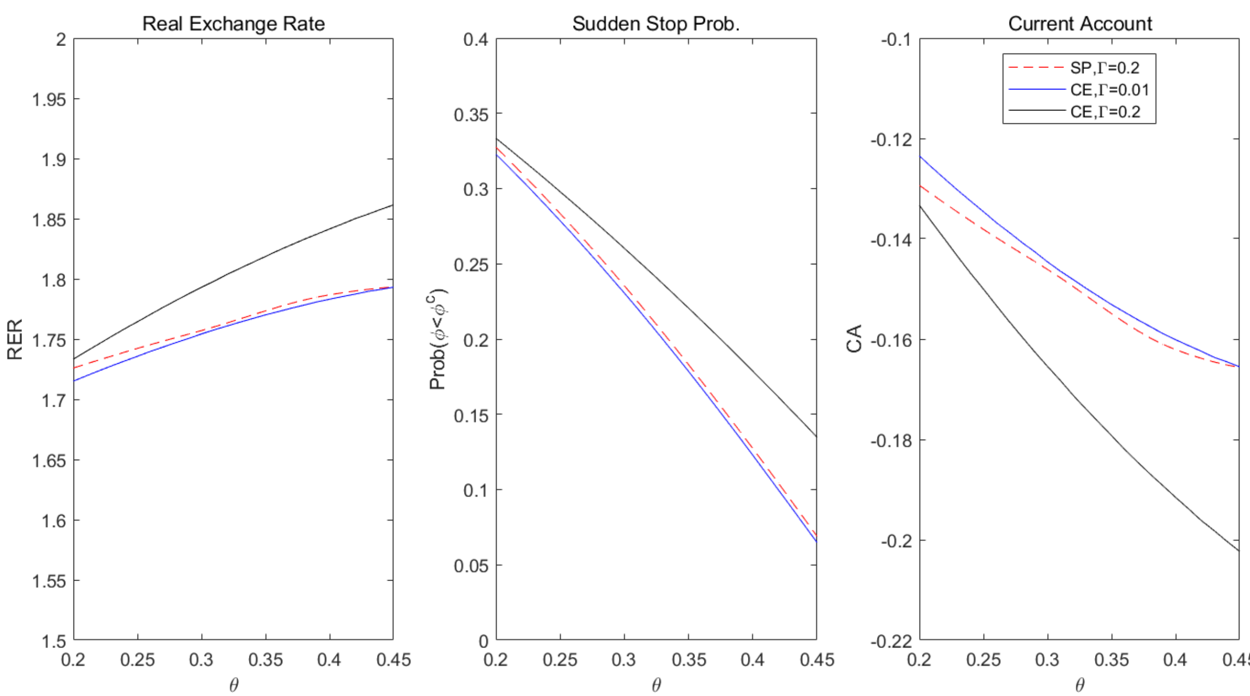
In the literature, it has been often argued that international reserve holdings at extraordinary large amounts cannot be justified by the precautionary view and therefore the large amounts of reserves are a byproduct of the export-oriented growth strategy; for example, some East Asian countries such as China, Thailand, or Malaysia. The underlying idea is that reserve holdings in such countries seem to be much more than needed for a precautionary purpose. Here we argue that the simple international reserve to GDP ratio is not a correct measure to identify a currency manipulation. If the apparent excessive reserve accumulation is caused by massive capital inflows, then the purpose of the reserve accumulation can be precautionary. Of course, we do not analyze or discuss the currency manipulation itself or try to examine whether certain EMEs manipulate their currencies in the spirit of mercantilism. We only aim at explaining why amounts of accumulated reserves are not a good "litmus" to test currency manipulation.[43]

The empirical facts about reserve accumulation that we find in Appendix and proposition 3 altogether imply that EMEs facing massive capital inflows make corresponding capital outflows in the form of reserves to maintain the macroeconomic balance. The purpose of reserve accumulation is to reallocate the resources from capital inflows so as to minimize the inefficiencies of capital outflows by private sectors and be better prepared for possible sudden stops in the future. All the motivation of reserve accumulation lies in the precautionary purpose. The social planner in our model does not have an idea of mercantilism since we do not put any ingredients related to the mercantilism, such as positive externalities from export sectors.

[43] For the theoretical exploration of currency manipulation, see Hassan et al. (2016). For empirical studies, see Dominguez (2019).

Figure 5 below well illustrates it. First, in the absence of reserve accumulation, the direct investment flows generate consumption booms. As θ increases, the households' saving increases, but not enough due to falling returns for the households, which appreciates the real exchange rate through more tradable goods consumption. Therefore, although the direct investments lower the probability of sudden stop, it falls slowly as the households cannot invest abroad enough. On the contrary, when the social planner accumulates reserves, more capital outflows are made and it keeps tradable goods consumption constant and hence real exchange as well. More importantly, thanks to the more overseas investments, the sudden stop probability falls much faster. As a result, the equilibrium with reserve accumulation is similar to little or no friction on capital outflows; that is, a very small Γ_s.

Figure 5: Reserve Accumulation and Currency Manipulation



Note: 1) All the parameter values are the same as the benchmark except for distribution of credit coefficient(=Beta(1.2,5)). 2) CE refers to competitive equilibrium without reserve accumulation by social planner. SP refers to a equilibrium with social planner optimal reserve accumulation.

In this regard, reserve accumulation is a way to "restore" the macroeconomic balance under little direct investment inflows or to achieve the balance under frictionless capital outflows. The real exchange rate might be a target for the planner since it measures over-consumption in the present, but the purpose of the intervention is to prevent "appreciations" of the currency, not to "depreciate" the currency. The current account ($y_0^T - c_0^T$) neither improves or becomes worsen as more reserves are accumulated, as opposed to a common prediction from the mercantilism view.

Such patterns of reserve accumulation in our model correspond to observations in Yeyati (2008) and Levy-Yeyati et al. (2013), which document that foreign exchange market interventions seem to aim at limiting domestic currency appreciations. Also, the patterns are in the same line with the famous finding in Calvo and Reinhart (2002) so-called "Fear of Floating" in a sense that real exchange rates in our model economy may look somehow managed in the eyes of outsiders to the economy. However, the motivation for reserve accumulation in our model lies in the precautionary purpose as it is designed so. Therefore, our model reconciles the precautionary view of reserve accumulation with empirical findings in the managed float literature. Back to our discussion of currency manipulation, our theory implies that evidence supporting the managed float exchange regime does not necessarily support the mercantilist view and the evidence can be aligned with the precautionary view.

The key analytical results in Proposition 1 and Proposition 3 are derived from the representative agent model. While the representative agent model captures our empirical findings in the last section in a simple way, the analytical results of the model cannot fully resemble rich empirical features we observe in the data. The representative agent model cannot borrow and lend abroad at the same time, however EMEs in reality of course borrow and lend abroad simultaneously. To accommodate such rich features into our model, we extended our baseline model to a heterogeneous agents model. The heterogeneous agents model is introduced in Appendix, and confirms key results and insights survive, and the mechanism can be even more powerful, in a more realistic environments where EMEs borrow and lend abroad simultaneously.

3 Empirical Analysis

In this section, we test the model implication with cross-country panel data. The bottom line of the previous section is that it is optimal for the countries to accumulate reserves in response to excessive capital inflows. And precisely, this is what the policymakers in emerging economies have been arguing they are doing. Contrary to the prediction of the classical international economic theory, many policymakers believe that capital inflows are expansionary.[44] In their view, capital inflows inflate asset prices, increase credit, and may contribute to future financial instability. The most favored policy instrument against the unwelcome capital inflows has been the FX intervention and reserve accumulation.

Recent empirical works support the emerging market policymakers' argument by providing evidence that capital inflows indeed have expansionary effects. They show that capital inflows may lead to credit easing, a local lending boom, and even result in resource misallocation (e.g. Di Giovanni et al., 2021, Gopinath et al., 2017). Furthermore, recent works in the international reserve literature show that reserve accumulation can indeed be a proper response against capital inflows because it can curb credit expansion. Hofmann et al. (2019) and Yun (2020) show that reserve accumulation can work against credit expansion by slowing down bank loan growth. Reinhart et al. (2016) and Choi et al. (2020) present evidence that reserve accumulation reduce firm leverage and investment.

Despite the rapid progress in the literature, however, it is still unknown whether the countries accumulate reserves mainly to mitigate the negative impact of capital inflows or to manipulate exchange rates, as some other countries have been arguing. In this section, we investigate data on international capital flows to examine whether countries accumulate FX reserves in response to capital inflows.

3.1 Sample

We analyze cross-country panel data for the massive reserve accumulation period in the emerging economies: between the Asian Financial Crisis and the Global Financial Crisis (1999q1 to 2008q2). Reserve accumulation took off after the Asian crisis among emerging economies, and the pace slowed down after the Global Financial Crisis. The main data for the analysis is sourced from the IMF International Financial Statistics (IFS). We obtain quarterly capital flows documented in the official balance of payment statistics from the IFS. We analyze gross capital inflows (non-resident investors' acquisition of domestic assets). We analyze emerging economies

[44] The classical Mundell-Fleming open economy model suggests that capital inflows are contractionary because they appreciate the currency and reduce exports and output.

only. From the country universe of the IFS dataset, we exclude advanced economies. Countries with less than 25 observations are also dropped because our analysis includes country-by-country time-series estimation. This sample definition provides us with a total of 33 countries.[45]

Table 2 provides summary statistics for our sample. The reserve accumulation data is obtained from the balance of payment statistics, and hence it contains only the transaction component of the international reserve fluctuations. We adjust the nominal value with the U.S. CPI to measure it in the 2010 U.S. dollar value. Since we make a cross-country comparison, we need to normalize both reserve accumulation and capital flows. To avoid a mechanical correlation between the two, we normalize the reserve accumulation with the population (hence, making it a per-capita variable) while normalizing the capital inflows with GDP. In doing the regressions, we winsorize capital flow variables at a three percent level to eliminate the impact of potential outliers. The dependent variable, per capita reserve accumulation, is winsorized at a one percent level.

Table 2: Descriptive Statistics

	obs.	mean	St.Dev.	p25	median	p75
Country level						
Reserve accumulation per capita	1,214	33.84	89.13	-4.10	15.49	55.58
Total K inflow/GDP (×100)	1,041	16.02	45.38	0.90	2.33	4.97
Direct investment/GDP (×100)	1,077	6.58	17.21	0.52	1.13	2.50
Portfolio inflow/GDP (×100)	1,063	1.73	6.07	-0.07	0.07	0.76
Other inflow/GDP (×100)	1,077	7.14	24.22	-0.13	0.76	2.18
ka_open	1,214	0.60	0.31	0.42	0.69	0.94
Global level						
VIX	92	2.95	0.33	2.64	2.97	3.20
World uncertainty index	92	1.38	0.22	1.27	1.36	1.45
Fuel price index	92	1.02	0.50	0.67	0.83	1.48

Notes: This table presents descriptive statistics for the variables included in the regressions. The sample period is from 1999q1 to 2008q2. 33 emerging economies are included in the sample. Reserve accumulation and other capital flows data are obtained from the IMF International Financial Statistics. ka_open is the financial market openness index of (Chinn and Ito, 2006). The world uncertainty index is from (Ahir et al., 2022). Fuel price index is sourced from the IMF.

[45] Armenia, Azerbaijan, Belarus, Bosnia and Herzegovina, Brazil, Bulgaria, Chile, Costa Rica, Croatia, Czech Republic, Ecuador, El Salvador, Estonia, Georgia, Hungary, Iceland, Israel, Korea, Latvia, Lithuania, Mexico, Moldova, North Macedonia, Paraguay, Philippines, Poland, Romania, Russia, Slovakia, Slovenia, Thailand, Turkey, Ukraine.

3.2 Reserve Accumulation and Financial Openness

The purpose of our empirical analysis is to examine whether the countries respond to excessive capital inflows by accumulating FX reserves, as our model prescribes. Many central banks answer that an important goal of FX intervention is to manage capital flows and credit spillovers (Patel and Cavallino, 2019), but it requires an in-depth investigation of the data.

We are considering capital inflows as the cause and the reserve as the effect. Positive associations between the two are expected from the data, but it could be a result of reverse causality or other third factors affecting both. There is a high degree of endogeneity. Many factors considered for FX reserve management are correlated with capital flows. Also, reserve accumulation may influence other private capital flows.

We get around the identification challenge by adding one more dimension to our investigation of the channel. We compare countries with different levels of financial openness when they receive different sizes of gross capital inflows. Previous studies show that reserve accumulation becomes more effective when it is combined with capital controls (e.g. Choi and Taylor, 2017). Other theoretical works also show that FX intervention is more effective in economies with more closed financial markets (or larger financial frictions) (e.g. Gabaix and Maggiori, 2015). Therefore, we posit that countries with less open financial markets would utilize the reserve accumulation more. In countries with highly open financial markets, the Ricardian equivalence in capital flows would prevail stronger, and reserve accumulation may not be an effective tool. Hence, we compare reserve accumulation responses of countries with different levels of financial openness in times of capital inflows. We use the Chinn-Ito index as a proxy for the financial market openness. This index measures a degree of capital account openness. It is a de-jure measure that counts the restrictions on cross-border financial transactions reported by the IMF. We use the normalized index, which ranges between zero and one (zero is for a closed economy, and one is for a fully open economy).

The model presented in the previous section also implies that the reserve response should be more pronounced for countries with less-open capital markets (higher Γ). It is because the reserve accumulation becomes more effective when the Ricardian equivalence of capital flows fails. Hence we are aiming to test the model implication in this dimension.

In the end, we will have reserve accumulation as the regressand, and gross capital inflows as the main regressor. To examine whether the reserve response is stronger for less-open countries, we interact the capital inflows with financial openness. This gives us the following specification:

$$\texttt{RA\_per\_capita}_{i,t} = \delta_i + \delta_t + \gamma \texttt{K/GDP}_{i,t} + \theta \texttt{ka\_open}_{i,t} + \phi \texttt{ka\_open}_{i,t} \times \texttt{K/GDP}_{i,t} + \varepsilon_{i,t} \tag{25}$$

where δ_i and δ_t are the country and quarter fixed effects, respectively.

Table 3 presents the baseline results. We are most interested in total gross capital inflows, but we also examine subcategories of it: direct investment flows, portfolio flows, and other flows (banking flows). First, in Column (1), we find a strong correlation between the gross capital inflows and reserve accumulation among the emerging economies. The coefficient means that we observe 0.8 dollars per person reserve accumulation for each percent of GDP capital inflows. In the next column, we examine whether this relationship is different for countries with different levels of financial openness, as our model predicts. We find that the interaction term coefficient is negative and significant while the capital inflow coefficient grows in size and remains significant. In the data, the financial openness index of the 25th percentile is 0.42, while the 75th percentile is 0.94. Hence it means the 25th percentile country accumulates 1.3 dollars per one percent of GDP capital inflows while the 75th percentile country accumulates only 0.2 dollars. Columns (3)-(5) report the analysis of capital flows with different asset types. We

Table 3: OLS Results

	(1)	(2)	(3)	(4)	(5)
Type of capital flows:	total	total	direct	portfolio	other
`K/GDP`	0.805***	2.216***	2.922*	1.584	3.640***
	(0.000)	(0.002)	(0.082)	(0.658)	(0.004)
`K/GDP` × `ka_open`		-2.129**	-3.745**	-0.678	-3.909*
		(0.032)	(0.017)	(0.874)	(0.064)
`ka_open`		12.77	7.440	-9.686	-7.365
		(0.555)	(0.741)	(0.719)	(0.786)
observations	1,041	1,041	1,077	1,063	1,077
number of countries	33	33	33	33	33
R-squared	0.079	0.107	0.069	0.055	0.087
country fixed effects	yes	yes	yes	yes	yes
quarterly fixed effects	yes	yes	yes	yes	yes

Notes: The sample period is from 1999q1 to 2008q2. The dependent variable is the per capita FX reserve accumulation. The top and bottom 1% of the dependent variable are winsorized. The top and bottom 3% of the capital flow variable are winsorized. Standard errors are clustered at the country level. Reported in the parenthesis are the corresponding p-values. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

find similar results from direct investment flows and other flows (banking flows) but not from portfolio flows.

3.3 Two-Stage Regressions

We augment the previous analysis further with two-stage regressions. Using a set of global push factors of capital inflows, we isolate exogenous parts of the gross capital inflows to each country. Since our sample consists of small open economies only, the global push factors are exogenous to domestic fundamentals.

In the first stage, we regress gross capital inflows on global push factors of capital flows: VIX, the World Uncertainty Index, and the fuel price index. We do the regression country by country and get predicted capital inflows series for each country. Precisely the first stage regression equation is as below:

$$\texttt{K/GDP}_{i,t} = \alpha_i + \beta_{1,i} \ln \texttt{VIX}_{t-1} + \beta_{2,i} \texttt{WUI}_{t-1} + \beta_{3,i} \texttt{fuel}_{t-1} + \varepsilon_{i,t} \qquad \text{for each } i \tag{26}$$

The predicted value of the regression (26) contains the push-factor driven exogenous capital inflows. In the second stage, we do the same analysis with the estimated exogenous capital inflows. Since our regressor is a regression-generated variable, we calculate standard error by the bootstrapping method.

Table 4 presents the two-stage regression results. Columns (1)-(2) report the results of total gross capital inflows as before. Compared with the previous OLS results, the sizes of coefficients shrink, and the statistical significance is also reduced a little. Overall, however, the main result stays the same. We find that reserve accumulation is positively associated with exogenous capital inflows, and it is more so for the countries with less-open capital accounts. Among different types of capital flows, the result from direct investment flows is pronounced. The coefficients size increases. In the two-stage exercise, we find significant results also from the portfolio flows. The result on banking flows, on the other hand, loses significance.

Table 4: Two-Stage Regression Results

	(1)	(2)	(3)	(4)	(5)
Type of capital flows:	total	total	direct	portfolio	other
`K/GDP`	0.580*	1.690**	4.523**	18.63***	2.468+
	(0.087)	(0.024)	(0.048)	(0.001)	(0.146)
`K/GDP` × `ka_open`		-1.407**	-3.749*	-21.64***	-2.403
		(0.044)	(0.088)	(0.000)	(0.157)
`ka_open`		14.75	18.88	23.50	8.110
		(0.466)	(0.373)	(0.210)	(0.621)
observations	1,181	1,181	1,181	1,181	1,181
number of countries	33	33	33	33	33
R-squared	0.054	0.067	0.065	0.087	0.058
country fixed effects	yes	yes	yes	yes	yes
quarterly fixed effects	yes	yes	yes	yes	yes
First-stage F-stat (mean)	6.21	6.21	4.20	1.66	5.29

Notes: The sample period is from 1999q1 to 2008q2. The dependent variable is the per capita FX reserve accumulation. The top and bottom 1% of the dependent variable are winsorized. The top and bottom 3% of the capital flow variable are winsorized. Standard errors are clustered at the country level. Reported in brackets are the p-values from bootstrapped standard errors. Standard errors are clustered at the country level. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

4 Conclusion

This paper provides a novel theory of reserve accumulation in EMEs. The model assumes frictions in gross capital outflows based on the observations that many EMEs have under-developed financial markets and often have restrictions on residents' investment abroad. When the economy receives large capital inflows for exogenous reasons, there should be corresponding gross capital outflows that help maintain macroeconomic balances. However, the private sector cannot invest overseas enough due to the frictions. In this environment, we show that a second-best can be achieved by public capital outflows (i.e., reserve accumulation).

We provide empirical evidence in support of the proposed theory. We investigate 33 EMEs from 1999 to 2008 in their reserve accumulation responses to the gross capital inflows. Consistent with the theory, we find that reserve accumulation is more significant in countries with more open capital markets. The results stay the same when we use exogenous capital inflows, estimated using global push factors of capital flows such as the VIX, the world uncertainty index, and fuel prices.

This paper provides important implications for the ongoing debate on reserve accumulation and currency manipulation. In our model, reserve accumulation is an optimal response when the economy receives direct investment inflows beyond a threshold. The country passively accumulates reserves in response to exogenous capital inflows. It works in a way that prevents its currency from being appreciated, but it is not the policy objective. This narrative corresponds to the empirical findings in Levy-Yeyati et al. (2013): EMEs intervene in the FX market to prevent currency appreciation, not to depreciate and gain competitiveness.

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Appendix

In this appendix, we provide the followings:

- omitted algebras and proofs (section A),
- several extensions of the baseline model (section B, C, D, and E),
- micro-foundations and justifications for our model assumptions (section F and G),
- omitted empirical investigations (section H).

A Omitted Algebras and Proofs

A.1 Proof of Lemma 1

First, we show the the object function of the social planner is concave in b_1.

Define $V \equiv u\left(c_0^T, y_0^N\right) + \mathbb{E}_0\left[\beta u\left(c_1^T, y_1^N\right) + \beta^2 u\left(c_2^T, y_2^N\right)\right]$. Then taking the derivative of V with repect to b_1 gives us

$$\frac{dV}{db_1} = -u_0^T + \beta \int_{\underline{\phi}}^{\overline{\phi}} \left[\left(1 + r^* - 2\Gamma_j b_1\right)\left(1 - \frac{db_2}{dw_1}\right)\right] u_1^T d\mathscr{F}_\phi + \beta^2 \int_{\underline{\phi}}^{\overline{\phi}} \left[\left(1 + r^* - 2\Gamma_j b_1\right)\frac{db_2}{dw_1}\left(1 + r^* - 2\Gamma_b b_2\right)\right] u_2^T d\mathscr{F}_\phi \quad \text{(A.1)}$$

where $w_1 = b_1(1 + r_1)$ becasue we have no reserve accumulation yet.

Since the social planner can choose b_2 for $\phi \in \left(\underline{\phi}, \phi^c\right)$ where ϕ^c is the cut-off for the social planner, which is different from (11), the equation (A.1) changes to

$$\frac{dV}{db_1} = -u_0^T + \beta \int_{\underline{\phi}}^{\overline{\phi}} \left(1 + r^* - 2\Gamma_j b_1\right) u_1^T d\mathscr{F}_\phi + \beta^2 \int_{\underline{\phi}}^{\phi^c} \left(1 + r^* - 2\Gamma_j b_1\right)\frac{db_2}{dw_1}\left[-u_1^T + \left(1 + r^* - 2\Gamma_b b_2\right) u_2^T\right] d\mathscr{F}_\phi \quad \text{(A.2)}$$

It is easy to see $\beta \int_{\underline{\phi}}^{\overline{\phi}} \left(1 + r^* - 2\Gamma_j b_1\right) u_1^T d\mathscr{F}_\phi$ is decreasing in b_1 since the return to the saving is derceasing (increasing borrowing rates in the borrowing). Hence, the first and second terms in the equation (A.2) decreases in b_1. In the third term, from (9) we can check $\frac{db_2}{dw_1} < 0$ and $\mid \frac{db_2}{dw_1} \mid$ is decreasing in b_1. ϕ^c obviously decreases in b_1. Since $b_2 \mid_{\phi < \phi^c}$ decreases in b_1, we can immediately see $u_1^T - (1 + r^* - 2\Gamma_b b_2) u_2^T$ decreases in b_1 for all $\phi \in \left(\underline{\phi}, \phi^c\right)$. Therefore, all the terms are decreasing in b_1. It implies $\frac{d^2 V}{db_1^2} < 0$.

Now we show the claims in the lemma. First, we show $b_1^h < b_1^{sp}$ if $b_1^h < 0$. To show $b_1^{priv} < b_1^{sp}$, we first derive the first order condition of b_1 for the social planner. b_1^{sp} is characterized as below.

$$-u_0^T + \beta \int_{\underline{\phi}}^{\overline{\phi}} \left[\left(1 + r^* - 2\Gamma_b b_1\right) \left(1 - \frac{db_2}{dw_1}\right) \right] u_1^T d\mathscr{F}_\phi$$

$$\beta^2 \int_{\underline{\phi}}^{\overline{\phi}} \left[\left(1 + r^* - 2\Gamma_b b_2\right) \frac{db_2}{dw_1} \left(1 + r^* - 2\Gamma_b b_1\right) \right] u_2^T d\mathscr{F}_\phi = 0 \quad \text{(A.3)}$$

where $w_1 = b_1 (1 + r_1) + R_1$. This can be represented by

$$-u_0^T + \beta \int_{\underline{\phi}}^{\overline{\phi}} (1 + r_1) u_1^T - \Gamma_b b_1 u_1^T d\mathscr{F}_\phi$$

$$-\left(1 + r^* - 2\Gamma_b b_1\right) \beta \int_{\underline{\phi}}^{\overline{\phi}} \frac{db_2}{dw_1} \left(u_1^T - \beta (1 + r_2 - \Gamma_b b_2) u_2^T\right) d\mathscr{F}_\phi = 0 \quad \text{(A.4)}$$

Suppose $b_1 = b_1^{priv}$ and $b_2 = b_2^{priv}$. Then we have $u_0^T = \beta \int_{\underline{\phi}}^{\overline{\phi}} (1 + r_1) u_1^T d\mathscr{F}_\phi$ and also $u_1^T = \beta (1 + r_2) u_2^T \; if \; \phi > \phi^c$. This gives us

$$-\beta \int_{\underline{\phi}}^{\overline{\phi}} \Gamma_b b_1 u_1^T + \beta^2 \left(1 + r^* - 2\Gamma_b b_1\right) \int_{\underline{\phi}}^{\overline{\phi}} \frac{db_2}{dw_1} \Gamma_b b_2 u_2^T d\mathscr{F}_\phi$$

$$-\beta \left(1 + r^* - 2\Gamma_b b_1\right) \int_{\underline{\phi}}^{\phi^c} \frac{db_2}{dw_1} \left(u_1^T - \beta (1 + r_2) u_2^T\right) d\mathscr{F}_\phi > 0 \quad \text{(A.5)}$$

If $\phi \in [\underline{\phi}, \phi^c)$, then $u_1^T - \beta (1 + r_2) u_2^T > 0$. Absolutely the remaining terms are all positive. It implies that $b_1^h < b_1^{sp}$.

Next, we show If $b_t^h > 0$, then there always exists $\gamma_0 \in (0, \infty]$ such that for $\Gamma_s \in \left(0, \gamma_0\right)$ $b_t^h < b_t^{sp}$. If $\gamma_0 \in (0, \infty)$, then there exists $\gamma_1 \in \left(\gamma_0, \infty\right)$ such that for $\Gamma_s \in \left(\gamma_0, \gamma_1\right)$ $b_t^h > b_t^{sp}$. Since $b_1^h > 0$,we replace Γ_b with Γ_s in front of b_1 in equation (A.2). This gives us

$$\frac{dV}{db_1} = -\beta \int_{\underline{\phi}}^{\overline{\phi}} \Gamma_s b_1 u_1^T + \beta^2 \left(1 + r^* - 2\Gamma_s b_1\right) \int_{\underline{\phi}}^{\overline{\phi}} \frac{db_2}{dw_1} \Gamma_b b_2 u_2^T d\mathscr{F}_\phi$$

$$-\beta \left(1 + r^* - 2\Gamma_s b_1\right) \int_{\underline{\phi}}^{\phi^c} \frac{db_2}{dw_1} \left(u_1^T - \beta (1 + r_2) u_2^T\right) d\mathscr{F}_\phi \quad \text{(A.6)}$$

First, notices $\frac{dV}{db_1}$ is continuous in Γ_s as long as $b_1 > 0$. All the variables are continuous and the mappings in $\frac{dV}{db_1}$ are continuous as well. Then we show

$$\lim_{\Gamma_s \downarrow 0} \frac{dV}{db_1} > 0 \;\; and \;\; \lim_{\Gamma_s \uparrow \infty} \frac{dV}{db_1} > 0$$

These properties come from $\lim_{\Gamma_s \downarrow 0} \Gamma_s b_1 = \lim_{\Gamma_s \uparrow \infty} \Gamma_s b_1 = 0$. $\lim_{\Gamma_s \downarrow 0} \Gamma_s b_1 = 0$ is obvious. $\lim_{\Gamma_s \uparrow \infty} \Gamma_s b_1 = 0$ can be easily shown by a contradiction. If $\lim_{\Gamma_s \uparrow \infty} \Gamma_s b_1 \neq 0$, then the gross return must be zero. Then the households can be better off by letting $b_1 = 0$. If $\Gamma_s b_1 = 0$, then the equation (A.6) will be

$$\frac{dV}{db_1} = \beta^2 \left(1 + r^*\right) \int_{\underline{\phi}}^{\overline{\phi}} \frac{db_2}{dw_1} \Gamma_b b_2 u_2^T d\mathscr{F}_\phi - \beta \left(1 + r^*\right) \int_{\underline{\phi}}^{\phi^c} \frac{db_2}{dw_1} \left(u_1^T - \beta (1 + r_2) u_2^T\right) d\mathscr{F}_\phi > 0 \quad \text{(A.7)}$$

It proves there always exists $\gamma_0 \in (0, \infty]$ such that for $\Gamma_s \in \left(0, \gamma_0\right)$ $b_t^h < b_t^{sp}$ since $\frac{dV}{db_1}$ is continuous in Γ_s.

Finally, we show if $\gamma_0 \in (0, \infty)$, then there exists $\gamma_1 \in \left(\gamma_0, \infty\right)$ such that for $\Gamma_s \in \left(\gamma_0, \gamma_1\right)$ $b_t^h > b_t^{sp}$. To show it, it is enough to show we may have $\frac{dV}{db_1} < 0$ for some Γ_s by again the continuity of $\frac{dV}{db_1}$ in Γ_s. In equation (A.6), we can manipulate the distribution of ϕ and relative values of y_1^T and y_s^T so that $\phi^c \to \underline{\phi}$ and $-b_2$ is small enougn to have $\beta \int_{\underline{\phi}}^{\overline{\phi}} \Gamma_s b_1 u_1^T > \beta^2 (1 + r^* - 2\Gamma_s b_1) \int_{\underline{\phi}}^{\overline{\phi}} \frac{db_2}{dw_1} \Gamma_b b_2 u_2^T d\mathscr{F}_\phi$. This holds as long as $\Gamma_s b_1 > 0$, and we showed $\frac{dV}{db_1} < 0$ for some Γ_s. This completes the proof.

A.2 Proof of Proposition 1

In the same way we did in the proof of lemma 1, we let b_1 and b_2 as functions of R_1. Taking a derivative with respect to R_1 gives us

$$\begin{aligned}
\frac{dV}{db_1} = &\left[-u_0^T + \beta\left(1 + \overline{r}\right)\mathbb{E}\left[u_1^T\right]\right] + \frac{db_1}{dR_1}\left(u_0^T - \left(1 + r^* - 2\Gamma_j b_1\right)\beta \int_{\underline{\phi}}^{\overline{\phi}} u_1^T d\mathscr{F}_\phi\right) \\
&+ \beta\left(\frac{db_1}{dR_1}\left(1 + r^* - 2\Gamma_j b_1\right) + \left(1 + \overline{r}\right)\right) \int_{\underline{\phi}}^{\overline{\phi}} \left[\frac{db_2}{dw_1}\left(u_1^T - \beta\left(1 + r^* - 2\Gamma_b b_2\right) u_2^T\right)\right] d\mathscr{F}_\phi + \\
&\frac{d\phi^c}{dR_1}\left[\left(\beta u\left(c_1^T\left(b_2\left(\phi^{c+}\right)\right)\right) + \beta^2 u\left(c_2^T\left(b_2\left(\phi^{c+}\right)\right)\right)\right) - \left(\beta u\left(c_1^T\left(b_2\left(\phi^{c-}\right)\right)\right) + \beta^2 u\left(c_2^T\left(b_2\left(\phi^{c-}\right)\right)\right)\right)\right]
\end{aligned}$$

We know $\beta u\left(c_1^T\left(b_2\left(\phi^{c+}\right)\right)\right) + \beta^2 u\left(c_2^T\left(b_2\left(\phi^{c+}\right)\right)\right) = \beta u\left(c_1^T\left(b_2\left(\phi^{c-}\right)\right)\right) + \beta^2 u\left(c_2^T\left(b_2\left(\phi^{c-}\right)\right)\right)$. Also we know $u_0^T - \beta(1 + r_1)\mathbb{E}\left[u_1^T\right] = 0$ and $u_1^T - \beta(1 + r_2)\mathbb{E}\left[u_2^T\right] = 0$ for $\phi \in \left(\phi^c, \overline{\phi}\right)$. Taking all of these and letting $\frac{dw_1}{dR_1} = \frac{db_1}{dR_1}\left(1 + r^* - 2\Gamma_j b_1\right) + \left(1 + \overline{r}\right)$ gives the equation (15), (16).

A.3 Derivation of the Optimal Tax

First, we show the borrowing of decentralized households under the optimal taxation is same as the direct choice of the social planner. The Euler equation of the households under the taxation will be

$$u_0^T(b_1, ;) = \beta(1 + r_1)(1 + \tau_d)\left(\int_{\underline{\phi}}^{\phi^c} u_1^T\left(b_1, b_{2,c}, ;\right) d\mathscr{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T\left(b_1, b_{2,u}, ;\right) d\mathscr{F}_\phi\right) \quad \text{(A.8)}$$

The solution of the planning is τ_d such that the equation (A.8) is ex-post identical to the equation (A.6), which characterizes the borrowing determined by the government. Of course, it implies $b_1^{priv}(\tau_d) = b_1^{sp}$. Ignoring the term $r_1\tau_d$, solving for τ_d such that the equation (A.8) is same as the equation (A.6) yields the characterization of the optimal taxation in the equation (A.8).

A.4 Proof of Proposition 2

The first statement in the proposition follows from the definition of the optimal tax. To prove the second and third statements, rewrting the first order conditions in the equations (18),(19) are as below.

$$-u_0^T + \beta u_1^T \left(1 + r^* - 2\Gamma_s b_1^{sp}\right) + \beta \int_{\underline{\phi}}^{\phi^c} \left(-u_1^T + \beta u_2^T (1 + r_2)\right) \frac{db_2}{dw_1} \left(1 + r^* - 2\Gamma_s b_1^{sp}\right) = 0 \quad \text{(A.9)}$$

$$-u_0^T + \beta u_1^T \left(1 + \overline{r}\right) + \beta \int_{\underline{\phi}}^{\phi^c} \left(-u_1^T + \beta u_2^T (1 + r_2)\right) \frac{db_2}{dw_1} \left(1 + \overline{r}\right) + \eta = 0 \quad \text{(A.10)}$$

where η is the multiplier of the non-zero constraint of reserve accumulation.

If $b_1^{sp} < 0$, then $1 + r^* - 2\Gamma_s b_1^{sp} > 1 + \overline{r}$. It implies $\eta > 0$. Hence, if If $b_{t+1}^{sp} < 0$ and $b_1^{sp} = b_{t+1}$ then $R^* = 0$

If $b_1^{sp} > 0$, then we may have $1 + r^* - 2\Gamma_s b_1^{sp} = 1 + \overline{r}$. It implies $b_1^{sp} = (r^* - \overline{r})/2\Gamma_s$.

Then the optimal reserve accumulation is

$$R_1 = w_1 - \frac{r^* - \overline{r}}{2\Gamma_s}$$

where w_1 is the optimal net foreign liquid assets chosen by the social planner.

The social planner problem of choosing b_1 and R_1 can be understood the two steps where the planner firstly chooses w_1 and then choose how to compose w_1 with b_1 and R_1. As one might expect, for $b_1 < \frac{r^* - \overline{r}}{2\Gamma_s}$ b_1 is always more efficient. For $b_1 > \frac{r^* - \overline{r}}{2\Gamma_s}$ vice versa.

A.5 Proof of Corollary 1

We formulate the problem to the problem the social planner determine the optimal w_1 and then compare b_1 and R_1.

For the planner who only use the optimal taxation

$$b_1^{sp}\left(1 + r^* - \Gamma_j b_1^{sp}\right) = w_1 \quad \text{(A.11)}$$

For the planner who only use the reserve accumulation

$$b_1^h \left(1 + r^* - \Gamma_j b_1^h\right) + R_1 = w_1 \quad \text{(A.12)}$$

where $j = b, s$.

Assume that $w_1 < 0$. Then obviously $b_1 < 0$ in the both of equations (A.11) and (A.12), and $b_1^h < b_1^{sp}$. Since $1 + r^* - \Gamma_b b_1^h > \overline{r}$, it is always better to reduce R_1 and raise (reduce) $b_1^h \left(-b_1^h\right)$. It implies $V(\tau) > V(R)$ if $b_1^{sp} < 0$.

Next now assume $w_1 > 0$. Let's compute the return to the two different policies. For the optimal taxation policy in equation (A.11)

$$r_1 = 1 + r^* - \Gamma_s b_1^{sp}$$

Similarly, we can compute the return to the EME in equation (17). Denote the return by $\widehat{r}_1$

$$\widehat{r}_1 = \frac{R_1 / \left(1+\overline{r}\right)}{R_1 / \left(1+\overline{r}\right) + b_1^h} \overline{r} + \frac{b_1^h}{R_1 / \left(1+\overline{r}\right) + b_1^h}, \left(r^* - \Gamma_s b_1^h\right)$$

For the same w_1, $b_1^h < b_1^{sp}$. It is trivial that for w_1 large enough, $\widehat{r}_1 > r_1$. This completes the proof.

A.6 Proof of Proposition 3

To prove the second statement we make a reasonable assumption

Assumption A1 . Let the LHS of equation (23) be $h^L\left(b_1\left(\theta, R\left(\theta\right)\right), \theta, R\left(\theta\right)\right)$ and the the RHS be $h^R\left(b_1\left(\theta, R\left(\theta\right)\right), \theta, R\left(\theta\right)\right)$. Then we assume

1. $| \frac{dh^L}{d\theta} |_R = | \frac{\partial h^L}{\partial \theta} + \frac{\partial h^L}{\partial b_1} \frac{\partial b_1}{\partial \theta} | > | \frac{dh^R}{d\theta} |_R = | \frac{\partial h^R}{\partial \theta} + \frac{\partial h^R}{\partial b_1} \frac{\partial b_1}{\partial \theta} |$
2. $\frac{\partial h^L}{\partial b_1} \frac{\partial b_1}{\partial R_1} + \frac{\partial h^L}{\partial R_1} > 0$

To see why the assumption A1 can easily hold, first see $\frac{dh^L}{d\theta}, \frac{dh^R}{d\theta} < 0$. Then b_1 chages the LHS directly, but affetct RHS through b_2,which makes the RHS less responsive to b_1. The second inequality easily holds for most of the parameter values including some exterme values.

Now we prove the statements in the proposition. To show the first statement, recall that we have $\frac{dh^L}{d\theta}, \frac{dh^R}{d\theta} < 0$. With $\theta = \tilde{\theta}$, $h^L\left(b_1, \theta\right) = 0$, while $h^R > 0$ since $b_2 < 0$ for all ϕ. It implies that at $\theta = \tilde{\theta}$ $R_1^* > 0$. Since $\frac{\partial b_1}{\partial \theta} > 0$, $R_1^* > 0$ for all $\theta > \tilde{\theta}$. Then we need to show there exists $\delta > 0$ such that $R_1^* > 0$ $\theta \in \left(\tilde{\theta} - \delta, \tilde{\theta}\right)$. See both h^L and h^R are continuous in θ and R_1. For δ small enough, $h^L\left(b_1\left(\tilde{\theta} - \delta, 0\right), \tilde{\theta} - \delta, 0\right)$ is positive, but smaller than h^R by the continuity of h^L and h^R. It implies $R_1^* > 0$ $\theta \in \left(\tilde{\theta} - \delta, \tilde{\theta}\right)$.

Second statement can easily follow from the assumption. The envelope condition yields

$$\frac{\partial h^L}{\partial \theta} + \frac{\partial h^L}{\partial b_1}\left(\frac{\partial b_1}{d\theta} + \frac{\partial b_1}{dR}\frac{dR_1}{d\theta}\right) + \frac{\partial h^L}{\partial R_1}\frac{dR_1}{d\theta} = \frac{\partial h^R}{\partial \theta} + \frac{\partial h^R}{\partial b_1}\left(\frac{\partial b_1}{d\theta} + \frac{\partial b_1}{\partial R}\frac{dR_1}{d\theta}\right) + \frac{\partial h^R}{\partial R_1}\frac{dR_1}{d\theta}$$

Then we must have $\frac{dR_1}{d\theta} > 0$ under the assumption A1.

Lastly, we prove the third and forth statements in the proposition. The resource constraints in tradable goods are

$$c_0^T\left(\theta, R_1\right) + b_1\left(\theta, R_1\right) + R_1 = \left(1-\theta\right) y_0^T + Q_0 \theta K$$
$$c_0^T\left(\tilde{\theta}, 0\right) + b_1\left(\tilde{\theta}, 0\right) = \left(1-\tilde{\theta}\right) y_0^T + Q_0 \tilde{\theta} K$$

Since $(1-\theta)\, y_0^T + Q_0\theta K = y_0^T + (Q_0 - A_0)\,\theta K$, we have

$$R = (Q_0 - A_0)\left(\theta - \tilde{\theta}\right) K + c_0^T\left(\tilde{\theta}, 0\right) - c_0^T\left(\theta, R_1\right) + b_1\left(\tilde{\theta}, 0\right) - b_1\left(\theta, R_1\right)$$

We know, by definition, $r^* - \overline{r} - \Gamma_s b_1\left(\theta, R_1\right)\left(1 - \frac{db_1(\theta, R_1)}{dR_1}\right) > 0$ and $r^* - \overline{r} - \Gamma_s b_1\left(\tilde{\theta}, 0\right)\left(1 - \frac{db_1(\tilde{\theta}, 0)}{dR_1}\right) = 0$. It implies $b_1\left(\tilde{\theta}, 0\right) > b_1\left(\theta, R_1\right)$

To show the last statement, notice that $\frac{\partial c_0^T}{\partial \theta} > 0$, $\frac{\partial c_0^T}{\partial R_1} < 0$ and $c_0^T\left(\tilde{\theta}, R_1^*\left(\tilde{\theta}\right)\right) < c_0^T\left(\theta, 0\right)$. Thus we have $c_0^T\left(\tilde{\theta}, R_1^*\left(\tilde{\theta}\right)\right) < c_0^T\left(\tilde{\theta}, 0\right)$. If $\frac{\partial c_0^T}{\partial \theta} + \frac{\partial c_0^T}{\partial R_1}\frac{dR_1}{d\theta} < 0$, then it is obvious. Even if $\frac{\partial c_0^T}{\partial \theta} + \frac{\partial c_0^T}{\partial R_1}\frac{dR_1}{d\theta} > 0$, there always exists $\delta > 0$ such that $c_0^T\left(\tilde{\theta} + \delta, R_1^*\left(\tilde{\theta} + \delta\right)\right) < c_0^T\left(\theta, 0\right)$. This completes the proof.

B The Model with Heterogeneous Agents

We introduce a heterogeneous agents model. Our goal in the model is to show how our main results in the representative agent model can survive in the new heterogeneous agents model rather than solve the heterogeneous model fully. We borrow some features from Korinek and Sandri (2016).

The environments of productions, international financial intermediations (IFIs), and direct investors are same as the baseline model. However, there are two heterogeneous agents in the small open economy: Borrower and Saver. We may think that savers are the group who receives direct investments; hence they sold their tradable goods capitals to foreign direct investors. The savers need to lend their tradable goods to the borrowers or invest abroad. Similarly, the borrowers can borrow from either IFIs or the savers. We denote the total borrowing of the borrowers by b_{t+1}^b and similarly the total saving of the savers by b_{t+1}^s. To differentiate the borrowings from IFIs from the total borrowings, we let $b_{t+1}^{b^*}$ be the borrowing from IFIs. Similarly, we define the overseas investment by the savers as $b_{t+1}^{s^*}$.

The social planner issues their own bonds to accumulate reserves. Let's denote it by b_{t+1}^g. Similarly with $b_{t+1}^{b^*}$, define $b_{t+1}^{g^*}$. The market clearing condition of the domestic funds market is as below.

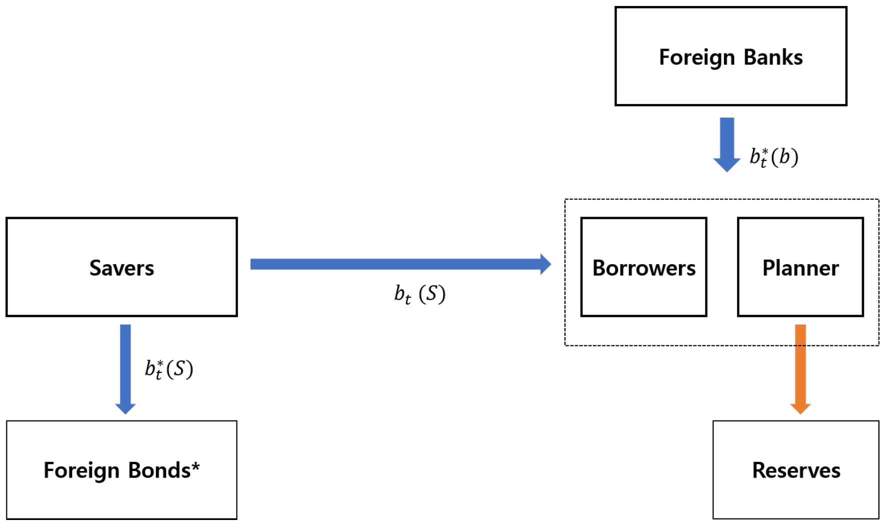
$$b_{t+1}^b - b_{t+1}^{b^*} + b_{t+1}^g - b_{t+1}^{g^*} + b_{t+1}^s - b_{t+1}^{s^*} = 0 \tag{A.13}$$

That is, the total demand for the tradable goods borrowing in the domestic market is $b_{t+1}^b - b_{t+1}^{b^*} + b_{t+1}^g - b_{t+1}^{g^*}$, while $b_{t+1}^s - b_{t+1}^{s^*}$ is the supply from the savers in the domestic market. The description of flows of funds is provided in Figure A.1 below.

To clear the market, we need one more market clearing condition, by which borrowers (savers) are indifferent between borrowing from IFIs and savers (lending abroad and lending to the borrowers). To have the condition, let's assume that the yields from investing abroad without the fee to IFIs are higher than the borrowing rates. The net returns to the savers are characterized in the same way, but now

$$r_{t+1} = -\Gamma_s b_{t+1}^{s^*} + r^{**} \tag{A.14}$$

Figure A.1: Flow of Funds



Similarly, for the borrowing rates from IFIs,

$$r_{t+1} = -\Gamma_b \left(b^{b^*}_{t+1} + b^{g^*}_{t+1} \right) + r^* \tag{A.15}$$

Since $b^{b^*}_{t+1} < 0$ and $b^{s^*}_{t+1} > 0$, for the market to be cleared, we need $r^{**} > r^*$[46].

In period 1, the credit constraint can bind and it is as follows.

$$-b^{b^*}_{t+1} \leq \phi_t \left((1-\theta_t)\, y^T_t + p_t y^N_t \right) \tag{A.16}$$

That is, the amount of the total external debt by the borrowers is constrained by the aggregate GDP. The credit constraint does not include $b^{g^*}_{t+1}$ since the social planner does not borrow abroad when the credit constraint binds[47]. Furthermore, during a sudden stop, nontradable goods are really cheap so that savers dispose of all their assets abroad to consume more nontradable good; the retrenchment in Forbes and Warnock (2012).[48] Hence, we can imagine that from period 1, the different groups of the agents merge into one big family so that the model backs to the representative agent model.

The return to reserves, b^g_{t+1} is low at $\overline{r} < r^*$. Thus, the planner has to collect tax from the agents to pay the extra interest rates, $r_{t+1} - \overline{r}$. We assume the tax is imposed optimally so that it does not make any extra terms in the first order condition of the reserve accumulation, which we introduce below. It turns out to be optimal to impose the tax on the savers under reasonable parameter values.

Taking all the changes, we derive the first-order condition for the optimal reserve accumulation in period 0.[49]

[46] The r^* is the required rate for an economy who have nearly do external debt in terms of gross. Hence it must be low.

[47] It is different in a more dynamic version of the model as it is explained in the next subsection.

[48] Such a nice retrenchment does not strictly hold in the reality due to risk hedging motives or low confidence of the savers about the economy.

[49] We assume the planner assign equal weights to each of the borrower and the saver.

$$\underbrace{\beta\Gamma_b\mathbb{E}\left[u^b_{1,T}\right]b_1^{b^*}\frac{d(b_1^{b^*}+b_1^{g^*})}{dR_1}}_{Higher\, r_1}+\underbrace{\left[u^s_{0,T}-\beta\left(1+\overline{r}\right)\mathbb{E}\left[u^s_{1,T}\right]\right]}_{Consumption\, Wedge\, at\, \overline{r}}=$$

$$\frac{dw_1}{dR_1}\left[\underbrace{\sum_{i=b,s}\int_{\underline{\phi}}^{\phi^c}\beta\frac{d\left(-b_2^i\right)}{dw_1}\left(u^i_{1,T}-\beta\left(1+r_2\right)u^i_{2,T}\right)d\mathscr{F}_\phi}_{Marginal\, Value\, of\, Borrowing}\underbrace{-\beta\Gamma_b\mathbb{E}\left[u^i_{2,T}b_2^i\frac{db_2^i}{dw_1}\right]}_{Lower\, r_2}\right]\underbrace{-\beta\Gamma_s\mathbb{E}\left[u^s_{1,T}\right]b_1^{s^*}\frac{db_1^{s^*}}{dR_1}}_{Higher\, r_1} \tag{A.17}$$

where $w_1=-b_1^{g*}\left(1+\overline{r}\right)+\left(b^{b^*}_{t+1}+b^{g^*}_{t+1}+b^{s^*}_{t+1}\right)(1+r_1)$, $R_1=-b_1^g$.
and $\frac{dw_1}{dR_1}=\left(r_1-\overline{r}\right)\frac{db_1^{g^*}}{dR_1}+(1+r_1)\left(\frac{db_1^{b^*}}{dR_1}+\frac{db_1^{s^*}}{dR_1}\right)-\left(b^{b^*}_{t+1}+b^{g^*}_{t+1}+b^{s^*}_{t+1}\right)\left(\Gamma_b\left(\frac{db_1^{b^*}}{dR_1}+\frac{db_1^{g^*}}{dR_1}\right)\right)$.
From (27), we see $b^b_{t+1}+b^g_{t+1}+b^s_{t+1}=b^{b^*}_{t+1}+b^{g^*}_{t+1}+b^{s^*}_{t+1}$. Then

$$\frac{dw_1}{dR_1}=\left(1+\overline{r}\right)+(1+r_1)\left(-1+\frac{db_1^b}{dR_1}+\frac{db_1^s}{dR_1}\right)-\left(b^{b^*}_{t+1}+b^{g^*}_{t+1}+b^{s^*}_{t+1}\right)\left(\Gamma_b\left(\frac{db_1^{b^*}}{dR_1}+\frac{db_1^{g^*}}{dR_1}\right)\right) \tag{A.18}$$

See $\frac{db_1^j}{dR_1}>0$ for $j=s,b$. Therefore, for Γ_b,Γ_s large enough, $\frac{dw_1}{dR_1}>0$. The mechanism of how reserve accumulation improves the net foreign asset position is analogous to the baseline model. Since it raises the borrowing rates and therefore returns to the savers as well, the interventions to accumulate reserves discourage the borrowing, but encourage the saving.

As one might expect, the mechanism in proposition 1 and proposition 3 applies. As direct investment capitals inflows, $-b_1^{b^*}$ decreases, but $-b_1^{s^*}$ increases. It reduces marginal costs of reserve accumulation while raising the benefits. Therefore, we have the following proposition.

Proposition 4. *The optimal reserve accumulation in the heterogeneous agents model is characterized as follows.*

1) It is characterized by the first-order condition

$$\left[r^{**}-\overline{r}-\Gamma_s b_1^{s^*}(\theta)+\Gamma_s b_1^{s^*}\frac{db_1^{s^*}(\theta)}{dR_1}\right]\mathbb{E}\left[u^s_{1,T}\right]+\Gamma_b b_1^{b^*}(\theta)\frac{d(b_1^{b^*}(\theta)+b_1^{g^*})}{dR_1}\mathbb{E}\left[u^b_{1,T}\right]=$$
$$\frac{dw_1}{dR_1}\left[\sum_{i=b,s}\int_{\underline{\phi}}^{\phi^c}\beta\frac{d\left(-b_2^i\right)}{dw_1}\left(u^i_{1,T}-\beta\left(1+r_2\right)u^i_{2,T}\right)d\mathscr{F}_\phi-\beta\Gamma_b\mathbb{E}\left[u^i_{2,T}b_2^i\frac{db_2^i}{dw_1}\right]\right] \tag{A.19}$$

2) There exist δ such that for $\theta>\theta^c-\delta$, $R_1^*>0$ and R_1^* increases in θ
3) There exists δ_1 such that $R_1^*>(Q_0-A_0)(\theta-\theta^c)K$ for $\theta\in(\theta^c,\theta^c+\delta_1)$.

Proof) See the discussion above.

Proposition 4 is analogous to proposition 3. Therefore, our analytics and insights in the baseline model survive and hold in the heterogeneous agents model. As one might expect, more direct investment capital inflows - higher θ - make more borrowers switch to savers and the increase in the share of savers in the economy force the planner to accumulate more reserves.

C Endogenous Direct Investments and Capital Price

We look at the extension where the direct investment and capital price are endogenous. Same as the last two extensions, we describe our new environment and then show why our key results do not change rather than solve the model fully. In the reality, direct investment in an EME depends on various factors such as locations, natural resources, or macroeconomic stabilities. Obviously, still we cannot reflect all the realistic features. We simply assume that the direct investment is an increasing function of the profitability of the investment. First, define

$$\pi^T = \frac{\sum_{t=0}^{2} M_0^t A_t}{Q_0}$$

where M_0^t is the discount factor of the investor. π^T is the gross return rate to the direct investment in the tradable goods sector capital. Then we can assume

$$\theta Q_0 K^T = F_T(\pi_T) \tag{A.20}$$

Hence, how much direct investors purchase the tradable goods capitals depends on the profitability. Of course, we assume $F' > 0$.

We also endogenize the capital price. To model the capital price, we can posit that the domestic capital market is perfectly competitive. Then the capital price must be

$$\begin{aligned} Q_0 &= \sum_{t=0}^{2} \beta^t \frac{u_t^T}{u_0^T} A_t \\ &= A_0 + \frac{A_1}{1+r_1} + \mathbb{E}\left[\frac{A_2}{(1+r_1)(1+r_2)}\right] \end{aligned}$$

We can easily see the reserve accumulation will impact the capital price through the changes in the interest rates; the borrowing rates and returns to the saving are the linear functions of the borrowing and saving, which are changed by reserve accumulation. Then from previous sections, once $b_1 > 0$ we expect r_1 increases in reserve accumulation, while r_2 changes in the opposite ways to the reserve accumulation, depending on whether the credit constraint binds or not. However, with plausible parameter values, we can expect the effect through r_1 dominates. Hence, we reasonably conjectures

$$\frac{\partial Q_0}{\partial R_1} < 0$$

Then of course, θ increases in reserve accumulation R_1 because the lower capital price boosts the profitability of the direct investment.

It provides two opposite different implications of reserve accumulations. First, the reserve accumulation by the social planner would have a sort of self-multiplication effect. As we saw in the last section, reserve accumulation increases in the amounts of direct investment. Here the reserve accumulation attracts more direct investments which in turn calls for more reserve accumulation. Second, in the other way, reserve accumulation discounts the capital price, which is unfavorable for the EME. However, such negative impacts, which might be small, could be offset by possible positive effects, which we abstract in our model. For example, any knowledge spillover effects can be probably more than offsetting the negative impacts. Overall, although it is hard to make a strong assumption since the environment in our model is not rich enough, the optimal reserve accumulation could rise once we let the direct investment and the capital price be endogenous.

Another strict restriction imposed in our model is that we exclude direct investments in the nontradable goods sector. Now we allow the investments in the nontradable goods capital. Let's denote the share of foreign direct investors in the domestic tradable goods and nontradable gods capital markets by θ^T and θ^N respectively. Direct investors interested in holding K^N also decide their investments based on the expected profitability. The crucial difference is the investors convert the returns of nontradable goods to tradable goods[50]. Hence the expected profitability of the nontradable goods capital investments is

$$\pi^N = \mathbb{E}_0\left[\frac{\sum_{t=0}^{2} M_0^t p_t A_t}{p_0 Q_0}\right]$$

Notice that the expected profit increases in the expected appreciation $\mathbb{E}\left[\frac{p_{t>0}}{p_0}\right]$. Also, from previous results, we can notice

$$\frac{d\left(\mathbb{E}\left[\frac{p_t}{p_0}\right]\right)}{dR_1} > 0 \ \ for \ t = 1,2$$

Intuitively, the reserve accumulation has effects of increasing net foreign assets in the future, which raise the price of the nontradable goods prices in the future, but lowers the price in the present. Therefore, the direct investment in nontradable goods sector increases in the reserve accumulation.

This is the mechanism examined in handful papers that studied the relationship between foreign direct investment and reserve accumulation. Matsumoto (2022) argued that EMEs accumulate reserves to attract more direct investments. In contrast, we provide the causality in the opposite direction: direct investments cause reserve accumulations. However, we also have a similar mechanism with Matsumoto (2022). Reserve accumulation causes the currency depreciation, while promising appreciations in the future, and therefore it is more profitable to invest in the EME. But, we do not believe that EMEs accumulate reserves to attract direct investments as we discussed in the literature review section[51]. More plausible scenario is that

[50] Such assumption is common in the local currency sovereign debt literature.

[51] Another difficulty in the explanation that reserve is accumulated to attract direct investments through currency depreciation is a possibility that EME policy authorities may depreciate their currency in the future to dilute "real liabilities" measured in foreign currencies. If currency valuations are truly important in making direct investment

EMEs facing lots of direct investment capital inflows make corresponding outflows in the form of reserve and the reserve accumulation again attracts more direct investments, which calls for even more reserve accumulation.

We close this section summarizing our findings in the claim below.

Claim. *Once we endogenize direct investment and the capital price, and allow nontradable goods sector capital investment as above, we have following properties.*

1) Reserve accumulation makes it more profitable for foreign direct investors to purchase domestic capitals since it discounts the current capital price through lower domestic interest rates or lower real exchange rates.

2) Therefore, reserve accumulation may attract more direct investments, which in turn incentives EMEs to accumulate even more reserves. As a result, we have a sort of loop mechanism, by which magnifies both of direct investment inflows and reserve outflows.

D Capital Controls over Direct Investment Flows

If reserve accumulation is a reaction to direct investment flows beyond a level above which households are forced to save in an inefficient way, one easy solution would be to limit the direct investments themselves. For example, EME governments may ban foreign investors from buying domestic assets or set a cap, above which foreign investors cannot buy more. Regardless of the difficulties in implementing these regulations, it is hard to analyze the optimal control over direct investment flows in this paper. Direct investment flows are viewed better than debt flows not just because it is more stable, but also there might be some technological spill-over effects. Such unobservable positive effects might be the same for equity portfolio investments; for example, it promotes the development of domestic stock markets. Those externalities may exist or not, and moreover, the quantitative importance is hard to measure. Since these are beyond the scope of our paper, we mute all these channels, through which the capital inflows positively impact EMEs. Interestingly, even without those externalities, it turns out that the social planner in our model economy wants to receive direct investments to some extent, which provides incentives to accumulate reserves.

To make the problem simple, let's suppose that the social planner can choose θ. Further, assume that the capital is priced by the stochastic discount factors of households. That is

$$Q_0 = \sum_{t=0}^{2} \left(\beta^t A_t \frac{u_t^T}{u_0^T} \right)$$

decisions, such a time-inconsistent incentive to depreciate currency in the future must significantly impugn the direct investors' interest so as to make the reserve accumulation less useful policy to attract direct investments. Such an ex-ante desire to depreciate currency is widely studied in the local currency sovereign debt literature; see Du et al. (2016), Engel and Park (2018), and Perez and Ottonello (2019). These papers concluded that desires to dilute local currency significantly limit the gain from local currency sovereign debts.

Hence there is no extra gain from direct investments in terms of price. Let θ^* be θ chosen by the social planner. Then FOC of θ^* is as follows.

$$\underbrace{\mathbb{E}\left[-\beta\Gamma_j b_1 u_1^T \frac{db_1}{d\theta} - \beta^2 \Gamma_b b_2 u_2^T \frac{db_2}{d\theta}\right]}_{Changes\, in\, r_1, r_2} - \underbrace{\int_{\underline{\phi}}^{\phi^c} \frac{db_2}{d\theta}\left(u_1^T - \beta(1+r_2) u_2^T\right)}_{Marginal\, Value\, of\, Borrowing} + \underbrace{\frac{dQ_0}{d\theta} u_0^T}_{Change\, in\, Q_0} = 0 \tag{A.21}$$

We interpret the equation (21) after introducing our second lemma driven from the equation (21).

Lemma 2. *Suppose* $\frac{\partial b_2}{\partial \theta} |_{\phi<\phi^c} < 0$ *but* $E\left[u_2^T b_2 \frac{\partial b_2}{\partial \theta}\right] < 0$. *Then with* $\theta = \theta^*, b_1^h > 0$.

Proof) It it obvious that all the three terms in equation (21) are positive under $b_1^h < 0$. *If* $b_1 > 0$, *then the first term* $\mathbb{E}\left[-\beta\Gamma_j b_1 u_1^T \frac{db_1}{d\theta}\right]$ *is negative. Therefore, for the equation (21) to hold,* b_1^h *must be positive.*

To give some intuition to equation (21) and lemma 2, first notice that all the terms in equation (21) are related to the externalities from borrowing or saving by households. Any borrowing or saving by households affects interest rates and real exchange rates in the future, of which the households do not take account. Since direct investments change b_1, it generates the same externalities as well through the changes in b_1. The first and second term are related to changes in r_1 and r_2 respectively, and the last term is the changes in borrowing under sudden stops through real exchange rates in the collateral constraint[52].

As stated in lemma 1 and the comparative statics in the last section, it is always $b_1^h < b_1^{sp}$ if $b_1^h < 0$ and $\frac{\partial b_1}{\partial \theta} > 0$. Then it is straightforward that more direct investment is beneficial since it lessens the overborrowing problem: Receiving more direct investment flows is desirable for the social planner as long as the economy is in the state of overborrowing. To give more economic interpretation, our model EME "transfers" resources from the future to the present for the consumption smoothing. However, because of the imperfect capital mobilities and credit constraint, the economy has trouble having transfer resources from the future. Direct investments do provide another way of transferring the resources while circumventing the frictions in the external borrowings. As a result, it is optimal to allow direct investors to purchase domestic capitals at least until the borrowing in the form of debt alters to a saving.

How is it related to reserve accumulation? The social planner in our model does not limit direct investments until households begin lending abroad. Also, we know that we may have the optimal reserve accumulation at positive amounts when households save. Hence if the social planner can do both reserve accumulation and control of direct investments, the social planner is likely to accumulate reserves. It is more likely if we add some positive externalities from direct investments, which we do not consider in this paper.

[52]Full characterization of $\frac{\partial b_2}{\partial \theta} |_{\phi<\phi^c}$ and conditions to guarantee the assumptions in lemma 1 are in appendix

E Reserve Accumulation of Saving Glut EMEs

In this section in the appendix, we remove one assumption that we have kept throughout this paper. Now we assume that our model EME saves even in period 1. We can think of an EME experiencing rapid aging or saving excessively due to some reasons[53]. In the data, some EMEs, in particular East Asian EMEs, have sizably positive net foreign asset positions. We show how our model can explain reserve accumulation in such EMEs.

To model EMEs that save in period 1, we assume $y_0^T > (1-\theta)\, y_1^T > (1-\theta)(1-\sigma)\, y_2^T$. Hence, we can posit $b_1, b_2 > 0$ and therefore there is no chance of sudden stop in period 1[54]. Interestingly, the planner in the EME might be incentivized to accumulate reserves even when $b_2 > 0$. The optimal reserve accumulation in period 1 is characterized by

$$r^* - \overline{r} - \Gamma_s b_2 \left(1 - \frac{d b_2}{d R_2}\right) = 0 \tag{A.22}$$

The equation (A.15) does not necessarily hold, but it holds if $R_2 > 0$. On the contrary, if equation (A.15) does not hold, we have $R_2 = 0$. As one might expect, $\frac{db_2}{dR_2}$ is almost invariant to R_2. Then, conditioning on $R_2 > 0$, the equation (A.20) almost pins down b_2. The intuition behind this results is rather straightforward. The return to private overseas investments (for the planner, not for the households) decrease in the investment amounts and it implies that the return to the investment beyond a certain level is even below the return to the reserve. Hence, the planner chooses reserve accumulation by a mean of national saving.

Now we characterize the reserve accumulation in period 0, while accommodating the results in equation (A.15). The optimal reserve accumulation is characterized by the equation (A.16) below.

$$\left[r^* - \overline{r} - \Gamma_s b_1(\theta)\left(1 - \left(\frac{\partial b_1(\theta)}{\partial R_1} + \frac{\partial b_1(\theta)}{\partial R_2}\frac{dR_2}{dR_1}\right)\right)\right]\mathbb{E}\left[u_1^T\right] = -\frac{dw_1}{dR_1^*}\beta\Gamma_s\mathbb{E}\left[u_2^T b_2 \frac{db_2}{dw_1}\right] \tag{A.23}$$

See the RHS is negative and as we saw in equation (A.15), b_2 is almost fixed once we have $R_2 > 0$. Once the value of RHS is highly invariant, then we need to have invariant LHS as well: the RHS pins down b_1 in the LHS; let's denote it as b_1^c. It implies that for EMEs to need external assets more than b_1^c, the extra demands for external assets beyond b_1^c is absorbed by reserve accumulation. To understand it, think of an EME that has current account surpluses and receive lots of direct investments. The EME needs to accumulate external assets, but accumulating external assets by private sectors accompanies nonnegligible inefficiencies. As we have repeated throughout this paper, in such a case the planner accumulate reserve to replace the inefficient overseas investment by private sectors. Hence, passive capital inflows (direct investments or equity portfolio investments) to such EMEs generate seemingly excessive reserve accumulation. Although we cannot make a strong claim because our model is not suitable for quantitative analysis, the result in this section potentially explain why some East Asian EMEs with high net positive foreign asset positions, for example Malaysia, Thailand, and China, hold high levels of international reserves from 30 to 40 percent of GDP.

[53] For example, low financial development in certain EMEs lead the EMEs to save excessively. See Caballero et al. (2008), Mendoza et al. (2009) and Maggiori (2017).

[54] It might be little extreme and such results stem from our modeling of sudden stop

F Micro-foundation for Social Cost of Tax Evasion Using Overseas Investment

In this section, we introduce a model of tax evasion using overseas investments. Purpose of introducing the simple model is to build microfoundations for the cost of overseas investments due to tax evasions.

The environments are same as the model in section 3, except that there is no credit constraint in period 2. This is for simplicity and to focus on what the model in this section describes and the public expenditures along with tax collection.

The government (or social planner) collects has to make public expenditures in period 2 and thus has to collect taxes in period 1. The tax is collected from all the tradable goods streams, i.e., tradable goods output and gross returns to the overseas investments in period 1. Let's say the tax rate on the tradable goods streams is t. One important assumption here is that the public expenditures in period 2 is determined by the amounts of collected taxes in period 2. Thus, the government expenditures are constrained by the amounts of collected tax. This is a realistic assumption if governments in EMEs cannot substantially raise tax rates and cannot borrow from either of domestic investors or foreign investors.

Another important assumption is that households can conceal of part of their incomes from overseas investments, and the portion they can conceal is proportional to the amounts of overseas investments. That is, if l is the fraction that households can conceal, then

$$l = \frac{1}{2}\gamma a$$

where a is the overseas investments. And the government expenditures are as follows.

$$G = \left[y_1^T + a(1 + r_1 - l) \right] t$$

Then the marginal decrease of government expenditures in overseas investments is γa, which is not taken account by the households as the households take the total tax collections and government expenditures as given.

More formally, if the utility from government expenditures is u_G, then

$$\frac{\partial V}{\partial a} = -u'_G \gamma a < 0$$

where V is the lifetime utility of households. Therefore, the overseas investments induce social cost overlooked by households. As discussed in section 3, adding some management costs of overseas investments results in a similar form with the specification in section 3, in which more overseas investments induce more rent extraction to foreign financial intermediary. In reality, overseas investments by residents in EMES, in particular direct investments, can be exploited for tax evasion and governments in EMEs might replenish the reduced tax revenues by imposing more taxes on domestic consumption and incomes. Of course, this should create more distortions, which are another form of social costs, which are also overlooked by households.

G Capital Outflows Restrictions in EMEs

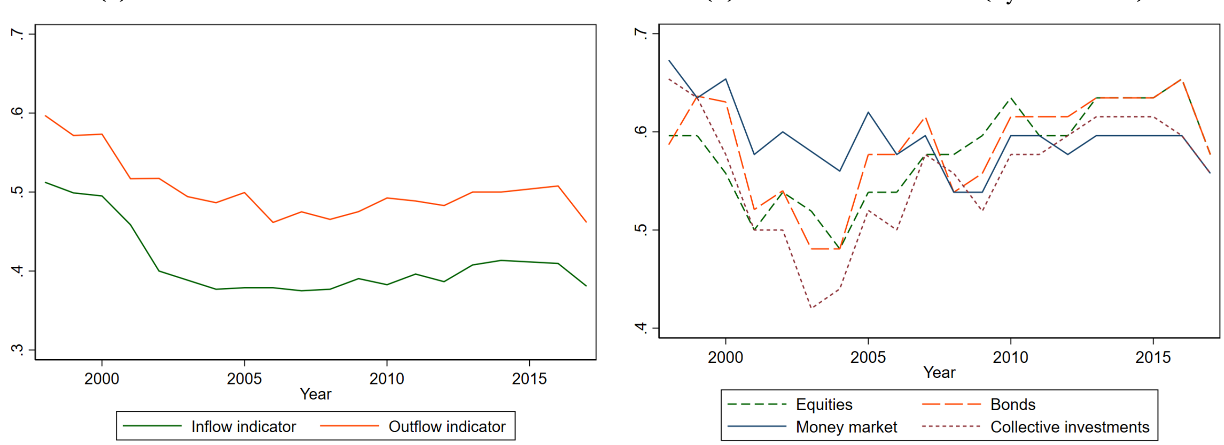
Our model assumes that there are frictions in the foreign investment of domestic agents so that private capital outflows do not sufficiently occur. This could be due to the under-development of the domestic financial sector. Another significant impediment to the gross capital outflows is the regulatory restriction. In this section of the appendix, we review the capital outflow restrictions in the EMEs.

Although there has been less attention compared to capital inflow restrictions, the restrictions on capital outflows have been significant in many EMEs. Based on the capital control data of Fernandez et al. (2016), we assess capital outflow restrictions in the 33 sample EME countries. Figure A.2 (a) presents the evolution of capital control restrictions on both inflows and outflows for our sample countries. While we observe strong co-movement of the two series over time, we find that there have been more restrictions on outflows than inflows. Fernandez et al. (2016) also note that this pattern of strict outflow restrictions is more noticeable for EMEs than advanced economies. Figure A.2 (b) shows capital outflow restrictions in EMEs by different asset classes. The outflow controls on money market instruments (which include bonds with a maturity of one year or less) have maintained a relatively high level.

To provide a detailed illustration of what happened in the capital flow restrictions, we focus on one country, Korea. Korea had strict restrictions on capital outflows by 2004. The Fernandez et al. (2016) measure averaged 0.64 during the 1998-2004 period. The inflow restriction index averaged 0.51 for the same period. It shows that there were more restrictions on outflow than on inflows. While foreign currency was scarce, it was important for economic development. The government needed to have strong control over outflows. Things changed after the 2000s. After opening up the capital account, the country received large capital inflows and experienced current account surpluses. To maintain the competitiveness of the industrial sector, the government lifted many of the restrictions on capital outflows and encouraged residents to invest abroad. The outflow measure fell down to 0.2 by 2005, and it averaged 0.11 from 2006 to 2017.

Figure A.2: Capital Outflow Restrictions

(a) Inflows and Outflows Restrictions (b) Outflows Restrictions (by asset class)



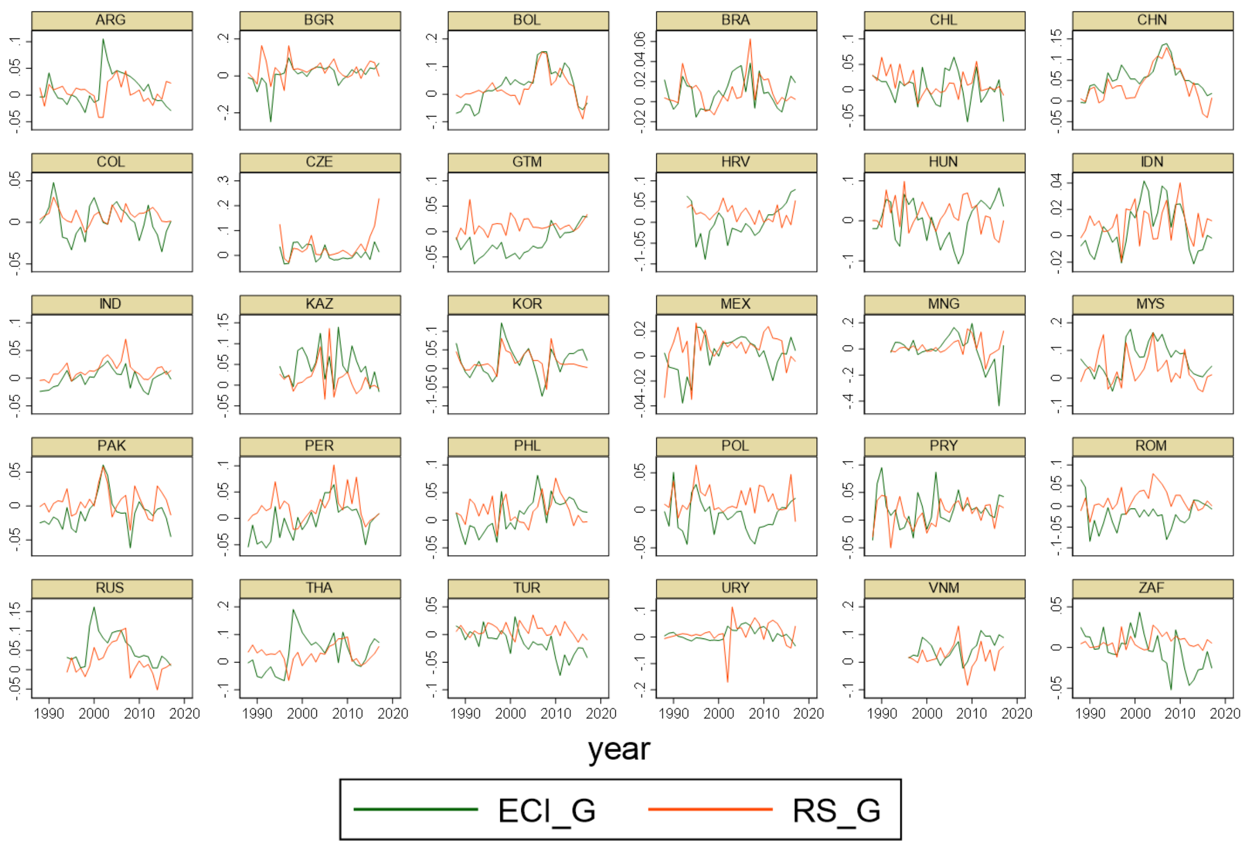
Notes: A higher value means strict restrictions. All values are averaged across the sample countries.
Source: *Capital Control Measures: A New Dataset*, Fernandez et al. (2016)

H Empirical Regularities

H.1 Omitted Figures

Figure A.3 plots the change in the reserve outflows-to-GDP and extra capital inflows-to-GDP for a different sample countries[55] during the period 1998-2017. For most sample countries, the two series show strong co-movement.

Figure A.3: Reserve Outflows and Extra Capital Inflows



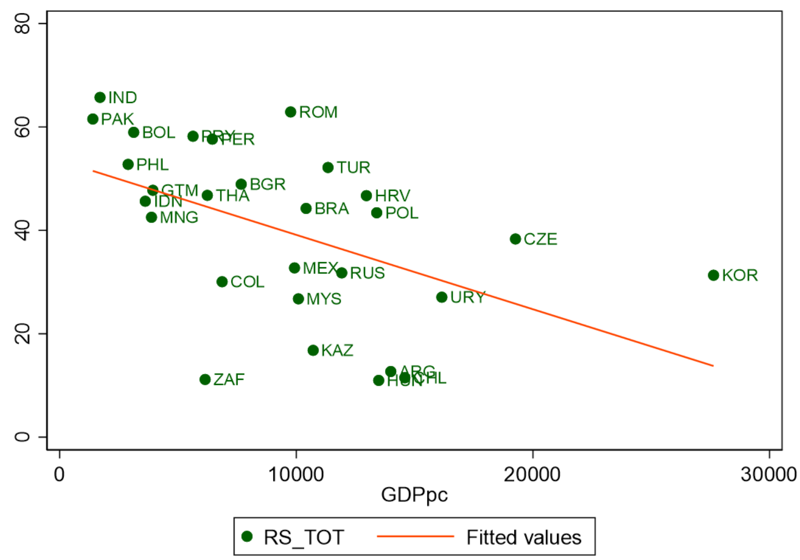
Note: All values are scaled by GDP.
Source : IMF BOP/IIP

Figure A.4 plots per capital GDP and the ratio of reserve asset to total external assets during the period 2013-2017. This Figure shows a negative association between the two variables. That is, the country with higher GDP per capita tends to have less international reserves of the total external assets. The relation might be explained by the following reasons. First, developed countries generally have more efficient financial systems, which suggests their institutional quality is superior enough to generate private capital outflows. Second, in the course of growth, an EME can reduce the reliance on public capital outflows. Korea is a good example in

[55] Argentina, Bolivia, Brazil, Bulgaria, Chile, Colombia, Croatia, Czech Republic, Guatemala, India, Indonesia, Kazakhstan, Korea, Malaysia, Mexico, Mongolia, Pakistan, Paraguay, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Uruguay, Vietnam (28 EMEs)

this regard. In 2003, the ratio of reserves to the total external assets in Korea was 60.1%. But, it has decreased to about 40% recently. On the flip side, it means the portion of private external assets has increased. This divergence between the private and public outflows may be caused by structural changes in private sector's overseas investment such as improved access to foreign financial assets due to technological advances.

Figure A.4: External Asset Structure and GDP per capita



Note: Y axis (*RS_TOT*) is defined as the ratio of reserve assets to the total external assets. X axis (GDP per capita) is dollar value. All values are averaged over 2013-2017.
Source : IMF BOP/IIP

H.2 Omitted Tables

Here, we list the regression results. A difficulty in our regression analysis is that our sample periods include both of the periods of reserve accumulation and the periods of reserve depletion. We want to exclude the periods of sudden stops such as the crises in Latin America in 2002 or other periods alike because we want to see how reserve "accumulation" is associated with different types of capital inflows; during a sudden stop, we should see reserve depletion with falling debt capital inflows and relatively stable FDI inflows, which might cause positive correlations of reserve outflows with debt inflows, but negative correlations with FDI inflows.

To handle the issue, we opt to run regressions in the sample period of 2003-07. This is to avoid the periods of the crises in the sample period.[56] The results of the regressions are provided in Table A.1 below, with the results of the whole sample periods, 1998-2017. As we ex-

[56]The currency crises in Malaysia and Indonesia in 1999, the sovereign debt crisis in Argentina in 2002 that propagated to other Latin America countries, and obviously the Global Financial Crisis in 2008 along with its subsequent turbulent periods such as tapering tantrum in 2013. Furthermore, 2003-07 was the time under the mood of the great moderation except for the subprime mortgage default in 2007, which was yet to propagate to emerging markets.

pected, almost all the coefficients of capital inflows are positive and significant regardless of our choice of sample periods.First, the coefficients of the equity portfolio are larger than the others and moreover the largenesses are statistically significant. Second, once we trim down the sample period to 2003-07, the coefficients of direct investment inflows become larger while the coefficients of debt inflows become smaller or insignificant.

Table A.1: Correlations with reserve flows

Dependent Variable:	Reserve outflows-to-GDP					
Sample Period:	1998-2017			2003-2007		
	(1)	(2)	(3)	(4)	(5)	(6)
FDI inflows	0.27***			0.60***		
	(0.090)			(0.061)		
EQ inflows		0.82***			1.33***	
		(0.186)			(0.111)	
DT inflows			0.31***			0.25***
			(0.083)			(0.036)
Current account	0.26***	0.16**	0.25***	0.42***	0.31***	0.31***
	(0.047)	(0.060)	(0.055)	(0.032)	(0.030)	(0.046)
Constant	0.01*	0.01***	0.01***	0.02***	0.04***	0.04***
	(0.003)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)
Observations	140	140	140	140	140	140
R-squared	0.204	0.192	0.264	0.247	0.306	0.209
Number of countries	28	28	28	28	28	28

Note: Panel Regressions with country and year fixed effects. All dependent variables are scaled by GDP. Driscoll and Kraay (1998) standard errors, *** p<0.01, ** p<0.05, * p<0.1. Data sourced from IMF BOP/IIP