Digital Currency and Privacy

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Abstract

We develop a monetary model in which a private company issues digital currency and uses payment data to estimate consumers' preferences. Sellers purchase preference information to produce goods that better match consumers' preferences. Due to reinforcing interactions between the value of preference information and trade volume, multiple equilibria (with and without digital currency) can exist. If multiple digital currencies circulate in the economy, the government can achieve a Pareto improvement by imposing a price ceiling on preference information. When left to market forces alone, socially efficient privacy utilization may not occur. The effects of the introduction of central bank digital currency on welfare depend on whether it can support socially efficient privacy utilization.

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1 Introduction

As our economy has become more digitalized, electronic payments have steadily increased over recent decades (see Stavins, 2017). Although electronic means of payment (henceforth, E-money), such as debit cards, Alipay, and PayPal, differ from traditional cash in many respects, one important difference is privacy: Cash retains user privacy, while digital currency transactions are collected by the company that operates the electronic payment system. Payment histories can indicate individual preferences for certain items, and this preference information, combined with users' personal information, can be used for marketing purposes and to design better goods that are more tailored to consumers' preferences. Thus, payment history data have commercial value and their importance has increased with advances in analytical technologies such as machine learning.

Although economic studies on digital currency have emerged recently since a surge in the Bitcoin price, the practice of using payment data of digital currency has received relatively little attention in academic areas. The following questions still need to be addressed: Under which conditions, does the E-money business — issuing digital currency and commercially using payment data of digital currency — exist in equilibrium and is it good or bad to a society? How does monetary policy affect the E-money business? How do equilibrium outcomes depend on the market structure of the E-money business? What are the effects of market interventions, such as price control and the introduction of central bank digital currency, on real allocations and welfare?

In this paper, we construct a money search model in which a private payment platform company issues E-money that is backed by government-issued cash, similar to Alipay, PayPal, and debit cards, to address the above questions. The company can estimate consumers' preferences using E-money transaction data and sell the preference information to sellers. A seller can then use the preference information to produce products that are customized to buyers' preferences, which increases total trade surplus in a pairwise meeting with a buyer. The precision of the preference information increases with the amount of payment data, and the company provides rewards for using E-money to attract more consumers to use it. Buyers incur disutility from providing private information, including payment histories, to the company and hence use E-money only if the rewards are higher than the disutility; otherwise, they use cash.

The additional surplus that the seller can obtain by preparing the production of customized goods with preference information is higher when the trade volume in pairwise meetings is high than when it is low. Thus, the value, and thereby the price, of preference information increases with the trade volume. Because trade volume decreases with the inflation rate, as is standard in money search models, an increase in the inflation rate decreases the price of preference information, which drives down the company's profits. In particular, if the inflation rate is sufficiently high, the company does not issue E-money because it cannot make positive profits from running the E-money business; thus, only government issued money is used as a medium of exchanges.

The buyers' demands for real balances that determine trade volume increase with the probability that buyers meet sellers who can produce customized goods. The probability of consuming customized goods, in turn, depends on whether the E-money business is in equilibrium. Because the price of preference information affects the company's profits and therefore the existence of the E-money business in equilibrium, the price of preference information indirectly affects the trade volume. Thus, there are reinforcing interactions between the trade volume and the price of preference information. Because of these interactions in the model economy, multiple equilibria can exist with different transaction patterns (with and without E-money).

The effects of the E-money business on social welfare depend on the relative size of the social benefits and costs of its operations. The social benefit is an increase in consumers' utility from consuming products that better match their individual preferences. The social costs include E-money users' disutility from providing private information to the company and the investment costs that sellers incur to prepare the production of customized goods. In the model economy, whenever the profit maximizing company runs the E-money business, the social benefit of the E-money business dominates its social costs. However, when left to market forces alone, socially efficient privacy utilization may not occur due to a wedge between the socially efficient and profitable uses of payment data.

An introduction of competition with free entry in the E-money business does not affect the trade volume and the existence of each type of equilibrium — whether E-money is circulated or not —, but it changes the division of the surplus from the E-money business among agents and welfare. Specifically, an increase in competition in the E-money business moves the surplus that the monopoly company enjoys to sellers. More importantly, there can be multiple Pareto-ranked equilibria with different numbers of E-money issuers: The more E-money issuers there are, the lower welfare with the higher price of preference information. The model shows that when multiple E-moneys circulate in the economy, the government can achieve a Pareto improvement by imposing a price ceiling on preference information.

Once the government introduces central bank digital currency (CBDC), which allows the government to estimate preference information by analyzing CBDC transaction data, into the economy, a unique equilibrium exists with privacy utilization. As a result, the government can increase welfare by issuing CBDC when socially efficient privacy utilization cannot be achieved in the private sector. However, if the seller's investment cost of preparing the production of customized goods and the inflation rate are sufficiently high, the introduction of CBDC can reduce welfare without supporting efficient privacy utilization.

Although the literature on privacy is extensive, relatively little attention has been given to privacy in monetary economics.[1] Kahn et al. (2005) investigate the role of money in providing transaction privacy in an economy where credit transactions reveal the identity of buyers to sellers. Garratt and van Oordt (2021) study the potential for sellers to exploit payment information to price discriminate and show that individual customers do not preserve their privacy in payments at the socially optimal level. More relatedly, Garratt and Lee (2020) show that payment data that can be used to design future goods drive the formation of a market monopoly. Guennewig (2021) shows that a firm that produces consumption goods issues its own digital currency to obtain information on consumers and does not accept currencies issued by other firms.

This paper contributes to the literature in two respects. First, the payment platform company obtains and commercializes the payment data in our model, in contrast to previous studies in which merchants obtain customers' private data.[2] This modeling structure allows us to analyze the economic conditions under which the E-money business is oper-

[1] A non-monetary model that is closely related to our model is Bergemann et al. (2021) in which a monopolist intermediary buys preference information directly from individual consumers and resells the information in a product market and shows the presence of informational externalities. We differ from their model as we focus on the special characteristics of digital currency industry and the effects of government policies. We refer to Acquisti et al. (2016) for a more comprehensive review of economic perspectives of privacy.

[2] The assumption that merchants observe consumers' private information is relevant to the case that a customer buys goods at an online store, such as Amazon, after entering personal information into the online store system. However, it does not capture the case of consumers buying goods at off-line stores using debit cards or credit cards. In that case, the customers' personal information that the stores can obtain is limited, while the payment platform company can still observe payment data and personal information of users.

ated, complementing previous studies. Second, because our model is based on Lagos and Wright (2005), it admits the analysis of the effects of conventional monetary policy on the economic uses of transaction data and how the introduction of CBDC affects the economy.[3]

Chiu and Koeppl (2020) investigate dynamic feedback loops between the data and activity sides of the platform and show that the monopoly platform company over-adopts payment services. In Chiu and Koeppl (2020), the platform company obtains consumer information similar to our model. However, in their model, the payment service is a technology that increases the probability that consumers have a trade, and there are no digital currencies and government issued money. In this sense, Chiu and Koeppl (2020) study the interaction between the trade volume and the value of preference information at the extensive margin, while we study this interaction at the intensive margin.

Our paper contributes to the emerging literature on CBDC. Barrdear and Kumhof (2016) show that the injection of CBDC into the economy increases GDP and contributes to the stabilization of the business cycle, and Williamson (2019) also emphasizes the positive effects of CBDC on welfare. Keister and Sanches (2021) investigate the welfare implications of targeted CBDC which competes only with physical currency or only with bank deposits. Chiu et al. (2020) find that introducing CBDC can promote competition in the deposit market and expand bank intermediation and output, while Fernández-Villaverde et al. (Forthcoming) show that the central bank may become the monopoly provider of deposits with CBDC, which might endanger maturity transformation. Keister and Monnet (2020) show that the introduction of CBDC can improve the effectiveness of regulatory policy. Schilling et al. (2020) find that the introduction of CBDC does not lead to efficiency, financial stability, and price stability simultaneously, and Williamson (2020) shows that the introduction of CBDC encourages bank panics but mitigates the damage caused by a panic. Kwon et al. (2020) and Wang (2021) study the effects of interest bearing CBDC on tax evasion.[4] These previous works explore the potential implications of CBDC as interest bearing central bank liability, while we study how cash-like CBDC can improve welfare

[3] Guennewig (2021) shows that the central bank loses its policy autonomy if digital currencies issued by private firms are used in the economy. However, he does not investigate how monetary policy affects uses of digital currency and transaction data.

[4] In Kwon et al. (2020) and Wang (2021), the government can monitor CBDC transactions and better impose tax when CBDC is used. However, these studies do not consider how payment data are used in the private sector for commercial purposes. Furthermore, the way CBDC transaction data is used differs. In our model, the government uses CBDC transaction data to provide information about aggregate economic conditions (consumers' preferences) to the private sector.

focusing on the privacy issue of CBDC.

Finally, this paper is related to the literature on digital currency. Chiu and Koeppl (2017) and Kang (2021) investigate double spending incentives in the Bitcoin system and the optimal design of cryptocurrency systems. Schilling and Uhlig (2018), Choi and Rocheteau (2019), and Pagnotta (2019) study cryptocurrency pricing in a monetary model where cryptocurrency can be held for a speculative motive.[5] Fernández-Villaverde and Sanches (2019) study cryptocurrencies as privately issued currencies by adding currency-providing entrepreneurs to the Lagos and Wright (2005) model and analyze whether currency competition can achieve price stability and efficient allocation. Kang and Lee (2019) study the competition between Bitcoin and central bank-issued money and investigate how monetary policy affects welfare and Bitcoin transactions. Chiu and Wong (2015) take a mechanism design approach to discuss how E-money helps to implement constrained efficient allocations. Carli and Uras (2021) investigate the role of E-money in household consumption smoothing and welfare. While these papers focus on analyzing the economic implications of technical features of digital currency, such as the blockchain technology of cryptocurrency, we focus on the privacy issue of digital currency payments.

The rest of the paper is organized as follows. Section 2 presents the environment of the model, and section 3 characterizes the equilibrium. In section 4, we conduct a welfare analysis. In section 5, we study the effects of competition in the E-money business on equilibrium allocations and welfare. We study the economic implications of introducing CBDC in section 6. Section 7 extends the baseline model by introducing heterogeneous agents, and section 8 concludes the paper. The omitted proofs are relegated to Appendix.

2 Environment

The basic framework of the model is based on Lagos and Wright (2005), with heterogeneous agents similar to those in Lagos and Rocheteau (2005) and Rocheteau and Wright (2005). Time is indexed by $t = 0, 1, 2, \ldots$, and each time period t is divided into three subperiods: morning (m), afternoon (a), and evening (e). A continuum of buyers and sellers exists, each with unit mass.

[5]For empirical studies on the value of cryptocurrencies, see Gandal and Halaburda (2014), Glaser et al. (2014), and Gandal et al. (2018).

Each buyer has preference, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - H_t - \mathbf{1}_p \delta + \upsilon(q_t) + \alpha u(x_t)],$$

and each seller has preference, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - H_t - c(h_{a,t}) - h_{e,t}].$$

Here, $\beta \in (0,1)$ is the discount rate, and X_t, q_t, and x_t are consumption in the morning, afternoon, and evening, respectively. H_t, $h_{a,t}$, and $h_{e,t}$ are labor supplies in the morning, afternoon, and evening, respectively. We assume that υ, u, and c are twice continuously differentiable with $\upsilon(0) = 0$, $\upsilon'' < 0 < \upsilon'$, $\upsilon'(0) = \infty$, $\upsilon'(\infty) = 0$, $u(0) = 0$, $u'' < 0 < u'$, $u'(0) = \infty$, $u'(\infty) = 0$, $c(0) = 0$, $c' > 0$, and $c'' > 0$. Here, $\alpha > 0$ is a parameter that affects the buyer's utility in the evening, $\delta > 0$ is a parameter that captures the buyer's disutility by forgoing privacy, the exact definition of which will be provided later, and $\mathbf{1}_p$ is an indicator function that takes the value of 1 if a buyer forgoes privacy and zero otherwise.

The production technology for consumption goods available to buyers and sellers allows the production of one unit of the perishable consumption good for each unit of labor supply in each subperiod. We call goods produced in the morning, afternoon, and evening morning goods, afternoon goods, and evening goods, respectively, and we set morning goods as the numeraire goods.

In the morning, there is a centralized Walrasian market in which all agents trade numeraire goods and assets. Buyers and sellers meet in large groups trading afternoon goods in a competitive market in the afternoon.[6] Finally, in the evening, there are bilateral meetings between buyers and sellers. In pairwise meetings, a buyer and a seller bargain over the terms of trade which are determined according to the bargaining solution of Kalai (1977), where the seller's bargaining power is $\theta \in (0,1)$.

Ideally, buyers would like to borrow output from sellers in the afternoon and evening markets and to repay loans in the next morning. Such credit arrangements are ruled out here because agents are anonymous and no device is available to record credit histories, which

[6]The specific market structure in the afternoon does not affect the main results of the model. For example, we obtain similar results with the assumption that there are bilateral meetings between buyers and sellers in the afternoon.

would allow the possibility of punishing someone who does not honor debt obligations. Consequently, any trades between buyers and sellers must occur on a quid-pro-quo basis through the use of a medium of exchanges.

There exists fiat money that is traded at price ϕ_t in terms of numeraire goods in the morning in period t. Money is supplied by the government at the beginning of the morning with a lump-sum transfer $\tau_t = (\gamma - 1)\phi_t M_{t-1}$ to each buyer. Thus, the money stock grows at a constant gross rate γ. We restrict attention to policies where $\gamma \geq \beta$ because there is no equilibrium if $\gamma < \beta$ as is standard in monetary models. Furthermore, when $\gamma = \beta$, we consider equilibrium obtained by taking the limit $\gamma \to \beta$.

At the beginning of the morning, buyers are subject to an idiosyncratic shock on the timing of consumption, which determines whether they consume early (in the afternoon) or late (in the evening). Let $\rho \in (0,1)$ denote the probability that a buyer goes to the afternoon market and a buyer goes to the evening market with probability $1 - \rho$. Note that this shock is realized at the beginning of the morning. Thus, buyers know which market they will go to in the subsequent period when they make decisions in the morning. We call buyers who go to the afternoon market early buyers and those who go to the evening market late buyers.

Individual preference in the evening In the model economy, $\mathscr{N} \in \mathbb{N}_+$ different tastes exist for evening goods, and a buyer has one of those tastes with probability $\frac{1}{\mathscr{N}}$. The evening taste is realized at the beginning of the evening, and buyers' evening tastes are their private information. If a buyer consumes customized goods tailored to his/her taste in the evening, then $\alpha = \alpha_H$; otherwise, and $\alpha = \alpha_L$, where $0 < \alpha_L < \alpha_H$. Given the value of α, we define the threshold values of trade volume in the evening market and real balances as follows:

$$x_i^* = u'^{-1}\left(\frac{1}{\alpha_i}\right) \text{ and } m_i^* = \frac{\theta \alpha_i u(x_i^*) + (1-\theta)x_i^*}{\beta} \tag{1}$$

for each $i \in \{H, L\}$.

We assume that a seller can produce a customized product tailored to a particular taste in a pairwise meeting only if the seller prepared the production of that product by incurring $\varsigma > 0$ units of labor in advance at the beginning of the evening. Furthermore, the set of evening tastes changes over time, although the number $\mathscr{N}$ is constant. For example, it is possible that a group of late buyers had a preference for pasta in the evening at $t-1$, but no buyers had a preference for pasta in the evening in the current period t. Specifically, we assume that there exists a collection of infinite numbers of different evening tastes, and in each period, $\mathscr{N}$ evening tastes are randomly chosen from that collection. Thus, the probability that a particular evening taste is realized in a given period is zero. Consequently, sellers will not prepare the production of a customized product tailored to a particular taste at the cost ς unless they know realized evening tastes in the current period.

In principle, a seller may contact some late buyers requesting information about their evening tastes at the beginning of the evening and prepare the production of customized goods based on the obtained information before having a random meeting. However, there is no way to verify whether the preference information obtained from an individual late buyer is correct. In particular, we assume that late buyers prefer to keep their personal information, including evening tastes, private. For example, late buyers incur some disutility from providing personal information to others due to privacy concerns, which will be discussed further later. Note that the buyer has a zero probability of meeting the seller to whom he/she provided the preference information because of the random matching process in the evening market. Thus, a late buyer always has an incentive to provide incorrect information to keep his/her privacy, so sellers cannot obtain information about realized evening tastes by asking individual late buyers.[7]

Digital currency and privacy In this economy, there is a company operating a payment system that supports online money transfers. In the baseline model, we assume that the company is monopoly, and in section 5, we introduce free entry into the online payment system market. The company issues electronic money (henceforth, E-money), and we assume that E-money must be backed by government issued money.[8] For example, the government can prohibit private sectors from issuing pure fiat money to maintain the efficacy of monetary policy, or agents may be reluctant to receive money issued by a private company unless it is backed by government issued money.[9]

[7] Note that even though a seller pays a late buyer for providing information about his/her evening taste to compensate the late buyer's disutility, the late buyer will always give incorrect information because his/her statement about the evening taste cannot be verified.

[8] A company can also support online transactions by issuing credit cards instead of E-money. However, the main implications from the model with the credit card business do not differ from the model with the E-money business.

[9] Chinese government, for example, forbids using any privately issued money in China, while it allows people to use Alipay and WeChat Pay that are basically backed by Renminbi. Furthermore, although it is not

Specifically, if an agent deposits money to the company by opening an account, then the company puts the same amount of E-money into the agent's account.[10] In this sense, E-money is equivalent to debit cards and Alipay in reality.[11] We assume that the company cannot restrict the number of users: Any agents can deposit money to the company and use E-money if they want to as long as the company runs its business. Finally, we assume that the company is owned by buyers and that the company distributes its profits to buyers in the morning.

Similar to sellers, the company cannot obtain the information about realized evening tastes by directly asking individual late buyers because late buyers always have incentives to provide incorrect information, that cannot be verified, to keep their privacy. However, a key feature in the model economy is that the company can observe and collect each individual's E-money transaction data. In particular, we assume that the company can obtain information about evening tastes realized in the current period by analyzing E-money transactions in the afternoon market. The preference information is correct with probability $\kappa(B) \in [0,1]$, where $B \in [0,\rho]$ is the mass of early buyers who use E-money in the afternoon. With probability $1-\kappa(B)$, the company obtains incorrect information. We assume that $\kappa(0) = 0$, $\kappa(\rho) = 1$, and $\kappa'(B) > 0$ for all $B \in [0,\rho]$.

Preference information is valuable to sellers because sellers can prepare the production of customized goods tailored to a specific evening taste realized in the current period, which could increase the total trade surplus in a pairwise meeting. Because sellers cannot directly obtain information about realized evening tastes from late buyers as explained above, the only way that sellers can prepare the production of customized goods tailored to evening tastes realized in the current period is to obtain preference information from the company.

Consequently, the company can use data on E-money transactions in the afternoon market for commercial purposes by selling preference information to sellers. Let φ_t denote the price of each taste information in period t in terms of morning goods in the next period

illegal to use cryptocurrencies for trades in countries, such as Japan, South Korea, and the U.S., cryptocurrencies are not widely used in retail transactions in those countries in contrast to debit cards and PayPal that are backed by government issued money.

[10]Technically, the company keeps money received from its users in its vault and manages a digital ledger that records changes in balances of all users and provides money to users when they withdraw it. We assume that the digital ledger is safe from attacks and manipulations.

[11]Although we assume a one-to-one exchange ratio between government issued money and E-money, equilibrium outcomes do not hinge on an exchange ratio as long as E-money is fully backed by government issued money.

$t+1$. We assume that sellers can commit to making payments for information purchases in the morning of the next period. For example, at the beginning of the evening in period t, a seller can purchase $n \in \{1, \ldots, \mathcal{N}\}$ number of preference information by promising to pay $\varphi_t n$ units of numeraire goods to the company in the morning in period $t+1$. However, the main implications do not change with the alternative assumption that sellers purchase preference information with money. Note that the company sells information about realized evening tastes in the current period, not buyers' identities, to sellers.

In reality, many people are inherently reluctant to reveal private information to others. We capture this feature in the model with disutility δ similar to Choi et al. (2019) and Garratt and van Oordt (2021): When a buyer opens an account at the company, the buyer incurs $\delta > 0$ units of fixed disutility in the morning to agree that the company can use his/her personal information, including payment histories, for commercial purposes. This disutility is associated with, for instance, privacy concerns such as data hacking or privacy costs originating from the agent's own taste for keeping privacy. Consequently, the company must compensate buyers for using E-money, such as the Bounty Payments program of PayPal and assorted benefits and subsidizations for purchasing prespecified goods provided by debit card issuers.

The reward can have two forms: fixed and proportional rewards. Note that evening transaction data have no value to the company. This implies that if the company provides any reward to late buyers, it must be a proportional reward to affect late buyers' trade volume in the evening market. Thus, the reward policy consists of one fixed reward and two proportional rewards. Specifically, the company can provide $R_{t+1} \geq 0$ units of numeraire goods to a buyer in the morning in period $t+1$ if the buyer used E-money for afternoon transactions in period t.[12] Next, the company subsidizes $\varkappa_a \in [0,1]$ and $\varkappa_e \in [0,1]$ fractions of the E-money payments in the afternoon market and evening market, respectively. For example, if the total E-money payment is \$100 and $\varkappa_a = 0.2$, then the early buyer and the company pay \$80 and \$20, respectively, to the seller.

Figure 1 summarizes the sequence of events in a representative period. Throughout the paper, the E-money business means the business of obtaining and selling evening taste

[12] Alternatively, we can assume that the company transfers R_{t+1} units money (or E-money) in terms of morning goods in the next period to an early buyer at the end of the afternoon in period t if he/she used E-money for afternoon transactions in period t. This alternative assumption raises the company's cost in equilibrium, but the main implications do not change.

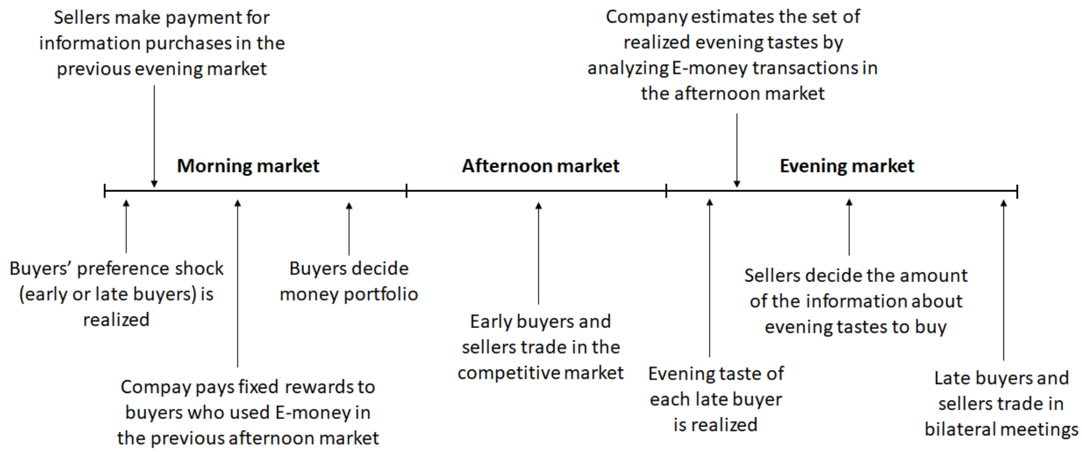

Figure 1: Timeline of a representative period

information by issuing E-money.

3 Equilibrium

In this section, we characterize the equilibrium of the model economy as follows. First, we study agents' value functions in each subperiod. Second, we study the optimal decisions of buyers, sellers, and the company. Third, we study market clearing conditions. Then, we characterize equilibrium.

3.1 Value functions

Morning market In the morning, agents consume numeraire goods, supply labor, and readjust their portfolios. Note that the company does not control the supply of E-money because all E-money is backed by money. The only difference between money and E-money is that transactions of E-money are observable to the company, while money transactions are anonymous, and the company may compensate buyers if they use E-money. In what follows, we call money supplied by the government P-money to emphasize that it is paper money rather than electronic money.

We define an indicator variable $\iota_t \in \{0,1\}$ that captures buyers' choices about keeping privacy as follows: $\iota_t = 1$ if a buyer used E-money in the afternoon in period t and $\iota_t = 0$

otherwise. Let $V^{b,early}_{m,t}(m_p, m_e, \iota_{t-1})$ denote the value function of an early buyer at the beginning of the morning in period t with m_p units of P-money, m_e units of E-money, and previous action ι_{t-1}. Then, by virtue of quasi-linearity of preferences, the value function $V^{b,early}_{m,t}(m_p, m_e, \iota_{t-1})$ is given as

$$\begin{aligned} V^{b,early}_{m,t}(m_p, m_e, \iota_{t-1}) &= m_p + m_e + \tau_t + \pi_t + R_t \iota_{t-1} \\ &\quad + \max_{m'_p, m'_e} \left\{ -\frac{\phi_t}{\phi_{t+1}}(m'_p + m'_e) - \mathbf{1}_{\{m'_e > 0\}} \delta + V^b_{a,t}(m'_p, m'_e) \right\}. \end{aligned} \tag{2}$$

Here, π_t is dividend payments from the company, which equals to the company's profit, m'_p and m'_e are the real balances of P-money and E-money, respectively, both in terms of numeraire goods in the next period, $\mathbf{1}_{\{m'_e > 0\}}$ is an indicator function that takes the value of 1 if $m'_e > 0$ and zero otherwise, and $V^b_{a,t}(m'_p, m'_e)$ is the value of the early buyer with portfolio (m'_p, m'_e) in the afternoon in period t. Note that the buyer incurs δ units of disutility from opening an account at the company to hold E-money. Similarly, the value function $V^{b,late}_{m,t}(m_p, m_e, \iota_{t-1})$ of a late buyer with portfolio (m_p, m_e) and previous action ι_{t-1} at the beginning of the morning in period t is given as

$$\begin{aligned} V^{b,late}_{m,t}(m_p, m_e, \iota_{t-1}) &= m_p + m_e + \tau_t + \pi_t + R_t \iota_{t-1} \\ &\quad + \max_{m'_p, m'_e} \left\{ -\frac{\phi_t}{\phi_{t+1}}(m'_p + m'_e) - \mathbf{1}_{\{m'_e > 0\}} \delta + V^b_{e,t}(m'_p, m'_e) \right\}, \end{aligned} \tag{3}$$

where $V^b_{e,t}(m'_p, m'_e)$ is the value of the late buyer in the evening with portfolio (m'_p, m'_e).

Next, the seller's value function in the morning in period t with portfolio (m_p, m_e) and the number of unpaid information purchases $n \in \{0, 1, \ldots \mathscr{N}\}$ in the previous evening, denoted by $V^s_{m,t}(m_p, m_e, n)$, is given as

$$V^s_{m,t}(m_p, m_e, n) = m_p + m_e - n\varphi_{t-1} + \max_{m'_p, m'_e} \left\{ -\frac{\phi_t}{\phi_{t+1}}(m'_p + m'_e) + V^s_{a,t}(m'_p, m'_e) \right\}, \tag{4}$$

where $V^s_{a,t}(m'_p, m'_e)$ is the value function of the seller with portfolio (m'_p, m'_e) in the afternoon in period t.

Afternoon market When sellers receive payments from buyers in the afternoon market, they are indifferent between P-money and E-money given a one-to-one exchange ratio between P-money and E-money. Thus, the price of afternoon goods cannot vary depending on the type of payment method. Let p_t denote the market price of goods in the afternoon in period t in terms of morning goods in the next period. Then, the buyer's value in the afternoon is given as

$$V^b_{a,t}(m_p, m_e) = \max_{q_p, q_e} \left\{ \upsilon(q_p + q_e) + \beta V^b_{m,t+1}(m_p - p_t q_p, m_e - p_t(1-\varkappa_a)q_e, \iota_t) \right\} \tag{5}$$

subject to

$$m_p - p_t q_p \geq 0 \tag{6}$$

$$m_e - p_t(1-\varkappa_a)q_e \geq 0, \tag{7}$$

where q_p and q_e are the quantities of afternoon goods purchased with P-money and E-money, respectively, $\iota_t = 1$ if $q_e > 0$ and $\iota_t = 0$ otherwise, and $V^b_{m,t+1}(m'_p, m'_e, \iota_t)$ is the buyer's value with asset portfolio (m'_p, m'_e) and the decision about keeping privacy ι_t at the beginning of the morning in period $t+1$ before the realization of the shock on the timing of consumption, i.e., $V^b_{m,t+1}(\cdot,\cdot,\cdot) = \rho V^{b,early}_{m,t+1}(\cdot,\cdot,\cdot) + (1-\rho) V^{b,late}_{m,t+1}(\cdot,\cdot,\cdot)$. Note that the early buyer only pays $p_t(1-\varkappa_a)q_e$ units of E-money to sellers to buy q_e units of afternoon goods because the company pays $p_t \varkappa_a q_e$ units of E-money to sellers for the buyer as rewards for using E-money.

In the afternoon in period t, sellers sell afternoon goods in a competitive market at price p_t in exchange for money. Thus, the seller's value in the afternoon is given as:

$$V^s_{a,t}(m_p, m_e) = \max_{q^s_p, q^s_e} \left\{ -c(q^s_p + q^s_e) + V^s_{e,t}(m_p + p_t q^s_p, m_e + p_t q^s_e) \right\}, \tag{8}$$

where q^s_p and q^s_e are the quantities of afternoon goods that the seller sells in exchange for P-money and E-money, respectively, and $V^s_{e,t}(m'_p, m'_e)$ is the seller's value function at the beginning of the evening with asset portfolio (m'_p, m'_e).

Evening market Late buyers and sellers are randomly matched in the evening. In a bilateral meeting between a buyer with portfolio (m_p, m_e) and a seller, the bargaining outcome is a triple (x, d_p, d_e) that specifies the quantity of evening goods x produced by the seller and P-money and E-money transfers (d_p, d_e) from the buyer to the seller in terms of morning goods in the next period. Furthermore, the late buyer's utility depends on the type of goods that the buyer consumes in the evening market through parameter α. Specifically, $\alpha = \alpha_H$ if the buyer consumes customized goods tailored to his/her evening taste and $\alpha = \alpha_L$ otherwise. Thus, the terms of trade depend on the buyer's asset portfolio but also on the seller's ability to produce customized goods.

As shown in (2) and (3), the buyer's value function in the morning is linear with respect to asset holdings. Thus, the buyer's trade surplus in a bilateral meeting is $\alpha_i u(x) - \beta d_p - \beta(1-\varkappa_e)d_e$ given the proportional reward $\varkappa_e$ and parameter α_i for $i \in \{H, L\}$. By the same reasoning, the seller's surplus is given as $-x + \beta(d_p + d_e)$. To determine the terms of trade (x, d_p, d_e), we use the proportional solution of Kalai (1977) with $\theta \in (0,1)$ as the bargaining power of a seller. The bargaining solution of Kalai (1977) maximizes the total trade surplus, $\alpha_i u(x) - x + \beta \varkappa_e d_e$, and requires the seller to receive a fraction θ of the total trade surplus, which gives $d_p + (1-\theta\varkappa_e)d_e = \frac{\theta\alpha_i u(x) + (1-\theta)x}{\beta}$ as the bargaining rule. Then, given $\varkappa_e$ and α_i for $i \in \{H, L\}$, the terms of trade (x, d_p, d_e) are obtained by solving the following maximization problem:

$$\max_{x, d_p, d_e} \{\alpha_i u(x) - x + \beta \varkappa_e d_e\} \tag{9}$$

subject to

$$d_p + (1-\theta\varkappa_e)d_e = \frac{\theta\alpha_i u(x) + (1-\theta)x}{\beta} \tag{10}$$

$$d_p \leq m_p \tag{11}$$

$$(1-\varkappa_e)d_e \leq m_e, \tag{12}$$

where (10) is the bargaining rule, and (11) and (12) are feasibility constraints.

Let $\left[\hat{x}_i(m_p, m_e), \hat{d}_{p,i}(m_p, m_e), \hat{d}_{e,i}(m_p, m_e)\right]$ denote the bargaining solution of the maximization problem (9) given (m_p, m_e) for each $i \in \{H, L\}$, and we characterize the specific form of the terms of trade later. Then, the buyer's value in the evening with asset portfolio

(m_p, m_e) is given as

$$
\begin{aligned}
V_{e,t}^b(m_p, m_e) = \omega_t & \left[\alpha_H u\left(\hat{x}_H(m_p, m_e)\right) - \beta\left\{\hat{d}_{p,H}(m_p, m_e) + \hat{d}_{e,H}(m_p, m_e)\right\}\right] \\
& + (1-\omega_t)\left[\alpha_L u\left(\hat{x}_L(m_p, m_e)\right) - \beta\left\{\hat{d}_{p,L}(m_p, m_e) + \hat{d}_{e,L}(m_p, m_e)\right\}\right] \\
& + \beta V_{m,t+1}^b(m_p, m_e, \iota_t),
\end{aligned} \tag{13}
$$

where ω_t is the probability that a buyer meets a seller who can produce customized goods tailored to the buyer's taste in a bilateral meeting and $\iota_t = 0$ because late buyers do not trade in the afternoon market. Note that late buyers have a meeting in the evening market with certainty because late buyers are the short side of the market.

When the company sells preference information at the beginning of the evening, a seller optimally chooses the amount of information $n \in \{0, \ldots, \mathcal{N}\}$ that he/she will buy at unit price φ_t. Based on the above arguments, the seller's value function at the beginning of the evening with asset portfolio (m_p, m_e) is given as

$$
\begin{aligned}
& V_{e,t}^s(m_p, m_e) \\
&= \max_{n \in \{0, \ldots, \mathcal{N}\}} \left\{
\begin{array}{c}
\frac{(1-\rho)\kappa(B)n}{\mathcal{N}} \int \left[-\hat{x}_H(\tilde{m}_p, \tilde{m}_e) + \beta\left(\hat{d}_{p,H}(\tilde{m}_p, \tilde{m}_e) + \hat{d}_{e,H}(\tilde{m}_p, \tilde{m}_e)\right)\right] dF_t(\tilde{m}_p, \tilde{m}_e) \\
+ \frac{(1-\rho)(\mathcal{N} - \kappa(B)n)}{\mathcal{N}} \int \left[-\hat{x}_L(\tilde{m}_p, \tilde{m}_e) + \beta\left(\hat{d}_{p,L}(\tilde{m}_p, \tilde{m}_e) + \hat{d}_{e,L}(\tilde{m}_p, \tilde{m}_e)\right)\right] dF_t(\tilde{m}_p, \tilde{m}_e) \\
-n\varsigma + \beta V_{m,t+1}^s(m_p, m_e, n)
\end{array}
\right\}
\end{aligned} \tag{14}
$$

where $\kappa(B)$ is the probability that the evening taste information is correct, ς is the labor input for preparing the production of each customized good, and $F_t(\tilde{m}_p, \tilde{m}_e)$ is the cumulative distribution of late buyers starting the evening in period t holding $m_p \leq \tilde{m}_p$ and $m_e \leq \tilde{m}_e$ units of real balances. In (14), $1-\rho$ is the probability that a seller has a meeting because there are $1-\rho$ mass of late buyers and unit mass of sellers in the evening market.

3.2 Agents' optimal decisions

In this subsection, we study the optimal behaviors of each economic agent in stationary equilibrium. By stationarity, we mean that all real quantities are constant over time, which implies that $\frac{\phi_t}{\phi_{t+1}} = \gamma$.

Buyers' choices In the morning, buyers determine the portfolio of real balances. Note that buyers will not carry any money into the next morning because the buyer's value functions in the morning are linear in money holdings and $\gamma \geq \beta$.

Early buyers optimally choose the composition of money portfolios considering the reward for using E-money and privacy costs. Specifically, from (2) and (5) - (7), we obtain the early buyer's problem as

$$\max_{q_p, q_e} \left\{ (\beta R - \delta)\mathbf{1}_{\{q_e > 0\}} - \gamma p(q_p + (1 - \varkappa_a)q_e) + \upsilon(q_p + q_e) \right\}, \tag{15}$$

where $\mathbf{1}_{\{q_e>0\}}$ is an indicator function that takes the value of 1 if $q_e > 0$ and zero otherwise. Note that early buyers will either use E-money or P-money given the fixed return, privacy cost, and $\varkappa_a \geq 0$.[13]

If the early buyer chooses to use E-money, then his/her surplus is

$$S_e^{early} = \max_{q_e} \{\beta R - \delta - \gamma p(1 - \varkappa_a)q_e + \upsilon(q_e)\} \tag{16}$$

and the first order condition for q_e is

$$\gamma p(1 - \varkappa_a) = \upsilon'(q_e). \tag{17}$$

On the other hand, if the early buyer chooses to use P-money, then his/her surplus is

$$S_p^{early} = \max_{q_p} \left\{ -\gamma p q_p + \upsilon(q_p) \right\}, \tag{18}$$

which gives

$$\gamma p = \upsilon'(q_p) \tag{19}$$

as an optimality condition. Then, if $S_e^{early} \geq S_p^{early}$, early buyers use E-money for afternoon transactions and use P-money otherwise.

Similar to early buyers, late buyers also either use P-money or E-money for evening transactions given the proportional reward $\varkappa_e \geq 0$ and fixed privacy cost δ. From (3) and (13), we can write the late buyer's surplus if he/she chooses to accumulate P-money in the

[13] If $\varkappa_a = 0$, then E-money and P-money are the same at the margin. In this case, we assume, without loss of generality, that early buyers use E-money if $\beta R \geq \delta$ and use P-money if $\beta R < \delta$.

morning as

$$S_p^{late} = \max_{m_p} \left\{ \begin{array}{c} -(\gamma-\beta)m_p + \omega\left[\alpha_H u\left(\hat{x}_H(m_p,0)\right) - \beta \hat{d}_{p,H}(m_p,0)\right] \\ +(1-\omega)\left[\alpha_L u\left(\hat{x}_L(m_p,0)\right) - \beta \hat{d}_{p,L}(m_p,0)\right] \end{array} \right\}, \tag{20}$$

where ω is the probability that a buyer meets a seller who can produce evening goods tailored to the buyer's evening taste. Then, the first-order condition for m_p is given as

$$\begin{aligned} \gamma-\beta = {} & \omega\left[\alpha_H u'\left(\hat{x}_H(m_p,0)\right)\frac{\partial \hat{x}_H(m_p,0)}{\partial m_p} - \beta\frac{\partial \hat{d}_{p,H}(m_p,0)}{\partial m_p}\right] \\ & +(1-\omega)\left[\alpha_L u'\left(\hat{x}_L(m_p,0)\right)\frac{\partial \hat{x}_L(m_p,0)}{\partial m_p} - \beta\frac{\partial \hat{d}_{p,L}(m_p,0)}{\partial m_p}\right]. \end{aligned} \tag{21}$$

Next, the late buyer's surplus if he/she chooses to hold E-money is given as

$$S_e^{late} = \max_{m_e} \left\{ \begin{array}{c} -(\gamma-\beta)m_e - \delta + \omega\left[\alpha_H u\left(\hat{x}_H(0,m_e)\right) - \beta \hat{d}_{e,H}(0,m_e)\right] \\ +(1-\omega)\left[\alpha_L u\left(\hat{x}_L(0,m_e)\right) - \beta \hat{d}_{e,L}(0,m_e)\right] \end{array} \right\}, \tag{22}$$

which gives

$$\begin{aligned} \gamma-\beta = {} & \omega\left[\alpha_H u'\left(\hat{x}_H(0,m_e)\right)\frac{\partial \hat{x}_H(0,m_e)}{\partial m_e} - \beta\frac{\partial \hat{d}_{e,H}(0,m_e)}{\partial m_e}\right] \\ & +(1-\omega)\left[\alpha_L u'\left(\hat{x}_L(0,m_e)\right)\frac{\partial \hat{x}_L(0,m_e)}{\partial m_e} - \beta\frac{\partial \hat{d}_{e,L}(0,m_e)}{\partial m_e}\right] \end{aligned} \tag{23}$$

as the first-order condition. Then, if $S_e^{late} \geq S_p^{late}$, late buyers use E-money for afternoon transactions and use P-money otherwise.

Sellers' choices In the morning, sellers spend all real balances of moneys to purchase numeraire goods, and do not carry any real balances into the next subperiods due to the money holding cost, given that $\gamma \geq \beta$, as can be verified by (4), (8), and (14).

In the afternoon, sellers optimally supply goods in the market, given the market price p. Specifically, from (4), (8), and (14), we can write the seller's problem in the afternoon market as

$$\max_{q_p^s, q_e^s} \left\{ -c(q_p^s + q_e^s) + \beta p(q_p^s + q_e^s) \right\},$$

which gives

$$c'(q_p^s + q_e^s) = \beta p \tag{24}$$

as the optimality condition.

In the evening, sellers decide the amount of information purchases. As shown in (21) and (23), all late buyers make the same choice for real balances given the probability ω. Let (m_p, m_e) be the equilibrium real balances of late buyers in the evening. From (4) and (14), the seller's problem of information purchase in the evening can be written as

$$\max_{n \in \{0,\ldots,\mathscr{N}\}} \left\{ n \left[\frac{(1-\rho)\kappa(B)D(m_p, m_e)}{\mathscr{N}} - \varsigma - \beta\varphi \right] \right\}, \tag{25}$$

where

$$\begin{aligned} D(m_p, m_e) = {} & \theta \left[\alpha_H u(\hat{x}_H(m_p, m_e)) - \hat{x}_H(m_p, m_e) + \beta \varkappa_e \hat{d}_{e,H}(m_p, m_e) \right] \\ & - \theta \left[\alpha_L u(\hat{x}_L(m_p, m_e)) - \hat{x}_L(m_p, m_e) + \beta \varkappa_e \hat{d}_{e,L}(m_p, m_e) \right] \end{aligned} \tag{26}$$

is the difference in the seller's trade surplus in the evening between when the seller can produce customized goods tailored to the buyer's taste and when he/she cannot.

There are $\mathscr{N}$ different evening tastes, preference information is correct with probability $\kappa(B)$, and a seller has a meeting with a probability $1-\rho$. Thus, the expected payoff from preparing the production of customized goods for a particular taste is $\frac{(1-\rho)\kappa(B)D(m_p,m_e)}{\mathscr{N}}$. The cost of preparing the production of customized goods is ς for each evening taste and the seller must pay φ units of numeraire goods in the morning of the next period to obtain the information about each taste. Combined together, the net expected payoff from preparing the production of goods tailored to each evening taste is $\frac{(1-\rho)\kappa(B)D(m_p,m_e)}{\mathscr{N}} - \varsigma - \beta\varphi$.

Assuming that sellers purchase preference information if they are indifferent, we obtain, from (25), the seller's optimal choice for the number of information purchases n as:

$$n = \begin{cases} \mathscr{N} & \text{if } D(m_p, m_e) \geq \frac{\mathscr{N}(\varsigma + \beta\varphi)}{(1-\rho)\kappa(B)} \\ 0 & \text{if } D(m_p, m_e) < \frac{\mathscr{N}(\varsigma + \beta\varphi)}{(1-\rho)\kappa(B)}. \end{cases} \tag{27}$$

Thus, sellers purchase all information about evening tastes or do not purchase any information in equilibrium.

Company's choices The company decides the reward policy $(R, \varkappa_a, \varkappa_e)$ for using E-money and the price of each preference information φ. Given the monopoly power, the company will set $(R, \varkappa_a)$ such that $S_e^{early} = S_p^{early}$ and set $\varkappa_e$ such that $S_e^{late} = S_p^{late}$. The next lemma describes the property of the optimal reward policy $(R, \varkappa_a, \varkappa_e)$ of the company.

Lemma 1 *The profit maximizing company does not provide any proportional rewards:* $\varkappa_a = \varkappa_e = 0$.

Proof. See Appendix. ■

The result of lemma 1 implies that the company only provids the fixed reward to attract early buyers to use E-money in the afternoon. Then, from (16) - (19), the company sets the fixed reward as

$$R = \frac{\delta}{\beta} \tag{28}$$

given the monopoly power whenever the company runs its business. Then, all early buyers use E-money, and, hence, the company obtains the correct preference information with certainty, i.e., $\kappa(B) = 1$. This result hinges on the assumption of the constant disutility δ across buyers. In section 7, we show that it is possible that $\kappa(B) \in (0,1)$ in equilibrium once we introduce heterogeneity in disutility δ.

Next, the company does not provide any rewards to late buyers for using E-money in the evening market as shown in lemma 1. Thus, late buyers will carry only P-money into the evening market to keep transaction privacy and do not use E-money, i.e., $m_e = 0$. The company takes late buyers' real P-money balance m_p as given and sets the price for information about each evening taste as

$$\varphi = \frac{(1-\rho)D(m_p, 0) - \mathscr{N}\varsigma}{\beta\mathscr{N}}, \tag{29}$$

using its monopoly power given the sellers' optimal choice described in (27) with $\kappa(B) = 1$.

Terms of trade in a pairwise meeting We close this subsection with a study of the terms of trade in the evening market. As shown above, late buyers do not hold E-money in the evening market, i.e., $m_e = 0$, which means $d_e = 0$ in the bargaining problem (9) by (12). Given this result, the next lemma characterizes the terms of trade in bilateral meetings in the evening market by solving the maximization problem (9).

Lemma 2 *Given late buyers' real balances (m_p, m_e) with $m_e = 0$ and parameter value α_i for $i \in \{H, L\}$ in a pairwise meeting, the terms of trade are given as follows:*

$$\left(\hat{x}_i(m_p,0), \hat{d}_{p,i}(m_p,0), \hat{d}_{e,i}(m_p,0)\right) = \begin{cases} (x_i^*, m_i^*, 0) & \text{if } m_p \geq m_i^* \\ \left(\Phi_i^{-1}(m_p), m_p, 0\right) & \text{if } m_p < m_i^* \end{cases} \tag{30}$$

where $\Phi_i(x) = \frac{\theta \alpha_i u(x) + (1-\theta)x}{\beta}$ for each $i \in \{H, L\}$.

Proof. See Appendix. ■

3.3 Market clearing conditions

In equilibrium, asset and goods markets must clear. First, because E-money is backed by P-money, the sum of demands for E-money and P-money should be equal to the supply of money coming from the government. Thus, we obtain

$$\gamma[\rho p(q_p + q_e) + (1-\rho)m_p] = \phi_t M_t,$$

as a market clearing condition in the money market. Second, buyers' demand for afternoon goods should be equal to the supply from sellers, which gives us

$$\rho q_p = q_p^s \text{ and } \rho q_e = q_e^s \tag{31}$$

as a market clearing condition in the afternoon market.

3.4 Equilibrium characterization

Late buyers only use P-money, and early buyers use either P-money or E-money in equilibrium. Thus, there are two relevant cases: 1) equilibrium in which all buyers use only P-money, and 2) equilibrium in which early buyers use E-money. We call the first equilibrium P-equilibrium and the second equilibrium E-equilibrium in what follows. As an intermediate step for equilibrium characterization, we first describe the properties of the function $D(m_p, m_e)$, defined in (26), given that late buyers do not use E-money in the next lemma.

Lemma 3 *For all* $m_p < m_H^*$, $\frac{\partial D(m_p,0)}{\partial m_p} > 0$, *and for all* $m_p \geq m_H^*$, $D(m_p, 0) = \overline{D}$, *where*

$$\overline{D} \equiv \theta \left[\alpha_H u\left(x_H^*\right) - x_H^*\right] - \theta \left[\alpha_L u\left(x_L^*\right) - x_L^*\right]. \tag{32}$$

Proof. See Appendix. ■

The main implication of lemma 3 is that an increase in the seller's trade surplus from being able to produce customized goods in a pairwise meeting increases with the late buyer's real balances that determine the trade volume in the evening market. Thus, sellers' incentives to purchase preference information rise with the trade volume in the evening market.

We now analyze equilibrium real allocations. Note, from (17), (19), (24), and lemma 1, that early buyers' demands and sellers' supplies in the afternoon market do not depend on the type of money that is used in the afternoon. Furthermore, late buyers use only P-money. Thus, in what follows, we drop the index $j \in \{e, p\}$ in q_j and m_j which specifies the type of money traded in each subperiod unless it causes any confusion.

From (17), (19), (24), and (31), we obtain

$$q = \widetilde{q}(\gamma), \tag{33}$$

where $\widetilde{q} : [\beta, \infty) \to \mathbb{R}_+$ is a decreasing function of γ determined by

$$\frac{\upsilon'(\widetilde{q}(\gamma))}{c'(\rho \widetilde{q}(\gamma))} = \frac{\gamma}{\beta}. \tag{34}$$

After finding equilibrium trade volume q, we obtain the equilibrium price of afternoon goods as $p = \frac{c'(\rho \widetilde{q}(\gamma))}{\beta}$ from (24) and (31).

In E-equilibrium, the company obtains correct preference information with certainty and sellers purchase all preference information. Thus, a late buyer meets a seller who can produce customized goods tailored to his/her taste in the evening with certainty, i.e., $\omega = 1$. On the other hand, $\omega = 0$ in P-equilibrium. Then, from (21) and (30), we obtain equilibrium real balance m and trade volume x in the evening as follows:

$$x = \widetilde{x}_H(\gamma) \text{ and } m = \widetilde{d}_H(\gamma) \text{ in E-equilibrium} \tag{35}$$

$$x = \widetilde{x}_L(\gamma) \text{ and } m = \widetilde{d}_L(\gamma) \text{ in P-equilibrium,} \tag{36}$$

where

$$\widetilde{x}_i(\gamma) \equiv u'^{-1}\left(\frac{\gamma(1-\theta)}{\alpha_i(\beta-\theta\gamma)}\right) \tag{37}$$

$$\widetilde{d}_i(\gamma) \equiv \frac{\theta\alpha_i u\left(\widetilde{x}_i(\gamma)\right) + (1-\theta)\widetilde{x}_i(\gamma)}{\beta} \tag{38}$$

are decreasing functions of γ for each $i \in \{H, L\}$.

In summary, we have the following proposition, whose proof is omitted, which describes economic outcomes in each equilibrium.

Proposition 1 *Given monetary policy γ, real allocations and prices are as follows:*

1) *In E-equilibrium, $q = \widetilde{q}(\gamma)$, $x = \widetilde{x}_H(\gamma)$, $m = \widetilde{d}_H(\gamma)$, $p = \frac{c'(\rho\widetilde{q}(\gamma))}{\beta}$, and $\varphi = \frac{(1-\rho)D(\widetilde{d}_H(\gamma),0) - \mathscr{N}\varsigma}{\beta\mathscr{N}}$.*

2) *In P-equilibrium, $q = \widetilde{q}(\gamma)$, $x = \widetilde{x}_L(\gamma)$, $m = \widetilde{d}_L(\gamma)$, and $p = \frac{c'(\rho\widetilde{q}(\gamma))}{\beta}$.*[14]

In money search models, as the money growth rate γ increases, the inflation rate and the money holding cost increase in a stationary equilibrium. Thus, buyers accumulate less money in the morning for transactions in subsequent subperiods, and trade volumes in the afternoon and evening markets fall, as shown in (33) - (38). A decrease in demand in the afternoon market decreases the market price of afternoon goods. Finally, given the results of lemma 3, the price of preference information φ decreases with γ because $\widetilde{d}'_H(\gamma) < 0$.

The existence of each type of equilibrium depends on whether the company can make positive profits from running its business. The company can make all early buyers use E-money and sell all preference information to all sellers by setting the reward as (28) and the information price as (29). Thus, the company's profit is given as

$$\pi = \frac{(1-\rho)D(m,0) - \mathscr{N}\varsigma - \rho\delta}{\beta}, \tag{39}$$

[14] In P-equilibrium, the company does not run its business, so there is no price of preference information φ.

where m is the late buyer's real P-money balances in the evening market in equilibrium. Then, it must be that $D(m,0) \geq \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$ for E-equilibrium to exist because the company has no incentive to run its business otherwise. On the other hand, it must be that $D(m,0) < \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$ for P-equilibrium to exist because, otherwise, the company can make non-negative profits from running its business. Based on these arguments, the next proposition characterizes the existence of each equilibrium.

Proposition 2 *Stationary monetary equilibrium exists as follows:*

1) *Suppose that* $\frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*,0)$. *Then, there exist* $\gamma_1 > \gamma_2 \geq \beta$ *such that i) for all* $\gamma \in [\beta,\gamma_1]$, *E-equilibrium exists, and ii) for all* $\gamma > \gamma_2$, *P-equilibrium exists.*

2) *Suppose that* $D(m_L^*,0) < \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho} \leq \overline{D}$. *Then, there exists* $\gamma_3 \geq \beta$ *such that i) for all* $\gamma \in [\beta,\gamma_3]$, *E-equilibrium exists, and ii) for all* $\gamma \geq \beta$, *P-equilibrium exists.*

3) *Suppose that* $\overline{D} < \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$. *Then, for all* $\gamma \geq \beta$, *P-equilibrium exists.*

Proof. See Appendix. ■

Proposition 2 shows how the value of $\frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$ and the inflation rate, γ, together determine the existence of each type of equilibria. Figure 2 depicts how the parameter space is subdivided with $\frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$ on the vertical axis and γ on the horizontal axis, illustrating graphically proposition 2.

The effects of parameters δ, ς, $\mathcal{N}$, and ρ on the type of equilibrium are straightforward. Here, δ is the buyers' disutility from forgoing privacy for using E-money, ς is the seller's investment cost to prepare the production of customized goods tailored to a particular taste in the evening, $\mathcal{N}$ is the number of different evening tastes, and $1-\rho$ is the probability that a seller meets a buyer in the evening market. Thus, if δ, ς, $\mathcal{N}$, and ρ are sufficiently high, as illustrated in the third case of proposition 2, the costs of obtaining and utilizing preference information are higher than the expected payoff; hence, the company cannot make non-negative profits from its business.

Next, proposition 2 shows that the company is more likely to run the E-money business when γ is low, while E-money is not circulated when γ is sufficiently high.[15] The intuitive

[15]On a related point, Berentsen et al. (2007) show that existence of a monetary equilibrium with credit requires some positive inflation, while E-money and government issued P-money co-exist when inflation is sufficiently low in our model.

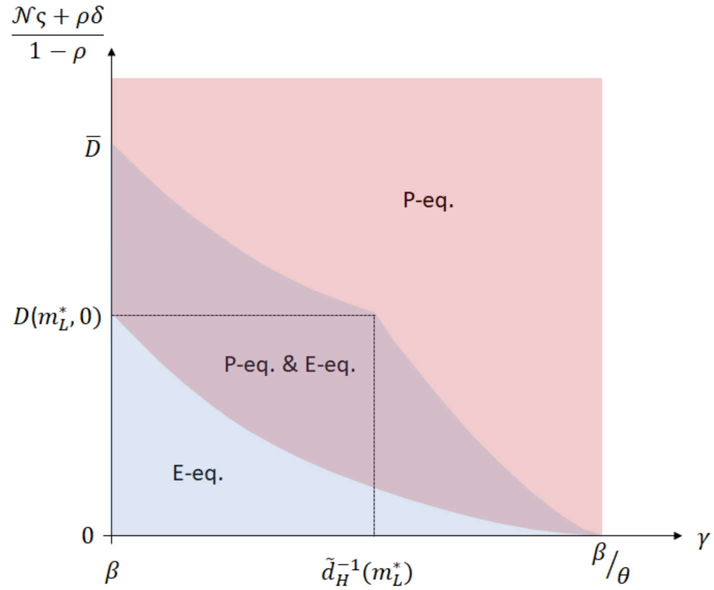

Figure 2: Typology of equilibria in $\left(\gamma, \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}\right)$-space

explanation for this result is in line with our earlier observation. In E-equilibrium, the late buyer's money holding, $m = \widetilde{d}_H(\gamma)$, decreases with γ for all $\gamma \geq \beta$. Furthermore, we show that $D(m,0)$ increases with m for all $m < m_H^*$ in lemma 3. Thus, $D(\widetilde{d}_H(\gamma),0)$ decreases with γ. As a result, as γ decreases, it is more likely that $D(\widetilde{d}_H(\gamma),0) \geq \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$, which is the necessary condition for the company to make non-negative profits and for E-equilibrium to exist. Similarly, for a sufficiently high γ, we have $D(\widetilde{d}_L(\gamma),0) < \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$, which is the necessary condition for the existence of P-equilibrium because $\widetilde{d}_L(\gamma)$ decreases with γ.

One noticeable result in proposition 2 is that the model can generate multiple stationary monetary equilibria: For all $\gamma \in (\gamma_2, \gamma_1]$ when $\frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*,0)$, or for all $\gamma \in [\beta, \gamma_3]$ when $D(m_L^*,0) < \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho} \leq \overline{D}$, both E-equilibrium and P-equilibrium can exist. The intuition for this finding is as follows. For intermediate inflation, if late buyers expect that they can buy customized goods in the evening, they bring sufficient money, which motivates sellers to buy preference information from the company. On the other hand, if late buyers expect that they cannot buy customized goods in the evening, they carry too little money to incentivize sellers to buy preference information, justifying late buyers' initial expectation. More technically, (38) shows that $\widetilde{d}_H(\gamma) > \widetilde{d}_L(\gamma)$ for all $\gamma \geq \beta$. Thus, if $\gamma_i' \geq \beta$ exists such that $D(\widetilde{d}_i(\gamma_i'),0) = \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$ for each $i \in \{H,L\}$, it must be that $\gamma_L' < \gamma_H'$, because $D(\widetilde{d}_i(\gamma),0)$

decreases with γ. Then, because E-equilibrium exists for $\gamma \leq \gamma'_H$, while P-equilibrium exists for $\gamma > \gamma'_L$, multiple equilibria exist when $\gamma \in [\gamma'_L, \gamma'_H)$.

4 Welfare analysis

In this section, we examine the model's normative properties in terms of social welfare and investigate the optimal monetary policy γ. To set the stage for welfare analysis, we define the sum of expected utilities in a steady state equilibrium across agents with equal weight as our welfare measure:

$$W = \rho \upsilon(q) - c(\rho q) + (1-\rho)\left[\alpha_i u(x) - x\right] - \mathbf{1}_{\{e=E\}}\left(\mathscr{N}\varsigma + \rho\delta\right), \tag{40}$$

where $i = H$ in E-equilibrium, $i = L$ in P-equilibrium, and $\mathbf{1}_{\{e=E\}}$ is an indicator function that takes the value of 1 if the economy is in E-equilibrium and zero otherwise. Specifically, using the results of proposition 1, we can express welfare as a function of the money growth rate γ as follows:

$$W_P(\gamma) = \rho \upsilon(\widetilde{q}(\gamma)) - c(\rho \widetilde{q}(\gamma)) + (1-\rho)\left[\alpha_L u(\widetilde{x}_L(\gamma)) - \widetilde{x}_L(\gamma)\right] \tag{41}$$

in P-equilibrium and

$$W_E(\gamma) = \rho \upsilon(\widetilde{q}(\gamma)) - c(\rho \widetilde{q}(\gamma)) + (1-\rho)\left[\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma)\right] - \left(\mathscr{N}\varsigma + \rho\delta\right) \tag{42}$$

in E-equilibrium.

How does the E-money business affect welfare? On the one hand, by estimating realized evening tastes from afternoon E-money transaction data, the E-money business supports late buyers to consume customized goods tailored to their tastes, which pushes up late buyers utility, i.e., $\alpha = \alpha_H$, and welfare. On the other hand, early buyers' disutilities $\rho\delta$ from forgoing privacy and sellers' investment cost of preparing the production of customized goods $\mathscr{N}\varsigma$ are social costs in E-equilibrium because they do not increase the consumption of agents. Thus, whether the E-money business contributes to welfare depends on which effect dominates the other effect. However, the next proposition shows that the economy achieves higher welfare with the E-money business whenever the profit maximizing company runs the E-money business.

Proposition 3 *Whenever the company can make non-negative profits from the E-money business,* $W_E(\gamma) > W_P(\gamma)$.

Proof. See Appendix. ■

The intuition for the results of proposition 3 is as follows. As γ increases, the trade volume in a pairwise meeting falls. Then, the positive effect of the E-money business on welfare through the term $\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma)$ in (42) falls, while the social cost of the E-money business, $\mathscr{N}\varsigma + \rho\delta$, is constant. Thus, the economy tends to have higher welfare with the E-money business, i.e., $W_E(\gamma) > W_P(\gamma)$, when γ and $\mathscr{N}\varsigma + \rho\delta$ are sufficiently low. Similarly, the company can make non-negative profits, which requires $D(\widetilde{d}_H(\gamma), 0) \geq \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$, only if $\mathscr{N}\varsigma + \rho\delta$ and γ are sufficiently low. In particular, whenever $D(\widetilde{d}_H(\gamma), 0) \geq \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$, we have $W_E(\gamma) > W_P(\gamma)$, detailed in the proof of proposition 3. Thus, the fact that the company can make non-negative profits from running the E-money business implies that the positive effect of the E-money business on welfare dominates the negative effect in the model economy. However, the results of proposition 3 do not imply that the company fully internalizes the positive effects of privacy utilization on welfare, as described in the next proposition.

Proposition 4 *If* $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} < \frac{\overline{D}}{\theta}$*, then there exists* $\gamma^* > \beta$*, such that* $W_E(\gamma^*) = W_P(\gamma^*)$ *and for all* $\gamma \in [\beta, \gamma^*]$*,* $W_E(\gamma) \geq W_P(\gamma)$*, with a strict inequality for* $\gamma < \gamma^*$*. Furthermore, if* $D(m_L^*, 0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq \overline{D}$*, then* $\gamma_3 < \gamma^*$*, and if* $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*, 0)$*, then* $\gamma_1 < \gamma^*$*, where* γ_1 *and* γ_3 *are defined in proposition 2.*

Proof. See Appendix. ■

The results of proposition 4 imply that the company does not always run the E-money business at a socially efficient level. For example, proposition 2 shows that E-equilibrium does not exist for all $\gamma \geq \beta$ if $\overline{D} < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} < \frac{\overline{D}}{\theta}$, because the company cannot make non-negative profits from the E-money business. However, $W_E(\gamma) > W_P(\gamma)$ for all $\gamma \in [\beta, \gamma^*)$. Similarly, when $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*, 0)$, the company runs the E-money business only if $\gamma \leq \gamma_1$, while the E-money business still improves welfare when $\gamma \in (\gamma_1, \gamma^*)$. Thus, when privacy utilization is left to market forces alone, socially efficient privacy utilization may not occur.

We now analyze the effects of monetary policy that determines γ on welfare. As shown in proposition 1, q and x decrease with γ in each type of equilibrium, and for all $\gamma > \beta$,

$q < q^*$, where q^* is such that $\upsilon'(q^*) = c'(\rho q^*)$, and $x < x_i^*$. As a result, welfare W decreases with γ in each type of equilibrium. Furthermore, as shown in proposition 2, a decrease in γ tends to change the equilibrium type from P-equilibrium to E-equilibrium, thereby discontinuously increasing welfare by supporting socially efficient privacy utilization. Consequently, welfare monotonically decreases with γ. Thus, optimal monetary policy is the Friedman rule as stated in the next proposition, whose proof is omitted.

Proposition 5 *Optimal monetary policy is the Friedman rule, i.e., $\gamma = \beta$.*

5 Competition in the E-money business

In the baseline model, we assume that the monopoly company runs the E-money business. To see how this assumption affects the main implications, we introduce competition into the market in the form of contestable markets.[16] Specifically, we introduce free entry into the E-money business: There exists a pool of an infinite number of companies, indexed by $l \in \mathbb{N}_+$, that can run the E-money business with their own E-money and they have the same technology of recovering preference information from afternoon transaction data. We assume that buyers must incur disutility δ for each account that he/she opens to use multiple E-moneys issued by different companies.

As in the baseline model, a company $l \in \mathbb{N}_+$ only provides the fixed reward, R_l, to early buyers to encourage them to use its E-money by the results of lemma 1 because providing proportional rewards distorts the buyer's decisions, creating an additional cost to the company. This implies that the total amount of buyers' real balances in each type of equilibrium is the same as in the baseline model, although the composition of the money portfolio can be different because early buyers use any E-moneys that give higher rewards for using them in the afternoon than the privacy cost, i.e., $\beta R_l \geq \delta$. Consequently, the equilibrium trade volumes in the afternoon and evening markets are the same as in the baseline model.

Given the early buyer's decision in the morning, any companies that actively run the E-money business set the reward as $R = \frac{\delta}{\beta}$, and those companies obtain the correct information about the evening tastes with certainty.[17] Let $\mathscr{L}$ denote the set of companies whose

[16] A discussion of contestable markets is in Baumol et al. (1982).

[17] As in the baseline model, we maintain the assumption that the probability that a company obtains the

E-money is used by early buyers in the afternoon. Sellers will buy preference information from a company that sells the information at the lowest price. Let $\mathscr{L}^*=\arg\min_{l\in\mathscr{L}}\varphi_l$, where φ_l is the price of preference information that a company $l \in \mathscr{L}$ posts in the market. Then, $\mathscr{L}^*$ is the set of active companies that run the E-money business in equilibrium. We assume that if $|\mathscr{L}^*| > 1$, where $|\mathscr{L}^*|$ is the cardinality of the set $\mathscr{L}^*$, sellers randomly choose one company in the set $\mathscr{L}^*$ to buy preference information.

Suppose that $|\mathscr{L}^*| > 0$, so the economy is in E-equilibrium, and let $\varphi_e = \min_{l\in\mathscr{L}}\varphi_l$. In E-equilibrium, each company provides $\frac{\delta}{\beta}$ units of reward to all early buyers and sells all preference information to $\frac{1}{|\mathscr{L}^*|}$ mass of sellers at the beginning of the evening. Thus, the profit of a company $l \in \mathscr{L}^*$ from running the E-money business is given as

$$\pi = \frac{\mathscr{N}\varphi_e}{|\mathscr{L}^*|} - \frac{\rho\delta}{\beta}. \tag{43}$$

The next lemma shows that a company makes zero profits from running the E-money business in equilibrium.

Lemma 4 *In the model with free entry into the E-money business, a company makes zero profit in E-equilibrium.*

Proof. See Appendix. ■

The results of lemma 4 and (43) imply that $\varphi_e = \frac{\rho\delta|\mathscr{L}^*|}{\beta\mathscr{N}}$ in E-equilibrium. For E-equilibrium to exist, sellers must have incentives to buy preference information, which requires, from (27), that

$$\varphi_e = \frac{\rho\delta\,|\mathscr{L}^*|}{\beta\mathscr{N}} \leq \frac{(1-\rho)D(m,0) - \mathscr{N}\varsigma}{\beta\mathscr{N}} \tag{44}$$

given the equilibrium real balances m of late buyers. Because $|\mathscr{L}^*| \geq 1$ in E-equilibrium, the necessary condition for E-equilibrium to exist is $D(m,0) \geq \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$. On the other hand, for P-equilibrium to exist, it must be that $D(m,0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$ in equilibrium because a pos-

correct preference information by analyzing afternoon transactions only depends on the mass of early buyers who use its E-money in the afternoon market. Alternatively, we can impose an additional assumption that there is a minimum quantity of E-money transactions that an individual early buyer must make in the afternoon market for a company to obtain a meaningful information from the payment data of the early buyer. However, the main results do not change unless the minimum level is sufficiently high.

itive number of companies can make non-negative profits from running its business otherwise. These results are exactly the same as those in the baseline model. Thus, proposition 2 characterizes the existence of each type of equilibrium in the model with the free entry condition in the E-money business.

However, the introduction of competition changes the number of E-money issuers and the price of preference information in E-equilibrium. Substituting equilibrium real balances $m = \widetilde{d}_H(\gamma)$ of late buyers in E-equilibrium into (44), we obtain $|\mathscr{L}^*| \leq \frac{(1-\rho)D(\widetilde{d}_H(\gamma),0)-\mathscr{N}\varsigma}{\rho\delta}$. Then, for any positive integer $|\mathscr{L}^*|$ that does not exceed $\frac{(1-\rho)D(\widetilde{d}_H(\gamma),0)-\mathscr{N}\varsigma}{\rho\delta}$, E-equilibrium exists. Given $|\mathscr{L}^*|$, the equilibrium price of preference information is given as $\varphi_e = \frac{\rho\delta|\mathscr{L}^*|}{\beta\mathscr{N}}$ from (44). Note that if $\frac{(1-\rho)D(\widetilde{d}_H(\gamma),0)-\mathscr{N}\varsigma}{\rho\delta} \geq 2$, then multiple E-equilibria exist with a different pair of $(|\mathscr{L}^*|, \varphi_e)$ in addition to the multiple equilibria issue (E-equilibrium and P-equilibrium) described in proposition 2.

How does the introduction of competition affect the division of surplus from the E-money business among agents and welfare in E-equilibrium? E-money issuers with the free entry condition make zero profits in contrast to the baseline model in which the monopoly company makes positive profits. The monopoly company's surplus, which is distributed to buyers, is passed on to sellers because sellers buy preference information at a lower price than in the baseline model. In particular, if $|\mathscr{L}^*| = 1$, then all profits that the monopoly company obtains in the baseline model go to sellers. In this case, welfare defined in (40) does not change because trade volumes q and x are the same as in the baseline model and only one company runs the E-money business.

We now consider the case with $\frac{(1-\rho)D(\widetilde{d}_H(\gamma),0)-\mathscr{N}\varsigma}{\rho\delta} \geq 2$, so multiple E-equilibria exist with a different pair of $(|\mathscr{L}^*|, \varphi_e)$. In this case, the seller's surplus decreases with the number of E-money issuers, $|\mathscr{L}^*|$, because the price of preference information φ_e increases with $|\mathscr{L}^*|$. The surplus that sellers lost with the higher price φ_e goes to early buyers because they receive reward $R = \frac{\delta}{\beta}$ from $|\mathscr{L}^*|$ companies. Specifically, (44) shows that if $|\mathscr{L}^*|$ increases by 1, the total payment, $\mathscr{N}\varphi_e$, that sellers pay to buy preference information increases by $\rho\frac{\delta}{\beta}$ which equals the aggregate amount of additional rewards that early buyers receive from a new E-money issuer. However, early buyers have to incur δ units of the privacy cost $|\mathscr{L}^*|$ times for using $|\mathscr{L}^*|$ different E-moneys. Thus, an increase in $|\mathscr{L}^*|$ does not lead to an increase in the early buyer's net surplus. It only reduces the seller's surplus and, hence, welfare.

The above analysis implies that when $\frac{(1-\rho)D(\widetilde{d}_H(\gamma),0)-\mathscr{N}\varsigma}{\rho\delta} \geq 2$, the economy has multiple Pareto-ranked E-equilibria with different numbers of active companies operating the E-money business. In particular, all agents are better off in E-equilibrium with a lower number of E-money issuers. Consequently, when multiple E-moneys circulate in the economy, the government can achieve a Pareto improvement by imposing the upper bound on the price of preference information. Specifically, if the government imposes the price ceiling such that $\varphi_e \leq \frac{\rho\delta}{\beta\mathscr{N}}$, then only one company runs the E-money business in E-equilibrium, achieving Pareto efficiency. This result is re-emphasized as the next proposition, whose proof is omitted.

Proposition 6 *If multiple E-moneys circulate in the economy, the government can achieve Pareto improvements by imposing the price ceiling on preference information as* $\varphi_e \leq \frac{\rho\delta}{\beta\mathscr{N}}$.

6 Central bank digital currency and privacy

Although traditional paper money is the major form of legal tender in most countries in the world, central banks in many countries are contemplating issuing central bank digital currency (CBDC).[18] Specifically, Boar et al. (2020) show that more than 80% of central banks in the world are engaging in some sort of CBDC work. Central banks must consider many issues before initiating CBDC, and one of these issues is user privacy.

To study how the introduction of CBDC affects real allocations and welfare in the model economy, we assume that the government issues CBDC in addition to P-money. By opening an account at the central bank, agents can use CBDC. There have recently been extensive studies on the potential implications of CBDC as interest bearing central bank liability in monetary economics. In particular, an increase in interest payments to CBDC in our model works as a decrease in the money growth rate γ, thereby raising welfare. Thus, to focus on the privacy issue, we assume that the central bank does not pay interest on CBDC. Agents can change P-money into CBDC, and vice versa, at a one-to-one exchange ratio at the central bank. Consequently, the sum of the aggregate amounts of P-money and CBDC grows at rate γ.

Technically, the central bank can design CBDC such that it protects user privacy from

[18] See Chapman and Wilkins (2019) and Kumhof and Noone (2018), for instance.

even the central bank, for instance, by applying cryptographic techniques. However, we assume that the government designs CBDC such that the government can observe transaction histories, because, otherwise, CBDC is equivalent to P-money. Thus, buyers incur δ units of disutility for using CBDC. Furthermore, we assume that the government can estimate realized evening tastes from analyzing afternoon transaction data using the same technology that a private company has: The government obtains correct evening taste information with probability $\kappa(B)$, where $B \in [0,\rho]$ is the mass of early buyers who use CBDC in the afternoon.

Because of privacy cost δ, the government must compensate buyers for using CBDC similar to a private company in the baseline model. Although a profit maximizing company does not provide proportional rewards, as shown in lemma 1, proportional rewards for using CBDC work as interest payments on CBDC and can raise trade volume. Thus, a proportional reward is a more efficient tool than a fixed reward to the government. However, to better compare the economy with and without CBDC in this subsection, we first assume that the government only uses fixed rewards as a private company does, and we later analyze the case in which the government can also use proportional rewards.

Given privacy cost δ, early buyers use CBDC in the afternoon market as long as the government provides the fixed reward $R \geq \frac{\delta}{\beta}$ to early buyers for using CBDC in the afternoon market. Without loss of generality, we assume that the government provides a fixed reward $R = \frac{\delta}{\beta}$ to early buyers for using CBDC in the afternoon market whenever the government issues CBDC. As a result, the government obtains the correct information about realized evening tastes by analyzing afternoon transactions.[19]

We assume that the government provides evening taste information to sellers for free, as most central banks in the world provide their research outcomes for free. This implies that when the government issues CBDC, a private company cannot sell preference information and thus cannot run the E-money business. However, if the government does not issue CBDC, then the private company can run the E-money business as long as it can make non-negative profits. We make these assumptions because if early buyers use both E-money and CBDC, then the introduction of CBDC creates welfare loss without changing real allocations by the similar reasoning described in section 5, and if buyers do not use CBDC,

[19]Note that providing fixed rewards to late buyers for using CBDC in the evening market does not change equilibrium trade volume, and it only creates privacy cost of late buyers. Thus, the government should not provide any fixed rewards to late buyers for using CBDC.

then the equilibrium outcomes are the same as those in the baseline model.

Given the fixed rewards, the introduction of CBDC does not change agents' problems and equilibrium allocations much. The key difference is that there exists a unique equilibrium with CBDC for all $\gamma \geq \beta$, because there is no incentive problem of making non-negative profits as a private company does. However, the introduction of CBDC does not necessarily imply higher welfare as stated in the next proposition.

Proposition 7 *Suppose that $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$. Then, welfare with CBDC is $W_P(\gamma) - \rho\delta$, while welfare without CBDC is $W_P(\gamma)$.*

Proof. See Appendix. ■

The results of proposition 7 imply that if $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$, then the introduction of CBDC reduces welfare. To obtain the intuition for these results, it seems worthwhile to discuss, in advance, about under which conditions, we have $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$. Note, from lemma 3 and (38), that $D(\widetilde{d}_H(\gamma),0)$ decreases with γ. Thus, if $\overline{D} < \frac{\mathscr{N}\varsigma}{1-\rho}$, then for all $\gamma \geq \beta$, $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$. On the other hand, if $\overline{D} \geq \frac{\mathscr{N}\varsigma}{1-\rho}$, then there exists γ^{**} such that $D(\widetilde{d}_H(\gamma^{**}),0) = \frac{\mathscr{N}\varsigma}{1-\rho}$ and for all $\gamma > \gamma^{**}$, $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$.

Thus, the condition that $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$ holds only if cost, $\mathscr{N}\varsigma$, of preparing the production of customized goods is sufficiently high and the seller's benefit from preparing the production of customized goods is sufficiently low due to high γ. Consequently, when $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$, sellers do not prepare the production of customized goods in the evening even though the government provides preference information for free. Thus, the real allocations are the same as those in P-equilibrium except that early buyers incur δ units of disutility. On the other hand, if the government does not issue CBDC, then the economy is in P-equilibrium because the private company cannot make non-negative profits from running the E-money business when $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$. Consequently, welfare is $W_P(\gamma) - \rho\delta$ with CBDC and is $W_P(\gamma)$ without CBDC. Thus, if $D(\widetilde{d}_H(\gamma),0) < \frac{\mathscr{N}\varsigma}{1-\rho}$, then, the government should not issue CBDC.

How does the introduction of CBDC affect welfare when $\frac{\mathscr{N}\varsigma}{1-\rho}$ and γ are sufficiently low such that $\frac{\mathscr{N}\varsigma}{1-\rho} \leq \overline{D}$ and $\gamma \leq \gamma^{**}$, and, hence, $\frac{\mathscr{N}\varsigma}{1-\rho} \leq D(\widetilde{d}_H(\gamma),0)$? In this case, sellers will use the preference information provided by the government to prepare the production of customized goods in the evening, as verified by (27) and (35). Thus, the real allocations are exactly the same as those in E-equilibrium so welfare is given as $W_E(\gamma)$. Furthermore, $\gamma^{**} >$

γ_i for each $i \in \{1,2\}$, where γ_1 and γ_2 are defined in proposition 2, by the construction of γ^{**}.[20] As a result, the economy with CBDC can support socially efficient privacy utilization more effectively eliminating the multiple equilibria issue and for a larger range of γ than the economy without CBDC.

Specifically, if $\gamma^* \leq \gamma^{**}$, where γ^* is defined in proposition 4, then for all $\gamma \in [\beta, \gamma^*]$, the government can always support the socially efficient privacy utilization by issuing CBDC. On the other hand, if $\gamma^{**} < \gamma^*$, then the economy achieves the efficient privacy utilization with CBDC for all $\gamma \in [\beta, \gamma^{**}]$, but the government should not issue CBDC for all $\gamma > \gamma^{**}$ because issuing CBDC only reduces welfare by the results of proposition 7.[21] However, even in the case with $\gamma^{**} < \gamma^*$, CBDC can still support an efficient privacy utilization that cannot be achieved without CBDC. For example, when $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*, 0)$, if $\gamma \in (\gamma_1, \gamma^{**}]$, the efficient privacy utilization exists with CBDC, while it does not occur without CBDC, because the private company cannot make non-negative profits.

In this subsection, we have assumed that the government only provides fixed rewards for using CBDC to better compare equilibrium outcomes with CBDC and the results in the baseline model. However, a proportional reward is a more efficient tool to the government because it can increase trade volumes, thereby reducing welfare costs from positive money holding costs, similar to interest payments on CBDC. Specifically, whenever the government wants to make early (late) buyers use CBDC in the afternoon (evening) market, it is socially optimal for the government to provide proportional rewards such that the trade volume in the afternoon (evening) market is efficient, and provides appropriate fixed rewards if necessary.[22] Thus, the introduction of CBDC can contribute to welfare by supporting efficient trades in addition to the channel that we discussed above — supporting efficient privacy utilization.

[20] Specifically, γ^{**} is determined by $D(\widetilde{d}_H(\gamma^{**}), 0) = \frac{\mathscr{N}\varsigma}{1-\rho}$, while γ_1 and γ_3 are determined by $D(\widetilde{d}_H(\gamma_i), 0) = \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$ for $i \in \{1,2\}$, detailed in the proof of proposition 2. Given that $D(\widetilde{d}_H(\gamma), 0)$ decreases with γ, it must be that $\gamma^{**} > \gamma_i$.

[21] Note that for all $\gamma > \gamma^{**}$, $D(\widetilde{d}_H(\gamma), 0) < \frac{\mathscr{N}\varsigma}{1-\rho}$ by the construction of γ^{**}. Thus, welfare is lower with CBDC than without CBDC as shown in proposition 7.

[22] Specifically, given the fixed privacy cost and proportional rewards, whenever a buyer decides to use CBDC, he/she will only use CBDC without holding any P-money. Then, it can be verified, from (9) - (12), (15), (17), (22) - (24), and (31), that the government can support the efficient quantity of trade in the afternoon and in the evening by subsidizing $\frac{\gamma-\beta}{\gamma}$ and $\frac{\gamma-\beta}{\gamma-\beta\theta}$ fractions of the CBDC payments in the afternoon market and the evening market, respectively, as proportional rewards.

7 Heterogeneous buyers

In the baseline model, we assume that the disutility from forgoing privacy is the same for all buyers. However, in reality, people could have different preferences for keeping personal information from others. To capture this, we extend the baseline model such that buyers are heterogeneous in terms of disutility δ from losing privacy. We assume that δ is uniformly distributed over $\left[\underline{\delta}, \overline{\delta}\right]$. In this section, we assume that the monopoly company issues E-money.

Introducing heterogeneity does not change the problems of buyers and sellers. Specifically, early buyers use E-money if $\beta R \geq \delta$ and use P-money otherwise.[23] Furthermore, (16) - (24), and (27) describe the optimal choices of buyers and sellers in the extended model. However, introducing heterogeneous δ changes the company's problem of deciding the level of reward R and market clearing conditions in the afternoon market.

Given that early buyers use E-money if $\beta R \geq \delta$, the $\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}$ mass of early buyers uses E-money in the afternoon for all $R \in \left[\frac{\underline{\delta}}{\beta}, \frac{\overline{\delta}}{\beta}\right]$.[24] The mass $\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}$, in turn, determines the probability of obtaining the correct information about the evening tastes as $\kappa\left(\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}\right)$. Then, from (27), we obtain the price of preference information as

$$\varphi = \frac{(1-\rho)\kappa\left(\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}\right)D(m,0)-\mathscr{N}\varsigma}{\beta\mathscr{N}},$$

where m is the equilibrium real balances of late buyers in the evening market, given the monopoly power of the company. The company provides R units of morning goods to $\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}$ mass of early buyers in the next morning and sells all preference information to all sellers in the evening. Then, the company's discounted profit is

$$\beta\pi = \max_{R\in\left[\frac{\underline{\delta}}{\beta},\frac{\overline{\delta}}{\beta}\right]} \left\{-\frac{\beta R\rho\left(\beta R-\underline{\delta}\right)}{\overline{\delta}-\underline{\delta}} + (1-\rho)\kappa\left(\frac{\rho\left(\beta R-\underline{\delta}\right)}{\overline{\delta}-\underline{\delta}}\right)D(m,0)-\mathscr{N}\varsigma\right\}, \tag{45}$$

[23] By the same reasoning of lemma 1, it is optimal for the company not to provide any proportional rewards in the extended model.

[24] If $\beta R < \underline{\delta}$, no early buyers use E-money and there is no reason for the company to set $\beta R > \overline{\delta}$.

which gives the first order condition as

$$-(2\beta R-\underline{\delta})+(1-\rho)\kappa'\left(\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}\right)D(m,0)-\lambda_1+\lambda_2=0, \tag{46}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are the Lagrange multipliers such that $\lambda_1 = 0$ if $R < \beta\overline{\delta}$ and $\lambda_2 = 0$ if $R > \beta\underline{\delta}$. Then, for E-equilibrium (P-equilibrium) to exist, it must be the case that $\beta\pi \geq 0$ ($\beta\pi < 0$).

Next, in the afternoon market, the $\frac{\rho(\overline{\delta}-\beta R)}{\overline{\delta}-\underline{\delta}}$ mass of buyers purchases goods with P-money, and the $\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}$ mass of buyers purchases goods with E-money. Thus, the market clearing conditions in the afternoon market are

$$\frac{\rho(\overline{\delta}-\beta R)}{\overline{\delta}-\underline{\delta}}q_p = q_p^s \text{ and } \frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}q_e = q_e^s. \tag{47}$$

Although market clearing conditions change, the equilibrium quantity of afternoon goods that an early buyer purchases is given as (33), which can be obtained from (17) with $\varkappa_a = 0$, (19), (24), and (47).

In the extended model, a late buyer can meet a seller who cannot produce customized goods with probability $1-\kappa\left(\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}\right)$ in E-equilibrium, although the company runs the E-money business, because only the $\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}$ mass of early buyers use E-money in the afternoon. Thus, welfare (40) must be adjusted as

$$\begin{aligned} W = \rho\upsilon(q) - c(\rho q) + (1-\rho)&\left\{\begin{array}{c} \kappa\left(\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}\right)[\alpha_H u(x_H)-x_H] \\ +\left(1-\kappa\left(\frac{\rho(\beta R-\underline{\delta})}{\overline{\delta}-\underline{\delta}}\right)\right)[\alpha_L u(x_L)-x_L] \end{array}\right\} \\ -\mathbf{1}_{\{e=E\}}\mathscr{N}\varsigma - \rho\int_{\underline{\delta}}^{\beta R}\frac{\delta}{\overline{\delta}-\underline{\delta}}d\delta, \end{aligned}$$

where x_H and x_L are the equilibrium trade volume in the evening market when a seller produces customized goods and non-customized goods, respectively, in a bilateral meeting.

To study equilibrium allocations and welfare in the extended model, we conduct a numerical analysis with functions $\upsilon(q) = \frac{q^{1-\eta}}{1-\eta}$, $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$, $c(q) = q^{\mu}$, and $\kappa(B) = \left(\frac{B}{\rho}\right)^r$, where $(\eta,\sigma) \in (0,1)^2$, $\mu > 1$, and $r > 0$. We set $\beta = 0.97$, $\eta = 0.7$, $\sigma = 0.5$, $\mu = 2$, $\rho = 0.5$, $\theta = 0.1$, $r = 0.3$, $\alpha_L = 1$, $\alpha_H = 1.2$, $\underline{\delta} = 0$, $\overline{\delta} = 0.1253$, and $\mathscr{N}\varsigma = 0.0125$.

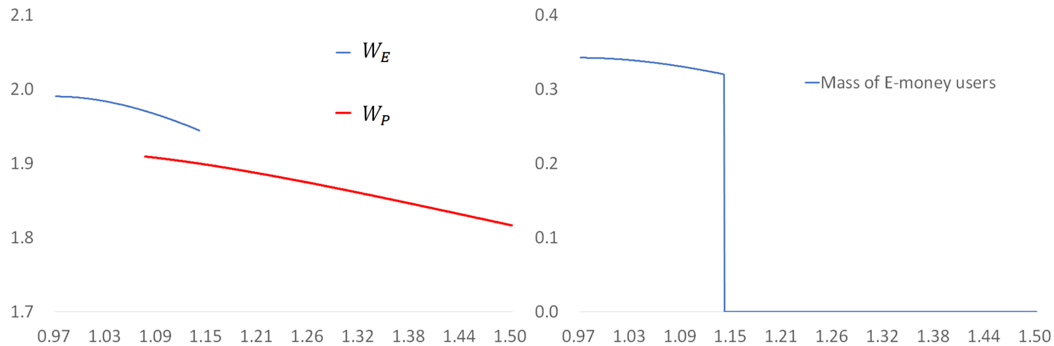

Figure 3: Mass of E-money users and weflare in the extended model

Figure 3 describes welfare and the mass of E-money users in the extended model. As shown in the figure, introducing heterogeneous disutility from providing personal information to the third party in the extended model does not change the main implications of the baseline model: E-equilibrium is more likely to exist with low γ, and the E-money business improves welfare whenever the company runs the business. An additional finding is that the mass of early buyers who use E-money in E-equilibrium decreases with γ in the extended model, while it is constant at ρ in the baseline model.

The economic mechanism for the results described in Figure 3 is similar to that for the baseline model. As γ increases, late buyers carry less real balances into the evening market, which reduces the trade volume and the seller's benefit $D(m,0)$ from preparing the production of customized goods. Given that κ is concave in the numerical exercise, a decrease in $D(m,0)$ reduces reward R, as determined by (46), whenever $R \in \left(\frac{\underline{\delta}}{\beta}, \frac{\overline{\delta}}{\beta}\right)$. This, in turn, decreases the mass of E-money users $\frac{\rho(\beta R - \underline{\delta})}{\overline{\delta}-\underline{\delta}}$. Furthermore, a decrease in $D(m,0)$ reduces the company's profit in (45), so E-equilibrium is more likely to exist when γ is low and P-equilibrium is more likely to exist when γ is high.

8 Conclusion

In this paper, we have developed a money search model in which the electronic payment platform company issues E-money and estimates consumers' preferences by analyzing E-money transaction data. Sellers purchase preference information to produce goods that better match consumers' preferences. We have shown that the company runs the business of issuing E-money and analyzing payment data only if inflation is sufficiently low. Socially efficient privacy utilization may not occur because of a wedge between the socially efficient and profitable uses of payment data. The introduction of central bank digital currency can increase or decrease welfare depending on whether it supports efficient privacy utilization. If there is no entry cost to the E-money business, there can exist Pareto-ranked multiple equilibria and the government can achieve a Pareto improvement by imposing a price ceiling on preference information.

Appendix: Omitted proofs

Proof of Lemma 1. We first show that $\varkappa_a = 0$. Substituting (17) and (19) into (16) and (18), respectively, we obtain

$$\begin{aligned} S_e^{early} &= \beta R - \delta - \gamma p(1-\varkappa_a)\upsilon'^{-1}(\gamma p(1-\varkappa_a)) + \upsilon(\upsilon'^{-1}(\gamma p(1-\varkappa_a))) \\ S_p^{early} &= -\gamma p \upsilon'^{-1}(\gamma p) + \upsilon(\upsilon'^{-1}(\gamma p)). \end{aligned}$$

Then, given the monopoly power, the company will set R and $\varkappa_a$ such that $S_e^{early} = S_p^{early}$, which gives

$$\begin{aligned} \beta R = \delta &- \left\{-\gamma p(1-\varkappa_a)\upsilon'^{-1}(\gamma p(1-\varkappa_a)) + \upsilon(\upsilon'^{-1}(\gamma p(1-\varkappa_a)))\right\} \\ &+ \left\{-\gamma p \upsilon'^{-1}(\gamma p) + \upsilon(\upsilon'^{-1}(\gamma p))\right\}. \end{aligned} \tag{48}$$

The company pays $p\varkappa_a q_e$ units of E-money, which is backed by P-money, to sellers in the afternoon as subsidies for buying goods with E-money in the afternoon market. Combined with the fixed reward, which is given by (48), the total cost of attracting each early buyer is given as

$$\delta + \left[\gamma p \upsilon'^{-1}(\gamma p(1-\varkappa_a)) - \upsilon(\upsilon'^{-1}(\gamma p(1-\varkappa_a))) - \gamma p \upsilon'^{-1}(\gamma p) + \upsilon(\upsilon'^{-1}(\gamma p))\right].$$

Note that the term in the square bracket is strictly positive for all $\varkappa_a > 0$ and zero with $\varkappa_a = 0$. Thus, it is optimal for the company to set $\varkappa_a = 0$.

We now prove that $\varkappa_e = 0$ by showing that a company's profit decreases with $\varkappa_e$. First, note that there is no reason for the company to provide any rewards to late buyers for using E-money in the evening market if early buyers do not use E-money in the afternoon market. Thus, we assume that early buyers use E-money. Furthermore, we assume that late buyers choose to use E-money for evening transactions, and hence, $m_p = 0$, to analyze how proportional reward $\varkappa_e$ affects a company's profit.

Given $m_p = 0$, we can re-write the bargaining problem, from (9) - (12), as

$$\max_{x,d_e} \left\{ \alpha_i u(x) - x + \frac{\varkappa_e}{1-\theta\varkappa_e} \left[\theta\alpha_i u(x) + (1-\theta)x\right] \right\} \tag{49}$$

subject to

$$\beta(1-\varkappa_e)d_e = \frac{1-\varkappa_e}{1-\theta\varkappa_e} \left[\theta\alpha_i u(x) + (1-\theta)x\right] \leq \beta m_e, \tag{50}$$

for each $i \in \{H, L\}$. Let $\hat{x}_i(0, m_e)$ and $\hat{d}_i(0, m_e)$ denote the solution to the above maximization problem given $i \in \{H, L\}$. Note that $\hat{x}_i(0, m_e)$ increases with $\varkappa_e$ whenever the constraint (50) binds.

By (27), the company will set the price of preference information as

$$\varphi = \frac{(1-\rho)\kappa(B)D(0,m_e) - \mathscr{N}\varsigma}{\beta\mathscr{N}}$$

given the monopoly power, and sell all preference information to all sellers. Then, the discounted revenue from selling preference information is given as

$$\beta\mathscr{N}\varphi = (1-\rho)\kappa(B)D(0,m_e) - \mathscr{N}\varsigma.$$

Next, the cost of running the E-money business consists of rewards. First, given the monopoly power and the finding that the company does not provide proportional rewards to early buyers, i.e., $\varkappa_a = 0$, the company sets the fixed reward as $R = \frac{\delta}{\beta}$ by (16) and (18) to attract early buyers to use E-money in the afternoon market. Second, the company subsidizes $\varkappa_e d_e$ units of E-money transfers in each bilateral meeting in the evening as proportional rewards to late buyers. From the above analysis, we obtain discounted net profit as

$$\beta\pi = (1-\rho)D(0,m_e) - \mathscr{N}\varsigma - \left[\rho\delta + (1-\rho)\gamma\varkappa_e d_e\right], \tag{51}$$

where we impose the condition that $\kappa(B) = 1$ because all early buyers use E-money in the afternoon market, i.e., $B = \rho$, given that $R = \frac{\delta}{\beta}$.

Next, late buyers will minimize idle E-money that is not used in the evening market given the linearity of the value function in the morning with respect to asset holdings. This implies that $(1-\varkappa_e)\hat{d}_{e,H}(0,m_e)=m_e$. Then, from (26) and the binding (50), we obtain

$$D(0,m_e)=-\hat{x}_H(0,m_e)+\frac{\beta m_e}{1-\varkappa_e}-\theta\left[\alpha_L u(\hat{x}_L(0,m_e))-\hat{x}_L(0,m_e)+\beta\varkappa_e\hat{d}_{e,L}(0,m_e)\right]. \tag{52}$$

Note that the term $\alpha_L u(\hat{x}_L(0,m_e))-\hat{x}_L(0,m_e)+\beta\varkappa_e\hat{d}_{e,L}(0,m_e)$ increases with $\varkappa_e$ by (49) and (50). Then, from the binding (50), (51), and (52), we obtain

$$\begin{aligned}&\frac{\partial(\beta\pi)}{\partial\varkappa_e}\\&=(1-\rho)\left\{-\frac{\partial\hat{x}_H(0,m_e)}{\partial\varkappa_e}-\frac{m_e(\gamma-\beta)}{(1-\varkappa_e)^2}-\theta\frac{\partial\left[\alpha_L u(\hat{x}_L(0,m_e))-\hat{x}_L(0,m_e)+\beta\varkappa_e\hat{d}_{e,L}(0,m_e)\right]}{\partial\varkappa_e}\right\}<0.\end{aligned}$$

Thus, it is optimal for the company to set $\varkappa_e=0$, which finishes the proof. ■

Proof of Lemma 2. Given $m_e=0$, it must be that $d_{e,i}=0$ for each $i\in\{H,L\}$, and the objective function (9) is given as $\alpha_i u(x)-x$. Take any $i\in\{H,L\}$ for α_i. Note that the objective function $\alpha_i u(x)-x$ is maximized when $x=x_i^*$, where $\alpha_i u'(x_i^*)=1$. Thus, if $x=x_i^*$ is feasible, then it must be the solution, and $x=x_i^*$ is feasible only if $d_{p,i}=\frac{\theta\alpha_i u(x_i^*)+(1-\theta)x_i^*}{\beta}\le m_p$ by the constraints (10) and (11). On the other hand, if $m_p<\frac{\theta\alpha_i u(x_i^*)+(1-\theta)x_i^*}{\beta}$, then $x=x_i^*$ is not attainable. In this case, it is optimal to use all P-money, i.e., $d_{p,i}=m_p$, to maximize the trade volume x given the bargaining rule (10), because the objective function $\alpha_i u(x)-x$ increases with x for all $x<x_i^*$. By combining the above two cases, we obtain the results of lemma 2. ■

Proof of Lemma 3. Note, from (30), that for all $m_p\ge m_H^*$, $\hat{x}_H(m_p,0)=x_H^*$ and $\hat{x}_L(m_p,0)=x_L^*$, and hence, $D(m_p,0)=\theta\left[\alpha_H u(x_H^*)-x_H^*\right]-\theta\left[\alpha_L u(x_L^*)-x_L^*\right]\equiv\overline{D}$ by (26). Suppose that $m_L^*\le m_p<m_H^*$. Then, from (26) and (30), we obtain

$$D(m_p,0)=\theta\left[\alpha_H u(\hat{x}_H(m_p,0))-\hat{x}_H(m_p,0)\right]-\theta\alpha_L\left[\alpha_L u(x_L^*)-x_L^*\right],$$

which strictly decreases with m_p because $\frac{\partial\hat{x}_H(m_p,0)}{\partial m_p}<0$ and $\hat{x}_H(m_p,0)<x_H^*$ for all $m_p<m_H^*$.

Finally, suppose that $m_p < m_L^*$. Then, (30) gives

$$\theta\alpha_L u(\hat{x}_L(m_p,0)) + (1-\theta)\hat{x}_L(m_p,0) = \theta\alpha_H u(\hat{x}_H(m_p,0)) + (1-\theta)\hat{x}_H(m_p,0) = \beta m_p. \tag{53}$$

Because $\alpha_H > \alpha_L$, it must be that $\hat{x}_L(m_p,0) > \hat{x}_H(m_p,0)$ to satisfy (53). Substituting (53) into (26) gives

$$D(m_p,0) = \hat{x}_L(m_p,0) - \hat{x}_H(m_p,0).$$

Taking partial derivative $D(m_p,0)$ with respect to m_p and using (53), we obtain

$$\frac{\partial D(m_p,0)}{\partial m_p} = \frac{\beta}{\theta\alpha_L u'(\hat{x}_L(m_p,0)) + 1 - \theta} - \frac{\beta}{\theta\alpha_H u'(\hat{x}_H(m_p,0)) + 1 - \theta},$$

which is positive because $\hat{x}_L(m_p,0) > \hat{x}_H(m_p,0)$. In summary, we obtain that $\frac{\partial D(m_p,0)}{\partial m_p} > 0$ for all $m_p < m_H^*$. ■

Proof of Proposition 2. We prove the proposition as follows: First, we analyze the necessary conditions for the existence of E-equilibrium. Second, we analyze the necessary conditions for the existence of P-equilibrium. Third, we show that $\gamma_2 < \gamma_1$ when $\frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho} < D(m_L^*,0)$.

Step 1 In E-equilibrium, the company must make non-negative profits given the late buyer's equilibrium real balances $(m_p,m_e) = (\widetilde{d}_H(\gamma),0)$. By substituting $m_p = \widetilde{d}_H(\gamma)$ into (39), we obtain the company's profit as

$$\pi = \frac{(1-\rho)D(\widetilde{d}_H(\gamma),0) - \mathcal{N}\varsigma - \rho\delta}{\beta}.$$

Thus, to have non-negative profits, it must be that $D(\widetilde{d}_H(\gamma),0) \geq \frac{\mathcal{N}\varsigma+\rho\delta}{1-\rho}$. Note, from (1), (37), and (38), that $\widetilde{x}_H(\gamma) < x_H^*$ and $\widetilde{d}_H(\gamma) < m_H^*$ for all $\gamma > \beta$. Then, from (26), (30), (37), and (38), we obtain

$$D(\widetilde{d}_H(\gamma),0) = \theta\left[\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma)\right] - \theta\left[\alpha_L u\left(\hat{x}_L(\widetilde{d}_H(\gamma),0)\right) - \hat{x}_L(\widetilde{d}_H(\gamma),0)\right]. \tag{54}$$

First, suppose that $\gamma \in \left[\beta, \widetilde{d}_H^{-1}(m_L^*)\right]$, which implies $\widetilde{d}_H(\gamma) \geq m_L^*$ and $\hat{x}_L(\widetilde{d}_H(\gamma),0) = x_L^*$

by (30).[25] Then, we obtain, from (54), that

$$D(\widetilde{d}_H(\gamma),0) = \theta\left[\alpha_H u\left(\widetilde{x}_H(\gamma)\right) - \widetilde{x}_H(\gamma)\right] - \theta\left[\alpha_L u\left(x_L^*\right) - x_L^*\right] \tag{55}$$

which decreases with γ. Evaluating (55) at $\gamma = \widetilde{d}_H^{-1}(m_L^*)$ gives

$$D(m_L^*,0) = \theta\left[\alpha_H u\left(\widetilde{x}_H(\widetilde{d}_H^{-1}(m_L^*))\right) - \widetilde{x}_H(\widetilde{d}_H^{-1}(m_L^*))\right] - \theta\left[\alpha_L u\left(x_L^*\right) - x_L^*\right].$$

Then, if $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*,0)$, for all $\gamma \in \left[\beta, \widetilde{d}_H^{-1}(m_L^*)\right]$, E-equilibrium exists because $D(\widetilde{d}_H(\gamma),0)$ in (55) decreases with γ. Now suppose that $D(m_L^*,0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$. From (37) and (55), we obtain $D(\widetilde{d}_H(\beta),0) = \overline{D}$, where $\overline{D}$ is defined in (32). Then, if $\overline{D} < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$, E-equilibrium cannot exist for all $\gamma \geq \beta$. On the other hand, if $D(m_L^*,0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq \overline{D}$, then there exists $\gamma_3 \in \left[\beta, \widetilde{d}_H^{-1}(m_L^*)\right)$ such that

$$\theta\left[\alpha_H u\left(\widetilde{x}_H(\gamma_3)\right) - \widetilde{x}_H(\gamma_3)\right] - \theta\left[\alpha_L u\left(x_L^*\right) - x_L^*\right] = \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}.$$

Then, for all $\gamma \in [\beta, \gamma_3]$, E-equilibrium exists.

Second, suppose that $\gamma > \widetilde{d}_H^{-1}(m_L^*)$, which implies $\widetilde{d}_H(\gamma) < m_L^*$. From (30) and (38), we obtain

$$\theta\alpha_L u\left(\hat{x}_L(\widetilde{d}_H(\gamma),0)\right) + (1-\theta)\hat{x}_L(\widetilde{d}_H(\gamma),0) = \theta\alpha_H u\left(\widetilde{x}_H(\gamma)\right) + (1-\theta)\widetilde{x}_H(\gamma) = \beta\widetilde{d}_H(\gamma).$$

Note that $\hat{x}_L(\widetilde{d}_H(\gamma),0) > \widetilde{x}_H(\gamma)$ because $\alpha_H > \alpha_L$. Because $\lim\limits_{\gamma\to\widetilde{d}_H^{-1}(m_L^*)} D(\widetilde{d}_H(\gamma),0) = D(m_L^*,0)$, if $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \geq D(m_L^*,0)$, then for all $\gamma > \widetilde{d}_H^{-1}(m_L^*)$, E-equilibrium cannot exist. Now assume that $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} < D(m_L^*,0)$ and define γ_1 such that

$$D(\widetilde{d}_H(\gamma_1),0) = \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}. \tag{56}$$

Given that $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} < D(m_L^*,0)$, it must be that $\gamma_1 > \widetilde{d}_H^{-1}(m_L^*)$. Then, for all $\gamma \in \left(\widetilde{d}_H^{-1}(m_L^*), \gamma_1\right]$, E-equilibrium exists, and for all $\gamma > \gamma_1$, E-equilibrium cannot exist.

[25] Note that $\widetilde{d}_H^{-1}(m_L^*) > \beta$, because $m_H^* > m_L^*$, as shown in (1), and $\widetilde{d}_H(\beta) = m_H^*$.

In summary, E-equilibrium exists if one of the following conditions holds:

1. $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*,0)$ and $\gamma \in [\beta, \gamma_1]$.

2. $D(m_L^*,0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq \overline{D}$ and $\gamma \in [\beta, \gamma_3]$.

Step 2 In P-equilibrium, the late buyer's real balance in the evening is $m_p = \widetilde{d}_L(\gamma)$ by (36). If the company runs the E-money business by setting reward R and information price φ as described in (28) and (29), respectively, then the company's profit is given as $\pi = \frac{(1-\rho)D(\widetilde{d}_L(\gamma),0)-\mathscr{N}\varsigma-\rho\delta}{\beta}$ by (39). In P-equilibrium, the company should not be able to make non-negative profits. Thus, it must be that $D(\widetilde{d}_L(\gamma),0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$ for P-equilibrium to exist.

Note, from (37) and (38), that $\widetilde{d}_L(\gamma)$ decreases with γ, and, hence, $D(\widetilde{d}_L(\gamma),0)$ decreases with γ by results of lemma 3. From (1), (37), and (38), we obtain $\widetilde{d}_L(\beta) = m_L^*$, and hence, $D(\widetilde{d}_L(\beta),0) = D(m_L^*,0)$. Thus, if $D(m_L^*,0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$, then for all $\gamma \geq \beta$, P-equilibrium exists. On the other hand, if $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*,0)$, then there exists $\gamma_2 \in \left[\beta, \frac{\beta}{\theta}\right]$ such that

$$D(\widetilde{d}_L(\gamma_2),0) = \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \tag{57}$$

because $\lim_{\gamma \to \frac{\beta}{\theta}} D(\widetilde{d}_L(\gamma),0) = 0$. Then, for all $\gamma > \gamma_2$, P-equilibrium exists.

In summary, P-equilibrium exists if one of the following conditions holds:

1. $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*,0)$ and $\gamma > \gamma_2$.

2. $D(m_L^*,0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$.

Step 3 We now show $\gamma_2 < \gamma_1$ when $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*,0)$. From (56) and (57), we obtain that

$$D(\widetilde{d}_H(\gamma_1),0) = D(\widetilde{d}_L(\gamma_2),0) = \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}.$$

Note that $\widetilde{d}_H(\gamma) > \widetilde{d}_L(\gamma)$ for all $\gamma \geq \beta$ by (38). Thus, it must be that $\gamma_1 > \gamma_2$ because $\widetilde{d}_i(\gamma)$ decreases with γ for each $i \in \{H, L\}$ and $\frac{\partial D(m_p,0)}{\partial m_p} > 0$ by the results of lemma 3.

By combining and reorganizing the results of steps 1 - 3, we obtain proposition 2. ■

Proof of Proposition 3. Substituting equilibrium real balance of late buyers $\widetilde{d}_H(\gamma)$ to (39), we obtain that the company's profit is non-negative if and only if $D(\widetilde{d}_H(\gamma),0) \geq \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$. Next, from (41) and (42), we obtain that $W_E(\gamma) \geq W_P(\gamma)$ if and only if $\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma) - [\alpha_L u(\widetilde{x}_L(\gamma)) - \widetilde{x}_L(\gamma)] \geq \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$. From (26), (30), (37), and (38), we obtain that

$$\begin{aligned}
D(\widetilde{d}_H(\gamma),0) &= \theta\left\{\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma) - \left[\alpha_L u\left(\hat{x}_L(\widetilde{d}_H(\gamma),0)\right) - \hat{x}_L(\widetilde{d}_H(\gamma),0)\right]\right\} \\
&\leq \theta\left\{\alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma) - [\alpha_L u(\widetilde{x}_L(\gamma)) - \widetilde{x}_L(\gamma)]\right\} \\
&< \alpha_H u(\widetilde{x}_H(\gamma)) - \widetilde{x}_H(\gamma) - [\alpha_L u(\widetilde{x}_L(\gamma)) - \widetilde{x}_L(\gamma)],
\end{aligned}$$

where we use the property that $\widetilde{x}_L(\gamma) \leq \hat{x}_L(\widetilde{d}_H(\gamma),0) \leq x_L^*$ because $\widetilde{d}_L(\gamma) < \widetilde{d}_H(\gamma)$ to obtain the first inequality. Then, whenever $D(\widetilde{d}_H(\gamma),0) \geq \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho}$, it must be that $W_E(\gamma) > W_P(\gamma)$, which finishes the proof. ■

Proof of Proposition 4. Suppose that $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} < \frac{\overline{D}}{\theta}$. Then, from (1), (32), (37), (41), and (42), we obtain

$$W_E(\beta) - W_P(\beta) = (1-\rho)\left\{\alpha_H u(x_H^*) - x_H^* - [\alpha_L u(x_L^*) - x_L^*]\right\} - (\mathscr{N}\varsigma + \rho\delta) > 0.$$

Note, from (37), that as γ goes to $\frac{\beta}{\theta}$, $\widetilde{x}_i(\gamma)$ goes to zero. Thus, $\lim\limits_{\gamma\to\frac{\beta}{\theta}}\{W_E(\gamma) - W_P(\gamma)\} = -(\mathscr{N}\varsigma + \rho\delta)$. Then, by Intermediate Value Theorem, the set

$$\Phi \equiv \left\{\gamma \in \left(\beta, \frac{\beta}{\theta}\right) : W_E(\gamma) - W_P(\gamma) = 0\right\}$$

is non-empty, because the function $W_E(\gamma) - W_P(\gamma)$ is continuous with respect to γ. By setting $\gamma^* = \min\Phi$, we obtain that for all $\gamma \in [\beta, \gamma^*]$, $W_E(\gamma) \geq W_P(\gamma)$, with a strict inequality for $\gamma < \gamma^*$.

Next, suppose that $D(m_L^*,0) < \frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq \overline{D}$. Then, for all $\gamma \in [\beta, \gamma_3]$, $W_E(\gamma) > W_P(\gamma)$ by the results of propositions 2 and 3. Thus, it must be that $\gamma^* = \min\Phi > \gamma_3$. Similarly, if $\frac{\mathscr{N}\varsigma+\rho\delta}{1-\rho} \leq D(m_L^*,0)$, then for all $\gamma \in [\beta, \gamma_1]$, $W_E(\gamma) > W_P(\gamma)$, and hence, it must be that $\gamma^* = \min\Phi > \gamma_1$ by the same argument. ■

Proof of Lemma 4. Note that it must be that $|\mathscr{L}^*| < \infty$ in any E-equilibrium, because profit π in (43) is negative otherwise. Suppose that there exists an E-equilibrium in which $\frac{\mathscr{N}\varphi_e}{|\mathscr{L}^*|} > \frac{\rho\delta}{\beta}$ and, hence, active companies make positive profits. Similar to pre-existing active companies, a new company can make all early buyers use its E-money by providing $R = \frac{\delta}{\beta}$ units of reward and can obtain the correct information about evening tastes. Then, if the new company sells its information at price $\varphi_e - \varepsilon$ with a sufficiently small $\varepsilon > 0$, then all sellers buy the evening taste information from that company. Then, the new company's profit is given as $\pi' = \mathscr{N}(\varphi_e - \varepsilon) - \frac{\rho\delta}{\beta}$ that is strictly positive given that $\frac{\mathscr{N}\varphi_e}{|\mathscr{L}^*|} > \frac{\rho\delta}{\beta}$. Thus, the new company has an incentives of running its own business, which is a contradiction. Thus, E-equilibrium in which $\frac{\mathscr{N}\varphi_e}{|\mathscr{L}^*|} > \frac{\rho\delta}{\beta}$ cannot exist. ■

Proof of Proposition 7. Suppose that $D(\widetilde{d}_H(\gamma), 0) < \frac{\mathscr{N}\varsigma}{1-\rho}$ and the government issues CBDC with the fixed reward $R = \frac{\delta}{\beta}$ to early buyers for using CBDC in the afternoon market. Then, all early buyers use CBDC in the afternoon and, hence, the government obtains the correct information about the realized evening tastes with certainty.

If late buyers expect that they can purchase customized goods in the evening with certainty, then they will carry $\widetilde{d}_H(\gamma)$ units of real money into the evening market. Then, according to (27), sellers prepare the production of customized goods in the evening only if

$$D(\widetilde{d}_H(\gamma), 0) \geq \frac{\mathscr{N}\varsigma}{1-\rho}.$$

Thus, given that $D(\widetilde{d}_H(\gamma), 0) < \frac{\mathscr{N}\varsigma}{1-\rho}$, sellers will not prepare the production of customized goods although the government provides the preference information for free. Thus, we obtain a contradiction. Next, suppose that late buyers expect that they cannot buy customized goods in the evening with certainty. Then, late buyers' real balance is $\widetilde{d}_L(\gamma)$. Because $\widetilde{d}_H(\gamma) > \widetilde{d}_L(\gamma)$ and $\frac{\partial D(m,0)}{\partial m} > 0$, we obtain $D(\widetilde{d}_L(\gamma), 0) < \frac{\mathscr{N}\varsigma}{1-\rho}$. Thus, sellers do not prepare the production of customized goods in the evening market, justifying late buyers' initial guess.

Consequently, if $D(\widetilde{d}_H(\gamma), 0) < \frac{\mathscr{N}\varsigma}{1-\rho}$, sellers will never prepare the production of customized goods even though the government provides the preference information for free. Then, equilibrium real allocations are the same as those in P-equilibrium except that early buyers incur δ units of disutility for providing private information to the government. Thus, welfare with CBDC, in this case, is given as $W_P(\gamma) - \rho\delta$.

Now suppose that the government does not issue CBDC. Then, the private company may run the E-money business. However, if $D(\widetilde{d}_H(\gamma), 0) < \frac{\mathscr{N}\varsigma}{1-\rho}$, then the company cannot make non-negative profits from running the E-money business as one can see from proposition 1 and (39). Thus, the economy is in P-equilibrium without CBDC and welfare is given as $W_P(\gamma)$. ■

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