System Wide Runs and Financial Collapse

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Abstract

This paper presents a model where system wide runs can be triggered by small shocks to asset values. When some financial institutions suffer an adverse shock to asset values and lenders cannot distinguish who has suffered the losses, healthier financial institutions can differentiate themselves from weaker firms by offering to borrow less at more favorable prices, when they roll over their short-term debt. To successfully separate, the healthy institutions must liquidate a fraction of their portfolio, which leads to a fire sale of assets. However, these fire sales worsen the balance sheet integrity of the firms. If the fire sale is too severe, even the otherwise healthy institutions turn insolvent, and this leads to the complete collapse of the financial system: a system wide run.

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1 Introduction

Losses from the subprime mortgage sector that triggered the financial crisis of 2008 were small compared to the catastrophic events that followed. According to former Fed chairman Ben Bernanke (2013), a complete wipe-out of all the subprime mortgages in the U.S. would be *"about equivalent to one bad day in the stock market: they were not very big."* The problem occurred, in his view, over the uncertainty regarding which institutions had suffered the brunt of the losses. An influential characterization for the crisis along these lines was presented by Gorton and Metrick (2012). They argued that the financial crisis was a system wide run, where problems in the subprime mortgage sector triggered a run on repos resulting in the panic of 2007-2008. However, the detailed investigations of the events during the crisis by both Copeland et al.(2014) and Krishnamurthy et al.(2014) cloud the notion of traditional bank runs on repos. Krishnamurthy et al.(2014), in particular, push back against the idea of traditional bank runs on repo and conclude that the phenomenon resembles a credit crunch that disproportionately affected a select number of dealers. Moreover, the link between these runs to the small shocks to asset values are unclear.

In this paper, I present a model of system wide runs, in which runs can be triggered by small shocks to fundamentals when there is uncertainty over who suffered the losses. When a fraction of financial institutions suffer an adverse shock to asset values and lenders cannot distinguish who has suffered the losses, healthier financial institutions can differentiate themselves from weaker firms by offering to borrow less at more favorable prices, when they roll over their short-term debt. To successfully separate, the healthy institutions must liquidate a fraction of their portfolio, which leads to a fire sale of assets. However, these fire sales worsen the balance sheet integrity of the firms. If the fire sale is too severe, even the otherwise healthy institutions turn insolvent, and this leads to the complete collapse of the financial system.

The key insight of this paper is that system wide runs can be caused by the incentives of financial institutions to separate and secure better terms on their debt. Institutions with different probabilities of default have divergent preferences over the combination of the amount of total debt they issue and the price of each unit of debt. Institutions with lower probability of default have a higher affinity for receiving a better price for each unit of debt, while institutions with higher probability of default prefer to obtain as much funding as possible. Therefore, when there is asymmetric information between borrowers and lenders regarding the financial health of the borrowers, the healthier institutions can credibly signal their type by reducing the total amount of borrowing in exchange for more favorable prices. In equilibrium, the liquidation of assets required to reduce borrowing results in large price discounts which implies that firms suffer a loss on their asset sales. However, as each individual institution is small, the institutions fail to internalize the decreases in asset value that follow from the collective reduction in borrowing.

Asset fire sales threaten the survival of the financial sector as a whole. If the fire sale discount is too severe, even otherwise healthy institutions may turn insolvent. As lenders are unwilling to lend to insolvent institutions, this results in a complete credit market collapse and consequent failure of the financial system – a system wide run. Thus, a system wide run in this model is fundamentally different from a collection of individual bank runs. System wide runs stem from the incentives of high quality borrowers to distinguish themselves from low quality borrowers whereas traditional bank runs arise from coordination failures of the lenders.

The model captures the sentiment during the crisis that it was not the losses per se but the uncertainty regarding who was suffering the losses that was most damaging. Furthermore, understanding the source of credit contraction is policy relevant. Because the credit contraction is initially driven by the costly signaling (by decreasing the total amount of debt) that allows financial institutions to differentiate themselves, a policy that eliminates the possibility of signaling and separation can be effective.[1]

This paper is related to several strands of literature on financial market failures and crises. The literature on bank runs starting from Diamond and Dybvig (1983), is large and diverse. Some notable papers include Uligh (2010) who considers uncertainty aversion, Goldstein and Pauzner (2005) who use global games techniques to derive unique equilibrium predictions depending on the fundamentals and He and Xiong (2012) that extends runs to dynamic settings. Martin et al.(2014) model bank runs of repo markets during the crisis. There is also a large literature regarding asset illiquidity, market freezes and funding difficulties that arise from asymmetric information problems. Stiglitz and Weiss (1981) show how credit may be rationed even with competitive lenders when there is asymmetric information between lenders and borrowers. Some recent papers in this vein include House and Masatlioglu (2015) and Kurlat (2013).

A closely related literature studies signaling and screening in credit markets with asymmetric information. The idea that, costly to acquire education serves as a signal for ability in the job market put forth by Spence (1973), has been adapted to financial contexts to study the role of "dissipative signals" that allow the borrowers to convey the quality of securities to potential lenders. Leland and Pyle (1977) present a model of capital structure where the entrepreneur's willingness to invest in their own projects serves as a signal of project quality. Bester (1985,1987) finds that when there is a cost to posting collateral, banks maybe able to screen applicants by conditioning rates as a function of collateral requirements. Milde and Riley (1988) find that banks may be able to screen borrowers when their return is a function of loan size by offering a loan schedule. These papers are similar to this paper in that they find mechanisms the good borrower can utilize to offer contractual terms that are

[1] In Appendix B of the online appendix, I study the effect of two potential measures: mandating participation in a government clearing house for loans, and implementing counter-cyclical borrowing limits. I find that mandating participation in a government clearing house for loans garners results very similar to that of a perfect information benchmark. I also find that the effect of counter-cyclical borrowing limits can be effective if the degree of counter-cyclicality in the borrowing constraints are sufficiently large.

unappealing to the bad borrower and thus allow the borrowers to be separated by their credit worthiness. To the best of my knowledge, this is the first application of this mechanism to financial crises.

This paper also draws from the literature regarding liquidity mismatch and the interaction between credit market conditions, asset market conditions and the health of financial institutions. Brunnermeier and Pedersen (2009) analyze the relationship between what they term as funding liquidity and market liquidity. They find that the two can be mutually reinforcing and lead to liquidity spirals. Diamond and Rajan (2005) find that bank failures can shrink the aggregate pool of liquidity and this shortage of liquidity can lead to a systemic crisis. Diamond and Rajan (2011) show that the anticipation of fire sales may decrease lending from potential buyers of assets.

The remainder of the paper is structured as follows. Section 2 describes the model and the main findings. Section 3 explores the properties of the model and Section 4 offers a discussion on the policy implications and other elements of the model. Section 5 concludes.

2 Model

The ensuing model describes a financial system where a small shock to the fundamental value of the asset can result in a collapse of the financial system. When there is a shock to the asset values, liquidity contractions occur endogenously and generate fire sales that threaten the solvency of all financial institutions. System wide runs may follow. In this section, I first describe the model environment and the decisions that each agent faces. I then discuss the equilibrium outcomes in the credit and asset markets.

2.1 Model Setup

There are three periods in the model, periods zero, one, and two, and there is no discounting. The financial system consists of three types of agents, broker-dealers, households and banks. They are all risk-neutral.

Broker-Dealers

In this model, the broker-dealers represent highly sophisticated financial institutions with the skills and expertise to exploit profitable, albeit complicated, investment opportunities. For example, they can be thought of as dealer banks investing in asset-backed securities and derivatives or hedge funds executing complex trading strategies.[2] In the model economy, there is a continuum of long-lived broker-dealers of measure one, who consume their entire wealth in period two.

In period zero, each broker-dealer is given the opportunity to purchase $a^H > 1$ units of risky assets for one unit of cash. These assets represent complex securities or portfolios that only broker-dealers can invest in. The expected payoff of one unit of the risky asset is arbitrarily close to one (the meaning of this will become clear below) and thus the expected return from the risky asset is a^H. Investing in risky assets is very profitable and this reflects the proprietary skills of the broker-dealers that can generate excess profits. Therefore, the broker-dealers will always choose to purchase the full amount (a^H units) of risky assets as this is optimal as long as the probability of a crisis is small. In period one, the broker-dealers can buy and sell the assets on a secondary market.

The return of the risky asset has the following structure. In period one, the state of the world is revealed. The economy can either be in a "normal" state with probability $1-\delta$ or in a "crisis" state with probability δ. In a normal state of the world, each unit of risky asset is revealed to give a deterministic payoff of one, which will be realized in period two. In a crisis

[2] See Duffie (2010) for a detailed description of dealer banks and the difficulties they faced during the financial crisis.

state, the risky assets is understood to have a stochastic return of R with distribution F on $[0, \infty)$. The expected return of the risky asset remains one. In addition, in a crisis state a small random subset α of broker-dealers are hit with a shock that wipe out a fraction $1-\phi$ of their assets. The realization of this shock is private information, and only the broker-dealers themselves are aware of the state of their asset values.

Thus in a crisis state, a fraction α of the broker-dealers (to be labeled the low type) hold $a^L \equiv \phi a^H$ units of assets and the remaining fraction $1-\alpha$ of broker-dealers (to be labeled the high type) hold a^H units of risky assets, all of which have a stochastic return of R that follows the distribution F. Furthermore, I assume that $\alpha > 0$ converges to zero. This not only simplifies the algebra but also highlights the amplification process and the fact that the magnitude of the aggregate shock can be arbitrarily small. A larger α will only reinforce the results that follow.

The broker-dealers have no initial wealth and must borrow from the households to finance their investment. Furthermore, the broker-dealers face an exogenous borrowing limit of d^{limit}. I assume that this borrowing limit is fixed and does not depend on the price or any other characteristic of the asset.[3] In addition, the borrowing limit is assumed to be greater than their initial debt level and thus will not be, in itself, a source of credit contraction.

One interpretation of the borrowing limit in this model can be as a form of regulation. For example, the Basel II accords regulate the amount of risky investments financial intermediaries could make given their quantity of capital. The Basel III accords introduce even more stringent constraints. Another potential explanation for the existence of the borrowing limit could be due to moral hazard concerns. Experts may have the incentive to steal the funds and disappear once the amount of debt becomes too large. Because the borrowing limit is fixed and essentially a free variable in the model, the source of the borrowing limit

[3] I explore the implications of alternative assumptions regarding the borrowing limit in Appendix C of the online appendix.

is not critical to the main arguments of the paper.

Households

The households are the source of financing for the broker-dealers and banks in this model. The households can be thought of as any entity that provides funding to the financial system, such as individual investors or intermediate institutions like money market funds. The households are risk-neutral.[4]

A continuum of short-lived households of measure one are born in period zero with an endowment of ω units of cash. They consume their entire wealth one period later in period one and exit the system. Similarly another cohort of short-lived households of measure one enter in period one and consume in period two. The households can either store cash at no discount or purchase short-term debt issued by broker-dealers or banks. They cannot invest in risky assets because they do not have the necessary expertise. Furthermore, they are only willing to lend through debt contracts as they do not have the capabilities to verify the returns of the risky assets and thus the cash flow of their counter party.[5]

The amount of resources that the households are endowed is large in aggregate. Specifically, ω is sufficiently large enough to fund all investment opportunities that arise in the model if the households so choose. Thus the households are competitive and are willing to lend as long as their expected return is greater than zero.

The structure of the households in the model imply that all lending is restricted to be through one period debt. To begin with, it is well known that financial institutions are funded through disproportionately large amounts of short-term debt.[6] Diamond (1991), for

[4] In Appendix D of the online appendix, I explore the consequences of risk averse households on the credit market equilibrium and the model outcome. While the specifics of the equilibrium may change with risk averse households, the main conclusions of the paper remain robust.

[5] While this model does not explicitly feature a cost to state verification, it is consistent with the costly state verification framework of Townsend (1979).

[6] Recent empirical evidence shows that the reliance on short-term funding is exacerbated during a crisis. For example, Krishnamurthy et al.(2014) find that the maturities in tri-party repo markets decrease dramatically as the crisis unfolds.

example, focuses on the debt maturity structure of financial institutions and their heavy reliance on short-term debt. In addition, recent work by Brunnermeier and Oehmke (2013) support the restriction to short-term debt in this model. They show that because of the possibility of debt dilution, creditors will rely entirely on short-term financing if the interim information that is released is mostly about the probability of default rather than the expected recovery in default. In this paper, all the information that is released has to do with the probability of default and does not change the recovery structure post-default. Furthermore, the household sector in this model is structured such that one can ex-ante rule out coordination failure among households as the source of credit contractions. By ruling out coordination failures among lenders, it becomes clear that the mechanism driving system wide runs is fundamentally different from those of traditional bank runs.[7]

Banks

Banks represent traditional commercial banks that borrow from the household sector and lend to firms. In the context of this paper, banks participate in the secondary market for risky assets in period one and, during the crisis state, provide liquidity to asset markets

[7] As stated, the non-overlapping generations structure of the households mainly serves two purposes: to restrict lending to one period debt and to abstract away from concerns of coordination failures via lenders. Long-lived households could raise an issue regarding the anonymity of lenders and borrowers. For example, if the households and broker-dealers were of the same order, and an individual household's decision can influence the funding situation of a broker-dealer, than the decision during the crisis for the households previously matched to a given broker-dealer is different from the decision of an outside investor. The matched household must take into account the impact their decision regarding whether or not to fund the broker-dealer going forward will have on its profits from debt issued in period zero. Such complications seem ancillary to the main arguments of the paper and thus were thought best to be avoided.

Alternatively, if one assumes that broker-dealers are "large" and that households are "small," each broker-dealer would need to be funded by a continuum of households and the funding decision of each individual household would have no bearing on the final outcome for the broker-dealer. If the lending instrument was also restricted to one-period debt, the financing decisions of broker-dealers and households would be almost identical to that of the current model. The difference is that each household must consider the decisions of other households when they are making their decision. Potentially, households may not be willing to fund a particular broker-dealer if they believe that other households will not lend either, even to a solvent broker-dealer. Thus, the belief of the households regarding the beliefs and actions of other households will become an important factor in equilibrium. Furthermore, in many cases, there may be an equilibrium in which households run due to coordination issues among lenders as is the case in traditional bank runs. While, this phenomenon may be of interest in itself, that was not the focus of this paper.

to determine the market price of the assets. This feature of the model is motivated by the findings of He et al.(2010) that find securitized assets such as asset-backed securities liquidated by broker-dealers and hedge funds ended up on the balance sheets of commercial banks during the financial crisis of 2008.

In the model there is a continuum of long-lived banks of measure one, each of whom consume in period two. The banks have no initial endowments and must borrow from the households to invest. The borrowing capacity of the banks is capped at B. In period zero, banks can invest in a continuum of safe projects that give a deterministic return in period two. The marginal return of the projects are captured by a decreasing function $I(\beta)$ where β is the amount invested in the projects. While the projects are safe, they are illiquid and cannot be sold for cash before they mature in period two. The banks do not have the opportunity to invest in risky financial assets in period zero. However, in period one they have the ability to participate in the secondary market for risky assets.

Now, consider the funding decision between banks and households. In period zero, the banks can borrow from the households at zero cost because the banks' projects have deterministic returns and the households who lend to them are competitive. In period one, a bank's cost of borrowing may depend on the state of the world. In a normal state, the borrowing cost is zero because the return of a risky asset is deterministic going forward. In a crisis state, the cost of funding may no longer be zero as the banks can choose to invest in risky assets with stochastic returns. However, for the sake of simplicity, I assume that the total return from a bank's safe project is enough to make bank debt risk-free even in a crisis state. Given β, if $\int_0^\beta I(\tilde{\beta})d\tilde{\beta} \geq B$, the bank's future return from safe projects are enough to guarantee full repayment of debt regardless of the return from risky assets. Bank debt is safe and thus banks should always be able to borrow at zero cost. This assumption does not change the qualitative implications of the results.[8]

[8]Note that this is equivalent to assuming that the banks begin with enough net worth to absorb the

The price of the risky asset in secondary markets is determined by supply and demand. In a normal state, the price of the risky asset must equal its expected return of one for markets to clear. In a crisis state, the price will depend on the amount of liquidity available from the banks and the amount of assets that need to be sold by the broker-dealers. Let $\rho \leq 1$ denote the price of the risky asset in a crisis state. In equilibrium, the banks correctly anticipate the future price of assets when they make their investment decision in period zero.[9]

Because the safe projects of the banks are illiquid, banks must consider all potential future profits when choosing the amount β to invest in safe projects in period zero. The marginal return from the safe project should equal the expected return from purchasing risky assets in period one. The banks choose β such that

$$I(\beta) = \delta(\frac{1}{\rho} - 1). \tag{1}$$

The left-hand-side of equation (1) describes the marginal return from investing in a safe project. The right-hand-side describes the expected return from purchasing risky assets in period one. With probability δ, the economy will be in a crisis state and the bank's expected profit per unit debt is $\frac{1}{\rho}$ minus the amount it must pay back, one. In the normal state, the expected profit from investing in risky assets is zero.

2.2 The Credit Market Equilibrium

I now describe the equilibrium in the credit market. First, I detail the interaction between the broker-dealers and the households in the credit market. Then I characterize the pooling, hybrid, and separating equilibria of the credit market in crisis during period one.

potential losses from investing in risky assets.

[9]The banks, as does all agents in the model, are fully aware of the parameters of the model, with the exception of the realization of the shock $1 - \phi$ to the asset value of select broker-dealers. Therefore, a rational bank can anticipate the fire sale price as they know the total available liquidity in the system and can anticipate the amount of assets that will need to be liquidated.

2.2.1 The Broker-dealers' and Households' Problems

To borrow from the households, the broker-dealers make a take-it-or-leave-it offer (d_t, q_t) to the households specifying the total quantity of debt they plan to issue d_t and the price per unit of that debt q_t. If accepted, a broker-dealer obtains cash $q_t d_t$ at date t and owes d_t at date $t+1$. The households can either accept and fund the broker-dealer or reject the offer and refuse to provide any funds. This feature of the credit market is designed to capture the fact that the lending market is competitive and the fact that there is no aggregate shortage of loanable funds in the model. It is also consistent with standard economic environments where agents can only earn excess profits if they have proprietary technology or skills.

In period zero, the broker-dealers need one unit of cash to invest in risky assets. They offer a price and quantity combination (d_0, q_0) such that $q_0 d_0$ equals one while allowing the households to break even. In period one, the broker-dealers must pay off their maturing debt d_0 and issue new debt to a fresh cohort of households. If the broker-dealers cannot pay off their maturing debt, they immediately default and their entire portfolio is liquidated. The proceeds from the liquidation are distributed to the households in proportion to the original debt. In a normal state in period one, the broker-dealers are able to borrow at the risk-free price of one as the return on the risky asset is henceforth deterministic. However, if the economy enters a crisis state, the equilibrium is more involved.

Suppose that the economy is in a crisis state and the price of the risky asset in the secondary market is $\rho \leq 1$. Then, a broker-dealer with a units of risky assets and d_0 units of maturing short-term debt has the expected payoff,

$$V(d_1, q_1) = \int_{\hat{R}}^{\infty} [(a - \frac{d_0 - q_1 d_1}{\rho})R - d_1] dF(R) \tag{2}$$

where $\hat{R} = \frac{\rho d_1}{q_1 d_1 + \rho a - d_0}$ denotes the minimum required return for the broker-dealer to be able to service their debt in full. The broker-dealer's offer (d_1, q_1) will seek to maximize (2), predicated on the households accepting the offer.

The expected return of households from buying one unit of debt from a broker-dealer who issues d_1 units of new short-term debt at price q_1 can be expressed as,

$$\pi(d_1, q_1) = (1 - F(\hat{R}))(1 - q_1) + \int_0^{\hat{R}} [\frac{1}{d_1}(a - \frac{d_0 - q_1 d_1}{\rho})R - q_1] dF(R). \tag{3}$$

If equation (3) is greater than zero, households accept the broker-dealer's offer and provide them with cash. If it is less than zero, households reject the offer. If equation (3) is exactly zero, households are indifferent between accepting and rejecting. In this case, I assume that households always accept the broker-dealer's offer.[10]

During a crisis state, there is asymmetric information in the credit market because households cannot distinguish between a high type broker-dealer with a^H units of risky assets and low type broker-dealers with only ϕa^H units. However, the terms of the broker-dealer's offer can serve as a signal to their type. The period one credit market between broker-dealers and households can be described as a signaling game where the debt contract (d, q) serves as a signal to the households.

The equilibrium in the period one market for credit in a crisis state, is a perfect Bayesian equilibrium that consists of the broker-dealers' offers, the households' actions of either accepting or rejecting the offer, and the households' beliefs regarding the broker-dealers' types given the offer. In addition, all equilibria must satisfy the Cho-Kreps intuitive criterion. Imposing this condition greatly reduces the set of potential equilibria and allows for interesting and insightful results. The intuitive criterion is the standard and most commonly used refinement of perfect Bayesian equilibria and imposes some structure on the off-equilibrium beliefs of the players. It says that when an agent deviates from the equilibrium play, and the deviation is such that for a certain type of agent it implies a lower payoff than the equilibrium

[10]This assumption is only meaningful in the case of hybrid equilibria. For pure strategy equilibria, this is an equilibrium outcome rather than an assumption.

payoff under any belief of the opposing player, it is unreasonable for the opposing player to believe that the deviator is of that type.

The credit market equilibrium can then be expressed as $\{(d_1^H, q_1^H), (d_1^L, q_1^L), (x, \mu)\}$ where (d_1^i, q_1^i) is the offer by the broker-dealer of type $i \in \{H, L\}$ and (x, μ) denotes the lender's action and belief respectively. The remainder of this section solves for this equilibrium.

2.2.2 Pooling and Hybrid Equilibria

While the credit market equilibrium can be either a pooling equilibrium, a separating equilibrium or a hybrid equilibrium, we show that all pooling and hybrid equilibria in which the broker-dealers can borrow can be ruled out by the intuitive criterion. First, consider pooling equilibria. In any pooling equilibrium, the high type and the low type broker-dealers should both offer the same debt contract. However, there is always a deviation from the pooling equilibrium, such that the high type receives a higher payoff from deviation then from his equilibrium action but the low type receives a lower payoff then he does from his equilibrium action, if the lender believes that the deviator is the high type. In turn, the lender must believe that the deviator is the high type and accept the offer.

Proposition 1. *All pooling equilibria in which the broker-dealers can borrow, fail the intuitive criterion.*

The proof is given in Appendix A. To see why Proposition 1 holds, consider the following lemma. Recall that equation (2) describes the broker-dealer's expected payoff.

Lemma 1 (Single-crossing property)**.** *Given d_0 and ρ, the slope of the indifference curve of the broker-dealer $\left(\frac{\partial q_1}{\partial d_1}\right)$ is increasing in the amount of risky assets owned by the broker-dealer.*

Lemma 1 implies a single-crossing property of the indifference curves of the broker-dealers. Intuitively, the property holds because the amount of risky assets held by the broker-dealers determine their probability of default in period two. Because the low type eventually defaults for a larger range of returns, the benefit they receive from a better price for their debt is smaller than that of the high type. The difference in the benefit from borrowing another unit of debt is mitigated by the fact that it must be paid back. Therefore, to decrease a unit of funding, the low type broker-dealers must be compensated with a greater increase in the price of debt than the high types.

Given Lemma 1, there is always a deviation where the total amount of debt d_1 is smaller than the pooling equilibrium offer, such that if the household accepts the offer, the high type broker-dealer benefits from the deviation but the low type broker-dealer does not. Furthermore, at least one such deviation exists for all quantities of debt less than the equilibrium offer. Then, at such a deviation point that is very close to the equilibrium offer, it is beneficial for the household to accept the deviation if they believe the deviator is the high type.

Figure 1 illustrates this point. For any pooling equilibria, high types can always deviate to a point above their own indifference curve but below that of the low type as depicted by the red diamond in Figure 1. As long as this deviation point is close enough to the pooling equilibrium point, the households will accept the offer. Thus, all pooling equilibria fail the Cho-Kreps intuitive criterion. Similar logic can be used to show that all hybrid equilibria fail the intuitive criterion as well.

Proposition 2. *All hybrid equilibria in which the broker-dealers can borrow, fail the intuitive criterion.*

The proof is given in Appendix A.

2.2.3 Separating Equilibria

Propositions 1 and 2 imply that any equilibrium of this model in which the broker-dealers can borrow, must be separating. The broker-dealer's type will be identifiable to the households in equilibrium. The interesting case occurs when the low type broker-dealers

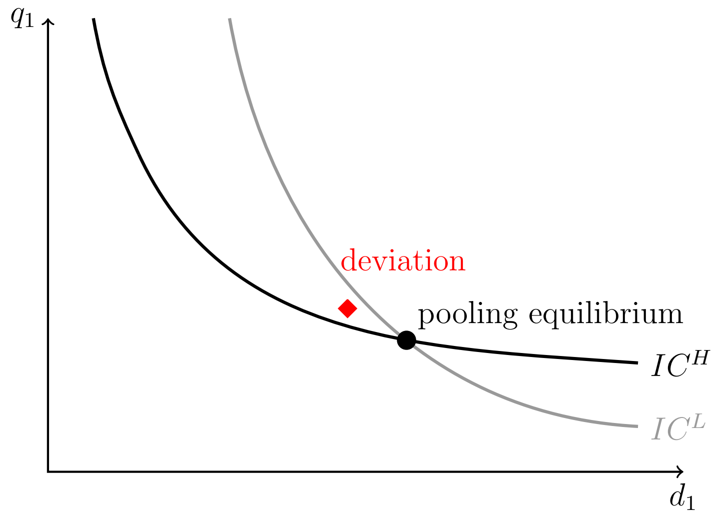


Figure 1: Proposition 1

are fundamentally insolvent ($\phi a^H < d_0$) but the high types could repay their maturing debt without borrowing if the price of the risky asset was equal to its expected value ($a^H > d_0$). This is the case if $\frac{1}{1-\delta} < a^H < \frac{1}{\phi}$, which I assume.

Before addressing the broker-dealer's problem, we must first analyze the quantities and prices of debt that the households are willing to accept in a separating equilibrium. Thus, consider the household's problem given that they are aware of the borrower's type. First, consider the conditions under which there exists an offer that the broker-dealer can make, that the household will be willing to accept. Note, that the household's expected payoff from equation (3) is concave in q_1 for a given d_1. Due to the concavity of π in q_1, there is a price $q^{max}(d_1)$ that maximizes the household's expected profit. From the first-order condition of equation (3), $q^{max}(d_1)$ solves

$$\rho = \int_0^{R^*} RdF(R) \tag{4}$$

where $R^* \equiv \frac{\rho d_1}{q^{max}(d_1)d_1+\rho a-d_0}$. Given, a, d_0 and d_1, an offer that the household is willing to accept will only exist if the expected return of the household in equation (3) is weakly greater than zero when $q_1 = q^{max}(d_1)$.

It is straightforward to show that if the liquidation value of the broker-dealer is positive ($\rho a - d_0 \geq 0$), the expected profit of the household is positive at $q^{max}(d_1)$ for any d_1. However, if the liquidation value is negative ($\rho a - d_0 < 0$), this is no longer true. Suppose that the liquidation value of the broker-dealer is negative. Then, the expected profit of the household will be positive at $q^{max}(d_1)$ when lending to a broker-dealer of type $i \in \{H, L\}$, only if d_1 is greater then:

$$d^i_{floor} \equiv \frac{d_0 - \rho a^i}{1 - F(R^*)}. \tag{5}$$

Equation (5) can be derived by plugging $q^{max}(d_1)$ in to equation (3) and solving for the minimum d_1 that makes π positive. Any offer where the total quantity of debt is less then d^i_{floor} will not be accepted. At d^i_{floor}, the only price of debt that is accepted is $q^{max}(d^i_{floor})$.

Now, consider the maximum price per unit debt (the worst offer) that the household is willing to accept from the high type broker-dealers given d_1. Denote this price as $q^H(d_1)$. At the maximum price, the household's expected profit should be zero. From equation (3) the household's zero profit condition is,

$$(1 - F(\hat{R})) + \frac{1}{\hat{R}} \int_0^{\hat{R}} R dF(R) = q_1 \tag{6}$$

where $\hat{R} = \frac{\rho d_1}{q_1 d_1 + \rho a^H - d_0}$. A solution to (6) only exists if the liquidation value of the broker-dealer is positive or $d_1 \geq d^H_{floor}$. In addition, because π is concave in q_1, there can be multiple solutions to equation (6) for a given d_1. However, $q^H(d_1)$ is the solution to equation (6) that is weakly greater than $q^{max}(d_1)$ and this, when it exists, is unique.

Finally, there is a price of debt below which the broker-dealer will default even if the households agree to accept the offer. Let $q^L_d(d_1)$ denote the price of debt below which the low type broker-dealer defaults. The expression for this price can be solved to be $q^L_d(d_1) = \frac{d_0 - \rho a^L}{d_1}$. At this price, the low type broker-dealer cannot repay its maturing liabilities even if it sells its entire portfolio of risky assets at price ρ.

Given the range of quantities and prices of debt the households are willing to accept, one can now study the broker-dealer's decision problem. To focus on the spillover of distress across institutions, I further assume that not only are the low type broker-dealers fundamentally insolvent, but that they would still be insolvent even if they could borrow to the full extent of the borrowing limit. Assuming $d^{limit} < (1 + \frac{1}{a^H - 1})(1 - \phi)$ satisfies this condition. With fire sales, this is a stronger assumption than fundamental insolvency, because a broker-dealer can improve its solvency by purchasing assets at discounted prices for profit. Without a borrowing limit, all broker-dealers, regardless of their current financial condition, could bail themselves out by purchasing infinite amounts of risky assets at fire sale discounts.

Now, consider the broker-dealer's problem. It is not obvious that the high type broker-dealer can borrow in equilibrium when there are fire sales and asymmetric information. The only sustainable equilibrium may be a pooling equilibrium where no broker-dealer can borrow. Nevertheless, I suppose, for now, that the high type broker-dealer can borrow in equilibrium and solve for what the equilibrium offer must be in such a case.

In any separating equilibrium, the low type broker-dealers will not be able to borrow and will always default. This implies that the low types have the incentive to pool with the high types as long as the terms of the high type's offer allows them to pay off their maturing debt. Thus any separating equilibrium offer of the high type cannot have price of debt greater than $q_d^L(d_1)$.

In addition, the high type's offer will have the price $q^H(d_1^*)$ for any equilibrium quantity d_1^*. To see why, suppose that in equilibrium the high type offers q_1 that is strictly less than $q^H(d_1^*)$. From Proposition 1, we know that the high type can deviate to a slightly smaller d_1 with a higher price, for which he is strictly better off and the low type will not mimic. Because q_1 is strictly smaller than $q^H(d_1^*)$, there is always such a deviation that the household will accept and the equilibrium unravels. This deviation is only possible when the price is less than $q^H(d_1^*)$. When the price is equal to $q^H(d_1^*)$, the household may not be willing to fund such a deviation. Thus, the high type must offer the maximum acceptable price $q^H(d_1)$ in equilibrium.

The equilibrium offer of the high type $(d_1, q^H(d_1))$ will be the solution to the following broker-dealer's constrained optimization problem:

$$\max_{d_1} V^H(d_1, q^H(d_1)) = \int_{\hat{R}}^{\infty} [(a^H - \frac{d_0 - q^H(d_1)d_1}{\rho})R - d_1]dF(R) \tag{7}$$

$$s.t. \qquad q^H(d_1) \leq q_d^L(d_1).$$

The solution to this optimization problem is described by the following proposition.

Proposition 3. *In any separating equilibrium in which the high type broker-dealers can borrow, their equilibrium offer is* $(d^*, q^H(d^*))$, *where* $q^H(d^*) = q_d^L(d^*)$.

Proposition 3 states that the high type broker-dealers will choose the maximum quantity of debt that the low type broker-dealer will not have the incentive to mimic, when the price of the debt is $q^H(d_1)$. In other words, $V^H(d_1, q^H(d_1))$ is weakly increasing in d_1. First, note that $q^H(d_1)$ is a decreasing function of d_1 when the liquidation value of the broker-dealer is positive and an increasing function of d_1 if the liquidation value is negative. Since $\pi(d_1, q^H(d_1)) = 0$, by the implicit function theorem:

$$\frac{dq^H(d_1)}{dd_1} = \frac{\rho a^H - d_0}{d_1^2} \frac{\int_0^{\tilde{R}} RdF(R)}{\int_0^{\tilde{R}} RdF(R) - \rho} \tag{8}$$

where $\tilde{R} = \frac{\rho d_1}{q^H(d_1)d_1 + \rho a^H - d_0}$. Because $q^H(d_1)$ is greater than $q^{max}(d_1)$, this means that $\tilde{R}$ is smaller than R^* and, by extension, $\int_0^{\tilde{R}} RdF(R) < \int_0^{R^*} RdF(R) = \rho$. Therefore, equation (8) is negative if $\rho a^H - d_0 > 0$ and positive otherwise. Thus, if the liquidation value of the broker-dealer is negative, V^H is increasing in d_1 because $q^H(d_1)$ is increasing in d_1. When the liquidation value of the broker-dealer is positive the same can no longer be said. Instead consider the following lemma.

Lemma 2. *$V^*(d_1)$ is increasing in d_1 for broker-dealers with positive liquidation value, where $V^*(d_1) \equiv V(d_1, q^H(d_1))$.*

It can be shown that $V^*(d_1)$ is increasing in d_1, by showing that $\frac{dV^*}{dd_1}(d_1) > 0$ as d_1 converges to infinity and that V^* is concave. Lemma 2 implies that high type broker-dealers prefer to maximize their borrowing quantity subject to the constraints, when they are separately identified. Thus, if a separating equilibrium exists in which the high type broker-dealer can borrow, their offer will be at the point where $q^H(d^*) = q^L_d(d^*)$. Figure 2 illustrates this point. The solid point indicates the high type broker-dealer's offer in a separating equilibrium. The low type broker-dealer always defaults and thus their equilibrium point is not depicted.

I end the discussion of the credit market equilibrium by noting that the separating equilibrium point, as shown in Figure 2, may not exist. This is the case if the debt floor of the high type broker-dealer d^H_{floor} exceeds the borrowing limit d^{limit}. Whether such a point exists will be decided by the interaction between the credit market and the asset market, and the prevailing fire sale price of the risky asset from this interaction. In the following section, I describe this interaction and characterize the full equilibrium of the model.

2.3 Interactions with the Asset Market

With an understanding of the credit market outcomes, one can now solve for the equilibrium of the full model that incorporates the interactions between the credit and asset markets. The credit market equilibrium is a perfect Bayesian equilibrium that satisfies the Cho-Kreps intuitive criterion, as described in the previous section, and the market for risky assets is a competitive market where prices are determined to clear the market.

Equilibrium: *The equilibrium consists of $\{\beta, (d_0, q_0), \rho, (s^H, s^L), (d^H_1, q^H_1), (d^L_1, q^L_1), (x, \mu)\}$:*

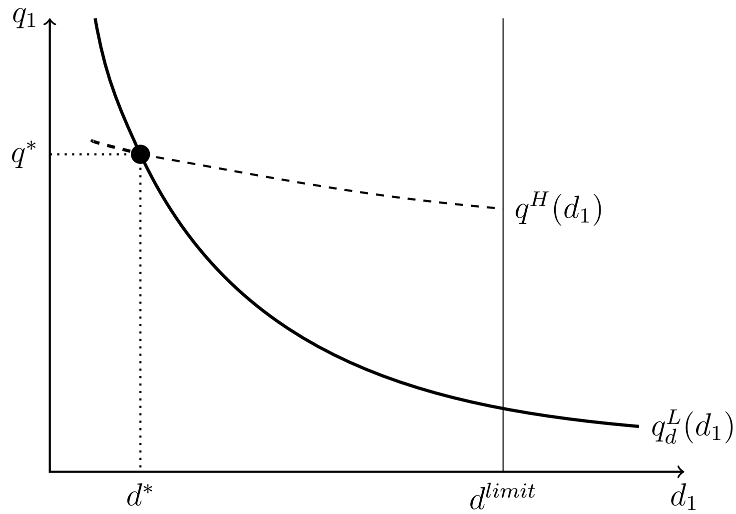


Figure 2: Separating equilibrium in the credit market

the amount invested in safe projects by banks in period zero, the broker-dealer's debt contract in period zero, the price of the risky asset in period one, the number of units of risky assets that need to sold by each type of broker-dealer, the debt contracts of high and low type broker-dealers, and the household's action and beliefs in the period one crisis state.

Note that the broker-dealers' and households' actions and the price of the risky asset in period one during a normal state is omitted. In this state, the price of the asset is equal to its expected return of one and the price per unit of the broker-dealer's debt must also be one. The quantity of debt each broker-dealer borrows is indeterminate.

Depending on the parameters of the model, two distinct equilibrium outcomes can prevail. In one outcome, the financial system suffers a large credit contraction in the crisis state but only the low type broker-dealers default in period one. The system as whole is able to withstand the crisis. In the other outcome, the financial sector suffers a system wide run and all broker-dealers of both types fail, resulting in a complete financial collapse. I describe each outcome in turn.

2.3.1 Credit Contraction

Whether or not the financial system can survive a crisis depends on the severity of the fire sales in the secondary market for risky assets. The price of the risky asset during a fire sale is determined by the supply and demand of assets in a competitive market. The demand for the assets depends on the available liquidity from the banks. As long as the price of the asset ρ is less than one, the banks will devote their entire remaining borrowing capacity to purchase the risky assets. The supply is determined by the broker-dealers funding conditions. Let s^L and s^H denote the number of assets sold by each type of broker-dealer respectively. Then, the market clearing condition for the risky assets is:

$$B - \beta = \rho(\alpha s^L + (1-\alpha)s^H). \tag{9}$$

$B - \beta$ is the remaining borrowing capacity of the banks after investing in period zero safe projects and $\alpha s^L + (1-\alpha)s^H$ is the number of units of the risky asset supplied by the broker-dealers. The price ρ must clear the market. Then, combining the bank's optimal investment decision described by equation (1) with the market clearing condition (9), the equilibrium price of the asset will be determined as the solution to the following equation:

$$\delta(\frac{1}{\rho} - 1) = I(B - \rho(\alpha s^L + (1-\alpha)s^H)). \tag{10}$$

A unique solution to this equation exists as long as $I(B - (\alpha s^L + (1-\alpha)s^H))$ is greater than zero.

Now, consider the supply of assets by each type of broker-dealer during a crisis. Suppose that, in equilibrium, the high type broker-dealers can borrow from the households. Then, by Propositions 1 and 2, the resulting equilibrium must be separating. As shown in Proposition 3, the high type broker-dealers are able to secure d^* of debt at rate $q^H(d^*)$, where d^* solves

$q^H(d^*) = q_d^L(d^*)$. The high types then need $d_0 - d^* q^H(d^*)$ additional units of cash to pay off their maturing debt. Since $q_d^L(d^*) = \frac{d_0 - \rho a^L}{d^*}$, this equates to ρa^L units of cash implying that they must sell $s^H = a^L = \phi a^H$ units of risky assets. On the other hand, the low types are unable to secure any funding and default. Thus they liquidate all of their remaining assets, supplying $s^L = a^L = \phi a^H$ units of risky assets. Then the total supply of assets equates

$$\alpha s^L + (1-\alpha)s^H = \phi a^H. \tag{11}$$

The supply of assets into the period one market for assets does not depend on the fraction of experts α who are hit with the idiosyncratic shock $1-\phi$. Even with α converging to zero, the number of assets that need to be sold can be large. Because high type broker-dealer's must liquidate a considerable amount of assets in order to separate, even a very small number of distressed broker-dealers can induce a large fire sale of assets.

Combining equations (10) and (11), the price of the asset will be determined as,

$$\delta(\frac{1}{\rho} - 1) = I(B - \rho\phi a^H). \tag{12}$$

Let ρ_c^* denote the solution to equation (12). ρ_c^* is an equilibrium price of the asset if the high type broker-dealer can borrow at the separating quantity of debt as assumed. They will be able to borrow if they satisfy the following condition:

Condition 1. $d_{floor}^{H,c} \leq (1-\phi)a^H R_c^*$

where $d_{floor}^{H,c} = \frac{1-\rho_c^* a^H}{1-F(R_c^*)}$ and R_c^* solves equation (4) for ρ_c^*.

Condition 1 is a solvency condition for the high type broker-dealers at their separating quantity of debt. The high type broker-dealer may not be able to borrow in equilibrium if they become insolvent at the separating quantity of debt. When the high type broker-dealers are forced to liquidate some of their assets in order to separate, they suffer significant losses for each unit of asset they sell. When the losses are severe, either because the discount is high or because they are forced to liquidate a large number of assets, the high type broker-dealers may become insolvent.

Condition 1 can be thought of as governing the size of the idiosyncratic shock $1 - \phi$. When the severity of losses that the distressed broker-dealers face is large, the high type broker-dealers do not need to liquidate as large a portion of their assets at a discounted price to separate and are more likely to remain solvent at the separating quantity of debt. The following proposition describes the equilibrium.

Proposition 4. *If Condition 1 holds, there is an equilibrium in which, during a crisis state, the broker-dealers experience a contraction in credit but the high type broker-dealers are able to survive. In a crisis, the high type broker-dealers' offer is determined by Proposition 3 and is accepted by the households. The low type broker-dealers default in period one, regardless of their offer.* $s^L = s^H = \phi a^H$ *and the price of the risky asset* ρ *is determined by equation* (12)*, and* β *by equation* (1)*. In period zero,* $(d_0, q_0) = (1,1)$ *as* α *converges to zero.*

2.3.2 System Wide Runs

When Condition 1 fails, there is no equilibrium outcome in which the high type broker-dealers can borrow during a crisis. The only possible equilibrium outcome is the one in which all broker-dealers default in a crisis and liquidate all their remaining assets. All lending to the broker-dealers are suspended and the broker-dealers suffer a system wide run.

Suppose that the high type broker-dealers are insolvent at the separating quantity of debt. Then, they can no longer borrow and must default. When broker-dealers default, they liquidate their entire portfolio of assets. Thus $s^L = a^L = \phi a^H$ and $s^H = a^H$. Furthermore, with α converging to zero, equation (10) can now be written as,

$$\delta(\frac{1}{\rho} - 1) = I(B - \rho a^H). \tag{13}$$

Let ρ_r^* denote the solution to equation (13). Because both high type broker-dealers and low type broker-dealers default during a crisis, the price of period zero debt will be determined as,

$$q_0 = 1 - \delta + \delta \frac{\rho_r^*}{d_0} a^H. \tag{14}$$

The quantity of period zero debt equals $d_0 = \frac{1}{q_0}$, as broker-dealers receive one unit of cash from households in period zero. Therefore,

$$d_0 = \frac{1}{1-\delta}(1 - \delta \rho_r^* a^H). \tag{15}$$

The high type broker-dealer is indeed insolvent at the separating quantity of debt when the price of the risky asset is ρ_r^*, if the following condition is satisfied:

Condition 2. $d_{floor}^{H,r} > (1-\phi) a^H R_r^*$

where $d_{floor}^{H,r} = \frac{1-\rho_r^* a^H}{1-F(R_r^*)}$ and R_r^* solves equation (4) for ρ_r^*. The price ρ_r^* is an equilibrium price only if Condition 2 is satisfied.[11] The following proposition describes the equilibrium.

Proposition 5. *When Condition 2 holds, there is an equilibrium in which the broker-dealers experience a system wide run. During a crisis, neither the high type broker-dealers nor the low type broker-dealers can borrow. All offers are rejected by the households and all broker-dealers default.* $s^L = \phi a^H$ *and* $s^H = a^H$. *The price of the risky asset* ρ *is determined by equation* (13)*, and* β *by equation* (1)*. In period zero,* d_0 *is determined by equation* (15) *and* q_0 *by equation* (14).

The results from the interactions between the asset and credit markets show that, because each individual borrower is small, institutions fail to internalize the decreases in asset value that follow from their attempts at separation. The motivation to separate leads to a collective

[11] Note that if Conditions 1 and 2 fail simultaneously an equilibrium may not exist. A sufficient condition for the existence of an equilibrium is $\frac{f(x)}{1-F(x)} \leq \frac{1}{x}$ for values $x > R_f^*$. This condition regarding the distribution of risky asset returns, guarantees that if Condition 1 fails then Condition 2 holds.

reduction in borrowing in equilibrium which exerts a negative externality in the form of fire sale discounts on assets sales. If fire sales are severe, this can lead to system wide runs where even high type broker-dealers default.

3 Properties of the Model

In this section I provide two numerical examples: first, an economy that suffers a credit contraction and second, an economy that suffers a system wide run.

3.1 Numerical Examples

Credit Contraction

Let the number of units of risky assets held by the high type experts be $a^H = 1.2$, the probability of a shock $\delta = 0.1$, and the size of the idiosyncratic shock $1 - \phi = 0.2$ implying $a^L = 0.96$. The borrowing limit is $d^{limit} = 1.05$. Suppose that the distribution of the return of the risky asset R follows a lognormal distribution with parameter values $\mu = -0.08$ and $\sigma^2 = 0.16$. Then, the distribution F of the risky asset return has a mean of one and a variance of 0.174. Let the marginal return function on the bank's safe project be $I(\beta) = 20e^{-4\beta} + 0.01$ and the borrowing capacity of the bank $B = 3.2$. Assume α converges to zero.

First, suppose that the broker-dealers cannot borrow in equilibrium, regardless of type. Then the equilibrium asset price solves equation (13). The equilibrium asset price is $\rho = 0.879$. The debt floor of the low type broker-dealer is $d^L_{floor} = 2.58$. However, the liquidation value of the high type broker-dealers is positive as $\rho a^H - d_0 = 0.061$, and will be able to borrow in equilibrium. Thus, Condition 2 fails and this cannot be an equilibrium of the model.

Instead, assume that the high type broker-dealers can borrow in equilibrium. In this case, the price of the risky asset is decided by equation (12). The implied price of the risky asset is $\rho = 0.90$. At this price the debt floor of the low type is $d^L_{floor} = 2.87$. This is higher than the debt limit, consistent with the assumption that the low type cannot borrow at any quantity and price. The liquidation value of the high type broker-dealers is positive as $\rho a^H - 1 = 0.074$, implying Condition 1 holds. This is an equilibrium of the model.

System Wide Runs

Now, consider the following economy. Let the borrowing capacity of the bank now be $B = 2.4$. All the other parameters of the model remain as before. Again, suppose that the high type broker-dealers can borrow in equilibrium. The price of the risky asset, as determined by equation (12), will be $\rho = 0.75$. At this price the debt floor of the low type is $d^L_{floor} = 1.97$. This is higher than the debt limit, consistent with the assumption that the low type cannot borrow at any quantity and price. However, in this example, Condition 1 fails. The debt floor of the high type is $d^H_{floor} = 0.72$ whereas the right side of Condition 1 is equal to 0.34. Therefore, this is not an equilibrium of the model.

Instead, assume that no broker-dealer can borrow in equilibrium. Then, the equilibrium asset price solves equation (13). The equilibrium asset price is $\rho = 0.684$. The debt floor of the low type broker-dealer is $d^L_{floor} = 1.92$ and the debt floor of the high type broker-dealer is $d^H_{floor} = 1.05$. The right-hand-side of Condition 2 is equal to 0.32 and thus Condition 2 holds. All broker-dealers default during the crisis. In period zero, $d_0 = 1.02$ and $q_0 = 0.98$. Banks invest $\beta = 1.58$ units of cash in safe projects in period zero. In equilibrium, there is a system wide run.

3.2 Comparative Statics

As can be seen in the numerical examples, the parameter values of the model will determine the equilibrium outcome of the model. The comparative statics of the model can be studied to gain a better understanding of the roles of various characteristics of the economy on the behavior of the financial system during a crisis.

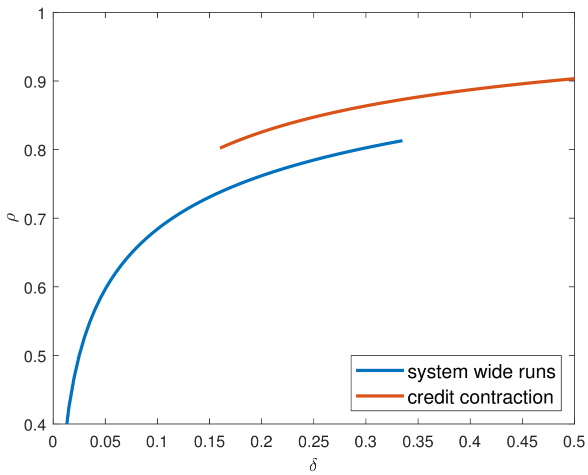
Panel (a) of Figure 3 illustrates the equilibrium price of the risky asset in the secondary market as a function of the ex-ante probability of the crisis.[12] The figure shows that the price of the asset is increasing in the ex-ante probability of the crisis. Furthermore, the financial system experiences a system wide run if the ex-ante probability of crisis is small whereas the system will suffer a credit contraction but the majority of broker-dealers will survive if the crisis is more anticipated.

This relationship between the ex-ante probability of a crisis and the equilibrium price of the risky asset holds because the probability of the crisis determines the amount of excess borrowing capacity the banks maintain to capitalize on a fire sale. With a greater probability of a crisis and thus a fire sale, the banks reserve a greater portion of their borrowing capacity for the possibility of a fire sale. This is akin to the liquidity hoarding behavior of financial institutions both reported empirically and observed in various models.[13] The focus of many models of liquidity hoarding is on the fact that this behavior decreases the lending activities of these institutions and depresses economic activity in the periods leading up to the crisis. This is also true in the current model. However, this paper also highlights a potential benefit to the liquidity hoarding behavior of banks. The extra liquidity that the banks hold may allow the financial system to survive a crisis that they otherwise could not.

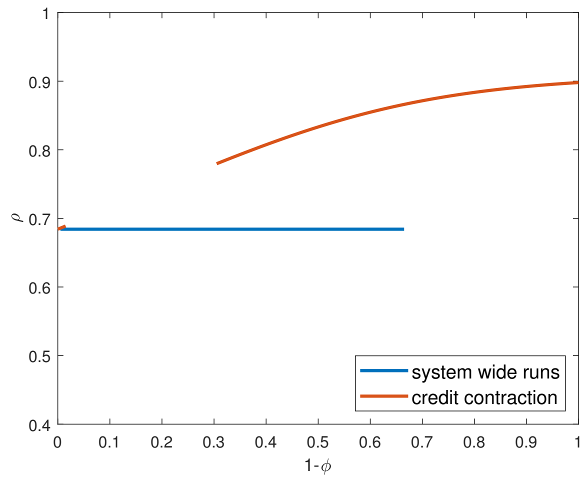
Panel (b) illustrates change in the equilibrium price of the asset as a function of the severity of the shocks to asset values. While the total loss in asset value due to the shock is not important, as evidenced by the fact that α does not affect the results, the depth of the difficulties that the distressed institutions face is very important for the equilibrium

[12]Throughout the remainder of this section, the values of the parameters other then that of interest, are as shown in the example economy with system wide runs.

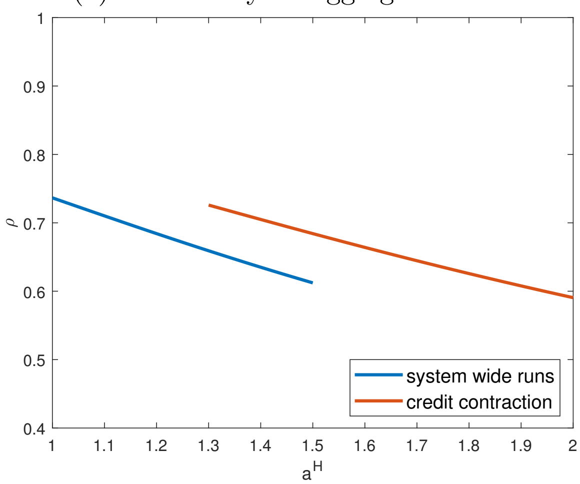
[13]For example, Acharya and Merrouche (2012) report evidence of liquidity hoarding by banks. Acharya and Skeie (2011) construct a model in which a bank hoards excess liquidity in anticipation of adverse asset shocks due to precautionary motives. Gale and Yorumazer (2013) construct a model in which both precautionary and speculative motives for liquidity hoarding exist.



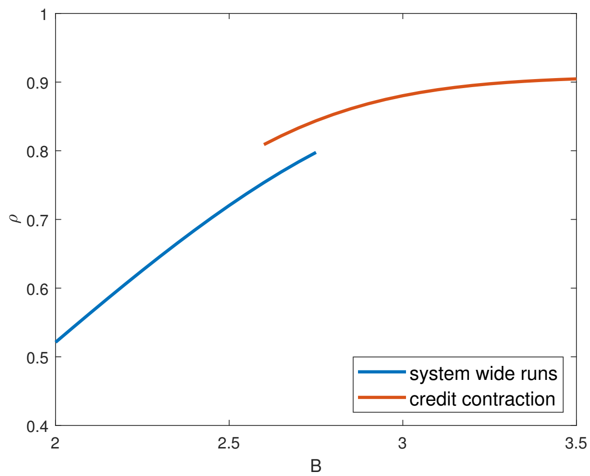
(a) Probability of Aggregate Shock



(b) Magnitude of Idiosyncratic Shock



(c) Profitability of Risky Assets



(d) Borrowing Capacity of Banks

Figure 3: Equilibrium fire sale prices for a range of parameter values.

outcome.

Surprisingly, the equilibrium outcome is worse for relatively small shocks. Panel (b) shows that, in the example economy, if the distressed institutions lose 20% of their overall value to a shock the financial system will suffer a system wide run. However, if the shock is greater than 30% of the value of the asset then there is an equilibrium outcome in which the high type broker-dealers avoid default. With shocks greater than 60%, there is no longer an equilibrium in which the system fails. When the shock is relatively small, the high type broker-dealers must decrease their borrowing quantity significantly in order to separate from the low type broker-dealers. This results in a significant credit contraction and a greater likelihood that the financial system fails. With larger shocks, the low type broker-dealers are left with a much smaller portfolio and readily default with a smaller contraction in credit.

Panels (c) and (d) illustrate the equilibrium price of the asset in response to varying degrees of slack in the financial system. In Panel (c), the parameter of interest is the period zero expected return of risky assets. When the return on assets are low, the system is vulnerable to a system wide run. The system is better able to withstand a crisis when the returns are high. The result is intuitive as the broker-dealers will have an easier time remaining solvent when their initial profit margins are large. Conditional on the equilibrium outcome, the equilibrium price is decreasing in asset returns. This is due to the fact that a larger a implies that there are more assets that need to be liquidated. In Panel (d), the parameter of interest is the total borrowing capacity of banks. As can be expected, when the total borrowing capacity of the banks increases, the equilibrium price increases.

As Figure 3 illustrates, this model exhibits both fragility and multiple equilibria. Brunnermeier and Pedersen(2009) define fragility as "the property that a small change in fundamentals can lead to a large jump in illiquidity." In their model, funding markets can be fragile if creditors cannot distinguish between falls in the fundamental value of the asset and deviations of prices from the fundamental value. This paper finds that financial markets can be fragile even when households can make that distinction. In addition, the model features multiple equilibria for some parameter regions. In these regions an equilibrium with system wide runs and an equilibrium with only credit contractions can both exist. Which outcome prevails, then, depends on the expected price of the risky asset in the secondary market.

The fragility and multiplicity properties of the model have some important ramifications. It suggests that when the economy is in a region with multiple equilibria or close to the boundaries of the parameter regions, even a subtle government intervention can have substantial effects. However, it also indicates that sometimes the effects of policy can be discontinuous and surprising. For example, when an economy is in a region that exhibits multiple equilibria, the policy authority may be able to change the equilibrium outcome by simply influencing the expectation of asset prices.

4 Discussions

4.1 Policy Implications

The findings of this paper suggest that policy makers maybe able to influence financial market outcomes through policies that either enforce pooling equilibria or facilitate separation of broker-dealers without large credit reductions.[14] For example, a government clearing house for loans may be able to curtail a financial meltdown by preventing all attempts at signaling and enforcing a pooling equilibrium in which all broker-dealers borrow under identical terms. For this policy to succeed, participation must be agreed upon ex-ante and mandatory during the crisis. If institutions can opt out, this may serve as a signal of quality in a manner similar to that of the mechanism of the original model. However, ex-ante participation would be optimal for all broker-dealers as their failure to internalize the costs of separation during the crisis is, in the end, very costly for all. Thus, institutions would voluntarily participate.

Alternatively, the policy of counter-cyclical borrowing limits could be a potential policy that allows separation without large and painful credit contractions. In Appendix B of the online appendix, I find that if the borrowing limit increases by a sufficient amount during the crisis, an equilibrium in which both the high type and low type broker-dealers can borrow exists. In this equilibrium, the degree of credit contraction is comparatively small. However, for such an equilibrium to exist, the debt limit during the crisis must be sufficiently large such

[14] I explore the effects of the polices mentioned in this section by comparing their induced outcomes to the outcome from a perfect information benchmark in Appendix B of the online appendix.

that the low type broker-dealers are able borrow to the point where they can buy enough risky assets at a discount to regain solvency.

4.2 Commitment to Quantity and Collateralized Lending

A question that is not addressed in the previous sections of this paper regards whether it is reasonable to believe that financial institutions can commit to their declared borrowing quantities. If institutions' funding outcomes were public information, commitment seems plausible. For example, when institutions borrow from commercial paper markets, the total quantity of debt they try to secure could be plainly visible. Admittedly, this does not preclude the institution from attempting to secretly secure funds in addition to their committed quantity.

Alternatively, financial institutions and their creditors could ensure commitment by hypothecating their assets as collateral. To ensure commitment to the quantity promised, the financial institutions must first sell some of their assets to acquire the funds required to pay-off their maturing debt, in excess of what they will eventually be able to borrow. Then they can provide their entire remaining portfolio as collateral and ensure commitment to their declared quantity. What is important is not the value of the collateral, which the creditors may not be able to value in the first place, but the fact that the institutions have no more collateral left to post. Repurchase agreements and asset-backed commercial paper markets are some examples of such collateralized lending arrangements.[15]

5 Conclusion

This paper offers a model of financial market failures and illustrates how system wide runs can be triggered by small shocks to asset values. When it is unclear to lenders which

[15] A detailed description of tri-party repos and their behavior during the crisis can be found in the paper by Krishnamurthy et al. (2014).

institutions suffered losses, firms can signal their type by offering to reduce borrowing quantities in exchange for better rates. I find that this mechanism leads to an overall credit contraction and a fire sale of assets, which in turn, deteriorates the balance sheets of all financial institutions. Institutions fail to internalize this cost of separation via fire sales. If fire sales are too severe, the balance sheets of otherwise health institutions deteriorate to the point of insolvency, and all lending to the financial system is suspended, leading to a system wide run.

Our findings sheds new light on potential policies that can mitigate financial crises and prevent financial collapse. Policies that enforce institutions to pool and accept identical terms on their loans can prevent the harmful side effects of separation. Alternatively polices that faciliate separation without large reductions in borrowing can also be effective. A government clearing house for loans, and counter-cyclical borrowing limits are examples of two such policies.

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Appendix A Proofs

Proposition 1. *All pooling equilibria in which the broker-dealers can borrow, fail the intuitive criterion.*

To prove Proposition 1, I first prove the following lemma. Note that the broker-dealer's expected payoff can be described by equation (2) and the household's expected payoff by equation (3).

Lemma 1. *Given d_0 and ρ, the slope of the indifference curve of the broker-dealer $\left(\frac{\partial q_1}{\partial d_1}\right)$ is increasing in the amount of risky assets owned by the broker-dealer.*

(proof) Let $V(d_1, q_1) = \bar{v}$, where $\bar{v}$ is some constant. By total differentiation, $\frac{dV}{dd_1}dd_1 + \frac{dV}{dq_1}dq_1 = 0$. Thus, the slope of the indifference curve is, $\frac{dq_1}{dd_1} = -\frac{\frac{dV}{dd_1}}{\frac{dV}{dq_1}}$. I show that $\frac{\partial}{\partial a}(\frac{dq_1}{dd_1}) > 0$. Note,

$$
\begin{aligned}
\frac{\partial}{\partial a}(\frac{dq_1}{dd_1}) &= -\frac{\partial}{\partial a}(\frac{q_1}{d_1}\frac{\int_{\hat{R}}^{\infty}(R-\frac{\rho}{q_1})dF(R)}{\int_{\hat{R}}^{\infty}RdF(R)}) \\
&= -\frac{1}{\int_{\hat{R}}^{\infty}RdF(R)}\frac{\partial\hat{R}}{\partial a}\frac{q_1}{d_1}f(\hat{R})\{\hat{R}[\frac{\int_{\hat{R}}^{\infty}(R-\frac{\rho}{q_1})dF(R)}{\int_{\hat{R}}^{\infty}RdF(R)} - 1] + \frac{\rho}{d_1}\}.
\end{aligned}
$$

The term $\{\hat{R}[\frac{\int_{\hat{R}}^{\infty}(R-\frac{\rho}{q_1})dF(R)}{\int_{\hat{R}}^{\infty}RdF(R)} - 1] + \frac{\rho}{d_1}\}$ can be rearranged to equal $\frac{\rho}{d_1}[1 - \frac{\int_{\hat{R}}^{\infty}\hat{R}dF(R)}{\int_{\hat{R}}^{\infty}RdF(R)}] > 0$. Furthermore, $\frac{1}{\int_{\hat{R}}^{\infty}RdF(R)} > 0$ and $\frac{\partial\hat{R}}{\partial a} = -\frac{\hat{R}^2}{d_1} < 0$. $\square$

Now, let $\pi^i(d_1, q_1)$ denote the expected payoff of the household given d_1 and q_1, under the belief that the broker-dealer is of type i. $V^i(d_1, q_1)$ denotes the expected payoff of the broker-dealer of type i. Let $C((d, q), r)$ denote a circle with radius r around the point (d, q). Then by Lemma 1, for any given (d^*, q^*) and r, there exists a point $(\tilde{d}, \tilde{q}) \in C((d^*, q^*), r)$

such that $\tilde{d} < d^*$ and the following inequalities both hold:

$$V^H(d^*, q^*) < V^H(\tilde{d}, \tilde{q}), \tag{16}$$

$$V^L(d^*, q^*) > V^L(\tilde{d}, \tilde{q}). \tag{17}$$

Suppose that such a point does not exist. Consider the indifference curves of the high type and low type broker-dealer that cross at (d^*, q^*). If the above point does not exist this implies that the indifference curve of the high type is above the indifference of the low type at any $d < d^*$ inside $C((d^*, q^*), r)$. However, this is a direct contradiction of Lemma 1.

Now, suppose that (d^*, q^*) is a pooling equilibrium offer. Then,

$$\pi(d^*, q^*) = (1 - \alpha)\pi^H(d^*, q^*) + \alpha\pi^L(d^*, q^*) \geq 0.$$

Since π is increasing in a,[16] $\pi^H(d^*, q^*)$ is strictly greater than $\pi^L(d^*, q^*)$ which implies that $\pi^H(d^*, q^*) > 0$. Because π is continuous in both d and q, there is an $\epsilon > 0$ small enough such that $\pi^H(d, q) > 0$ for all $(d, q) \in C((d^*, q^*), \epsilon)$.

Therefore, there is always a deviation from any pooling equilibrium offer (d^*, q^*) such that the inequalities (16) and (17) hold and the households accept under the belief that the deviator is the high type. Thus, all pooling equilibria fail the Cho-Kreps intuitive criterion. $\square$.

Proposition 2. *All hybrid equilibria in which the broker-dealers can borrow, fail the intuitive criterion.*

(proof) Without loss of generality, suppose that multiple actions of (d_1, q_1) are played in a hybrid equilibrium in which the broker-dealers can borrow. Suppose (d^*, q^*) and (d^{**}, q^{**}) are two such equilibrium actions. Let $d^{**} > d^*$. Because households always accept when they

[16] $\frac{d\pi}{da} = \frac{1}{d_1} \int_0^{\hat{R}} R dF(R) > 0$

are indifferent (by assumption), the households accept all equilibrium offers with probability one. Then, for households to accept, the high type broker-dealers must play all equilibrium actions with non-zero probability. Thus, (d^*, q^*) and (d^{**}, q^{**}) lie on the same indifference curve of the high type broker-dealer on a $d - q$ plane. Since, the broker-dealers' indifference curves have the single-crossing property by Lemma 1, the low type broker-dealers always prefer (d^{**}, q^{**}) over (d^*, q^*). This implies only high type broker-dealers play (d^*, q^*).

First, suppose that $q^H(d_1)$ is decreasing. Note that Lemma 2 (proven below) implies that the slope of the indifference curve of the high type broker-dealer is always steeper than the slope of the curve $q^H(d_1)$ at all points on $q^H(d_1)$ $(|\frac{\frac{dV^H}{dd_1}}{\frac{dV^H}{dq_1}}| < |\frac{dq^H(d_1)}{dd_1}|$ for all $q_1 = q^H(d_1))$. Otherwise $V^*(d_1)$ cannot be increasing. Consider the low type broker-dealer's indifference curve that crosses the point (d^{**}, q^{**}). Let $(\tilde{d}, \tilde{q})$ denote the point that this indifference curve crosses the curve $q^H(d_1)$. Because the indifference curve of the low type is steeper than the indifference curve of the high type, $\tilde{d} > d^*$.

Consider a deviation by the high type broker-dealer to the point $(d^* + 0.5 * (\tilde{d} - d^*), q^H(d^* + 0.5 * (\tilde{d} - d^*))$. If the household believes the deviator is of the high type, the household will accept this offer. According to the intuitive criterion, the household should believe that the deviator is the high type. By Lemma 2, the high type broker-dealer prefers $d^* + 0.5 * (\tilde{d} - d^*), q^H(d^* + 0.5 * (\tilde{d} - d^*))$ over (d^*, q^*). Because the indifference curve of the low type broker-dealer is steeper than $q^H(d_1)$, they prefer (d^{**}, q^{**}) over $d^* + 0.5 * (\tilde{d} - d^*), q^H(d^* + 0.5 * (\tilde{d} - d^*))$. The low types can always get at least $V^L(d^{**}, q^{**})$ in equilibrium. Thus the deviation is strictly dominated by the equilibrium play for the low types.

Now, suppose that $q^H(d_1)$ is increasing. Again, consider the low type broker-dealer's indifference curve that crosses the point (d^{**}, q^{**}) and let $(\tilde{d}, \tilde{q})$ denote the point that this indifference curve crosses the curve $q^H(d_1)$. Because the indifference curve of the low type is steeper than the indifference curve of the high type, $\tilde{d} > d^*$. Again, consider a deviation by the high type broker-dealer to the point $(d^* + 0.5 * (\tilde{d} - d^*), q^H(d^* + 0.5 * (\tilde{d} - d^*))$.

If the household believes the deviator is of the high type the household will accept this offer. The household should believe that the deviator is the high type. The high type broker-dealer prefers $(d^* + 0.5 * (\tilde{d} - d^*), q^H(d^* + 0.5 * (\tilde{d} - d^*)))$ over (d^*, q^*). Because the indifference curve of the low type broker-dealer is steeper than $q^H(d_1)$, they prefer (d^{**}, q^{**}) over $(d^* + 0.5 * (\tilde{d} - d^*), q^H(d^* + 0.5 * (\tilde{d} - d^*)))$. The deviation is strictly dominated by the equilibrium play for the low types. Thus, all hybrid equilibria fail the intuitive criterion. $\square$

Lemma 2. *$V^*(d_1)$ is increasing in d_1 for experts with positive liquidation value, where $V^*(d_1) \equiv V(d_1, q^H(d_1))$.*

(proof) I show that $V^*(d_1)$ is increasing in d_1 indirectly by showing that $\frac{dV^*}{dd_1}(d_1) > 0$ as d_1 goes to infinity, and that V^* is concave.

V^* is increasing if its first derivative, shown below, is greater than zero:

$$\frac{dV^*}{dd_1} = \frac{1}{\rho}\int_{\tilde{R}}^{\infty}[(q^H(d_1) + d_1 q^{H'}(d_1))R - \rho]dF(R) \tag{18}$$

where $q^{H'}(d_1) = \frac{dq^H(d_1)}{dd_1}$. From equation (6) we can derive the expression for $q^{H'}(d_1)$. For the sake of convenience, let $A \equiv \int_0^{\tilde{R}} RdF(R)$ and denote q_1^∞ as the value of $q^H(d_1)$ as d_1 converges to infinity. Now, if d_1 converges to infinity, $d_1 q^{H'}(d_1)$ converges to zero because $d_1 \cdot q^{H'}(d_1) = \frac{\rho a^H - d_0}{d_1}\frac{A}{A-\rho}$. Since equation (6) holds for $q^H(d_1)$, $A - \rho = -\frac{\rho}{q_1^\infty}(1 - F(\frac{\rho}{q_1^\infty}))$ which is a constant. Therefore, $\lim_{d_1 \to \infty} \frac{dV^*}{dd_1}(d_1) = \int_{\frac{\rho}{q_1^\infty}}^{1}[\frac{q_1^\infty}{\rho}R - 1]dF(R)$, which is greater than zero.

V^* is concave if its second derivative is less than zero. Note,

$$\frac{d^2V^*}{dd_1^2} = \frac{1}{\rho}\int_{\tilde{R}}^{\infty}[2q^{H'}(d_1) + d_1 q^{H''}(d_1)]RdF(R) - \frac{d\tilde{R}}{dd_1}(\frac{q^H(d_1) + d_1 q^{H'}(d_1)}{\rho}\tilde{R} - 1)f(\tilde{R}), \tag{19}$$

$$q^{H''}(d_1) = \frac{d^2q^H(d_1)}{dd_1^2} = -2\frac{\rho a^H - d_0}{d_1^3}\frac{A}{A-\rho} + \frac{\rho a^H - d_0}{d_1^2}\frac{-\rho}{(A-\rho)^2}\frac{dA}{dd_1} \tag{20}$$

where $\frac{dA}{dd_1} = \frac{d\tilde{R}}{dd_1}\tilde{R}f(\tilde{R})$ and $\frac{d\tilde{R}}{dd_1} = \frac{\rho}{q^H(d_1)d_1+\rho a^H-d_0}(1 - \frac{d_1 q^H(d_1)}{q^H(d_1)d_1+\rho a^H-d_0}) > 0$. From equation (20),

$$2q^{H\prime}(d_1) + d_1 q^{H\prime\prime}(d_1) = \frac{\rho a^H - d_0}{d_1}\frac{-\rho}{(A-\rho)^2}\frac{d\tilde{R}}{dd_1}\tilde{R}f(\tilde{R}) \tag{21}$$

Substituting (21) into (19),

$$\frac{d^2V^*}{dd_1^2} = \frac{d\tilde{R}}{dd_1}\tilde{R}f(\tilde{R})\frac{\rho a^H - d_0}{d_1}\frac{\rho - 1}{(A-\rho)^2} < 0$$

The inequality follows because $\frac{d\tilde{R}}{dd_1} > 0$ when the liquidation value is positive. Thus, since V^* is concave and increasing as d_1 goes to infinity, V^* is increasing. $\square$