Online Appendix For: System Wide Runs and Financial Collapse

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Appendix B Policy Implications

In this section, I consider the effects of various policies on the outcome of the model. There are two distinct approaches to studying the effects of a policy. One can assume an ex-post approach and suppose that the government intervention is unexpected by the agents. In the midst of an unexpected crisis, this may be the appropriate way to evaluate policy. However, if a policy is to be considered an appropriate course of action in a more general sense, it may be more appropriate to consider a situation in which the policy intervention is anticipated ex-ante. This approach can incorporate the fact that the expectation of policy intervention may alter agent behavior ex-ante. That is the approach I take here.

There are some difficulties in comparing the desirability of the outcomes. The difficulty arises from the fact that there is no natural concept of social welfare in the model. One option would be to define a social welfare function balancing the welfare considerations of the various agents in the model. That is not the approach I take. Instead I compare the equilibrium outcomes of economies in which the policies have been implemented against the equilibrium outcome of a perfect information economy.[1]

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[1] Nevertheless, in a model with no natural benchmark, comparing policy outcomes to the perfect infor-

I find that the policy that mandates the participation of all broker-dealers in a clearing house for loans during a state of crisis can garner outcomes that are very close to the outcome of the perfect information economy. A counter-cyclical borrowing limit can be effective if the degree of counter-cyclicality in the constraint is sufficiently large. Lastly, I include a brief discussion about the potential effect of quantitative easing on the financial system during a financial crisis.

B.1 Perfect Information Benchmark

In the perfect information case, the equilibrium outcome will depend on the borrowing limit. First, let β_0 denote the investment quantity of the bank such that $I(\beta_0) = 0$ (if $I(\beta) > 0$, $\beta_0 = B$). Then, if the borrowing limit is high, such that $(1 - d^{limit} q^H(d^{limit}))$ is less than $B - \beta_0$, the price of the risky asset on the secondary market will equal its expected value of one. Since the price of the risky asset is equal to the expected value of the asset, there is no surplus to be gained from buying or selling the assets.

Because the expected profit from purchasing risky assets is zero, the banks will fully invest in safe projects in period zero to the point that $\beta = \beta_0$. The broker-dealers are indifferent to the amount of debt they issue in a crisis. In equilibrium, the high type broker-dealers can borrow any amount $(d_1, q^H(d_1))$ where $d_1 q^H(d_1)$ is between $1 - (B - \beta_0)$ and one. The low type broker-dealers will be unable to borrow in a crisis and will default.

On the other hand, if $(1 - d^{limit} q^H(d^{limit}))$ is greater than $B - \beta_0$, the high type broker-

mation benchmark seems reasonable as, in models that *do* feature potential efficiency gains from reducing informational pressures, the perfect information case is often the first best outcome. In addition, the perfect information benchmark may have some qualities, that have not been explicitly studied in this paper, that could be appealing to policy makers. For example, concerns about moral hazard and over-investment from expectations of bailouts may make it desirable for insolvent firms to fail during a crisis, as long as it does not result in the collapse of the system as a whole. Furthermore, as insolvent firms fail during the crisis, households do not invest in these firms in period one to face expected losses in period 2. From the perspective of regulators, this may also be a desirable quality as they are liable to criticism over failing to protect investors from ex-ante faulty investments.

dealer will borrow to the limit d^{limit}. The price of the asset will be determined such that

$$\delta(\frac{1}{\rho} - 1) = I(B - (d_0 - d^{limit} q^H(d^{limit}))) \tag{B.1}$$

In either case, the high type broker-dealers are able to fully repay their period zero debt whereas low type broker-dealers default, so the price of period zero debt is given as

$$q_0 = 1 - \delta\alpha(1 - \frac{\phi a^H}{d_0}) \tag{B.2}$$

Because the broker-dealer's purchase the risky assets for one unit of cash in period zero, the quantity of period zero debt will equal $d_0 = \frac{1}{q_0}$. Therefore,

$$d_0 = \frac{1 - \delta\alpha\phi a^H}{1 - \delta\alpha} \tag{B.3}$$

As α converges to zero, (d_0, q_0) converges to (1,1).

The perfect information outcome of the example economy in Section 3.1 with system wide runs, yields the latter case. At the debt limit of 1.05 the price of debt will be 0.86. The high type broker-dealers will be able to secure 0.90 in funds and will need to make up the difference by selling their assets. Nevertheless, the amount of funds that need to be obtained by selling assets is comparatively small and the fire sale price will be determined to be 0.89. The high type broker-dealers will be able to avoid default in period one.

Ideally, the perfect information benchmark could be achieved if the policy authority could mandate the truthful disclosure of financial status for all financial institutions. However, implementing such a policy may prove difficult. Does the authority have the expertise to swiftly and accurately assess the health of financial institutions? Experience during the financial crisis suggests otherwise. Without the technical capacity to promptly assess the state of the financial institutions, it is unclear as to how the governing authority can force them to truthfully reveal their financial status. It seems unlikely that the institutions would report their financial difficulties, and even less likely that potential creditors would trust the announcements.

B.2 Government Clearing House for Loans

While it may be difficult to achieve the perfect information outcome through policy channels, it may be possible to attain an outcome that is very close. Because credit contraction stems not from the household's refusal to extend credit but from the incentive of the high type broker-dealers to separate, it may be possible to improve the equilibrium outcome by preventing all attempts at signaling. If the government could enforce a pooling equilibrium by not allowing broker-dealers the opportunity to signal their type through their debt contracts, all broker-dealers would be better off.

The government could sustain pooling equilibria by mandating all broker-dealers to participate in a government clearing house for debt. During a crisis, all broker-dealers would only be allowed to borrow through a government clearing house. The government would choose a total quantity and unit price of debt that is the same for all broker-dealers. The households can either choose to provide funds to the clearing house or choose not to. As long as terms that the government chooses allow the households to break even in expectation, the households will participate willingly.

It is important that the participation in the clearing house is mandatory during the crisis. If a broker-dealer is able to opt out of participating in the midst of a crisis, this can serve as a signal similar to that of the original model. The pooling equilibrium will become unsustainable. Broker-dealers cannot be allowed to decide on participation after their type has been privately revealed.

Nevertheless, the idea of mandating all broker-dealers to participate in this clearing house is not far fetched. From a period zero perspective, all broker-dealers benefit from the implementation of this policy. As long as they are not allowed to opt in at a future date, they would all agree to participate in this policy in period zero. Participation could be achieved voluntarily.

This policy can be very effective. In fact, if $I(B) \leq 0$ and the fraction of low type broker-dealers α converges to zero, a well implemented policy would result in an aggregate outcome that is arbitrarily close the outcome in the perfect information case. Let $\pi^H(d_1, q_1)$ be the expected profit of the households of equation (3) when a equals a^H and $\pi^L(d_1, q_1)$ the expected profit when a equals a^L. Consider the following proposition.

Proposition 6. *Let $\{\beta^{PI}, \rho^{PI}, (d_0^{PI}, q_0^{PI}), (d_1^{H,PI}, q_1^{H,PI})\}$ denote the respective equilibrium outcome in the perfect information economy and $\{\beta^{CH}, \rho^{CH}, (d_0^{CH}, q_0^{CH}), (d_1^{CH}, q_1^{CH})\}$ the equilibrium outcome with a government clearing house for debt where (d_1^{CH}, q_1^{CH}) is the debt contract of the clearing house determined by the government.*

Let $\alpha > 0$ converge to zero. Then, if $I(B) \leq 0$, there exists an $\eta > 0$ for all $\epsilon > 0$ such that, if $|d_1^{CH} - d_1^{H,PI}| < \eta$, $|q_1^{CH} - q_1^{H,PI}| < \eta$ and $(1-\alpha)\pi^H(d_1^{CH}, q_1^{CH}) + \alpha\pi^L(d_1^{CH}, q_1^{CH}) \geq 0$, then $|\rho^{CH} - \rho^{PI}| < \epsilon$, $|\beta^{CH} - \beta^{PI}| < \epsilon$, $|d_0^{CH} - d_0^{PI}| < \epsilon$ and $|q_0^{CH} - q_0^{PI}| < \epsilon$.

(proof) First, note that $d_0^{CH} = q_0^{CH} = 1$ since $(1-\alpha)\pi^H(d_1^{CH}, q_1^{CH}) + \alpha\pi^L(d_1^{CH}, q_1^{CH}) \geq 0$ implies that all period zero debt is repaid. If $1 - d^{limit}q^H(d^{limit}) \leq B - \beta_0$, $d_0^{PI} = q_0^{PI} = 1$ as well and if $1 - d^{limit}q^H(d^{limit}) > B - \beta_0$, both d_0^{PI} and q_0^{PI} converge to zero from equations (17) and (18) as α converges to zero.

Also note that $d_1^{PI}q_1^{PI} - \xi < d_1^{CH}q_1^{CH} < d_1^{PI}q_1^{PI} + \xi$ where $\xi = \eta(d_1^{PI} + q_1^{PI}) + \eta^2$ since $d_1^{PI}q_1^{PI} - \eta(d_1^{PI} + q_1^{PI}) + \eta^2 < d_1^{CH}q_1^{CH} < d_1^{PI}q_1^{PI} + \eta(d_1^{PI} + q_1^{PI}) + \eta^2$. Suppose that $1 - d^{limit}q^H(d^{limit}) < B - \beta_0$. Then $\rho^{PI} = 1$ and $d_1^{PI}q_1^{PI} > 1 - (B - \beta_0)$. Then, there exists $\xi > 0$ small enough such that $d_1^{CH}q_1^{CH} > 1 - (B - \beta_0)$. Denote this ξ as $\hat{\xi}_1$. Then, $\rho^{CH} = 1$. If $\rho^{CH} < 1$, $\beta^{CH} < \beta_0$ and markets could not clear. Since $\rho^{CH} = 1$, $\beta^{CH} = \beta^{PI} = \beta_0$.

(2) Suppose that $1 - d^{limit} q^H(d^{limit}) \geq B - \beta_0$. Then, $\beta^{PI} = B - (d_0^{PI} - d_1^{PI} q^{PI})$ and $\beta^{CH} = B - (d_0^{CH} - d_1^{CH} q^{CH})$ implying $|\beta^{PI} - \beta^{CH}| < \xi$. Then because $I(\cdot)$ is continuous, for any $\psi > 0$, there exists a $\xi > 0$ such that $|I(\beta^{PI}) - I(\beta^{CH})| < \psi$. Then since $\rho(\frac{1}{\rho} - 1) = I(\beta)$ in this case, $|\rho^{PI} - \rho^{CH}| = |\frac{I(\beta^{PI})}{\delta + I(\beta^{PI})} - \frac{I(\beta^{CH})}{\delta + I(\beta^{CH})}| < \frac{\psi}{\delta + min(I(\beta^{PI}, I(\beta^{CH})))}$. Then there exists an η such that $\psi < \epsilon$. $\square$

Even if the ideal conditions do not hold the policy seems to be reasonably effective. In other words, even when $I(B) > 0$ and (d_1^{CH}, q_1^{CH}) is not arbitrarily close to $(d_1^{H,PI}, q_1^{H,PI})$, as long as the government chooses reasonable terms the aggregate outcome should be a vast improvement over the alternative of having no policy.

For example, suppose that in the example economy of Section 3.1 with system wide runs the government chooses $d_1^{CH} = 1$ and $q_1^{CH} = 0.85$. Then the implied price of the asset ρ^{CH} will be 0.89, considerably higher than the 0.75 of the benchmark economy. The households will be willing to accept these terms because in equilibrium their expected profit from each unit of debt purchased is 0.004, which is strictly greater than zero. The high type broker-dealers will not default in period one and the financial system will not suffer a system wide run.

B.3 Counter-Cyclical Borrowing Limits

Recently there has been much debate about the role of borrowing limits, mostly in the form of capital requirements, as a regulatory measure against rapid liquidity contractions in the financial sector. Imposing capital conservation buffers has been suggested as a method of implementing counter-cyclical borrowing limits.

It turns out that, within the framework of this paper, counter-cyclical borrowing limits can be effective if the borrowing limit increases enough for the low type broker-dealers to borrow even when identified. An equilibrium potentially exists where the low type broker-

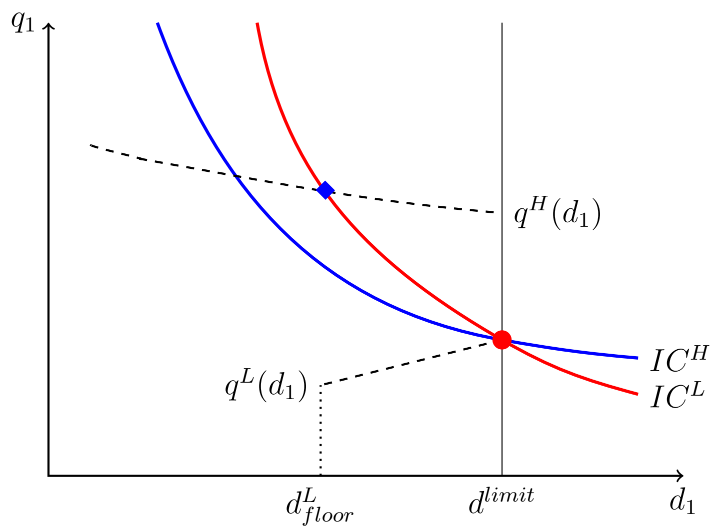

Figure 1: A potential credit market equilibrium with counter-cyclical borrowing limits.

dealer borrows up to the borrowing limit at the maximum price the household will accept. The high type broker-dealer borrows less, but at a more favorable price. Figure 4 illustrates the credit market outcome of this equilibrium. The red circle represents the equilibrium contract of the low type broker-dealer and the blue diamond represents the contract of the high type broker-dealer. The debt contract of the high type is such that the low type is exactly indifferent between the two contracts.

However, for this equilibrium to exist, the increase in the borrowing limit must be substantial. With higher debt limits the supply of assets in fire sale markets decrease and the fire sale discount for the assets will become smaller. As the potential profit margins decrease the low type broker-dealers need to purchase still larger quantities of the assets to become solvent. If the debt limit does not increase beyond the debt floor of the low type broker-dealers the policy will have no effect.

Applying the policy to the example economy shows that the debt limit must increase significantly for the policy to be effective. Numerically solving the model shows that the debt limit must be greater than approximately 2.7 to be effective. If the debt limit is below

2.7, the policy has no effect and the equilibrium will be as in the example economy. When the borrowing limit is equal to $d^{limit} = 2.7$ the prevailing fire sale price is $\rho = 0.88$. The debt floor of the low type broker-dealer is 2.67 and thus they can borrow in equilibrium. They will borrow up to the debt limit of 2.7 and the price per unit debt will equal 0.60. The high type broker-dealer will borrow a nominal amount of 0.85 for the price 0.86.

The analysis above shows that a policy of counter-cyclical borrowing limits can be an effective measure against systemic failures in financial systems. However, it also cautions that for the policy to take effect the magnitude of the counter-cyclicality must be large. A moderate and mechanical approach to counter-cyclical borrowing limits may result in a ineffective policy.

B.4 Quantitative Easing

The analysis of this paper also suggests an effect of quantitative easing that is not often discussed. Proposition 6 states that if banks are unconstrained in period zero ($I(B) \leq 0$), the government can achieve a perfect information outcome by implementing a government clearing house for loans. This result is predicated on banks being unconstrained because if $I(B) > 0$, even a very small amount of risky assets needing to be liquidated can generate a discrete drop in the price of the risky asset. Quantitative easing, or the act of providing banks with excess reserves and thus excess liquidity, is equivalent to increasing B in this model. Increasing B to the point that the banks are unconstrained, can improve the outcome of this policy intervention.

The result need not be confined to this model. Providing ample liquidity to banks may be beneficial as it may allow banks to provide liquidity in various asset markets and help curtail harmful fire sales and stabilize asset prices during a crisis. Even if certain frictions inhibit an immediate and direct effect, quantitative easing may still be useful in conjunction with various other government policies geared toward stabilizing asset markets. The analysis of this paper suggests that even if quantitative easing cannot achieve much on its own, it can help maximize the effects of various crisis management policies.

Appendix C Alternative Borrowing Constraints

In Section 2 of the main text, the borrowing limit does not bind explicitly during the crisis or the normal state. In this section, I consider alternative specifications of the borrowing limit to gain a better understanding of the borrowing limit and its implications. Due to the bang-bang nature of the results, there are only a few possible outcomes depending on the tightness of the borrowing limit. Thus, I review the potential outcomes in each case.

I first consider the alternative case where the borrowing limit becomes tighter during the crisis period. The borrowing limit must become tighter to the point where it is less than the separating quantity of debt ($d^{limit} < d^*$; where $q^H(d^*) = q_d^L(d^*)$ as defined in Proposition 3) for the borrowing limit to be constraining on the final outcome. Otherwise, the borrowing limit does not alter the equilibrium outcome. If the debt limit does bind, then the equilibrium offer of the high types will be $(d^{limit}, q^H(d^{limit}))$ in the separating equilibrium. Then, in the period 1 asset market, if the parameter values are such that a credit contraction is an equilibrium outcome, the quantity of assets sold by the high type broker-dealers will be,

$$s^H = \frac{1}{\rho}(d_0 - d^{limit} q^H(d^{limit})). \tag{C.1}$$

The low types will liquidate their entire portfolio. Thus,

$$\alpha s^L + (1-\alpha)s^H = \alpha\phi a^H + (1-\alpha)\frac{1}{\rho}(d_0 - d^{limit} q^H(d^{limit})) > \phi a^H \tag{C.2}$$

The last inequality in (C.2) shows that the amount of assets sold in the secondary market is greater than the case in which the debt limit does not bind. As α converges to zero, the price of assets will be determined as,

$$\delta\Big(\frac{1}{\rho}-1\Big)=I(B-(d_0-d^{limit}q^H(d^{limit}))). \tag{C.3}$$

For both the credit contraction case and the case with system wide runs, the fire sale price will be lower than that of Section 2 of the main text.

The case in which the borrowing limit is larger during the crisis is covered in detail in Appendix B.3, when analyzing the effect of counter-cyclical borrowing limits. If the borrowing limit is high enough during the crisis for the low type broker-dealers to become solvent by purchasing assets at fire sales discounts, there exists a separating equilibrium in which both types of broker-dealers can borrow. However, the equilibrium outcome remains as in Section 2 if the borrowing limits are greater during the crisis but not high enough for low types to buy their way out of insolvency by purchasing assets at a discount.

Lastly, the borrowing limit could also be tighter during the normal states (in both period zero and period one), relative to the crisis period. If the borrowing limit decreases to the point that it is less than one during the normal periods, the total amount of investment in period 0 would decrease. In the normal state in period one, the debt of the broker-dealers would get rolled over. During the crisis, the outcome would depend on whether the borrowing limit is large enough for the low type broker-dealers to become solvent, in which case an outcome analogous to that in Appendix B.3 will be possible. Otherwise, the equilibrium during the crisis will remain as shown in Section 2 of the main text.

Appendix D The Credit Market Equilibrium with Risk Averse Households

In order to gain a better understanding of the role that the risk neutrality assumption of households plays in deriving the main results, I explore the credit market equilibrium with risk averse households. Suppose that households are risk averse and that they have the following utility function

$$u(x)$$

where x is the return on investment and $u' > 0, u'' < 0$. u is continuous and differentiable. The expected return from buying one unit of debt from a broker-dealer who issues d_1 units of new short-term debt at price q_1 can be expressed as

$$\pi(d_1, q_1) = (1 - F(\hat{R}))u(1 - q_1) + \int_0^{\hat{R}} u([\frac{1}{d_1}(a - \frac{d_0 - q_1 d_1}{\rho})R - q_1])dF(R). \tag{D.1}$$

If equation (D.1) is greater than $u(0)$, households accept the broker-dealer's offer and provide them with cash. If it is less than $u(0)$, households reject the offer.

D.1 Pooling and Hybrid Equilibria

The results for the pooling and hybrid equilibria do not change with risk averse households. This is because the household's decision problem is not central to Propositions 1 and 2. The households only need to be accounted for when considering whether households will accept a deviating offer.

First, consider pooling equilibria. As in the risk neutral case, there is always a deviation from the pooling equilibrium such that the high type receives a higher payoff then he would from his equilibrium action but the low type receives a lower payoff then he does from his equilibrium action if the lender believes that the deviator is the high type. As before, by the intuitive criterion the lender must believe that the deviator is of the high type and accept the offer.

Proposition 1'. *All pooling equilibria in which the broker-dealers can borrow, fail the intuitive criterion when households are risk averse.*

(Proof) The proof of Proposition 1' is the same as the proof of Proposition 1 once one redefines $\pi^i(d_1, q_1)$ with risk averse households. Note that the statement and proof of Lemma 1 is unchanged with risk averse households as the lemma does not concern households. Now, let $\pi^i(d_1, q_1)$ denote the expected utility of the risk averse household given d_1 and q_1, under the belief that the broker-dealer is of type i. $V^i(d_1, q_1)$ is the expected payoff of the broker-dealer of type i. Let $C((d, q), r)$ denote a circle with radius r around the point (d, q). Then, as before, by Lemma 1, for any given (d^*, q^*) and r, there exists a point $(\tilde{d}, \tilde{q}) \in C((d^*, q^*), r)$ such that $\tilde{d} < d^*$ and,

$$V^H(d^*, q^*) < V^H(\tilde{d}, \tilde{q}), \tag{D.2}$$

$$V^L(d^*, q^*) > V^L(\tilde{d}, \tilde{q}). \tag{D.3}$$

Now, suppose that (d^*, q^*) is a pooling equilibrium offer. Then,

$$\pi(d^*, q^*) = (1 - \alpha)\pi^H(d^*, q^*) + \alpha\pi^L(d^*, q^*) \geq 0.$$

$\pi^H(d^*, q^*)$ is strictly greater than $\pi^L(d^*, q^*)$ which implies that $\pi^H(d^*, q^*) > 0$. Because π is continuous in both d and q, there is an $\epsilon > 0$ small enough such that $\pi^H(d, q) > 0$ for all $(d, q) \in C((d^*, q^*), \epsilon)$. Therefore, there is always a deviation from any pooling equilibrium offer (d^*, q^*) such that the inequalities (D.2) and (D.3) hold and the households accept under the belief that the deviator is the high type. Thus, all pooling equilibria fail the Cho-Kreps intuitive criterion. $\square$.

Next, I show that Proposition 2 also holds with risk averse households.

Proposition 2'. *All hybrid equilibria in which the broker-dealers can borrow, fail the intuitive criterion when households are risk averse.*

(Proof) Without loss of generality, suppose that multiple actions of (d_1, q_1) are played in a hybrid equilibrium in which the broker-dealers can borrow. Suppose (d^*, q^*) and (d^{**}, q^{**}) are two such equilibrium actions. Let $d^{**} > d^*$. Because households always accept when they are indifferent (by assumption), the households accept all equilibrium offers with probability one. Then, for households to accept, the high type broker-dealers must play all equilibrium actions with non-zero probability. Thus, (d^*, q^*) and (d^{**}, q^{**}) lie on the same indifference curve of the high type broker-dealer on a $d - q$ plane. Since, the broker-dealers' indifference curves have the single-crossing property by Lemma 1, the low type broker-dealers always prefer (d^{**}, q^{**}) over (d^*, q^*). This implies only high type broker-dealers play (d^*, q^*).

First, suppose that $q^H(d_1)$ is decreasing. Note that at d^{**}, $q^H(d^{**}) > q^{**}$. Otherwise, households would not accept this offer and (d^{**}, q^{**}) could not be an equilibrium of this game. Because $q^H(d_1)$ is continuous, this implies that at some points between d^* and d^{**}, $q^H(d_1)$ is above the high type's indifference curve crossing (d^*, q^*) and (d^{**}, q^{**}). Let $(\tilde{d}, \tilde{q})$ denote some point between d^* and d^{**}, where $q^H(d_1)$ is above the high type's indifference curve crossing (d^*, q^*) and (d^{**}, q^{**}) and below indifference curve of the low type's crossing (d^{**}, q^{**}). Such a point exists by the single crossing properties of the broker-dealers shown in Lemma 1. Thus, $\tilde{q} < q^H(\tilde{d})$, above the high type's indifference curve, and below the low type's indifference curve for some $\tilde{d} \in (d^*, d^{**})$. Consider a deviation by the high type broker-dealer to this point. If the household believes the deviator is of the high type, the household will accept this offer. According to the intuitive criterion, the household should believe that the deviator is the high type.

Now, suppose that $q^H(d_1)$ is increasing. Consider the low type broker-dealer's indifference curve that crosses the point (d^{**}, q^{**}) and let $(\hat{d}, \hat{q})$ denote the point that this indifference curve crosses the curve $q^H(d_1)$. Because the indifference curve of the low type is steeper than the indifference curve of the high type, $\hat{d} > d^*$. Consider a deviation by the high type broker-dealer to the point $(d^* + 0.5 * (\hat{d} - d^*), q^H(d^* + 0.5 * (\hat{d} - d^*))$. If the household believes the deviator is of the high type, the household will accept this offer. The household should believe that the deviator is the high type. The high type broker-dealer prefers $(d^* + 0.5 * (\hat{d} - d^*), q^H(d^* + 0.5 * (\hat{d} - d^*)))$ over (d^*, q^*). Because the indifference curve of the low type broker-dealer is steeper than $q^H(d_1)$, they prefer (d^{**}, q^{**}) over $(d^* + 0.5 * (\hat{d} - d^*), q^H(d^* + 0.5 * (\hat{d} - d^*)))$. The deviation is strictly dominated by the equilibrium play for the low types. Thus, all hybrid equilibria fail the intuitive criterion. $\square$

D.2 Separating Equilibria

Propositions 1' and 2' show that any equilibrium of this model in which the broker-dealers can borrow must be separating with risk averse households. The broker-dealer's type will be identifiable to the households in equilibrium. Thus, consider the lender's problem given they are aware of the borrower's type. As in the main text, I assume $\frac{1}{1-\delta} < a^H < \frac{1}{\phi}$.

First, note that the household's expected payoff from equation (D.1) is concave in q_1 for a given d_1 even with risk averse utility functions. Due to the concavity of π in q_1, there is a price $q^{max}(d_1)$ that maximizes the household's expected profit. From the first-order condition of equation (D.1), $q^{max}(d_1)$ solves,

$$\frac{\partial \pi}{\partial q_1} = 0 \quad \Leftrightarrow \quad u'(1 - q_1)(1 - F(R^*)) = \int_0^{R^*} u'\Big(\frac{R}{R^*} - q_1\Big)\Big(\frac{R}{\rho} - 1\Big) dF(R) \tag{D.4}$$

where $R^* \equiv \frac{\rho d_1}{q^{max}(d_1) d_1 + \rho a - d_0}$.

Now, consider the maximum price per unit debt (the worst offer) that the household will be willing to accept from the high type broker-dealers given d_1. Denote this price as $q^H(d_1)$ (if it exists). At the maximum price the household's expected profit should be zero. From equation (D.1) the household's zero profit condition is

$$\pi(d_1, q_1) = (1 - F(\hat{R}))u(1 - q_1) + \int_0^{\hat{R}} u\Big(\frac{R}{\hat{R}} - q_1\Big) dF(R) = 0 \tag{D.5}$$

where $\hat{R} = \frac{\rho d_1}{q_1 d_1 + \rho a^H - d_0}$. Because π is concave in q_1, there can be multiple solutions to equation (D.5) for a given d_1. However, $q^H(d_1)$ is the solution to equation (D.5) that is weakly greater than $q^{max}(d_1)$ and this is unique. Lastly, let $q_d^L(d_1)$ denote the price of debt below which the low type broker-dealer defaults even if the households agree to accept the offer. I maintain the assumption $d^{limit} < (1 + \frac{1}{a^H - 1})(1 - \phi)$ that implies that the low type broker-dealers would still be insolvent even if they could borrow to the full extent of the borrowing limit.

Suppose that the high type broker-dealer can borrow in equilibrium. In any separating equilibrium, the low types have the incentive to pool with the high types as long as the terms of the high type's offer allows them to pay off their maturing debt. Thus, any separating equilibrium offer of the high type cannot have price of debt greater than $q_d^L(d_1)$. In addition, for any equilibrium quantity d_1^*, the high type's offer will have the price $q^H(d_1^*)$. Thus, as in the risk neutral case, the equilibrium offer of the high type $(d_1, q^H(d_1))$ will be the solution to the broker-dealer's constrained optimization problem,

$$\max_{d_1} V^H(d_1, q^H(d_1)) = \int_{\hat{R}}^{\infty} [(a^H - \frac{d_0 - q^H(d_1)d_1}{\rho})R - d_1] dF(R) \tag{D.6}$$

$$s.t. \qquad q^H(d_1) \leq q_d^L(d_1)$$

The exact solution to this optimization problem changes with risk averse households. Therefore, Proposition 3 of the main text does not hold with risk averse households.

To see why Proposition 3 no longer holds, consider the following. First, note that $q^H(d_1)$ is still a decreasing function of d_1 when the liquidation value of the broker-dealer is positive and an increasing function of d_1 if the liquidation value is negative. Since $\pi(d_1, q^H(d_1)) = 0$, by the implicit function theorem:

$$\frac{dq^H(d_1)}{dd_1} = \frac{\rho a^H - d_0}{d_1^2} \frac{\int_0^{\tilde{R}} u'\big(\frac{R}{\tilde{R}} - q_1\big) dF(R)}{\int_0^{\tilde{R}} u'\big(\frac{R}{\tilde{R}} - q_1\big)\big(\frac{R}{\rho} - 1\big) dF(R) - (1 - F(\tilde{R}))u'(1 - q_1)} \tag{D.7}$$

where $\tilde{R} = \frac{\rho d_1}{q^H(d_1)d_1 + \rho a^H - d_0}$. Because $q^H(d_1)$ is greater than $q^{max}(d_1)$, this means that $\tilde{R}$ is smaller than R^* and by extension $\int_0^{\tilde{R}} u'\big(\frac{R}{\tilde{R}} - q_1\big)\big(\frac{R}{\rho} - 1\big) dF(R) - (1 - F(\tilde{R}))u'(1 - q_1) \leq 0$. Therefore, equation (D.7) is negative if $\rho a^H - d_0 > 0$ and positive otherwise. Thus, if the liquidation value of the broker-dealer is negative, V^H is increasing in d_1 because $q^H(d_1)$ is increasing in d_1.

When the liquidation value of the broker-dealer is positive, in contrast to the risk neutral case, we can no longer be sure that $V^*(d_1)$ is increasing in d_1. Loosely speaking, if $\frac{dq^H(d_1)}{dd_1}$ is flatter (less negative) then the indifference curve of $V^*(d_1)$ on a $q_1 - d_1$ plane, $V^*(d_1)$ is increasing in d_1. When households become more risk averse, then $\frac{dq^H(d_1)}{dd_1}$ will likely become steeper (more negative), which means that it is no longer guaranteed that $V^*(d_1)$ is increasing in d_1. Consider the motivations of the household. Note that households do not capture the upside of returns but bear all the downside. Therefore, with regard to their lending quantity d_1, holding prices constant, households do not care whether the amount they lend reduces the default probability of the broker-dealer because they are indifferent between marginal default and non default. Instead, households care about recovery value. When the liquidation value of the broker-dealer is negative ($\rho a^H - d_0 < 0$), broker-dealers default even at relatively high values of R, which implies increasing d_1 will improve their recovery value in expectation, as broker-dealers purchase assets at fire sale discounts. When the liquidation value of the broker-dealer is positive ($\rho a^H - d_0 \geq 0$), broker-dealers default only at low values of R, which implies that increasing d_1 does not improve recovery value. Thus, with risk averse households, the $\frac{dq^H(d_1)}{dd_1}$ curve becomes steeper because households need to be compensated more for uncertainty, and increasing a unit of d_1 becomes more costly in terms of increasing the promised interest rate represented by lowering q_1.

Nevertheless, this does not change the main results of the paper. The equilibrium offer in a separating equilibrium where the high type broker dealers can borrow, now will simply be the solution to equation (D.6). The remainder of the results will simply follow as with the risk neutral case, although the equations may become more complex. Thus, any point below $q_d^L(d_1)$ that maximizes (D.6) will be the equilibrium offer in the separating equilibrium.