Asymmetric volatility connectedness among G7 Stock Markets

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ABSTRACT

This paper investigates asymmetries in volatility connectedness among the G7 stock markets. Using daily realized semi-volatility indices, obtained from intra-day data, we provide ample evidence for the asymmetric volatility connectedness. We find that the impact of bad volatility strictly dominates that of good volatility in generating connectedness across financial markets. In particular, the global financial crisis, the European debt crisis and the recent COVID-19 pandemic have witnessed most influential episodes of volatility connectedness. We also discuss that the effect of the US stock market on other countries has been largely due to bad volatility.

Keywords: Asymmetric connectedness, Realized semi-volatility, G7 stock markets, VAR, Variance decomposition

JEL Classification: C32, G15

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1. Introduction

There is a long tradition in finance that stock prices tend to fall simultaneously, whereas they rise independently. A large body of finance literature has also discussed that volatility tends to rise (or fall) in response to "bad" (or "good") news. This type of empirical phenomena is often referred to as asymmetric or leverage volatility.[1] As asymmetric patterns in transmission mechanism of financial markets have an important implication on portfolio diversification and risk management strategies, the presence of asymmetric spillover may pose a challenge for investors. Hence, there have been many attempts to capture asymmetric connectedness across financial markets.

The GARCH-type models have been widely used as a formal econometric approach to empirical analysis on measuring volatilities in financial markets. As for the asymmetric volatility correlation, the EGARCH model of Nelson (1991) and the GJR specification of Glosten *et al*. (1993) have long been adopted in financial literature. As discussed in Bekaert and Wu (2000) and Wu (2001), the presence of asymmetric volatility is most apparent during stock market crashes when significant increases in market volatility are often led by a big drop in stock prices.

In multivariate framework, Cappiello *et al*. (2006) proposed asymmetric generalized dynamic conditional correlation model to explain asymmetric conditional correlations and variances. A few examples of empirical studies on the asymmetric volatility transmission include Koutmos and Booth (1995), Booth *et al*. (1997), Lee and Hong (2009), and Gjika and Horváth (2013).

While the GARCH-type model has been most often used to estimate volatility of financial data, it is not useful in capturing the spillover dynamics in multivariate framework. Recently, Diebold and Yilmaz (2012, 2014) developed the connectedness methodology, which is a unified framework for conceptualizing and empirically measuring the network connectedness at a variety of levels. Hence, many authors have employed the methodology in

[1] Black (1976) is considered as the seminal work on this issue. Other papers on the asymmetric effect include Christie (1982), French *et al*. (1987), Engle and Ng (1993), Karolyi and Stulz (1996), Bekaert and Wu (2000), Longin and Solnik (2001), Ang and Chen (2002), Bae *et al*. (2003), and Hong *et al*. (2007).

investigating connectedness across various markets and countries.

For instance, Tsai (2014) and Yarovaya et al. (2016) discussed the connectedness among stock markets, and Antonakakis (2012) and Chang (2013) applied this approach to the forex markets. Claeys and Vašíc ˇ ek (2014) and Ahmad et al. (2018) considered the bonds market, and Lee and Lee (2018, 2019a) examined the housing markets. In addition, the connectedness across different asset-class markets is also discussed in Diebold and Yilmaz (2014), Liow (2015), and Lee and Lee (2019b), among others. These studies mainly suggested that connectedness in return or volatility is time-varying and crisis sensitive.

Given that the last decades have experienced large perturbations in the financial markets, such as the global financial crisis and the European debt crisis, a significant body of literature has emerged on the connectedness dynamics across financial markets. While the presence of asymmetric volatility in financial data has been well recognized in the literature since the seminal work of Black (1976), asymmetries in volatility connectedness still remain in an early stage. As discussed in Garcia and Tsafack (2011), a proper quantification of such asymmetries is highly relevant to portfolio selection and risk management strategies.

Meanwhile, the availability of high-frequency data has opened new avenues for volatility analysis on financial markets. For instance, Andersen and Bollerslev (1998) introduced a robust measure for the actual market volatility, called the realized volatility. Barndorff-Nielsen *et al.* (2010) proposed realized semi-variance that decomposes the volatility measures into good and bad volatilities caused by positive and negative returns, respectively. Segal *et al*. (2015) defined bad (good) uncertainty as the volatility that is associated with negative (positive) innovations to quantities such as in output and return. Using the Diebold-Yilmaz connectedness approach, several studies investigated asymmetry in volatility connectedness across the financial markets. Baruník *et al*. (2016) examined asymmetries in volatility spillovers that emerge from good and bad volatilities. Based upon the spillover asymmetry measures, Baruník *et al*. (2017) presented evidence for dominating asymmetries of bad volatility over good news in spillovers across the major forex markets. Caloia *et al.* (2018) examined five EMU stock markets, and Wang and Wu (2018) and Xu *et al.* (2019) discussed the asymmetric relationship between oil and stock markets.

In this paper, we investigate the asymmetric volatility connectedness among the G7 stock markets. In particular, we assess the magnitude of asymmetric connectedness measures and the dynamic patterns of their transmission mechanisms. This work is related to BenSaïda (2019), which also examined the asymmetric connectedness among the G7 stock markets. However, we use high frequency realized measures, whereas BenSaïda (2019) inferred the volatility measure from the GJR-GARCH model. The use of high frequency data might improve the estimation of dynamic volatilities, and the availability of realized measures can provide more accurate forecasts (Hansen and Lunde, 2011). In fact, realized semi-variance (RS) measures are a more accurate estimator for current latent volatilities than those derived from the GARCH-type model.

We proceed as follows. Section 2 discusses the methodology employed in this study. Section 3 describes basic characteristics of the data. Section 4 presents the empirical results, and discusses their implications. A brief summary and concluding remarks are provided in Section 5.

2. Empirical methodology

In this section, we briefly discuss the asymmetric connectedness methodology. We first introduce the concept of realized variance and semi-variances measures. We then explain the connectedness indices, and describe how to estimate asymmetries in volatility connectedness.

2.1 Realized variance and semi-variance

Following Andersen and Bollerslev (1998), the realized variance (RV) can be defined as the sum of intraday squared returns, which can be derived as follows:

$$RV = \sum_{i=1}^{n} {r_i}^2, \tag{1}$$

where r_i denotes the intraday returns at five-minute intervals.

In order to analyse the asymmetric effects in volatility, Barndorff-Nielsen *et al.* (2010) introduced the measure of realized semi-variance (RS), which can separate positive and negative movements in financial time series. The positive and negative realized semi-variances (RS^+ and RS^-) are defined as follows:

$$RS^{+} = \sum_{i=1}^{n} \mathrm{I}(r_i \geq 0) r_i^2 \tag{2}$$

$$RS^{-} = \sum_{i=1}^{n} \mathrm{I}(r_i < 0) r_i^2, \tag{3}$$

where $I(\cdot)$ denotes the indicator function. The positive and negative realized semi-variances provide the information on upside opportunity and downside risk of the underlying variable, respectively.

Note that the sum of positive and negative realized semi-variances is always equal to the realized variance (i.e., $RV_t = RS_t^{+} + RS_t^{-}$). By using the realized semi-variances, we can estimate the volatility connectedness measures due to good or bad volatilities, and then quantify asymmetries in volatility connectedness across different financial markets.

2.2 Connectedness approach

In order to measure connectedness, we use the generalized variance decomposition approach as discussed in Diebold and Yilmaz (2012, 2014). The main advantage of the generalized method is to obtain the connectedness indices which are robust to the variable ordering. For a covariance stationary m-variable VAR (p) process:

$$RV_t = \sum_{i=1}^{p} \Phi_i RV_{t-i} + \varepsilon_t \quad \text{with } \varepsilon_t \sim (0, \Omega),$$

we have a moving average representation:

$$RV_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i}.$$

Here $m \times m$ coefficient matrices A_i are derived as: $A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + \cdots + \Phi_p A_{i-p}$ with $A_0 = I_m$ and $A_i = 0$ for $i < 0$.

The h-step-ahead forecast error variance decompositions are computed as:

$$\theta_{ij} = \frac{\omega_{jj}^{-1} \sum_{k=0}^{h-1} (e_i' A_k \Omega e_j)^2}{\sum_{k=0}^{h-1} (e_i' A_k \Omega A_k' e_i)}, \tag{4}$$

where $\mathbf{\Omega}$ is the variance matrix for the error vector ε_t, ω_{jj} is the variance of ε_{jt}, and e_i is the selection vector with i^{th} element unity and zero otherwise. Since $\sum_{j=1}^{m} \theta_{ij} \neq 1$, we normalize each entry by the row sum: [2]

$$\tilde{\theta}_{ij} = \frac{\theta_{ij}}{\sum_{j=1}^{m} \theta_{ij}}. \tag{5}$$

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[2] Although this row normalization scheme may lead to inaccurate measures of the net connectedness, as discussed in Caloia *et al*. (2018), it is most often used for interpretative purposes.

By construction, it holds that $\sum_{j=1}^{m} \tilde{\theta}_{ij} = 1$. Equation (5) represents a pairwise directional connectedness $\tilde{\theta}_{ij}$, from market j to market i (at horizon h), from which we can derive various connectedness measures. By denoting $\tilde{\theta}_{ij}$ as $C_{i\leftarrow j}$, we can explicitly indicate the direction of connectedness. We are also interested in the net pairwise directional connectedness, defined as:

$$C_{ij} = C_{i\leftarrow j} - C_{j\leftarrow i} \tag{6}$$

Next, the total directional connectedness has two measures: “from” and “to”, which can be obtained as the off-diagonal row sum and column sum, respectively. The total directional connectedness received from others to i can be defined as:

$$C_{i\leftarrow\bullet} = \sum_{\substack{i,j=1 \\ j\neq i}}^{m} \tilde{\theta}_{ij} \quad . \tag{7}$$

Similarly, the total directional connectedness to others from i can be computed as:

$$C_{\bullet\leftarrow \mathrm{i}} = \sum_{\substack{i,j=1 \\ j\neq i}}^{m} \tilde{\theta}_{ji} \; . \tag{8}$$

Sometimes, we are also interested in net total directional connectedness defined as the difference between the “to” and “from” others:

$$C_{\mathrm{i}} = C_{\bullet\leftarrow \mathrm{i}} - C_{i\leftarrow\bullet} . \tag{9}$$

The total connectedness is the ratio of the sum of the off-diagonal elements of the variance decomposition matrix to the sum of all its elements.

$$C_m = \frac{\sum_{\substack{i,j=1 \\ i\neq j}}^{m} \tilde{\theta}_{ij}}{\sum_{i,j=1}^{m} \tilde{\theta}_{ij}} = \frac{\sum_{\substack{i,j=1 \\ i\neq j}}^{m} \tilde{\theta}_{ij}}{m} \tag{10}$$

2.3 Asymmetric volatility connectedness

In order to estimate the asymmetric connectedness measures due to good and bad volatilities, we use the decomposed RV_t indices: positive and negative semi-variances (RS_t^{+} and RS_t^{-}). In this case, the asymmetric connectedness can be obtained in two ways. First, we can estimate two separate VAR models for positive and negative semi-variances, as examined in Baruník *et al.* (2016). Second, we can use a single VAR system that combines both positive and negative semi-variances, as discussed in Baruník *et al.* (2017).

(1) *Connectedness asymmetry measures (CAM)*

In order to capture the degree of asymmetries for individual market i, we can use the directional connectedness asymmetry measure (CAM) as discussed in Baruník *et al*. (2017). The **directional CAM** for individual market can be defined as the difference in responses to good and bad volatility shocks from market (or country) i to other markets.

$$CAM_{\bullet\leftarrow \mathrm{i}} = C^{+}_{\bullet\leftarrow \mathrm{i}} - C^{-}_{\bullet\leftarrow \mathrm{i}} \qquad (\text{for } i = 1,\cdots,m) \tag{11}$$

Here, $C^{+}_{\bullet\leftarrow \mathrm{i}}$ and $C^{-}_{\bullet\leftarrow \mathrm{i}}$ indicate the total directional connectedness to others from i for good and bad volatilities, respectively. This measure can be used to examine asymmetries in volatility connectedness for a given market.

We can also quantify asymmetries in volatility connectedness for the whole system, by using the **total directional CAM**, which can be defined as the difference between volatility connectedness measures due to positive and negative returns from all markets (or countries) in the VAR system:

$$CAM_{m} = \sum_{i=1}^{m}(C^{+}_{\bullet\leftarrow \mathrm{i}} - C^{-}_{\bullet\leftarrow \mathrm{i}}). \tag{12}$$

The total directional CAM_m is useful in characterizing the asymmetric pattern in volatility connectedness for the whole market system under investigation. For instance, the case for $CAM_m = 0$ denotes that RS^+ and RS^- have the same degrees of connectedness with no asymmetric effects. Otherwise, there are connectedness asymmetries. In particular, a negative CAM_m indicates that the volatility connectedness from bad news is larger than that from good news.

(2) *Total connectedness for semi-variance*

When we use a single VAR system, by stacking both positive and negative semi-variances, we need to adjust the total connectedness measure in equation (10), as proposed in Baruník *et al*. (2017). That is, besides the main diagonal elements ($i = j$), we need to exclude the cases for $|i - j| = m$, which denote own market connectedness between good and bad volatilities.

$$C_{2m} = \frac{\sum_{\substack{i,j=1, i\neq j \\ |i-j|\neq m}}^{2m} \tilde{\theta}_{ij}}{2m} \tag{13}$$

This measure of total connectedness represents the degree of connectedness across different financial markets, when the realized variance (RV) is decomposed into RS^+ and

RS^- series.

3. Data and descriptive statistics

We use daily observations on realized semi-variances, obtained from intra-day returns for the G7 stock markets: S&P 500 (US), S&P/TSX (Canada), FTSE 100 (UK), CAC 40 (France), DAX (Germany), FTSE MIB (Italy), and Nikkei 225 (Japan). The data span from May 2, 2002 to August 31, 2021, with a total of 5029 daily observations, available on the Oxford-Man Institute's Quantitative Finance Realized Library.[3] The Realized Library provides 5-minute sampled realized variance and semi-variances. When the market indices are not available due to holidays, the same indices as the previous days are used.

Figure 1 displays the time series plots of the realized volatility for the G7 stock markets. We can see highly persistent patterns in volatility dynamics with huge jumps in all volatilities around the global financial crisis and the recent COVID-19 pandemic.

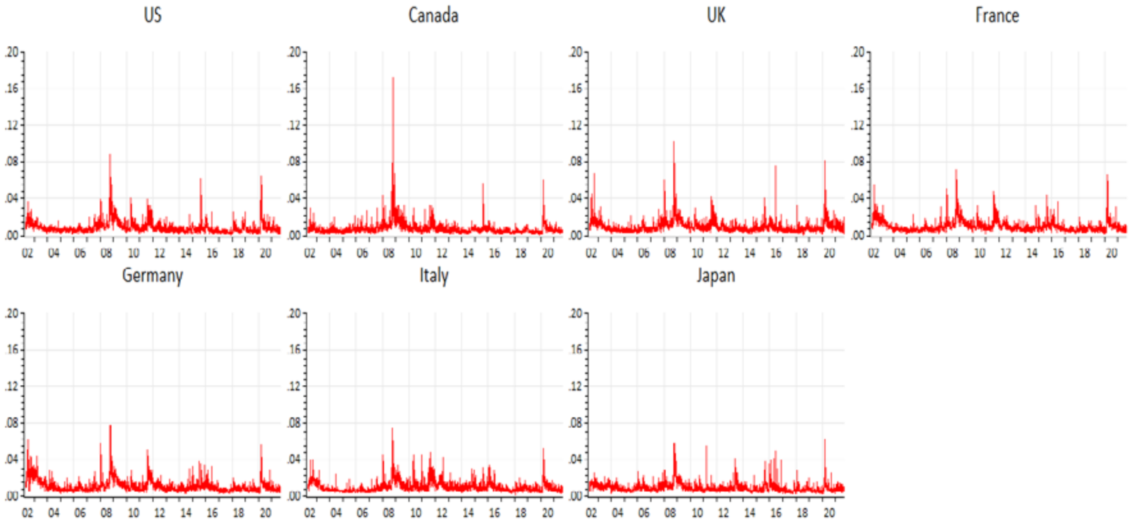

Figure 1. Time series plot of daily realized volatility.

Notes: This figure displays the time variations for G7 stock market volatilities from May 2, 2002 to August 31, 2021.

Following Andersen *et al*. (2003), we use the log-transformation to obtain approximate Gaussian measures. Table 1 presents the descriptive statistics for the log realized volatility, positive and negative log semi-volatilities. Notice that most of the volatility

[3] https://realized.oxford-man.ox.ac.uk

series seem to follow approximate Gaussian processes. As expected, the standard deviations of negative semi-variances are higher than those of positive semi-variances for all the G7 stock markets.

Table 1. Descriptive statistics for log RV, log RS^+, and log RS^-

	US	Canada	UK	France	Germany	Italy	Japan
Panel A: $log\ RV$							
Mean	-10.001	-10.319	-9.718	-9.562	-9.475	-9.605	-9.803
Max	-4.860	-3.527	-4.547	-5.274	-5.136	-5.245	-5.560
Min	-13.618	-13.403	-13.529	-12.352	-12.394	-14.083	-13.091
Std. dev	1.172	1.105	1.037	1.008	1.040	0.985	0.935
Skewness	0.491	0.904	0.634	0.480	0.540	0.446	0.465
Kurtosis	3.474	4.511	3.636	3.278	3.389	3.155	3.802
Panel B: $\frac{1}{2} log\ RS^+$							
Mean	-5.364	-5.535	-5.229	-5.141	-5.100	-5.171	-5.276
Max	-2.734	-1.787	-2.975	-2.927	-2.756	-3.064	-3.220
Min	-7.107	-7.081	-7.198	-6.555	-6.612	-7.246	-7.076
Std. dev	0.589	0.560	0.527	0.505	0.519	0.494	0.480
Skewness	0.520	0.874	0.632	0.489	0.555	0.459	0.422
Kurtosis	3.557	4.465	3.566	3.358	3.527	3.245	3.705
Panel C: $\frac{1}{2} log\ RS^-$							
Mean	-5.405	-5.582	-5.275	-5.148	-5.113	-5.163	-5.283
Max	-2.824	-2.682	-2.360	-2.883	-2.802	-2.778	-2.954
Min	-7.656	-7.225	-7.037	-6.827	-6.617	-7.588	-6.916
Std. dev	0.636	0.608	0.572	0.534	0.558	0.526	0.511
Skewness	0.432	0.801	0.635	0.419	0.473	0.346	0.417
Kurtosis	3.321	4.171	3.551	3.119	3.184	3.070	3.686

4. Empirical results

In this section, we first estimate the usual symmetric connectedness measures, using the RV_t indices of the G7 stock markets. We then extend the framework to examine asymmetries in volatility connectedness across different financial markets. In order to assess time-varying aspects of the asymmetric connectedness, we also employ the rolling-sample estimation with 250-day windows.

4.1 Full-sample analysis

(1) *Symmetric volatility connectedness analysis*

Table 2 presents the estimation result on the full-sample (symmetric) volatility connectedness among the G7 stock markets. The results are based on VAR(5) model, selected by the Schwarz information criterion, and ten-day ahead forecast error variance decompositions are used.

Table 2. Symmetric volatility connectedness table

	US	Canada	UK	France	Germany	Italy	Japan	From
US	39.29	15.99	9.82	12.68	11.56	8.75	1.92	60.71
Canada	20.86	45.49	8.67	9.22	7.56	6.70	1.50	54.51
UK	16.18	10.26	30.50	16.69	13.78	10.63	1.96	69.50
France	13.30	7.91	11.84	27.65	20.08	17.71	1.51	72.35
Germany	12.58	6.95	10.13	21.42	31.30	15.76	1.86	68.70
Italy	11.22	7.10	8.24	20.60	16.51	35.18	1.14	64.82
Japan	12.39	8.24	6.47	7.29	7.12	4.92	53.58	46.42
To	86.53	56.45	55.17	87.89	76.61	64.48	9.88	437.01
Net	25.82	1.95	-14.32	15.54	7.91	-0.34	-36.54	**62.43%**

As for the diagonal elements of the connectedness matrix in Table 2, the Japanese stock market shows the highest own variance share (53.58%), followed by the Canadian market (45.49%). Notice also that Japan shows the lowest "to" and "from" connectedness (9.88% and 46.42%, respectively). These results suggest that the Japanese market plays a very limited role in generating connectedness among the G7 stock markets.

As regards the off-diagonal elements concerning the pairwise connectedness measures, European countries such as France, Germany, and Italy show relatively high connectedness with each other. The highest pairwise connectedness measures are observed between France and Germany (21.42% and 20.08%), which are similar to those presented in Diebold and

Yilmaz (2015, Table 4.5). These results indicate a relatively strong tie between the two neighbouring stock markets. In terms of "to" connectedness, the French stock market turns out to show the highest connectedness (87.89%), followed by the US market (86.53%). The "from" connectedness of France is also the highest (72.35%), followed by UK (49.50%).

The net total directional connectedness measures ("to" - "from" others) vary substantially across countries. The US market has the highest net connectedness (25.82%), followed by the French market (15.54%), indicating that these countries are net transmitters of stock market volatilities. Contrarily, UK and Japan show negative net connectedness measures (-14.42% and -36.54%, respectively). The total connectedness is 62.43%, indicating that 37.57% of the variations are due to idiosyncratic shocks. Overall, the connectedness among the G7 markets seems quite high. Similar observations were discussed in Diebold and Yilmaz (2015), although they examined the connectedness measures of financial markets from different combinations of countries and asset classes.

(2) *Semi-volatility connectedness analysis*

In order to analyse the asymmetric connectedness, we first estimate two separate VAR models for the decomposed RV indices (i.e., positive and negative semi-variances: RS^+ and RS^-). Table 3 presents the estimation results on semi-volatility connectedness measures for the G7 stock markets. The results are based on VAR(5) model for the forecast-error variance decompositions with ten-day horizon.

We can see that the difference between the total connectedness measures of good and bad volatilities is noticeable, which indicates the presence of the asymmetries in volatility connectedness. Such a difference might be called the total CAM (**connectedness asymmetric measure**). Here, the negative value of CAM_m = -10.20% (55.58% - 65.38%) indicates that cross-market linkages tend to strengthen when stock markets are under downside risk, compared to the case when stock markets show upside variation. This interesting result cannot be observed when we use only the realized volatility indices. Given the (symmetric) total connectedness measure (62.43%) in Table 2, we can conjecture that the total connectedness for positive volatility (58.58%) tends to be over-estimated, whereas that for negative volatility (65.38%) is under-estimated, when the potential asymmetries are not properly considered.

Table 3. Semi-volatility connectedness table

	US	Canada	UK	France	Germany	Italy	Japan	From
Panel A : RS^{+}								
US	41.30	12.76	6.25	15.14	12.29	10.16	2.10	58.70
Canada	20.67	45.90	6.15	10.21	7.68	7.33	2.06	54.10
UK	15.70	7.67	32.92	17.77	13.35	10.57	2.04	67.08
France	12.60	4.90	8.43	32.33	20.96	19.79	0.99	67.67
Germany	11.59	4.19	7.13	23.53	35.23	17.10	1.24	64.77
Italy	9.88	4.08	5.26	23.00	16.82	40.41	0.55	59.59
Japan	10.85	5.32	3.18	7.26	6.76	4.77	61.86	38.14
To	81.28	38.92	36.40	96.91	77.86	69.71	8.97	410.05
Net	22.57	-15.18	-30.68	29.24	13.09	10.13	-29.17	**58.58%**
Panel B : RS^{-}								
US	36.10	16.91	11.69	12.76	11.76	9.04	1.73	63.90
Canada	20.14	40.16	10.87	10.47	8.74	8.17	1.44	59.84
UK	14.97	11.26	28.70	17.18	14.00	11.40	2.48	71.30
France	12.99	9.22	14.76	26.15	19.17	16.24	1.47	73.85
Germany	12.79	8.53	13.14	20.63	28.45	14.77	1.70	71.55
Italy	11.49	8.93	11.69	19.26	16.02	31.58	1.03	68.42
Japan	12.02	9.41	9.24	6.87	6.71	4.56	51.20	48.80
To	84.39	64.25	71.40	87.17	76.40	64.19	9.86	457.65
Net	20.50	4.41	0.10	13.32	4.85	-4.23	-38.95	**65.38%**

(3) *Asymmetric volatility connectedness analysis*

The result on the semi-volatility connectedness in Table 3 is useful in examining asymmetries in volatility connectedness. However, the analysis does not consider asymmetries in cross-market connectedness (i.e., between good and bad volatility). In order to address this issue, we estimate the asymmetric volatility connectedness by combining both positive and negative semi-variances in a single VAR system, as discussed by Baruník *et al.* (2017).

Based upon the asymmetric volatility connectedness in Table 4, we can distinguish how good and bad volatilities of individual market propagate across other markets. The total asymmetric connectedness measure is 62.44%, which is quite similar to the total symmetric connectedness measure (62.43%) in Table 2. Notice here that besides the main diagonal elements, the cases for $|i-j| = m$ are also excluded, as they indicate own market connections between good and bad volatilities. All excluded numbers are highlighted in bold, and we sum (*2m* - *2*) numbers for every column.

Table 4. Asymmetric volatility connectedness table

		RS^+							RS^-							
		US	CAN	UK	FRA	GER	ITA	JP	US	CAN	UK	FRA	GER	ITA	JP	From
RS^+	US	19.46	4.71	1.64	4.23	3.11	2.47	0.22	**21.54**	10.80	8.48	8.27	7.78	5.76	1.54	59.00
	Canada	9.22	29.07	2.24	3.08	2.22	1.85	0.32	12.12	**18.94**	5.86	5.47	4.40	4.11	1.10	51.99
	UK	4.44	2.16	19.79	6.37	4.59	3.02	0.38	10.84	7.60	**12.97**	10.52	8.60	7.33	1.39	67.24
	France	3.19	1.05	2.48	15.05	7.89	7.58	0.08	9.61	6.44	9.46	**14.57**	11.55	9.80	1.25	70.38
	Germany	2.77	0.91	2.12	9.28	16.49	6.23	0.11	9.33	6.02	8.41	12.44	**15.45**	8.92	1.52	68.06
	Italy	2.13	0.75	1.16	8.67	5.62	18.95	0.02	8.40	6.11	7.59	12.28	10.09	**17.20**	1.03	63.85
	Japan	2.30	0.96	0.63	1.93	1.70	1.27	36.43	8.86	6.35	6.03	4.80	4.55	2.83	**21.36**	42.21
RS^-	US	**6.37**	1.35	0.42	2.68	1.88	1.50	0.05	31.69	14.31	10.12	10.53	9.97	7.57	1.57	61.94
	Canada	3.05	**3.88**	0.54	2.16	1.17	1.47	0.03	17.85	35.54	9.40	8.81	7.95	6.95	1.22	60.59
	UK	2.11	0.65	**1.89**	5.07	3.32	3.44	0.03	12.64	9.05	24.44	14.01	11.77	9.34	2.25	73.67
	France	1.90	0.52	1.07	**7.44**	4.52	4.78	0.05	10.44	7.19	11.57	21.02	15.44	12.76	1.32	71.54
	Germany	1.49	0.26	0.62	5.46	**6.63**	3.88	0.04	10.38	7.00	10.63	16.60	23.31	12.13	1.55	70.06
	Italy	1.22	0.33	0.48	4.80	2.81	**9.15**	0.02	9.55	7.19	9.32	15.37	13.42	25.33	1.01	65.51
	Japan	1.75	0.64	0.12	1.87	1.31	1.35	**9.17**	10.34	7.51	7.84	5.83	5.88	3.73	42.66	48.17
To		35.57	14.28	13.52	55.59	40.13	38.83	1.36	130.37	95.58	104.68	124.93	111.39	91.23	16.75	874.19
Net		-23.43	-37.71	-53.72	-14.79	-27.93	-25.02	-40.85	68.43	34.99	31.01	53.39	41.33	25.72	-31.42	**62.44%**

The "to" connectedness shows that the effects of bad volatilities are much larger than those of positive volatilities for all the G7 stock markets. Note also that the net connectedness measures of all good volatilities are minus, whereas those of bad volatilities are plus except for Japan.[4] Table 4 indicates a limited role of the Japanese stock market among the G7 countries, which is consistent with the symmetric connectedness analysis in Table 2. The US bad volatility shows the highest net connectedness measure (68.43%). These results can serve as further evidence that bad volatility dominates good volatility in most financial markets.

In order to directly compare the result in Table 4 with the connectedness indices in Table 2, we aggregate the measures in Table 4 at country level, by summing up each country' s good and bad connectedness measures. In this case, we normalize each row entry by two. Table 5 presents the cross-country connectedness measures. Given that the overall estimated measures in Table 5 seem quite close to those in Table 2, the result suggests that the usual symmetric connectedness approach may lead to upward (or downward) bias for positive (or negative) volatility.

Table 5. Cross-country volatility connectedness table

	US	Canada	UK	France	Germany	Italy	Japan	From
US	39.53	15.59	10.33	12.85	11.37	8.65	1.69	60.47
Canada	21.12	43.71	9.02	9.76	7.87	7.19	1.34	56.29
UK	15.02	9.73	29.55	17.98	14.14	11.56	2.02	70.45
France	12.57	7.60	12.29	29.04	19.69	17.46	1.35	70.96
Germany	11.99	7.10	10.89	21.89	30.94	15.58	1.61	69.06
Italy	10.65	7.19	9.27	20.56	15.97	35.32	1.04	64.68
Japan	11.62	7.73	7.31	7.22	6.72	4.59	54.81	45.19
To	82.97	54.93	59.10	90.26	75.76	65.03	9.05	437.10
Net	22.50	-1.36	-11.35	19.30	6.70	0.35	-36.14	**62.44%**

4.2 Dynamic analysis

The full-sample analysis in the previous subsection provides an "average" aspect of

[4] Note here that the Japanese own market connectedness between good and bad volatilities displays dominating bad-to-good directional connectedness (21.36%) over good-to-bad connectedness (9.17%) in Table 4. However, the net connectedness of bad volatility for Japan is negative (-31.42%), because Japan is the largest net recipient of stock market volatility among the G7 countries, as presented in Table 2.

connectedness for the whole sample period. However, the connectedness may vary over time depending on the economic conditions. The advantage of dynamic analysis is to monitor how the degrees of connectedness fluctuate, as evidenced from the GFC and the COVID-19 pandemic which propagated across international financial markets. In this subsection, we provide a dynamic analysis by estimating 250-day rolling sample windows with ten-day forecast horizon.

(1) *Total connectedness*

Figure 2 presents the time-varying pattern of the total volatility connectedness, obtained from 250-day rolling-window samples. The symmetric and asymmetric total connectedness measures are based upon the approaches to Table 2 and Table 4, respectively. We can see that the two connectedness measures move very close to each other with almost the same trends.

As expected, both of the total connectedness indices have rapidly increased around the Lehman Brothers bankruptcy in September 2008. They have since decreased gradually until 2015, although they displayed a few humps during some major economic events: the European debt crisis, evolved from bailouts of Greece in May 2010, the US credit rating downgrade in August 2011, and the Bernanke shock in May 2013. The total connectedness measures display slight rises during 2015 and 2016, with increasing uncertainties concerning increases in the US federal funds rate and the Chinese stock market crash. The two indices also experienced a significant jump around the outbreak of the COVID-19 pandemic in March 2020.

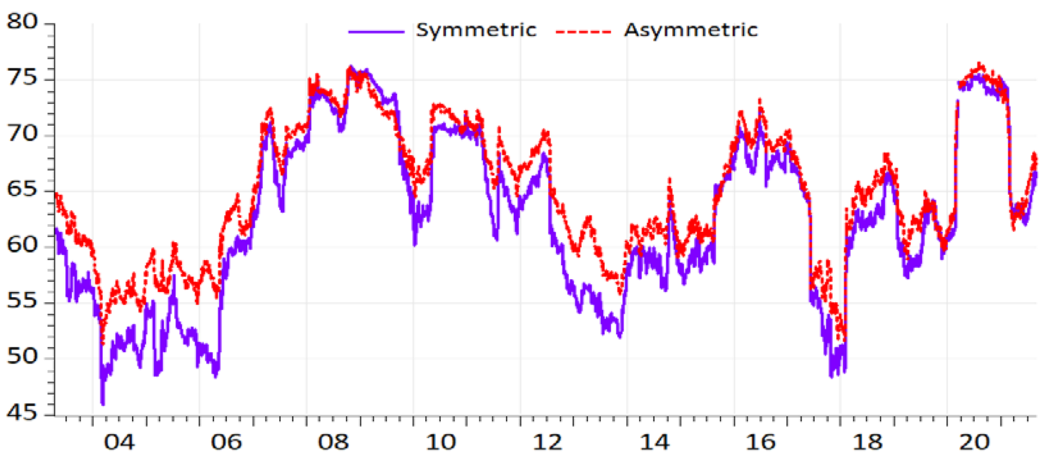

Figure 2. Dynamic symmetric and asymmetric total connectedness

Next, we decompose the asymmetric total connectedness into good-to-good, good-to-bad, bad-to-good, and bad-to-bad connectedness measures. In this analysis, we can decompose the 14×14 matrix in Table 4 into four 7×7 submatrices, so that the sum of four components is equal to the asymmetric total connectedness. Such a decomposition is quite useful in identifying the contribution of each volatility (*i.e.*, good and bad) to the overall asymmetric connectedness.

First, the upper-left 7×7 submatrix in Table 4 can be viewed as the good-to-good connectedness ($C_{G\text{-}to\text{-}G} = \frac{1}{14}\sum_{i,j=1,i\neq j}^{7}\tilde{\theta}_{ij}$). Second, the lower-left 7×7 submatrix is concerned with the good-to-bad connectedness ($C_{G-to-B} = \frac{1}{14}\sum_{i=8}^{14}\sum^{7}_{\substack{j=1,\\ |i-j|\neq 7}} \tilde{\theta}_{ij}$). Similarly, the bad-to-good and bad-to-bad connectedness measures are calculated as: $C_{B-to-G} = \frac{1}{14}\sum_{i=1}^{7}\sum^{14}_{\substack{j=8,\\ |i-j|\neq 7}} \tilde{\theta}_{ij}$, $C_{B-to-B} = \frac{1}{14}\sum_{i,j=8,i\neq j}^{14}\tilde{\theta}_{ij}$, respectively.

Figure 3 displays the decomposed asymmetric total connectedness indices. We can see that the connectedness measure for bad-to-bad volatility is the highest, whereas the good-to-good volatility connectedness is the lowest. This observation suggests that bad volatility contributes much more to the total connectedness than good volatility.

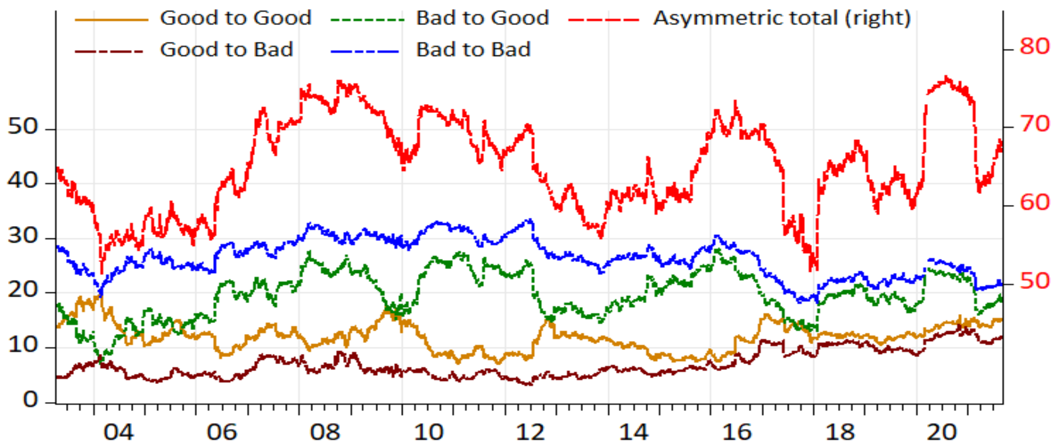

Figure 3. Decomposition of asymmetric total connectedness.

Note: The good-to-good, bad-to-good, good-to-bad, and bad-to-bad measure, scaled on the left axis, are decomposed from the asymmetric total connectedness in Table 4. The asymmetric total connectedness is on the right axis.

(2) *Connectedness asymmetric measure (CAM)*

We now examine asymmetric features of the connectedness, using the CAM (connectedness asymmetric measure) introduced in Section 2.3. We can first define the CAM as the difference in the total connectedness between good and bad volatilities, obtained from separate VARs in Table 3. Figure 4 displays the time series plot of the estimated CAM together with the total good and bad connectedness graphs. It appears that the total bad connectedness measures are relatively higher than those of the total good connectedness.

Accordingly, most of the estimates for the CAM are negative, except during the period from mid-2008 to mid-2009. This observation suggests that bad volatility strictly dominates good volatility in generating connectedness among the G7 stock markets. These results seem quite different from those discussed in BenSaïda (2019), which presented negative values for the CAM only during the GFC and the European debt crisis (from 2007 to spring 2012). In this paper, we find much stronger evidence for asymmetric effects of bad volatility over good volatility.

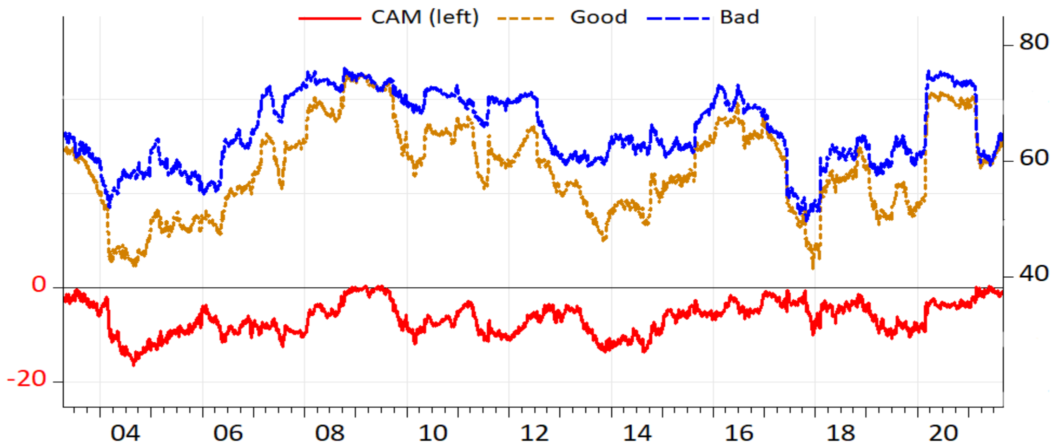

Figure 4. Connectedness asymmetric measure obtained from separate VAR.

Note: The total good and bad connectedness lines, scaled on the right axis, are obtained from the semi-volatility connectedness analysis as in Table 3. The estimated CAM is scaled on the left axis.

Figure 4 is based on two separate VAR models for positive and negative semi-variances. As the above approach does not consider the interaction between good and bad volatilities, the result cannot fully capture the asymmetrical effects. Hence, we next investigate the degree of asymmetries in volatility connectedness, by using a single VAR system for both positive and negative semi-variances.

Figure 5 displays the time-varying pattern of the CAM series, obtained from the combined VAR as in Table 4. We can first notice large negative values of the CAM during most periods, suggesting that bad volatility strictly dominates good volatility. Unlike Figure 4, however, the CAM reaches its lowest values during the GFC and the EDC periods around 2008 and 2012. In this case, as the graphs in Figure 5 are obtained from a single VAR containing additional good-to-bad and bad-to-good elements, they are not directly comparable with those in Figure 4. In order to make a fair comparison, we recalculate the CAM estimates using the difference between the sums of good-to-good (upper-left submatrix) and bad-to-bad (lower-right submatrix) elements in Table 4.

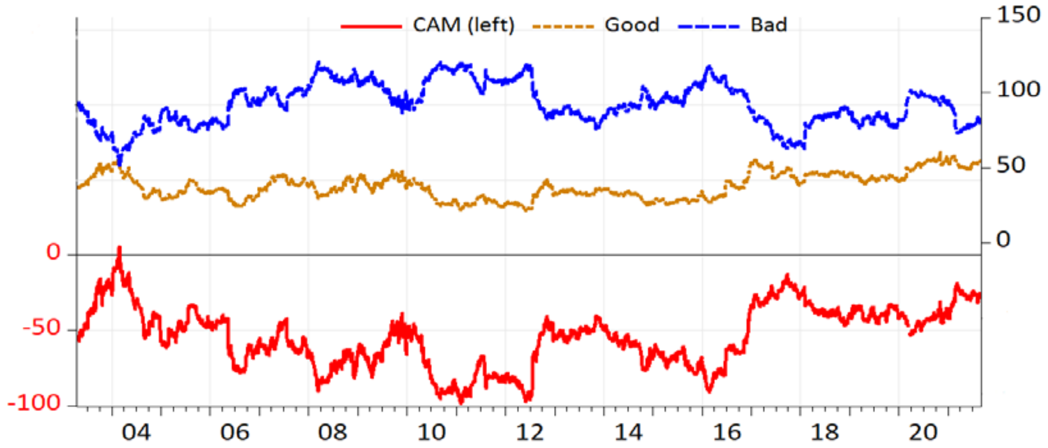

Figure 5. Connectedness asymmetric measure obtained from combined VAR.

Note: The total good and bad connectedness lines are obtained from the asymmetric volatility connectedness analysis as in Table 4. See note to Figure 4.

Figure 6 presents the decomposed CAM indices, obtained from good-to-good and bad-to-bad semi-variances as in Figure 4. The decomposed CAM displays almost the same trend as the CAM indices in Figure 5, although its scale is reduced by about half. Note also that the decomposed CAM in Figure 6 shows quite different time-varying patterns with a larger amplitude than the CAM series in Figure 4. In this case, we expect that the asymmetrical effects are better captured by a single combined VAR than separate VAR models. Wang and We (2018) discussed that the CAM is useful in examining whether the markets are in an optimistic or pessimistic mood. In particular, the episodes of the GFC and the EDC provide good evidence for significant negative values of the CAM with dominating pessimistic mood.

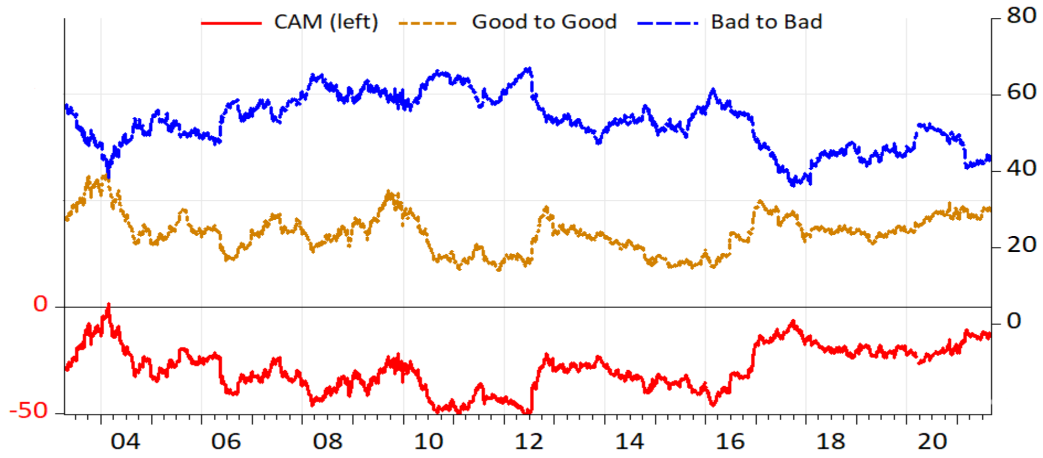

Figure 6. Connectedness asymmetric measures from good-good *vs* bad-bad decompositions

Note: See note to Figure 5.

(3) *Directional connectedness asymmetric measures for individual markets*

We can also investigate the dynamics of the directional connectedness asymmetric measures for individual markets, which are presented in Figure 7. The graphs are derived from a single VAR model as in Table 4.[5] Similarly to Figure 6, each country shows negative values for the directional CAM for most periods, indicating that the connectedness measures for bad volatility strictly dominate those for good volatility during the sample period. In particular, the magnitude of the directional CAM of the US is largest among the G7 stock markets, whereas Japan shows a much smaller directional CAM than other G7 stock markets.

[5] For comparison, the directional connectedness asymmetric measures obtained from separate VAR models are presented in the Appendix.

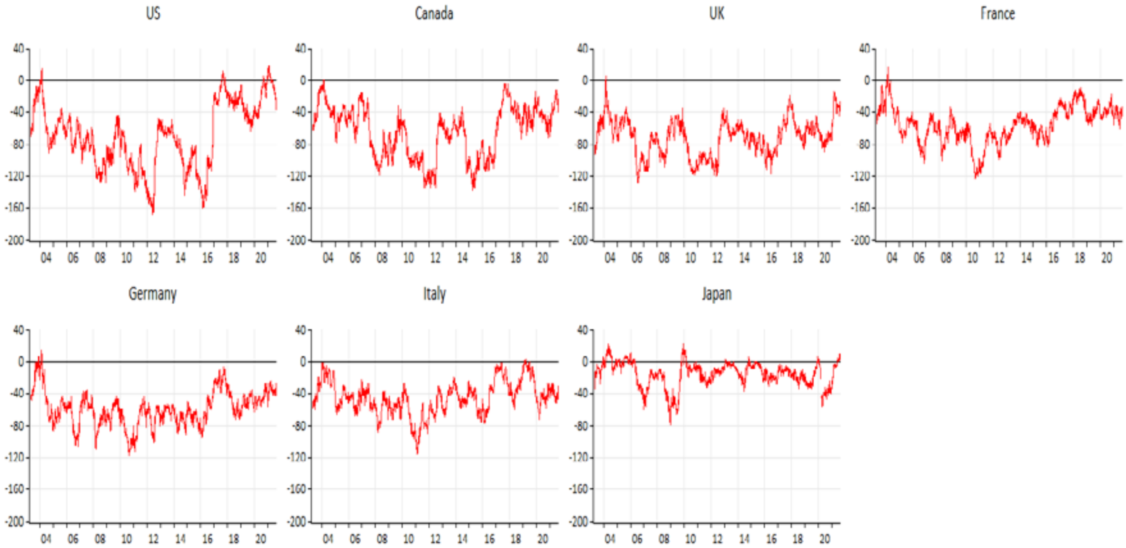

Figure 7. Directional connectedness asymmetric measure for individual markets.

(4) *Net connectedness of US*

Given that the US market plays the most important role in the world financial market, this subsection examines the net connectedness of the US market. In particular, we focus on how good and bad volatilities of the US market propagate to other countries. Figure 8 presents the net directional connectedness measure of the US. In Figure 8, net good and bad connectedness measures are calculated from combined VAR model as: $(\sum_{j=2,j\neq 8}^{14}\tilde{\theta}_{j1} - \sum_{j=2,j\neq 8}^{14}\tilde{\theta}_{1j})$ and $(\sum_{j=2,j\neq 8}^{14}\tilde{\theta}_{j8} - \sum_{j=2,j\neq 8}^{14}\tilde{\theta}_{8j})$, respectively. Here, the sum of net good and bad connectedness is the net total directional connectedness. It is clearly noticeable that the net bad connectedness mainly accounts for most of the net total connectedness, whereas the net good connectedness is negative for most periods.

This result suggests that the spillover effects propagate from bad volatility rather than good volatility. In particular, the US bad volatility is the major source of volatility connectedness among the international financial markets.

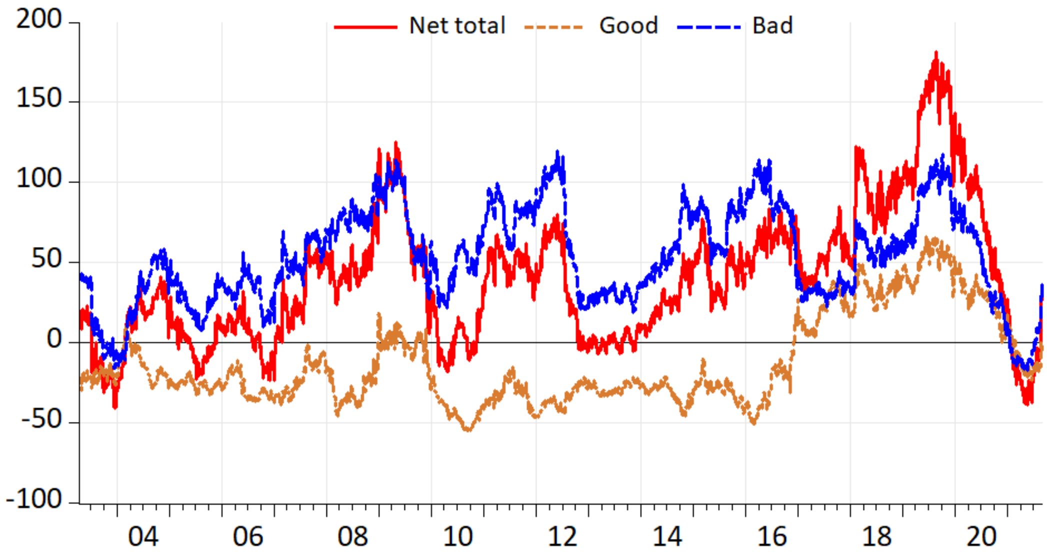

Figure 8. Net directional connectedness measures of the US stock market

5. Conclusion

This paper investigates asymmetric volatility connectedness among the G7 stock markets. In particular, we investigate the magnitude of asymmetries in volatility connectedness and their transmission mechanisms. The basic findings can be summarized as follows. First, we confirm that the effects of bad volatility strictly dominate those of good volatility in generating connectedness across financial markets. Second, the result on the full-sample symmetric volatility connectedness suggests that the US stock market is the dominant net transmitter of volatility shocks to other stock markets. Third, the asymmetric connectedness analysis, based on the semi-variances, also emphasizes the dominant role of US in the world financial markets. We also present evidence that the influence of US shocks on other countries is mainly due to bad volatility rather than good volatility.

The dynamic analysis suggests that both the symmetric and asymmetric connectedness measures fluctuate substantially over time. The observation that the connectedness measures displayed sharp peaks around the GFC, EDC and COVID-19 pandemic periods, for instance, also indicates that the connectedness across financial markets is time-varying and crisis-sensitive. By decomposing the total connectedness measures, we present further evidence for bad volatility contributing more to the total connectedness than good volatility, which is consistent with earlier results in this area that negative shocks lead to larger impacts on other markets than positive ones.

The results from the connectedness asymmetric measure (CAM) also provide evidence that bad volatility plays a dominant role over good volatility in generating volatility connectedness across the G7 financial markets. In particular, the impact of US volatility shocks on other countries is largely triggered by bad volatility rather than good volatility. These findings suggest that bad volatility is the major factor in the transmission mechanism of the global systemic risk. As asymmetries in transmission mechanism of financial markets may pose a challenge for investors, the results in this paper have an important implication on risk management strategies for portfolio diversification.

Data references

Oxford-Man Institute's Quantitative Finance Realized Library, (https://realized.oxford-man.ox.ac.uk)

References

Andersen, T.G., and Bollerslev, T. (1998). Answering the skeptics: Yes, Standard volatility models do provide accurate forecasts, *International Economic Reviews* 38, 885-905.

Ahmad W., Mishra, A.V., and Kevin, J.D. (2018). Financial connectedness of BRICS and global sovereign bond markets, *Emerging Markets Review* 37, 1-6.

Andersen, T.G., Bollerslev, T., Diebold, F.X., and Labys, P. (2003). Modelling and forecasting realized volatility, *Econometrica* 71, 579-625.

Ang, A., and Chen, J. (2002). Asymmetric correlation of equity portfolios, *Journal of Financial Economics* 63(3), 443-494.

Antonakakis. N. (2012). Exchange return co-movements and volatility spillovers before and after the introduction of Euro, *Journal of International Financial Markets, Institutions and Money* 22, 1091-1109.

Bae, K.-H., Karolyi, G.A., and Stulz, R.M. (2003). A new approach to measuring financial contagion, *Review of Financial Studies* 16, 717-763.

Barndorff-Nielsen, O., Kinnebrock, S., and Shephard, N. (2010). Measuring Downside Risk-Realised Semivariance. in Bollerslev, T., Russell, J., and Watson, M. (Eds.). *Volatility and Time Series Econometrics*, Oxford University Press.

Baruník, J., Kočcenda, E., and Vácha, L. (2016). Asymmetric connectedness on the U.S. stock market: Bad and good volatility spillovers, *Journal of Financial Markets* 27, 55-78.

Baruník, J., Kočcenda, E., and Vácha, L. (2017). Asymmetric volatility connectedness on the forex market, *Journal of International Money and Finance* 77, 39-56.

Bekaert, G., and Wu, G. (2000). Asymmetric volatility and risk in equity markets, *Review of Financial Studies* 13, 1-42.

BenSaïda, A. (2019). Good and bad volatility spillovers: An asymmetric connectedness, *Journal of* Financial Markets 43, 78-95.

Black, F. (1976). Studies of stock price volatility changes. *Proceedings of the American Statistical Association, Business and Economic Statistics Section*, 177-181.

Booth, G.G., Martikainen, T., and Tse, Y., (1997), Price and volatility spillovers in Scandinavian stock markets, *Journal of Banking and Finance* 21, 811-823.

Caloia, F.G., Cipollini, A., and Muzzioli, S. (2018). Asymmetric semi-volatility spillover effects in EMU stock markets, *International Review of Financial Analysis* 57, 221-230.

Cappiello, L., Engle, R.F., and Sheppard, K., Asymmetric dynamics in the correlations of global equity and bond returns, *Journal of Financial Econometrics* 4, 537-572.

Chang, B.-K. (2013). Return and volatility spillovers in East Asian currency markets - A generalized variance decomposition analysis, *Journal of Korea Trade* 17, 53-76.

Christie, A. (1982) The stochastic behavior of common stock variances, *Journal of Financial Economics* 10, 407-432.

Claeys, P., and Vašícˇek, B. (2014). Measuring bilateral spillover and testing contagion on sovereign bond markets in Europe, *Journal of Banking and Finance* 46, 151-165.

Diebold, F.X., and Yilmaz. K. (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers, *International Journal of Forecasting* 28, 57-66.

Diebold, F. X., and Yilmaz. K. (2015). Financial and macroeconomic connectedness: A network approach to measurement and monitoring. New York, NY: Oxford University Press.

Diebold, F.X., and Yilmaz. K. (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms, *Journal of Econometrics* 182, 119-134.

Engle, R.F., and Ng, V. (1993). Measuring and testing the impact of news on volatility, *Journal of Finance* 48,1749-1778.

French, K., Schwert, G., and Stambaugh, R. (1987). Expected stock returns and volatility, *Journal of Financial Economics* 19, 3-29.

Garcia, R., and Tsafack, G. (2011). Dependence structure and extreme comovements in international equity and bond markets, *Journal of Banking and Finance* 35, 1954-1970.

Gjika, D., and Horváth, R. (2013). Stock market comovements in Central Europe: Evidence from the asymmetric DCC model, *Economic Modelling* 33, 55-64.

Glosten, L.R., Jagannathan, R., and Runkle, D.E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance* 48,1779-1801.

Hansen, P.R., and Lunde, A. (2011). Forecasting volatility using high frequency data. in Clements, M. P., and Hendry D.F. (Eds.) *The Oxford Handbook of Economic Forecasting*, Oxford University Press.

Hong, Y., Tu, J., and Zhou, G. (2007). Asymmetries in stock returns: Statistical tests and economic evaluation, *Review of Financial Studies* 20, 1547-1581.

Karolyi, G., and Stulz, R.M., (1996). Why do markets move together? An investigation of U.S.-Japan stock return comovements, *Journal of Finance* 51, 951-986.

Koutmos, G., and Booth, G.G. (1995). Asymmetric volatility transmission in international stock markets, *Journal of International Money and Finance* 14, 747-762.

Lee, H.S., and Hong, W.H. (2009). International transmission of swap market movements: The U.S., Korea, and China, *Asia-Pacific Journal of Financial Studies* 38, 723-744.

Lee, H.S., and Lee, W.S. (2018). Housing market volatility connectedness among G7 countries, *Applied Economics Letters* 25, 146-151.

Lee, H.S. and Lee, W.S. (2019a). Cross-regional connectedness in the Korean housing market, *Journal of Housing Economics* 46, 10154. (doi.org/10.1016/j.jhe.2019.101654)

Lee, H.S. and Lee, W.S. (2019b). Network connectedness among Northeast Asian financial markets, *Emerging Markets Finance and Trade*, (doi.org/10.1080/1540496X.2019.1668267)

Liow, K.H. (2015). Volatility spillover dynamics and relationship across G7 financial markets, *North American Journal of Economics and Finance* 33, 328-365.

Longin, F., and Solnik, B. (2001). Extreme correlation of international equity markets, *Journal of Finance* 56, 649-676.

Nelson, D.B. (1991). Conditional heteroscedasticity in asset returns: A new approach, *Econometrica* 59, 347-370.

Segal, G., Shaliastovich, I., and Yaron, A. (2015). Good and bad uncertainty: Macroeconomic and financial market implications, *Journal of Financial Economics* 117, 369-397.

Tsai, I.-C. (2014). Spillover of fear: Evidence from the stock markets of five developed countries, *International Review of Financial Analysis* 33, 281-288.

Yarovaya, L., Brzeszczynski, J., and Lau, C.K.M. (2016). Intra- and inter-regional return and volatility spillovers across emerging and developed markets: Evidence from stock indices and stock index futures, *International Review of Financial Analysis* 43, 96-114.

Wang, X., and Wu, C. (2018). Asymmetric volatility spillovers between crude oil and international financial markets, *Energy Economics* 74, 592-604.

Wu, G. (2001). The determinants of asymmetric volatility, *Review of Financial Studies* 14, 837-589.

Xu, W., Ma, F., Chen, W., and Zhang, B. (2019). Asymmetric volatility spillovers between oil and stock markets: Evidence from China and the United States, *Energy Economics* 80, 310-320.

Appendix

In this appendix, we present the directional CAM graphs obtained from separate VAR models, which is comparable with those in Figure 7. In this case, a little different picture emerges from Figure 7. First, the degree of negativity in Figure A7 is not so significant for most countries as observed in Figure 7. Second, France, Germany, and Italy show more positive estimates for the CAM, whereas significantly negative estimates are observed for all countries (except Japan) in Figure 7. As discussed in the main test, however, we should expect that the asymmetrical effects are better captured by a combined VAR than separate VAR models.

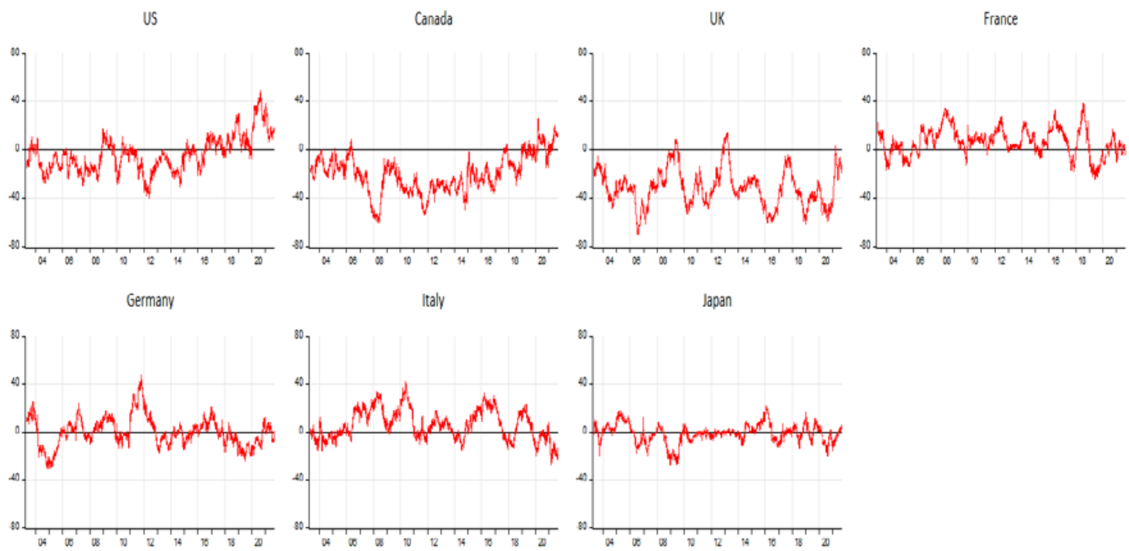

Figure A7. Directional CAM for individual markets obtained from separate VAR models