# A Comment on Signaling by Underpricing in the IPO Market

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**Abstract**

In a classical paper, Allen and Faulhaber (1989) provided a signaling explanation for underpricing in the IPO price. Their main insight is that good-type firms find it optimal to signal their type by underpricing their initial issue of shares, and investors know that only the best can recoup the cost of this signal from subsequent issues. In this note, we argue that their proof of the main result is not complete because they only checked part of the incentive compatibility conditions without showing that their outcome is robust against all possible deviations. We show that a good firm always has an incentive to deviate to raise the IPO price slightly from its equilibrium price if the price is the only signaling device. This implies that there is no separating equilibrium, that is, signaling by underpricing does not occur in equilibrium in the case of one-dimensional signal. If the firm can choose the equity fraction to be sold as well as the price, however, a high-type (good-type) firm can signal its high profitability by choosing a low fraction of equity. In this case, a high-type firm still engages in underpricing in the sense that it sets a lower IPO price than the real value of the firm, but underpricing cannot be a signal because both types choose the same price in equilibrium.

Key Words: IPO, separating equilibrium, signaling, underpricing

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# 1 Introduction

A company often announces initial public offering (IPO) when it decides to raise funds through sale of securities or shares for the first time to the public. The main purpose of IPO is to raise capital for the future growth of the company. The offering price is usually determined not only by many quantitative factors including future profitability and cash flow etc. but also by a strategic motive.

It is well known that IPO prices are often underpriced.[1] Underpricing is the practice of listing IPO at a price below its real value in the stock market. Many explanations for this anomaly have been offered. Among others, Allen and Faulhaber (1989) provided a signaling explanation.[2] Their main insight is that firms with good prospects find it optimal to signal their type by underpricing their initial issue of shares, and investors know that only the best can recoup the cost of this signal from subsequent issues.

In this note, we argue that their proof of the main result is not complete because they only checked part of the incentive compatibility conditions without showing that their outcome is robust against all possible deviations. We show that a good firm always has an incentive to deviate to raise the IPO price slightly from its equilibrium price if the price is the only signaling device. This implies that there is no separating equilibrium, that is, signaling by underpricing does not occur in equilibrium in the case of one-dimensional signal. If the firm can choose the equity fraction to be sold as well as the price, however, a high-type (good-type) firm can signal its high profitability by choosing a low fraction of equity. In this case, a high-type firm still engages in underpricing in the sense that it sets a lower IPO price than the real value of the firm, but underpricing cannot be a signal because both types choose the same price in equilibrium.

# 2 Simple Benchmark Model

We closely follow the central assumptions of Allen and Faulhaber (1989). Basically, our model is a reduced version of their model.

[1] Evidence of underpricing is well documented, especially in Ibbotson (1975).

[2] Allen and Faulhaber (1989) borrow the insight from Ibbotson (1975).

There is a firm and investors in the IPO market. The total number of shares outstanding of the firm is normalized to one. The firm is going to offer a certain fraction $\alpha$ of its equity to the public (homogeneous investors) in an IPO at some IPO price $p$ to acquire the capital needed for its new project which is denoted by $K(>0)$. We assume that $\alpha p \geq K$ because the IPO sales revenue must finance $K$. We will call this financing constraint (FC).

Let $\pi$ be the firm's future profit (its future value).[3] The future profit is either high ($H$) or low ($L$) with $H > L > 0$. We assume that the true value of $\pi$ is known to the firm but not known to the investors. The investors only know the prior probability that $\pi = H$ which will be denoted by $\theta \in (0,1)$.[4] We assume that the investors are risk-neutral in the face of uncertainty about the firm's type.

The interaction between the firm and the investors goes as follows. In the first period, the firm offers the IPO price $p$ to sell the fraction $\alpha$ of its equity, and then the investors decide whether to buy the shares (invest) or not. After that, $\pi$ is realized in the second period. We can interpret $\pi$ as the stock price equivalently, since the total number of shares is one. We assume no dividend.[5]

The payoff of the firm's owner is $\alpha p + (1-\alpha)\pi$ if $\alpha$ of the equity is sold, while it is $L$ if it is not sold. We are assuming that the high-type firm's innovation by the new project succeeds with probability one ($\lambda = 1$) in the notation of Allen and Faulhaber (1989). Note that both types of the firm get $L$ because even a high type cannot implement the profitable project if it cannot ensure the required capital by IPO.[6] The payoff of the investors is $\alpha(\pi - p)$ if they

[3]Since our model is a reduced version of Allen and Faulhaber (1989)'s multi-period model, our $\pi$ corresponds to their $V$. That is, all the relevant information about future earnings flow in $V$ is compressed in $\pi$.

[4]Our model follows a lottery interpretation of an incomplete information game. Alternatively, we can interpret our model as having infinitely many firms of which the proportion $\theta$ is $H$ type. This is called a random-vector interpretation. It is well known that the two interpretations are analytically equivalent. See the classical article of Harsanyi (1967).

[5]In Allen and Faulhaber (1989), dividends play a crucial role in computing the firm's value. In our model, we assume that the firm's value is realized in the second period without assuming dividends.

[6]This assumption follows the spirit of Allen and Faulhaber (1989) who assumes that a bad firm can never be a good firm by innovations. The only difference is that a good firm remains a good firm certainly if it succeeds in financing in our model, whereas it can become a bad firm with some probability even if it succeeds in financing in their model.

invest by buying the shares at the price $p$ and is zero if they do not invest.

# 3 No Separating Equilibrium in the Benchmark Model

In this section, we analyze the benchmark model. Our analysis will focus on the possibility that a separating equilibrium in which signaling by underpricing occurs exists. We will use the (weak) Perfect Bayesian Equilibrium (wPBE) as our main equilibrium concept. Roughly speaking, wPBE is defined by a strategy profile and a belief satisfying that (i) the strategy profile is sequentially rational given the belief in the sense that at each information set, each player chooses the optimal strategy given the other player's strategy and the belief, and (ii) the belief must be weakly consistent with the equilibrium strategy profile in the sense that the belief must be updated according to Bayes' law whenever it is possible.[7]

To figure out the configuration of the possible separating equilibrium, we will resort to the first best outcome under complete information.

## Complete Information Case

The equilibrium can be found by backward induction. Since this is a complete information game, it suffices to find the subgame perfect equilibrium in this dynamic game.

It is clear that given any IPO price $p$, the investors accept the price offer (buy the shares at the price) if $p \leq \pi$.

Now, consider the firm's pricing decision. Let $p^*(\pi)$ be the equilibrium price of type $\pi$. Taking the investors' decisions into account, the firm will choose the IPO price $p$ which is the maximal price that the investors will accept. Therefore, the equilibrium prices under full information must be $p^*(H) = H$ and $p^*(L) = L$. It is also clear that the $H$ type prefers this outcome to no investment outcome which is obtained when it offers $p > H$, because $\alpha H + (1-\alpha)H = H > L$, while the $L$ type is indifferent between investing and not investing because $\alpha L + (1-\alpha)L = L$.

[7] For the formal definition of wPBE, see Mas-Colell et al. (1995).

# Incomplete Information Game

Our main interest in this section is whether a separating equilibrium in which an IPO price signals the value of the firm is possible. We denote the separating equilibrium price of the high type and the low type by $p_H$ and $p_L$ respectively, and the investors' posterior belief updated after observing $p$ by $\hat{\theta}(p)$. Instead of $\hat{\theta}$, we may denote by $\pi^e$ the perceptions of investors about $\pi$. That is, $\pi^e \equiv \hat{\theta}H + (1-\hat{\theta})L$, so $\pi^e = H$ if $\hat{\theta} = 1$ and $\pi^e = L$ if $\hat{\theta} = 0$.

If the true value of the firm is not known to the investors, a low type wants to pretend to be a high type because he could sell his shares at a higher price by doing so. However, if the true value of the firm is revealed in the second period and is fully reflected in $\pi$ regardless of the IPO price $p$, a low type gains nothing in the second period by pretending a high type. If $p_H > p_L$, a low type can successfully imitate the high type by offering $p_H$ which will be always accepted. So, it cannot be an equilibrium. If $p_H < p_L$, a low type loses by pretending to be a high type. In this case, he has no incentive to deviate from $p_L$. So, a necessary condition for a separating equilibrium is that $p_H < p_L$.

Suppose $p_H < p_L$, i.e., a high type underprices in equilibrium. It is easy to see that the low-type firm's separating equilibrium price is not distorted, i.e., $p_L = p^*(L)$, because the low type would deviate to $p^*(L)$ if $p_L \neq p^*(L)$, under the most pessimistic belief, because $p^*(L)$ is the best price for the low-type firm among the prices such that $\hat{\theta}(p) = 0$. The equilibrium price of the high-type firm must satisfy the following incentive compatibility condition of the type:

$$\alpha p_H + (1-\alpha)H \geq \max\{L, \alpha p + (1-\alpha)H\}. \tag{1}$$

Inequality (1) is the incentive compatibility condition of the high type. It requires that a high-type firm has no incentive to deviate to any other price than its equilibrium price $p_H$. The right hand side of (1) is the high type's payoff when it deviates from $p_H$. If it deviates to $p > L$, investors do not buy the shares, so its payoff is just $L$. If it deviates to $p \leq L$, investors invest and thus the firm's payoff is $\alpha p + (1-\alpha)H$.[8]

It is not difficult to see that this incentive compatibility condition cannot be compatible with the optimal decision of the investors. Since $p_H < L$, the high-type firm would deviate

[8] If $p_H < p_L = L$, the incentive compatibility condition of a low type is trivially satisfied: $L \geq \alpha p_H + (1-\alpha)L$.

to $p \in (p_H, L)$ because such a price would always be accepted by the investors. The investors would infer from this price that the firm must be a low type, but finds the price still attractive enough to buy the shares because the price is too low ($p < L$);[9] hence, no IPO underpricing in equilibrium. To summarize, we have

**Proposition 1.** *There exists no separating equilibrium in this model with the one-dimensional signal (IPO price).*

*Proof.* The proof is immediate from the argument that any deviation to $p \in (p_H, L)$ is profitable for a high-type firm. □

Some may suspect that this result is an artifact of the assumption that only pure strategies are available. What if we allow mixed strategies of the investors? Unfortunately, it turns out that separation (semi-separating equilibrium) is not possible even with mixed strategies.

Let $r(p) \in [0, 1]$ be the probability that investors accept the IPO price $p$. It is true that the incentive of a high-type firm to deviate to $p \in (p_H, L)$ could be deterred if we allow mixed strategies of the investors so as to make the probability that the investors accept $p$ strictly less than one. However, the mixed strategy $r(p) < 1$ for any $p \in (p_H, L)$ cannot be optimal for the investors because they always strictly prefer investing at the price $p$ to not investing, because $\alpha(\pi^e - p) > 0$ for any $p \in (p_H, L)$ and for any belief $\pi^e$. As long as the investors always buy the shares at $p \in (p_H, L)$ with probability one, the high type always deviates to $p$ from the equilibrium price $p_H (< L)$.

# 4 Model of Two-Dimensional Signals

So far, we assumed that the fraction of shares that is sold at the market for IPO is exogenously fixed. In this section, we consider an extended model in which the firm can choose the fraction

[9] In Allen and Faulhaber (1989), the strategy of investors and their posterior belief off the equilibrium path are not clearly defined. We presume that their definition of investors' strategy is the same as ours from their phrase "investors will not pay more for the firm than its value to them." Then, the investors' decisions depend on the assumption on the off-the-equilibrium belief because it determines the expectation of the future value of the firm. What we have shown is that investors are willing to pay the price $p < L$ under the most pessimistic belief or even under any belief. Taking this optimal decision of the investors into account, a high-type firm will deviate to such a price $p \in (p_H, L)$; hence, no separating equilibrium.

to be sold as well as the IPO price.

This model is motivated by the following observation. The main reason why no separating equilibrium exists in the previous model is that a high-type firm can always profitably deviate by increasing the IPO price slightly ($p > p_H$) which will be still accepted by the investors. However, if the firm loses something by increasing the price, it may not profitably deviate to $p \in (p_H, L)$. For example, if the firm must sell more shares at a higher price and keep less shares, a high type may not deviate from a low IPO price $p_H$, and accordingly, the underpriced IPO price $p_H(< L)$ may be a separating equilibrium price. Below, we will investigate this possibility.

Let $(\alpha_H, p_H)$ and $(\alpha_L, p_L)$ be the equilibrium pair of choices of the high-type firm and the low-type firm respectively where $p_H < p_L = p^*(L)$. Note that the decision of the investors remain unaffected for any $\alpha_H$ and $\alpha_L$, i.e., the investors buy the shares $\alpha_H$ at price $p_H$ if $p_H \leq H$ and if $p \neq p_H$, they buy the fraction $\alpha$ if shares if $p < L$ for any $\alpha$. This implies that $p_H(< L)$ can never be an equilibrium price of a high-type firm even in this model, because a high type can always profitably deviate to $(\alpha_H, p)$ with $p \in (p_H, L)$, insofar as the investors' decision is unaffected for any $\alpha_H$. This implies that it must be that $p_H = p_L = L$ in a separating equilibrium. That is, a high-type firm can signal its type only by the fraction of shares to be sold in the IPO market, i.e., $\alpha_H \neq \alpha_L$. Accordingly, $\hat{\theta}(\alpha_H, L) = 1$ and $\hat{\theta}(\alpha_L, L) = 0$ in equilibrium. Again, we impose the most pessimistic off-the-equilibrium belief, i.e., $\hat{\theta}(\alpha, p) = 0$ for any $(\alpha, p) \neq (\alpha_H, L)$.

The equilibrium fractions of equity $\alpha_H$ and $\alpha_L$ must satisfy two incentive compatibility conditions. Let $V(\alpha, p; \pi)$ be the payoff of $\pi$-type firm when it chooses $\alpha$ and $p$. Then, the incentive compatibility conditions require (i) $V(\alpha_H, p_H; H) \geq V(\alpha, p, ; H)$ for any $(\alpha, p)$, and (ii) $V(\alpha_L, p_L; L) \geq V(\alpha_H, p_H; L)$. To elaborate, we have

$$(i)\ \alpha_H p_H + (1 - \alpha_H)H \geq \max\{L, \alpha p + (1 - \alpha)H\}, \forall (\alpha, p) \neq (\alpha_H, L), \tag{2}$$

$$(ii)\ L \geq \alpha_H p_H + (1 - \alpha_H)L, \tag{3}$$

where $p_H = L$.

Inequality (2) is the incentive compatibility condition for the high-type firm and inequality (3) is the incentive compatibility condition for the low-type firm. The right hand side of (3) is the low type's payoff when it imitates the high type by choosing $\alpha_H$. It is easy to see

that (3) is trivially satisfied for any $\alpha_H \neq \alpha_L$ if $p_H = L$. In inequality (2), $\alpha p + (1-\alpha)H$ is the high-type firm's payoff when $p < L$ so that its deviant offer $(\alpha, p)$ is accepted by the investors, and $L$ is his payoff when $p > L$ so that $(\alpha, p)$ is rejected. Since it is clear that $\alpha_H L + (1-\alpha_H)H > L$, it suffices to consider the case that $p < L$.

We know that (FC) imposes s lower bound for $\alpha_H$ because $\alpha_H p_H = \alpha_H L \geq K$ implies that $\alpha_H \geq \frac{K}{L} \equiv \overline{\alpha}_H$. Since $V(\alpha_H, L; H)$ is decreasing in $\alpha_H$, setting $\alpha_H$ as low as possible, i.e., $\alpha_H = \bar{\alpha}_H$ is the optimal equity fraction of the high type. Now, any deviation $(\alpha, p)$ also has to satisfy (FC) condition requiring that $\alpha p \geq K$. Since $p < L$, it implies that $\alpha > \alpha_H$. Then, it is easy to see that inequality (2) is satisfied for any $(\alpha, p)$ such that $\alpha > \alpha_H$ and $p < L$, i.e.,

$$\alpha_H L + (1-\alpha_H)H \geq \alpha p + (1-\alpha)H.$$

This implies that a high type has no incentive to deviate from $p_H$, either.

**Proposition 2.** *If the IPO firm can chooses the IPO price and the fraction of equity to be sold in the IPO market, there exists a separating equilibrium in which the high-type firm chooses* $(\bar{\alpha}_H, L)$ *and the low-type firm chooses* $(\alpha_L, L)$ *with* $\alpha_L > \bar{\alpha}_H$.

*Proof.* The proof is immediate from the above argument. □

This proposition implies that a high-type firm underprices in equilibrium. Underpricing in this paper does not mean that the high-type firm's price is lower than the low-type firm's price. It means that the high-type firm's price is lower than its first-best price, i.e., $p_H < p^*(H)$. However, this underpricing in equilibrium is possible only by the accompanying choice in the equity fraction to be sold in the IPO market. If a firm sells a smaller fraction of equity in the IPO market, it signals a high profitability of the firm. Note that a high-type firm cannot signal by underpricing, because both types choose the same low IPO price so that the public cannot tell by the IPO price. Although a high-type firm engages in underpricing in equilibrium, it signals by choosing a low equity fraction to be sold, not by underpricing. Then, how can a high-type firm signal its type with the same price as a low-type firm? Since the cost of selling the shares is higher for a high-type firm who knows that its future value will be higher, it will sell a lower fraction of equity which cannot imitated by a low-type firm who prefers increasing the monetary revenue by selling a higher fraction of equity. This is

consistent with the insight of Myers and Majluf (1984) that equity financing may be a bad signal of the firm's profitability.

At this point, it is important to compare this with the result of Allen and Faulhaber (1989). Their incentive compatibility conditions of a good firm and a bad firm are shown in their (9a) and (9b) as follows;

$$R_G(p_0, \lambda) \geq R_G(V_0(0), 0), \tag{4}$$

$$R_B(p_0, \lambda) \geq R_B(V_0(0), 0). \tag{5}$$

In these inequalities, $\lambda$ is the probability that a good firm remains to be good after innovation. As we put in Footnote 3, we assume that $\lambda = 1$. Also, $p_0$ is the IPO price and $V_0(0)$ is the value of the firm when $\hat{\theta} = 0$. Inequality (4) compares the firm's payoff when it chooses the good type's equilibrium price ($p_0$) and the bad type's equilibrium price ($V_0(0)$). However, they did not check whether the left hand side of (4) is not less than the good firm's payoff when it deviates to another price $p' \neq V_0(0)$, as we checked in (1) and (2). In fact, we showed in this paper that a good type (a high type) always has an incentive to slightly increase the IPO price to $p' = p_0 + \epsilon$ for $\epsilon > 0$. Such a deviation $p' \in (p_0, V_0(0))$ is always accepted and so it is profitable for a good firm, insofar as $p_0 < V_0(0)$. Some may think that the profitability of the deviation depends on the off-the-equilibrium belief of the deviation when the investors observe $p'$. However, as we argued in Proposition 1 of this paper, it is profitable even under the most pessimistic belief, implying that it is profitable regardless of the belief.

To close this section, it is worthwhile to note Allen and Faulhaber (1989)'s remark that a necessary condition for separation to occur is $0 < \lambda < 1$. They argue that if "a good firm could somehow signal its type" and $\lambda = 1$ as we assumed in this paper, the investors do not need further observations of dividend outcomes whatever to tell whether it is a good firm. It may be correct that they do not need information of dividend outcomes, but even if $\lambda = 1$, the benefit of the signal differs across types insofar as the additional capital increases the future profitability only for the high type, i.e., the probability of successful innovations for a high type and a low type ($\lambda = 1$ vs. $\lambda = 0$) differs. Contrary to the argument of Allen and Faulhaber (1989), a separating equilibrium is possible in our model, although we assume that $\lambda = 1$.

# 5 Conclusion

In a simple model, we showed that a good firm signals its type not by underpricing of IPO price but by its choice of the amount of equity to be sold in the market, although it engages in underpricing. We believe that this result may add some useful insight to understand the IPO market.

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