Time series relation between risk and return in the stock market: Value versus growth stocks

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<Abstract>

This paper studies the risk and return time series relation of value minus growth stocks. Using the Euler equation, we show that an increase in risk lowers return first before it raises (expected) return. An important implication is that the temporal effect of time-varying risk on the value minus growth return is reverting with a time lag and negligible when aggregated in time. However, there exists a (steady-state) long-run risk effect on the value minus growth return. We show that the alpha effect in the value premium literature is the long-run (market) risk effect in our model, indicating that the value premium is not an anomaly in the stock market. Using the Korean stock market data, we find that the long-run risk effect depends on the frequency of rebalancing value and growth stocks. Our analysis supports the empirical findings in the literature that value stocks earned a higher average return than growth stocks over a long horizon, but growth stocks outperformed value stocks for some periods. (e.g., Capaul et al., 1993; Asness et al. 2000).

Keywords: Value minus growth return; time-varying risk effect; long-run risk effect; reverting risk and return relation; frequency of rebalancing

JEL Classification: G10; E44

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1. Introduction

A fundamental theorem in finance tells us that an increase in risk causes a decrease in asset prices. Relying on the Euler equation, we show that time-varying risk causes the fluctuation in asset returns. We then study the risk and return time series relation concerning the value versus growth stocks. In doing so, we form an investment portfolio in which we sell growth stocks short and buy value stocks and examine the risk and return time series relation of the long-short portfolio. The value minus growth portfolio strategy is sometimes called the value strategy in the literature.

Value stocks are relatively cheap, but growth stocks are relatively expensive when we compare their prices to each other concerning some firm values such as book values or earnings. These divergences of the firm's prices may be due to the differences in growth prospects or riskiness between value stocks and growth stocks. Traditionally in the literature, value stocks are considered to be riskier than growth stocks. Fama and French (1992 and 1996) attributed the value stock risk factor to the value firm's financial distress, partly responsible for the excess stock return in their three-factor model. Petkova and Zhang (2005) found that value stocks tended to covary positively, and growth stocks tended to covary negatively with a market risk premium. Zhang (2005) and Bai et al. (2019) ascribed the value stock risk to the asymmetric adjustment cost of assets in place of the value firms, especially in bad times. Ai and Kiku (2013) showed that growth options were less risky than value assets because growth options acted as a hedge against risks in assets in place.

Empirical studies in the literature reported that value firms earned a higher average return than growth firms. However, growth firms outperformed value firms for some periods, which led the investors to doubt the value strategy (Capaul et al., 1993, and Asness et al., 2000). There exist two competing explanations for the value premium in the literature. One is the alpha effect of the CAPM which is known as the value premium puzzle in the literature (e.g., Fama and French, 1992). The other is the beta effect of the conditional CAPM in the literature (e.g., Ang and Chen, 2007; Bai et al., 2019). The purpose of our study is twofold: First, the alpha effect in the literature is a (steady-state) long-run risk effect in our model and the long-run risk is a market risk, indicating that the value premium is not an anomaly. Second, growth stocks could outperform value stocks for some periods because the time-varying risk effect is reverted with a time lag.

One of the basic notions in finance is a trade-off between risk and return: A riskier asset (e.g., value stock) earns a higher average return than a less risky asset (e.g., growth stock). The risk and return trade-off between value stocks and growth stocks are valid across firms at a cross-sectional analysis or when risk is invariant with respect to time. What if the risk is time-varying and the risk and return relation reverts over time from negativity contemporaneously to positivity with a time lag? In other words, what if an increase in risk lowers return first before it raises (expected) return?

This paper's fundamental premise is that time-varying risk causes much of the stock market's return fluctuations and is responsible for the reverting risk and return time series relation. Because of this reverting relation, growth firms may outperform value firms for some periods and vice versa for other periods even though value firms earn a higher average return in the long-run than growth firms.

Specifically, we hypothesize that risk is negatively related to return contemporaneously and positively related to (expected) return with a time lag both at the portfolio level and at the stock market's aggregate level. The time-varying risk effect on return is negligible when aggregated in time. However, there exists a long-run risk effect on return. This long-run risk effect is similar to the long-run consumption risk effect across book-to-market sorted portfolios introduced by Ai and Kuku (2013). We model this reverting relation and the long-run relation under a unifying framework of finance's fundamental pricing model to back up this hypothesis . Using the Korean stock market data, we test the hypothesis.

Our empirical analysis shows that the frequency of rebalancing value and growth stocks does matter for the value strategy's investment performance. We find that investors earned a significantly higher accumulated return throughout 2000-2020 in the Korean stock market when we rebalance value and growth stocks bi-annually and yearly than monthly and quarterly. Even though the time-varying risk effect on the value minus growth return is negligible when aggregated in time, the long-run risk effect is significant and quite close to the value minus growth return's unconditional mean rebalanced bi-annually and yearly. Investors would be better off if they let stocks walk through their way to earn a return for several months before rebalancing the portfolio. We conjecture because it takes time to compensate the investors for holding risky assets in the stock market.

We measure the aggregate risk in three different metrics: market variance, the individual variance of typical stock, and the ratio of these two variances. The ratio is a fraction of non-diversifiable risk relative to the total individual risk of the typical stock. We call this ratio a fractional non-diversifiable risk, which Park and Fang (2021) thoroughly discussed. It is also similarly known as correlation risk in the literature (e.g., Pollet and Willson, 2010; Adrian et al., 2018). Risk at the portfolio level is its portfolio beta multiplied by the aggregate risk in the stock market. The division of market variance between fractional non-diversifiable risk and individual risk is to see which one is the primary driver in explaining the variation in return in our regression.

Park and Fang (2021) derived risk and return time series relation at the aggregate level. The relation's sign reverted with a passage of time from negativity contemporaneously to positivity with a time lag. This reverting relation was, though, derived piecewise based on two separate theorems in finance. Park and Fang (2021) derived the negative relation using the simple dividend model for pricing stock. On the other hand, the positive relation is based on the notion of a trade-off between risk and return on an investment portfolio. The nature of the return in these two relations is different: The return in the contemporaneous relation is ex-post while the return in the relation with a time lag is ex-ante. In this paper, we derive the reverting relation under a unifying framework of the Euler equation. By doing so, we hope that our derivation becomes more amenable to scientific knowledge.

Our study is related to one strand of the literature. That is a conditional version of the CAPM, which several researchers applied explicitly to the value strategy (e.g., Petkova and Zhang, 2005; Bodie et al., 2010; Gubellini, 2014). The literature focused on the variation in portfolio beta or market risk premium for the causes of time-varying risk. The variation in beta or market risk premium was, in turn, related to some other economic variables such as the dividend yield, the term premium, the default premium, and the nominal 1-month Treasury-bill yield. Our study is different from the above literature in a meaningful way. The literature dealt with only the notion of a trade-off between risk and return, which corresponded to the positive risk and return relation with a time lag in our analysis. However, when one analyzes the time series relation of risk and return as opposed to the cross-sectional relation at a given point in time, it is necessary to deal with the lagged positive relation together with the contemporaneous negative relation because we are looking at the fluctuation in return on a continuum of time.

Our paper contributes to the literature in three respects: First, it shows that the alpha effect in the literature is a (steady-state) long-run risk effect in our model, indicating that the value premium is not an anomaly in the stock market. Second, the time-varying risk effect is reverted with a time lag. Hence growth stocks could outperform value stocks for some periods even though value stocks earned a higher average return than growth stocks. Third, we report that the frequency of rebalancing value and growth stocks matters for the accumulated return on value strategy.

We organize our paper as follows: In Section 2, we rework the Euler equation to derive risk and return time series relation. We differentiate the effect on return between the time-varying risk effect and the long-run risk effect in our model. Section 3 describes the data and presents summary statistics. We show in Figure 1 that the rebalancing frequency matters for the accumulated return on value strategy. Section 4 is on estimation in which our regression equations are specified in the first part (Sub-section 4.1), we report our empirical results in the second part (Sub-section 4.2). Section 5 provides robustness checks on our results, assuming alternative estimates of variance terms in the regression equations. In Section 6, we control the expected payoff effect on return to check whether the ceteris paribus effect of risk on return remains unaltered. Section 7 highlights our contributions to the literature and makes some suggestions for further research. Finally, we conclude our remarks in Section 8.

2. The Model

In finance, the fundamental asset pricing model is based on the Euler equation. The Euler equation states that an asset's price is, in equilibrium, determined as an expected value of the product of the stochastic discount factor (SDF) and its future payoff. The SDF is often referred to as a pricing kernel in the asset pricing literature. To derive the risk and return time series relation, we transform the Euler equation's price term into the return term concerning its relation to risk.

We write the Euler equation introduced in the literature as (footnote 1)

$$P_t = E_t(m \times X_{t+1}), \tag{1}$$

where

E_t= an expectation operator taken at time t,

P_t= the price of an asset at time t,

m= stochastic discount factor (SDF),

X_{t+1}= the payoff of the asset at time $t+1$.

When the representative investor solves his (her) expected utility maximization problem, the SDF is determined as the marginal rate of substitution of consumption at time $t+1$, $c(t+1)$ for consumption at time t, $c(t)$. It is then discounted at the investor's time preference factor. Assuming that the investor's utility is a function of consumption, i.e., $u(c)$, the SDF can be expressed as $\delta[\frac{u'(c_{t+1})}{u'(c_t)}]$, where δ is the investor's time preference parameter and u' is the first-order derivative of u with respective to c, i.e., du/dc. The parameters associated with the SDF are those of the investor's risk aversion and time preference.

When the underlying asset is riskless, we can obtain from (1) that

R_f= 1/m, where R_f is a risk-free gross rate. Rearranging (1), we have that

$$\begin{aligned} P_t &= E_t(m)E_t(X_{t+1}) + Cov_t(m, X_{t+1}) \\ &= \left(\frac{1}{R_f}\right) E_t(X_{t+1}) + \delta Cov_t(\frac{u'_{t+1}}{u'_t}, X_{t+1}), \end{aligned} \tag{2}$$

where Cov_t is a conditional covariance where an expectation is taken at time t

The investor's risk-averse utility function is concave in c. Hence, the sign of $Cov_t(\frac{u'_{t+1}}{u'_t}, X_{t+1})$ is strictly negative under the concavity of $u(c)$, i.e., $u''(c) < 0$. Therefore, equation (2) implies that a risky asset at time t is worth less than the riskless asset for the equal (expected) payoff at time $t+1$.

Moving backward (1) by one period,

$$P_{t-1} = E_{t-1}(mX_t) = (1/R_f)X_t \ . \tag{3}$$

We obtain (3) because all variables in (3) are ex-post (non-random) at time t. In that event, the Euler equation collapses to a pricing kernel with certainty.

Dividing (2) by (3), we have that

$$R_t = E_t\left(\frac{X_{t+1}}{X_t}\right) + R_f Cov_t\left(m,\ \frac{X_{t+1}}{X_t}\right), \tag{4}$$

where $R(t)$ is a gross return at time t, P_t/P_{t-1}. We assume that all payoffs are paid in dividend at $t = 1,2,$ and hence $X_{t+1}\ /\ X_t$ is equal to P_{t+1}/P_t. In that event, we can rewrite (4) as

$$R_t = E_t(R_{t+1}) + R_f Cov_t(m,\ R_{t+1}). \tag{5}$$

The next step is to relate R_t and $E_t(R_{t+1})$ to risk. We can write the second covariance term in (5) as

$$R_f Cov_t(m,\ R_{t+1}) = \delta R_f\left(\frac{1}{u'_t}\right) Cov_t(u'_{t+1},\ R_{t+1}). \tag{6}$$

Now we want to show that $Cov_t(u'_{t+1},\ R_{t+1})$ is a negative function of the conditional (time-varying) variance of R_{t+1}. For simplicity, let's assume a quadratic utility function for the investor, i.e., $u = R - 2bR^2$, where $b > 0$. (footnote 2) Here we express the investor's utility as a function of the gross return, R since a higher R means a greater c. In that event, we can write that

$$\begin{aligned} Cov_t(u'_{t+1}, R_{t+1}) &= E_t(u'_{t+1}R_{t+1}) \\ &= E_t R_{t+1} - 2bE_t({R_{t+1}}^2). \end{aligned} \tag{7}$$

Given that the investor is risk-averse, i.e., $b > 0,$ equations (5) and (7) mean that R_t is negatively related to the conditional variance of R_{t+1}. Putting together (5), (6), and (7), we will have the following relation that R_t is negatively related to the conditional variance of R_{t+1}:

$$R_t = E_t(R_{t+1}) - \gamma Var_t(R_{t+1}), \tag{8}$$

where $\gamma = 2b(\delta)(R_f)(\frac{1}{u'_t}) > 0$. We rearrange (8) as

$$E_t(R_{t+1}) - R_t = \gamma Var_t(R_{t+1}). \tag{9}$$

Equation (9) dictates that an increase in the time-varying risk causes a more considerable discrepancy between $E_t(R_{t+1})$ and R_t. Knowing that a more significant risk lowers P_t as we can see from (2), equation (9) implies that at a predetermined P_{t-1} and a given expected payoff, $E_t(X_{t+1})$, a decline in P_t means a lower R_t, and a higher $E_t(R_{t+1})$. In sum, the Euler equation states that other things (e.g., future payoff) being equal, an increase in risk lower return contemporaneously and raises future expected return. Time-varying risk is, by nature, transitory and moves around a long-run risk level. Equation (9) is our central premise that time-varying risk is responsible for the stock returns' fluctuation around its long-run level, other things being equal.

We wish to illustrate this reverting risk and return time series relation by discussing the stock price movements during the recent financial crisis periods. When the investor observes that a systemic shock such as the Covid-19 pandemic hits the market, he or she perceives that the future payoff, X_{t+1} will be considered more uncertain. This increase in risk today has an instantaneous negative impact on the current stock price P_t and lowers the stock return R_t contemporaneously. At the same time, a lowered price, P_t means a higher expected return unless investors change their future payoff prospect in the long-run. Investors may expect that the economy would return to a normal state, albeit with more significant uncertainty, when the Covid-19 pandemic becomes under control in the end. The higher expected return is, in equilibrium, required to compensate the investors for holding the stocks in a financial or health crisis. Judging from the past financial crisis experiences, we found that the price fell first and then recovered. Time-varying risk indeed causes the return fluctuation in the stock market.

We define a long-run steady state in which risk and expected return are invariant with time and constant. There is a particular relationship between them at a long-run steady-state (see Appendix A for proof):

$$E(R) = R_f + \gamma Var\ (R), \tag{10}$$

where $E(R)$ is an unconditional mean return, and $Var(R)$ is an unconditional variance of return that would prevail at a long-run steady state. Equation (10) states that at a long-run steady-state, a risky asset earns a higher average return than a safe asset over a long horizon, and the excess return depends on the asset's riskiness. Equation (10) is similar to a static version of the CAPM derived from the Euler equation in Chaigneau (2011).

We want to derive risk and return time series relation at a portfolio level: Value stocks versus growth stocks. Subscripts v and g refer to value stock portfolio and growth stock portfolio, respectively. The Euler equation (1) holds for value and growth portfolios, respectively. Therefore, we can derive the relations, (9) for v and g, respectively such that

$$E_t\left[R_{v,g}(t+1)\right] - R_{v,g}(t) = \gamma Var_t\left[R_{v,g}(t+1)\right], \quad \text{for v and g, respectively.} \qquad (11)$$

Define the value and growth portfolio betas, β_v and β_g as the ratios of R_v and R_s to R, respectively. In such cases, we can have that

$$E_t[R_v(t+1)] - R_v(t) = \gamma \times {\beta_v}^2 Var_t(R_{t+1}), \qquad (12)$$

and

$$E_t[R_g(t+1)] - R_g(t) = \gamma \times {\beta_g}^2 Var_t(R_{t+1}). \qquad (13)$$

Now consider a value minus growth portfolio, $v - g$. Subtracting (13) from (12), it immediately follows that

$$E_t[R_{v-g}(t+1)] - R_{v-g}(\mathrm{t}) = \gamma \times ({\beta_v}^2 - {\beta_g}^2) Var_t(R_{t+1}) \qquad (14)$$

Equations (12) and (13) state that the value and growth stock portfolio returns are negatively related to the aggregate stock market risk, and at the same time, their expected returns are positively related to the aggregate stock market risk. The sensitivity of return to risk depends on their portfolio beta's magnitude. Equation (14) indicates that the same reverting relation holds for the value minus growth portfolio.

At a long-run steady-state, we will have the same risk and return relation as in (10) concerning the value, the growth, and the value minus growth portfolios, respectively:

$$E(R_v) = R_f + (\gamma \times \beta_v{}^2) Var(R), \tag{15}$$

$$E(R_g) = R_f + (\gamma \times \beta_g{}^2) Var(R), \tag{16}$$

and

$$E(R_{v-g}) = R_f + \gamma \times (\beta_v{}^2 - \beta_g{}^2) Var(R). \tag{17}$$

Ai and Kuku (2013) introduced the differential effect in the long-run consumption risk across book-to-market sorted portfolios. Their analysis is comparable to our long-run risk effect on the return of value and growth stocks. They studied a cross-section of equity returns while ours is on time-series relation between risk and return.

3. Data and Descriptive Statistics

This section describes the data used in our empirical analysis and presents some figures and statistics to understand the risk and return time series relation for value and growth stocks. Our data consists of stock market data and financial data. Stock market data are the KOSPI 200 firms' daily stock prices and the KOSPI 200 index. Financial data are the KOSPI 200 firms' year-end book values and quarter-end operating and net profits. The data run from January of 2000 to December of 2020. We collect our data from DataGuide web (http://www.dataguide.co.kr), which is serviced by a private data vending company, FnGuide Inc. in Korea.

How frequently value and growth stocks portfolios are rebalanced does matter for the return performance. We do portfolio rebalancing as follows. First, all firms' book-to-market ratios are updated using the market prices on the first business day of each month. Second, based on the rankings of the month's book-to-market ratios of the KOSPI 200 firms, the top 20 stocks (top deciles) are assigned value stocks, and the bottom 20 stocks (bottom deciles) are assigned growth stocks every month. Last, for the monthly, quarterly, bi-annually, and yearly rebalanced portfolios, we newly form value stock portfolios and growth stock portfolios at the beginning of the months, quarters, half years, and years, respectively January of 2000 to December of 2020.

The rebalancing frequencies chosen in the previous studies were quite diverse from the quarterly frequency (e.g., Asness et al., 2000) and the bi-annual frequency (Capual et al.,

1993) to the yearly frequency (e.g., Fama and French, 1992).

Figure 1 shows the accumulated returns on the value minus growth stocks for the monthly, quarterly, bi-annually, and yearly rebalanced portfolios from January of 2000 until December of 2020. If value firms earn a higher average return than growth firms, we expect that the accumulated return on the value minus growth portfolio steadily grows over time. As seen in Figure 1, the accumulated return steadily increases for the bi-annual and the yearly rebalanced portfolios. This steadily growing accumulated return on value minus growth portfolio is quite similar to those documented in other countries such as the U.S., the U.K., and Japan by Capual et al. (1993). They studied bi-annually rebalanced value and growth indexes. However, the accumulated return declines modestly around 2008 for the quarterly rebalanced portfolio and sharply around 2007 for the monthly rebalanced portfolio. These observations for the monthly and the quarterly rebalanced portfolios indicated that growth firms outperformed value firms for prolonged periods. They are also consistent with Asness et al.'s (2000) findings that the value stocks performed poorly relative to the growth stocks for some periods. Asness et al. (2000) studied quarterly rebalanced portfolios.

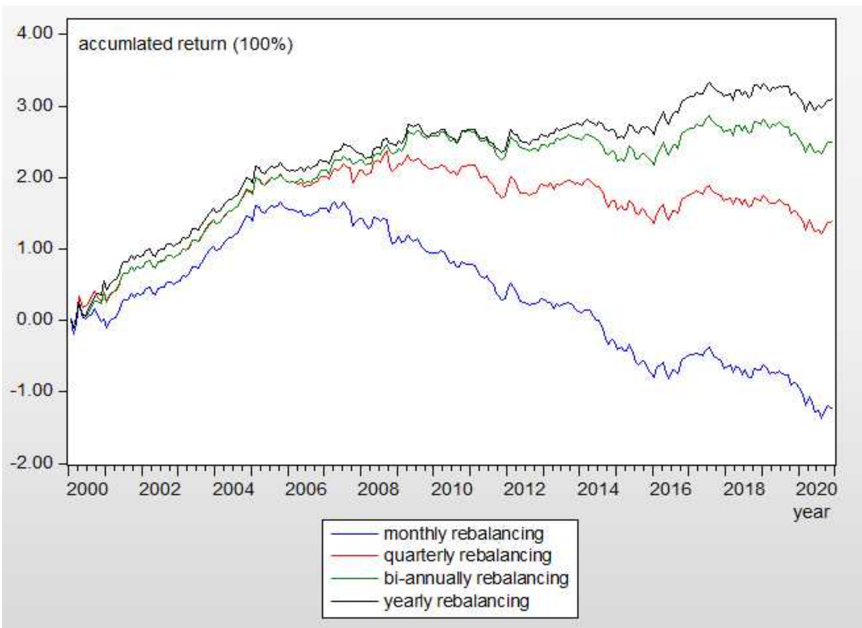

Figure 1. Accumulated return on value minus growth portfolios

Note: The book-to-market ratios are updated based on the current market prices each month. The monthly, quarterly, bi-annually, and yearly rebalancing are done on the first business days of the months, quarters, half years, and years, respectively. Returns are equally weighted portfolios' and daily compounded.

Table 1 shows the summary statistics of the portfolios' monthly returns of the value, the growth, the value minus growth stocks, and the KOSPI 200 index for the sample period of January 2000 – December 2020. KOSPI 200 is value-weighted while the value stocks and the growth stocks portfolios are equally weighted. We show the statistics for the monthly, quarterly, bi-annually, and yearly rebalanced portfolios in Panels A, B, C, and D of Table 1, respectively. As we can see from Table 1, the value returns are higher. However, the growth returns are lower as we rebalance the portfolios less frequently, i.e., from monthly to quarterly, from quarterly to bi-annually, and from bi-annually to yearly. The yearly-rebalanced value minus growth portfolio earned the highest mean monthly return of 1.20% (Panel D of Table 1). The monthly, quarterly, and bi-annually rebalanced portfolios yielded -0.5%, 0.6%, and 1.0%, respectively.

Table 1 shows that the standard deviation of the monthly return ranks from highest to lowest in the order of the value, the growth, the value minus growth, and the KOSPI 200 irrespectively of the rebalancing frequency. Even though the average return varies according to how often we rebalance the portfolios, the risk (standard deviation) does very little. When evaluating the value minus growth portfolios' relative performance in terms of the Sharp ratio, the yearly- rebalanced value minus growth portfolio ranks the best. Its Sharp ratio (computed as return/standard deviation) is 0.192. Compared to the Sharp ratio of 0.068 of KOSPI 200, it is almost three times as high as the KOSPI 200's. The Sharp ratios of the monthly, quarterly, and bi-annually rebalanced portfolios are -0.067, 0.081, and 0.150, respectively.

The mean BM ratios of the value stocks and the growth stocks are 5.838 and 0.259, respectively, when we rebalance the value and growth stocks monthly (Panel A of Table 1). The corresponding value spread (= value stock BM ratio minus growth stock BM ratio) is 5.579. The value spreads are slightly different when we rebalance the value and growth stocks quarterly, bi-annually, or yearly (Panels B, C, and D of Table1).

Table 1 Summary statistics of monthly returns of value, growth, and value minus growth

Panel A: Value and growth stocks are rebalanced monthly

	Return				Sharp ratio				BM ratio		
	Kospi 200	Value	Growth	Value-growth	Kospi 200	Value	Growth	Value-growth	Value	Growth	Value-Growth
Mean	0.004	-0.004	0.000	-0.005	0.068	-0.048	0.006	-0.067	5.838	0.259	5.579
Maximum	0.208	0.237	0.282	0.293					28.338	0.545	27.793
Minimum	-0.235	-0.537	-0.475	-0.218					1.762	0.116	1.575
Standard Deviation	0.064	0.091	0.084	0.072					6.245	0.107	6.154

Panel B: Value and growth stocks are rebalanced quarterly

	Return				Sharp ratio				BM ratio		
	Kospi 200	Value	Growth	Value-growth	Kospi 200	Value	Growth	Value-growth	Value	Growth	Value-Growth
Mean	0.004	0.000	-0.005	0.006	0.068	0.004	-0.062	0.081	5.823	0.262	5.562
Maximum	0.208	0.270	0.282	0.293					27.926	0.554	27.372
Minimum	-0.235	-0.537	-0.475	-0.218					1.762	0.116	1.575
Standard Deviation	0.064	0.091	0.084	0.069					6.240	0.107	6.149

Panel C: Value and growth stocks are rebalanced bi-annually

	Return				Sharp ratio				BM ratio		
	Kospi 200	Value	Growth	Value-growth	Kospi 200	Value	Growth	Value-growth	Value	Growth	Value-Growth
Mean	0.004	0.003	-0.007	0.010	0.068	0.031	-0.085	0.150	5.794	0.272	5.522

Maximum	0.208	0.318	0.282	0.232					27.954	0.741	27.214
Minimum	-0.235	-0.478	-0.397	-0.186					1.762	0.116	1.575
Standard Deviation	0.064	0.090	0.083	0.066					6.253	0.117	6.156

Panel D: Value and growth stocks are rebalanced yearly

	Return				Sharp ratio				BM ratio		
	Kospi 200	Value	Growth	Value-growth	Kospi 200	Value	Growth	Value-growth	Value	Growth	Value-Growth
Mean	0.004	0.004	-0.008	0.012	0.068	0.048	-0.097	0.192	5.710	0.283	5.427
Maximum	0.208	0.318	0.282	0.232					26.588	1.022	25.565
Minimum	-0.235	-0.466	-0.393	-0.186					1.713	0.133	1.495
Standard Deviation	0.064	0.090	0.083	0.064					6.219	0.122	6.117

Note: Portfolio returns are monthly returns throughout January of 2000 to December of 2020. The Sharp ratios of the KOSPI 200, the value, the growth, and the value minus growth stocks are computed by taking the ratios of the portfolio's returns to their standard deviations, respectively. The respective BM ratios are updated based on the current market prices every month.

We compute Sharp ratio by taking the ratio of return to standard deviation. Value spread is value stock BM ratio minus growth stock BM ratio.

Table 2 shows the correlation of the monthly return between value, growth, and value minus growth, and KOSPI 200, respectively. As we can see in Panels A, B, C, and D of Table 2 for the monthly, quarterly, bi-annually, and yearly rebalanced portfolios, the returns of the value and the KOSPI 200 index, and the growth and the KOSPI 200 index are highly correlated to each other. However, the correlation of the value minus growth and the KOSPI 200 index is very low (e.g., 0.066 in Panel D), and their return tends to move independently of one another. The correlation between the value and the value minus growth portfolios is

0.464 (in Panel D). The correlation between the growth and the value minus growth portfolios is -0.274(in Panel D), respectively. It appears that the value minus growth return is more driven by the value stocks than the growth stocks. Panels A, B, and C of Table 2 also show that the correlations between the value, the growth, the value minus growth, and the KOSPI 200 portfolios differ little by the rebalancing frequency. At this point, it is interesting to note that the rebalancing frequency matters for the mean returns of the portfolios but not for their standard deviations and correlations.

Table 2. Correlation of monthly return of value, growth, and value minus growth

Panel A: Value and growth stocks are rebalanced monthly

	KOSPI200	Value	Growth	Value-growth
KOSPI200	1.000	0.788	0.765	0.104
Value	0.788	1.000	0.665	0.490
Growth	0.765	0.665	1.000	-0.325
Value-growth	0.104	0.490	-0.325	1.000

Panel B: Value and growth stocks are rebalanced quarterly

	KOSPI200	Value	Growth	Value-growth
KOSPI200	1.000	0.786	0.777	0.092
Value	0.786	1.000	0.691	0.479
Growth	0.777	0.691	1.000	-0.304
Value-growth	0.092	0.479	-0.304	1.000

Panel C: Value and growth stocks are rebalanced bi-annually

	KOSPI200	Value	Growth	Value-growth
KOSPI200	1.000	0.777	0.786	0.077

Value	0.777	1.000	0.714	0.474
Growth	0.786	0.714	1.000	-0.278
Value-growth	0.077	0.474	-0.278	1.000

Panel D: Value and growth stocks are rebalanced yearly

	KOSPI200	Value	Growth	Value-growth
KOSPI200	1.000	0.780	0.796	0.066
Value	0.780	1.000	0.725	0.464
Growth	0.796	0.725	1.000	-0.274
Value-growth	0.066	0.464	-0.274	1.000

Note: Magnitudes are Pearson pairwise correlations between KOSPI 200, value stocks, growth stocks, and value minus growth stocks, respectively. They are computed using daily closing prices.

Park and Fang (2021) decomposed market variance into the product of the variance of typical stock and the ratio of these two variances such that $\sigma_t{}^2(M) = \bar{\sigma}_t{}^2(i) \times (\sigma_t{}^2(M)/\bar{\sigma}_t{}^2(i))$, where $\sigma_t{}^2(M)$ and $\bar{\sigma}_t{}^2$(i) stand for the market variance and the variance of the typical stock, respectively. Our market portfolio, M is KOSPI 200. $\bar{\sigma}_t{}^2$(i) is an average variance of individual stocks for *i* = 1, 2...M.

Table 3 shows the summary statistics of our three risk metrics, $\sigma_t{}^2(M)$, $\bar{\sigma}_t{}^2$(i) and the ratio of these two variances. $\sigma_t{}^2(M)$ and $\bar{\sigma}_t{}^2(i)$ are the variances in month t and computed using GARCH(1,1). We see from Table 3 that the means of $\sigma_t{}^2(M)$, $\bar{\sigma}_t{}^2$(i) and $\sigma_t{}^2(M)/\bar{\sigma}_t{}^2(i)$ are 0.005, 0.024, and 0.178, respectively. When we convert those two variances into annual standard deviations, they are approximately 0.24 and 0.53, respectively. The risk of the market portfolio is much less than the risk of the typical stock. These numbers are quite close to the standard deviations estimated in Elton and Gruber (1977).

Another crucial thing to note is that these risks vary much over time. The market variance's standard deviation is far greater than its mean, and a maximum of the variations in the variance is 0.087 (= 0.087-0.000), which is a 28% change from month to month. This time-varying risk of the market portfolio is due to the time-varying nature of both individual risk and the fractional non-diversifiable risk, i.e., the ratio of two variances.

Table 3. Summary statistics of three risk metrics

	${\sigma_t}^2(M)$	${\bar{\sigma}_t}^2$(i)	ratio
Mean	0.005	0.024	0.178
Maximum	0.087	0.168	0.636
Minimum	0.000	0.005	0.012
Std. Dev.	0.008	0.023	0.110
Observations	252	252	252

Note: ${\sigma_t}^2(M)$, ${\bar{\sigma}_t}^2$(i) are the variance of the daily log return of KOSPI 200, and the average variance of the daily log returns of individual stocks, respectively. The ratio ${\sigma_t}^2(M)/{\bar{\sigma}_t}^2$(i) measures the fraction of the non-diversifiable risk relative to the individual risk of typical stock, and is called the fractional non-diversifiable risk (Park and Fang, 2021). They are estimated using GARCH (1,1).

Table 4 shows the correlation of three risk metrics at times t and $t-1$. The market variance is highly correlated to both the individual risk and the fractional non-diversifiable risk contemporaneously. They are 0.729 and 0.679, respectively. The high correlation is because the market variance is simply the product of the individual risk and the fractional non-diversifiable risk. On the other hand, the correlation between the individual risk and the fractional non-diversifiable risk is relatively low, i.e., 0.242. The correlation between ${\sigma_t}^2(M)$ and ${\sigma_{t-1}}^2(M)$, and the correlation

between ${\bar{\sigma}_t}^2$(i) and ${\bar{\sigma}_{t-1}}^2$(i) are somewhat modest to high, ranging from 0.389 to 0.462, indicating a heteroscedasticity problem.

Table 4. Correlation of three risk metrics

	${\sigma_t}^2(M)$	${\sigma_{t-1}}^2(M)$	${\bar{\sigma}_t}^2$(i)	${\bar{\sigma}_{t-1}}^2$(i)	$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(\mathrm{i})}$	$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(\mathrm{i})}$
${\sigma_t}^2(M)$	1.000	0.462	0.729	0.395	0.679	0.389

${\sigma_{t-1}}^2(M)$	0.462	1.000	0.364	0.726	0.411	0.677
${\bar{\sigma}_t}^2$(i)	0.729	0.364	1.000	0.388	0.242	0.238
${\bar{\sigma}_{t-1}}^2$(i)	0.395	0.726	0.388	1.000	0.305	0.236
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(\mathrm{i})}$	0.679	0.411	0.242	0.305	1.000	0.479
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(\mathrm{i})}$	0.389	0.677	0.238	0.236	0.479	1.000

Note: ${\sigma_t}^2(M)$, ${\bar{\sigma}_t}^2$(i) are the variance of the daily log return of KOSPI 200, and the average variance of the daily log returns of individual stocks, respectively. Magnitudes are Pearson pairwise correlations between three risk metrics at times t and t-1.

4. Estimation

(4.1) Specification

This section, we will specify our regression equations to test the risk and return time series relation in the stock market with a particular interest in understanding value versus growth stocks' behavior. The empirical finance literature showed that stock return time series exhibited heteroscedasticity represented in such models as GARCH(1,1) and TARCH(1,1), where the variance of X_{t+1} depends on the variance of X_t Whitelaw (1994) also found that return led variance. Taking these into consideration, we will replace the expected variance at time t, $Var_t(R_{t+1})$ by the contemporaneous time-varying variance, $Var\,(R_t)$ in (9) for our specification. All other equations follow suit as $Var_t(R_{t+1})$ is replaced by $Var\,(R_t)$.

We define net returns, r_t and r_{t+1} as $\mathrm{R_t} - 1$ and $\mathrm{R_{t+1}} - 1$, respectively. When we subtract ones from both $E_t(R_{t+1})$, and R_t terms, we have that $E_t(R_{t+1}) - R_t = E_t[(R_{t+1}) - 1] - [R_t - 1] = E_t(r_{t+1}) - r_t$. We also know that the variance of gross return is equal to the variance of net return since $Var\,(R_t) = Var\,[R_t - 1] = Var\,(r_t)$. Accordingly, we will use r_t and r_{t+1} instead of R_t and R_{t+1} as our dependent variables throughout our regression analysis. Risk is usually related to net return in the stock market in the empirical literature. In the following regressions, we compute r_t using log return, i.e., ln (S_t/S_{t-1}), where S_t is the closing stock price at the end of month t.

Estimation of expected return, $E_t(r_{t+1})$ is a challenging issue in the literature. (footnote 3)

We assume that a realized future return is the sum of the expected return and a random error term for simplicity.

There are several ways to estimate $Var(r_t)$ of monthly return in month t: Sample variance, GARCH(1,1) and TARCH(1,1). We compute month t sample variance by taking an equally weighted average of daily squared returns within month t. We compute GARCH(1,1) and TARCH(1,1) variances in month t as of the final business day of month t. Consequently, more weights are given to recent days in the case of GARCH(1,1) and TARCH(1,1), respectively. Equal weight is given to each day in sample variance. Our baseline is GARCH(1,1) estimation of $Var(r_t)$. However, we will use other estimations too, where appropriate.

As for the independent variables of our regression equations, we will use three risk metrics discussed in Section 3 i.e., market variance $\sigma_t{}^2(M)$, the variance of typical stock $\bar{\sigma}_t{}^2(i)$, and the ratio of these two variances. As discussed in Park and Fang (2021), we want to determine whether the fractional non-diversifiable risk is more responsible for the risk and return time series relation than the individual risk at the portfolio levels of value and growth stocks.

Based on our discussion above and in Section 2, we can specify our regression equations as follows:

$$r_t = a_0 + a_1\sigma_t{}^2(M) + a_2(\sigma^2{}_{t-1}(M)) + \varepsilon_t, \tag{18}$$

$$r_t = b_0 + b_1\bar{\sigma}_t{}^2(i) + b_2\bar{\sigma}_{t-1}{}^2(i) + b_3\frac{\sigma_t{}^2(M)}{\bar{\sigma}_t{}^2(i)} + b_4\frac{\sigma^2{}_{t-1}(M)}{\bar{\sigma}_{t-1}{}^2(i)} + \varepsilon_t. \tag{19}$$

where ε_t is a random error term.

The constants in (18)-(19) pick up the long-run risk effect on return as presented in (10), (15), (16), and (17), respectively. The other terms capture the time-varying risk effect on return, which reverts with a time lag. The time-varying risk represents a transitory deviation from the long-run risk. We expect that $a_0 > 0$, $a_1 < 0$ and $a_2 > 0$ in (18), and $b_0 > 0$, $b_1 < 0$, $b_2 > 0$, $b_3 < 0$ and $b_4 > 0$ in (19). We rename (18) and (19) as models 1 and 2, respectively. We run regressions of (18) and (19), using KOSPI 200, value stocks, growth stocks, and value minus growth stocks as our dependent variables. We rebalance value stocks and growth stocks monthly, quarterly, bi-annually, and yearly, respectively, and the regression results are reported accordingly in the following section.

(4.2) Empirical Results

First, we analyze the regression results of the risk and return time series relation at the aggregate stock market level, i.e., KOSPI 200. As seen in Table 5, all coefficients are significant ($p<0.05$) and have expected signs in model 1. Indeed, the risk is negatively related to risk contemporaneously and positively to risk with a time lag. In terms of goodness-of-fit, model 2 has a higher adjusted R-squared of 0.167 than model 1's adjusted R-squared of 0.086. The coefficients of fractional non-diversifiable risk at time t and $t-1$ are highly significant ($p<0.01$) in model 2. However, the individual risk term loses its significance in model 2. Fractional non-diversifiable risk is the primary driver in explaining the variation in return. Park and Fang (2021) reported the same result for the sample period of 1995-2017 in the Korean, the U.S., and the U.K. stock markets.

Next, we analyze regression at the portfolio level: Value versus growth stocks. As our preliminary checks have revealed in Figure 1 and Tables 1 and 2, the rebalancing frequency does matter for the return performance of value stocks relative to growth stocks. Tables 6, 7, 8, and 9 report the regression results when we rebalance value and growth stocks monthly, quarterly, bi-annually, and yearly, respectively. Panels A, B, and C of Tables 6-9 show the regression results when we use value stocks, growth stocks, and value minus growth stocks as the regression equation's dependent variables.

Table 6 reports the regression results when we rebalance value stocks and growth stocks monthly. In terms of goodness-of-fit, the regression results are relatively good for the value stocks in which the adjusted R-squared is from 0.156 (model 1) to 0.216 (model 2). For the growth stocks, the adjusted R-squared is from 0.075 (model 1) to 0.138 (model 2). For the value minus growth stocks, the adjusted R-squared is relatively low, i.e., 0.023 (model 1) to 0.031 (model 2). It is probably because the variation in the value minus growth return contains a more significant amount of noise than the variation in the value return alone or the growth return alone. All coefficients have expected signs and statistically significant except for the individual risk variable in model 2. At this juncture, it deserves mentioning that the constant terms are not statistically significant for the value minus growth stocks in models 1 and 2 (Panel C). It indicates that even though the time-varying risk effect on the value minus growth return is statistically significant, the long-run risk effect is not so when we rebalance value and growth stocks monthly. This regression result seems to verify the observation that the accumulated return on the monthly-rebalanced value minus growth portfolio did not grow steadily over time, as shown in Figure 1.

Table 7 reports the regression results when we rebalance value and growth stocks quarterly. The regression results for the quarterly rebalanced portfolios are similar to those for the monthly rebalanced portfolios in adjusted R-squares and the individual coefficients' significance. The time-varying risk effect is reverting and significant, but the long-run risk effect is not significant for the value minus growth stocks.

Tables 8 and 9 report the regression results when we rebalance value and growth stocks bi-annually and yearly, respectively. As per the time-varying risk effect on return, the results are similar to those reported in Tables 6 and 7. The time-varying risk effect is reverting and significant. When we test the long-run risk effect on value minus growth return in terms of model 1, the effects (measured by constant terms) are significant for the bi-annually rebalanced portfolio ($p<0.05$) and the yearly-rebalanced portfolio ($p<0.01$). The constant term magnitudes are 0.11 and 0.12 for the bi-annually- and yearly- rebalanced portfolios, respectively (Panel Cs of Table 8 and 9). These magnitudes are approximately equal to the unconditional means of the monthly returns for the bi-annually-rebalanced and the yearly-rebalanced portfolios of value minus growth stocks (Panels C and D of Table1). However, the long-run risk effects on value minus growth return are not significant when we rebalance value and growth stocks monthly or quarterly. This result seems to verify that the accumulated return on value minus growth stocks grew steadily for the bi-annually- and yearly portfolios, but not for the monthly- and quarterly portfolios, as shown in Figure 1.

Our model predicts that the time-varying risk effect is reverting with a time lag, and hence the temporal effect is negligible when aggregated in time. This prediction will be correct if the unconditional mean return is almost entirely due to the long-run risk effect. Our regression results have shown these as the magnitudes of constant terms were quite close to the unconditional mean returns for the bi-annually- and yearly- rebalanced portfolios.

Table 5 Regression results of (18)-(19)

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.010	2.517	0.013	0.021	2.552	0.011
$\sigma^2(M)$	-4.073	-3.274	0.001			
${\sigma_{t-1}}^2(M)$	3.034	2.174	0.031			
${\bar{\sigma}_t}^2(i)$				0.379	0.421	0.674
${\bar{\sigma}_{t-1}}^2(i)$				-0.934	-1.038	0.300
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.332	-4.361	0.000
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.319	3.776	0.000
R-squared		0.094		R-squared		0.181
Adjusted R-squared		0.086		Adjusted R-squared		0.167
Durbin-Watson stat		2.030		Durbin-Watson stat		2.129

Note: Models 1 and 2 are as follows:

$$\text{Model 1: } r_t = a_0 + a_1{\sigma_t}^2(M) + a_2({\sigma^2}_{t-1}(M)) + \varepsilon_t, \tag{18}$$

$$\text{Model 2: } r_t = b_0 + b_1{\bar{\sigma}_t}^2(i) + b_2{\bar{\sigma}_{t-1}}^2(i)$$

$$+b_3\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)} + b_4\frac{{\sigma^2}_{t-1}(M)}{{\bar{\sigma}_{t-1}}^2(i)} + \varepsilon_t. \tag{19}$$

The dependent variables are the monthly returns on KOSPI 200. Variances (explanatory variables) are estimated using GARCH(1,1). We use HAC standard errors and covariance (Bartlett kernel, Newey-West fixed) to correct for heteroscedasticity of regression residuals.

Table 6. Regression results of (18)-(19) when value and growth stocks are rebalanced monthly

Panel A. Value stocks

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.004	0.719	0.473	0.025	2.331	0.021
$\sigma^2(M)$	-7.705	-5.253	0.000			

$\sigma_{t-1}{}^2(M)$	5.973	3.806	0.000			
$\bar{\sigma}_t{}^2(i)$				-0.187	-0.140	0.889
$\bar{\sigma}_{t-1}{}^2(i)$				-1.196	-1.047	0.296
$\frac{\sigma_t{}^2(M)}{\bar{\sigma}_t{}^2(i)}$				-0.482	-4.135	0.000
$\frac{\sigma_{t-1}{}^2(M)}{\bar{\sigma}_{t-1}{}^2(i)}$				0.519	5.810	0.000
R-squared		0.163		R-squared		0.229
Adjusted R-squared		0.156		Adjusted R-squared		0.216
Durbin-Watson stat		2.033		Durbin-Watson stat		2.066

Panel B. Growth stocks

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.006	1.132	0.259	0.030	2.562	0.011
$\sigma^2(M)$	-5.045	-3.775	0.000			
$\sigma_{t-1}{}^2(M)$	3.949	2.750	0.006			
$\bar{\sigma}_t{}^2(i)$				-1.970	-0.905	0.366
$\bar{\sigma}_{t-1}{}^2(i)$				0.263	0.169	0.866
$\frac{\sigma_t{}^2(M)}{\bar{\sigma}_t{}^2(i)}$				-0.245	-1.909	0.057
$\frac{\sigma_{t-1}{}^2(M)}{\bar{\sigma}_{t-1}{}^2(i)}$				0.333	3.587	0.000
R-squared		0.083		R-squared		0.152
Adjusted R-squared		0.075		Adjusted R-squared		0.138
Durbin-Watson stat		2.145		Durbin-Watson stat		2.236

Panel C. Value minus growth stocks

Model1	Model 2

Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	-0.002	-0.344	0.731	-0.005	-0.395	0.693
$\sigma^2(M)$	-2.660	-1.971	0.050			
${\sigma_{t-1}}^2(M)$	2.024	2.153	0.032			
${\bar{\sigma}_t}^2(i)$				1.783	0.912	0.363
${\bar{\sigma}_{t-1}}^2(i)$				-1.459	-0.898	0.370
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.236	-1.850	0.066
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.186	1.853	0.065
R-squared		0.031		R-squared		0.047
Adjusted R-squared		0.023		Adjusted R-squared		0.031
Durbin-Watson stat		1.949		Durbin-Watson stat		1.923

Note: Models 1 and 2 are as follows:

$$\text{Model 1: } r_t = a_0 + a_1{\sigma_t}^2(M) + a_2({\sigma^2}_{t-1}(M)) + \varepsilon_t, \tag{18}$$

$$\text{Model 2: } r_t = b_0 + b_1{\bar{\sigma}_t}^2(i) + b_2{\bar{\sigma}_{t-1}}^2(i)$$

$$+b_3\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)} + b_4\frac{{\sigma^2}_{t-1}(M)}{{\bar{\sigma}_{t-1}}^2(i)} + \varepsilon_t. \tag{19}$$

The dependent variables in Panels A, B, and C are the monthly returns on value stocks, growth stocks, and value minus growth stocks. Variances (explanatory variables) are estimated using GARCH(1,1). We use HAC standard errors and covariance (Bartlett kernel, Newey-West fixed) to correct for heteroscedasticity of regression residuals.

Table 7. Regression results of (18)-(19) when value and growth stocks are rebalanced quarterly

Panel A. Value stocks

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.009	1.463	0.145	0.030	2.830	0.005
$\sigma^2(M)$	-7.519	-5.142	0.000			

${\sigma_{t-1}}^2(M)$	5.821	3.728	0.000			
${\bar{\sigma}_t}^2(i)$				-0.240	-0.183	0.855
${\bar{\sigma}_{t-1}}^2(i)$				-1.198	-1.079	0.282
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.469	-4.179	0.000
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.513	6.001	0.000
R-squared		0.157		R-squared		0.224
Adjusted R-squared		0.150		Adjusted R-squared		0.211
Durbin-Watson stat		2.042		Durbin-Watson stat		2.088

Panel B. Growth stocks

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.003	0.516	0.606	0.033	2.988	0.003
$\sigma^2(M)$	-5.462	-4.245	0.000			
${\sigma_{t-1}}^2(M)$	3.907	2.781	0.006			
${\bar{\sigma}_t}^2(i)$				-1.899	-0.945	0.346
${\bar{\sigma}_{t-1}}^2(i)$				-0.246	-0.172	0.863
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.262	-2.168	0.031
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.368	4.233	0.000
R-squared		0.097		R-squared		0.182
Adjusted R-squared		0.090		Adjusted R-squared		0.169
Durbin-Watson stat		2.098		Durbin-Watson stat		2.222

Panel C. Value minus growth stocks

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.006	1.204	0.230	-0.002	-0.210	0.834
$\sigma^2(M)$	-2.058	-1.626	0.105			
${\sigma_{t-1}}^2(M)$	1.914	2.174	0.031			
${\bar{\sigma}_t}^2(i)$				1.659	0.957	0.340
${\bar{\sigma}_{t-1}}^2(i)$				-0.952	-0.664	0.507
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.206	-1.820	0.070
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.144	1.598	0.111
R-squared		0.022		R-squared	0.040	
Adjusted R-squared		0.014		Adjusted R-squared	0.024	
Durbin-Watson stat		2.021		Durbin-Watson stat	2.026	

Note: Models 1 and 2 are as follows:

Model 1: $r_t = a_0 + a_1{\sigma_t}^2(M) + a_2({\sigma^2}_{t-1}(M)) + \varepsilon_t,$ (18)

Model 2: $r_t = b_0 + b_1{\bar{\sigma}_t}^2(i) + b_2{\bar{\sigma}_{t-1}}^2(i)$

$$+b_3\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)} + b_4\frac{{\sigma^2}_{t-1}(M)}{{\bar{\sigma}_{t-1}}^2(i)} + \varepsilon_t. \quad (19)$$

The dependent variables in Panels A, B, and C are the monthly returns on value stocks, growth stocks, and value minus growth stocks. Variances (explanatory variables) are estimated using GARCH(1,1). We use HAC standard errors and covariance (Bartlett kernel, Newey-West fixed) to correct for heteroscedasticity of regression residuals.

Table 8 Regression results of (18)-(19) when value and growth stocks are rebalanced bi-annually

Panel A. Value stocks

	Model1	Model 2

Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.010	1.592	0.113	0.030	2.793	0.006
$\sigma^2(M)$	-6.978	-5.338	0.000			
${\sigma_{t-1}}^2(M)$	5.574	3.716	0.000			
${\bar{\sigma}_t}^2(i)$				0.340	0.264	0.792
${\bar{\sigma}_{t-1}}^2(i)$				-1.661	-1.593	0.112
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.488	-4.354	0.000
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.533	6.135	0.000
R-squared		0.137		R-squared		0.225
Adjusted R-squared		0.130		Adjusted R-squared		0.212
Durbin-Watson stat		2.031		Durbin-Watson stat		2.090

Panel B. Growth stocks

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	-0.001	-0.113	0.910	0.024	2.366	0.019
$\sigma^2(M)$	-5.365	-4.028	0.000			
${\sigma_{t-1}}^2(M)$	4.134	2.682	0.008			
${\bar{\sigma}_t}^2(i)$				-1.419	-0.818	0.414
${\bar{\sigma}_{t-1}}^2(i)$				-0.189	-0.143	0.886
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.291	-2.539	0.012
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.357	3.916	0.000
R-squared		0.096		R-squared		0.164

Adjusted R-squared	0.089	Adjusted R-squared	0.150
Durbin-Watson stat	2.129	Durbin-Watson stat	2.222

Panel C. Value minus growth stocks

	Model 1			Model 2		
Variable	Coeffi-cient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.011	2.210	0.028	0.006	0.690	0.491
$\sigma^2(M)$	-1.612	-2.200	0.029			
${\sigma_{t-1}}^2(M)$	1.441	2.117	0.035			
${\bar{\sigma}_t}^2(i)$				1.759	1.371	0.172
${\bar{\sigma}_{t-1}}^2(i)$				-1.472	-1.336	0.183
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.197	-2.026	0.044
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.176	2.140	0.033
R-squared		0.014		R-squared		0.044
Adjusted R-squared		0.006		Adjusted R-squared		0.028
Durbin-Watson stat		2.047		Durbin-Watson stat		2.051

Note: Models 1 and 2 are as follows:

Model 1: $r_t = a_0 + a_1{\sigma_t}^2(M) + a_2({\sigma^2}_{t-1}(M)) + \varepsilon_t,$ (18)

Model 2: $r_t = b_0 + b_1{\bar{\sigma}_t}^2(i) + b_2{\bar{\sigma}_{t-1}}^2(i)$

$$+b_3\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)} + b_4\frac{{\sigma^2}_{t-1}(M)}{{\bar{\sigma}_{t-1}}^2(i)} + \varepsilon_t. \quad (19)$$

The dependent variables in Panels A, B, and C are the monthly returns on value stocks, growth stocks, and value minus growth stocks. Variances (explanatory variables) are estimated using GARCH(1,1). We use HAC standard errors and covariance (Bartlett kernel, Newey-West fixed) to correct for heteroscedasticity of regression residuals.

Table 9 Regression results of (18)-(19) when value and growth stocks are rebalanced yearly

Panel A. Value stocks

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.012	1.899	0.059	0.031	2.963	0.003
$\sigma^2(M)$	-6.869	-5.514	0.000			
${\sigma_{t-1}}^2(M)$	5.441	3.950	0.000			
${\bar{\sigma}_t}^2(i)$				0.358	0.277	0.782
${\bar{\sigma}_{t-1}}^2(i)$				-1.686	-1.630	0.104
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.481	-4.370	0.000
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.527	6.214	0.000
R-squared		0.135		R-squared		0.223
Adjusted R-squared		0.128		Adjusted R-squared		0.210
Durbin-Watson stat		2.042		Durbin-Watson stat		2.090

Panel B. Growth stocks

	Model1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	-0.001	-0.132	0.895	0.023	2.308	0.022
$\sigma^2(M)$	-5.363	-3.995	0.000			
${\sigma_{t-1}}^2(M)$	3.959	2.340	0.020			
${\bar{\sigma}_t}^2(i)$				-1.524	-0.898	0.370
${\bar{\sigma}_{t-1}}^2(i)$				-0.076	-0.059	0.953
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.277	-2.455	0.015
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.337	3.623	0.000
R-squared		0.096		R-squared		0.157
Adjusted R-squared		0.089		Adjusted R-squared		0.143

Durbin-Watson stat	2.168	Durbin-Watson stat	2.263

Panel C. Value minus growth stocks

	Model 1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.012	2.694	0.008	0.008	0.898	0.370
$\sigma^2(M)$	-1.506	-2.034	0.043			
${\sigma_{t-1}}^2(M)$	1.482	1.736	0.084			
${\bar{\sigma}_t}^2(i)$				1.882	1.550	0.123
${\bar{\sigma}_{t-1}}^2(i)$				-1.610	-1.527	0.128
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.204	-2.137	0.034
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.190	2.312	0.022
R-squared		0.014		R-squared		0.052
Adjusted R-squared		0.006		Adjusted R-squared		0.036
Durbin-Watson stat		2.097		Durbin-Watson stat		2.083

Note: Models 1 and 2 are as follows:

$$\text{Model 1: } r_t = a_0 + a_1{\sigma_t}^2(M) + a_2({\sigma^2}_{t-1}(M)) + \varepsilon_t, \tag{18}$$

$$\text{Model 2: } r_t = b_0 + b_1{\bar{\sigma}_t}^2(i) + b_2{\bar{\sigma}_{t-1}}^2(i)$$

$$+b_3\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)} + b_4\frac{{\sigma^2}_{t-1}(M)}{{\bar{\sigma}_{t-1}}^2(i)} + \varepsilon_t. \tag{19}$$

The dependent variables in Panels A, B, and C are the monthly returns on value stocks, growth stocks, and value minus growth stocks. Variances (explanatory variables) are estimated using GARCH(1,1). We use HAC standard errors and covariance (Bartlett kernel, Newey-West fixed) to correct for heteroscedasticity of regression residuals.

5. Robustness checks

As robustness checks on the long-run risk effect, we use sample variance and TARCH(1,1), respectively, to estimate the variance terms in our regression equation. We check only for the case of the yearly-rebalanced value minus growth portfolio because the long-run risk effect is more relevant for that portfolio, given the results shown in Figure 1 and Table 9. The regression results are shown in Table 10, using sample variance and TARCH(1,1), respectively. When we test the long-run risk effect in terms of model 1, the constant term is significant either using sample variance ($p<0.01$) or TARCH ($p<0.01$). The magnitudes of the constant are 0.015 using sample variance and 0.014 using TARCH. These are close to the estimated constant using GARCH(1,1). Hence, the long-run risk effect is robust concerning the estimation method of variance.

Table 10 Regression results: Sample variance versus TARCH(1,1)

	Sample variance						TARCH(1,1)					
	Model 1			Model 2			Model 1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.015	3.285	0.001	0.016	2.010	0.046	0.014	2.999	0.003	0.011	1.327	0.186
$\sigma^2(M)$	-1.018	-1.614	0.108				-1.345	-1.726	0.086			
${\sigma_{t-1}}^2(M)$	0.550	0.830	0.407				1.069	1.300	0.195			
${\bar{\sigma}_t}^2(i)$				0.231	0.953	0.342				1.253	0.883	0.378
${\bar{\sigma}_{t-1}}^2(i)$				-0.059	-0.231	0.817				-1.062	-0.820	0.413
$\frac{{\sigma_t}^2(M)}{{\bar{\sigma}_t}^2(i)}$				-0.144	-2.481	0.014				-0.148	-1.491	0.137
$\frac{{\sigma_{t-1}}^2(M)}{{\bar{\sigma}_{t-1}}^2(i)}$				0.103	2.101	0.037				0.126	1.426	0.155
R-squared		0.014		R-squared		0.054	R-squared		0.012	R-squared		0.030
Adjusted R-squared		0.006		Adjusted R-squared		0.039	Adjusted R-squared		0.004	Adjusted R-squared		0.014
Durbin-Watson stat		2.068		Durbin-Watson stat		2.101	Durbin-Watson stat		2.092	Durbin-Watson stat		2.105

Note: Models 1 and 2 are as follows:

$$\text{Model 1: } r_t = a_0 + a_1\sigma_t{}^2(M) + a_2(\sigma^2{}_{t-1}(M)) + \varepsilon_t, \tag{18}$$

$$\text{Model 2: } r_t = b_0 + b_1\bar{\sigma}_t{}^2(i) + b_2\bar{\sigma}_{t-1}{}^2(i)$$

$$+b_3\frac{\sigma_t{}^2(M)}{\bar{\sigma}_t{}^2(i)} + b_4\frac{\sigma^2{}_{t-1}(M)}{\bar{\sigma}_{t-1}{}^2(i)} + \varepsilon_t. \tag{19}$$

The dependent variables, r_t is the monthly return on value minus growth stocks in yearly rebalanced portfolios. Variances (explanatory variables) are estimated using sample variance and TARCH(1,1), respectively. We use HAC standard errors and covariance (Bartlett kernel, Newey-West fixed) to correct for heteroscedasticity of regression residuals.

6. Expected payoff and return

So far, our empirical analysis has been on the simple relation between risk and return. However, as can be seen in (4), return is also affected by expected payoff growth and risk. Using (8), let us rewrite (4) as

$$R_t = E_t\left[\frac{X_{t+1}}{X_t}\right] - \gamma\, Var_t\,[R_{t+1}]. \tag{20}$$

Equation (20) requires that we control the effect of expected payoff growth to assess the ceteris paribus effect of risk on return in our regression. Considering the effect of expected payoff on return, we revise our specifications, (18) and (19) as follows:

$$r_t = a_0 + a_1x_{t+1} + a_2\sigma_t{}^2(M) + a_3\left(\sigma^2{}_{t-1}(M)\right) + \varepsilon_t, \tag{21}$$

$$r_t = b_0 + b_1x_{t+1} + b_2\bar{\sigma}_t{}^2(i) + b_3\bar{\sigma}_{t-1}{}^2(i) + b_4\frac{\sigma_t{}^2(M)}{\bar{\sigma}_t{}^2(i)} + b_5\frac{\sigma^2{}_{t-1}(M)}{\bar{\sigma}_{t-1}{}^2(i)} + \varepsilon_t. \tag{22}$$

where r_t is again the net return at time t, and x_{t+1} is the expected payoff (net) growth over the period of time t to time t+1. We use specifications (20) and (21) for the value minus growth portfolio.

We have drawn (20) and (21) from the single Euler equation, (1). The constant terms and the time-varying risk terms in (20) and (21) pick up the partial effects of risk, holding the expected payoff x_{t+1} fixed. These risk effects (i.e., the constant long-run risk effect and the time-varying risk effect) work through by depressing the current stock price P_t, given the expected payoff x_{t+1}. The expected payoff growth term x_{t+1} in (20) and (21) picks up the partial effect of the expected payoff x_{t+1}, holding the risk level fixed.

We measure x_t by an equally weighted average of the operating profit growth and the net profit growth of the value and growth firms, respectively. The operating profit is earnings before interest and taxes (EBIT), while the net profit is after interest and taxes. Since x_{t+1} is an expected value of future profit growth, we estimate x_{t+1} assuming an AR(2) process for x_t. (footnote 4)

Table 11 reports the summary statistics of monthly x_t of the value firms and the growth firms for the period of 2000 – 2020. The statistics are computed using both the operating profit growth and the net profit growth for x_t. Both are nominal figures, which are not price-deflated. Profit data are available on a quarterly basis. We convert x_t from quarterly to monthly data by an intra-quarter interpolation. The value and growth firms are yearly re-balanced.

Certain firms (e.g., low BM ratio's firms) are named "growth firms" because profits of those firms are supposed to grow faster in the future. Contrary to this conventional wisdom, Table 11 shows that the value firms performed a higher profit growth than the growth firms over the sample period of 2000-2020 in the Korean market. The mean growth rate is 4.77% for the value firms and 1.55% for the growth firms when x_t is the operating growth. The mean growth rate is 10.48% for the value firms and -7.02% for the growth firms when x_t is the net profit growth. The difference in sample means may not be a good measure of the difference in expected growth between the value firms and the growth firms, due to either irrational expectations or rare events as pointed out in Pastor and Veronesi (2007). However, it is not an exceptional observation that the value firms exhibited faster growth rates than the growth firms in the Korean market. Chen (2017) reported that in yearly rebalanced portfolios, dividends of value stocks had grown faster than those of growth stocks in the U.S. market.

Table 11 Summary statistics of monthly profit growth of value and growth firms

	Operating profit growth		Net profit growth	
	Value	Growth	Value	Growth
Mean	0.0477	0.0155	0.1048	-0.0702
Maximum	5.9846	0.3212	5.9966	0.2960
Minimum	-7.9161	-0.5372	-2.9233	-5.0968
Std. Dev.	1.2784	0.0959	1.0177	0.4840

Note: Operating profit is earnings before interest and taxes (EBIT). Net profit is earnings after interest and taxes. The firms in Korea release profit data on a quarterly basis. We converted the quarterly data into the monthly figures using an intra-quarter interpolation.

Now we want to check the robustness of our results when we control for the effect of expected payoff growth on return in our regression. Table 12 reports the regression results of (20) and (21) for the yearly- rebalanced value minus growth portfolio. We choose this yearly-rebalanced portfolio because the long-run risk effect of which is more significant as discussed before. Table 12 show the results when the operating profit growth and the net profit growth are used for x_t, respectively. The variance terms are estimated using sample variance. As seen in Table 12, the constant coefficients are positive and significant in model 1 (p<0.01) and model 2 (p<0.1). The magnitudes are pretty close to the unconditional mean of value minus and growth return as shown in Panel D of Table 1. The constant terms in (20) and (21) pick up the long-run risk effect. The time-varying risk effect is reverted with a time lag in models 1 and 2, i.e., $a_2 < 0$ and $a_3 > 0$ in (20), and $b_4 < 0$ and $b_5 > 0$ in (21). The fractional non-diversifiable risk variable in model 2 is significant at time t (p<0.05) and time t-1 (0.05), but the other risk coefficients are not significant. The operating and the net profit growth variables are not significant both in models 1 and 2. Table 12 and Panel C of Table 9 show that the R-squareds do not increase when we include the profit growth variables as an additional explanatory variable to explain the variation in the value minus growth return. An upshot is that the effects of the long-run risk and the time-varying risk remain unaltered and are robust when we control for the effect of expected payoff growth in our regression of the return on risk.

Table 12 Regression results of (20) and (21)

	Operating Profit Growth						Net profit Growth					
	Model 1			Model 2			Model 1			Model 2		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	0.0148	3.3045	0.0011	0.0151	1.8509	0.0654	0.0144	3.1592	0.0018	0.0148	1.8373	0.0674
x_t	-0.0025	-0.3727	0.7097	0.0004	0.0662	0.9472	0.0012	0.1939	0.8464	0.0024	0.4023	0.6878
$[\sigma^2(M)]$	-1.0333	-1.4070	0.1607				-0.9584	-1.4795	0.1403			
$[\sigma_{t-1}{}^2(M)]$	0.6322	0.9075	0.3650				0.5770	0.8637	0.3886			
$[\bar{\sigma}_i{}^2]$				0.2540	0.9816	0.3273				0.2598	1.0540	0.2929
$[\bar{\sigma}_{i-1}{}^2]$				-0.0007	-0.0029	0.9977				0.0046	0.0185	0.9853
$[\frac{\sigma^2(u)}{\bar{\sigma}_i{}^2}]$				-0.1430	-2.4537	0.0148				-0.1440	-2.4873	0.0135
$[\frac{\sigma_{t-1}{}^2(u)}{\bar{\sigma}_{i-1}{}^2}]$				0.0966	1.9770	0.0492				0.0958	1.9630	0.0508
	R-squared		0.0140	R-squared		0.0539	R-squared		0.0133	R-squared		0.0546
	Adjusted R-squared		0.0020	Adjusted R-squared		0.0345	Adjusted R-squared		0.0012	Adjusted R-squared		0.0353
	Durbin-Watson stat		2.0378	Durbin-Watson stat		2.0732	Durbin-Watson stat		2.0287	Durbin-Watson stat		2.0694

Note: Models 1 and 2 are as follows:

Model 1: $$r_t = a_0 + a_1 x_{t+1} + a_2 \sigma_t{}^2(M) + a_3(\sigma^2{}_{t-1}(M)) + \varepsilon_t, \quad (21)$$

Model 2: $$r_t = b_0 + b_1 x_{t+1} + b_2 \bar{\sigma}_t{}^2(i) + b_3 \bar{\sigma}_{t-1}{}^2(i) + b_4 \frac{\sigma_t{}^2(M)}{\bar{\sigma}_t{}^2(i)} + b_5 \frac{\sigma^2{}_{t-1}(M)}{\bar{\sigma}_{t-1}{}^2(i)} + \varepsilon_t. \quad (22)$$

r_t is the monthly returns on value minus growth stocks. x_{t+1} is the expected growth of the operating (net) profit growth, over the period of t to t+1. We rebalance value stocks and growth stocks yearly. Variances (explanatory variables) in month t are estimated using sample variance, i.e., the sum of the daily squared returns in month t. We use HAC standard errors and covariance (Bartlett kernel, Newey-West fixed) to correct for heteroscedasticity of regression residuals.

7. Contribution and suggestion

There are two value premium arguments in the literature: The first is that the value premium is an anomaly in the stock market (referred to as the value premium puzzle in the literature) because it is an alpha effect, but not a beta effect of the CAPM. This argument was backed by the empirical studies in which the data over the post-1963 period were used (e.g., Fama and French, 1992). The second one is that the value premium is not an anomaly because it is a beta effect but not an alpha effect of the conditional CAPM, which accounts for the time-varying beta or market risk premium. This argument was supported by the empirical studies in which the data over a long horizon of 1926-2001 were used (e.g., Ang and Chen, 2007; Bai, et al., 2019).

Our study has shown that the value premium was not an anomaly because it was a market risk effect. In this respect, our model follows suit the conditional CAPM argument in the literature. However, our model is significantly different from the literature in accounting for the cause of value premium in terms of market risk effect. In our model, the value premium was due to the constant long-run risk effect, while it was due to the time-varying risk effect in the literature. In our model, the time-varying risk effect was reverted with a time lag, and hence the temporal effect was negligible when aggregated in time. It is because the first negative effect of risk on return negates the second positive effect of risk. In the conditional CAPM literature of value premium, the first negative effect of the time-varying risk on return was omitted. On a continuum time basis, the risk does not raise (expected) return without lowering it first. This first fall and rise later phenomena in the stock market were apparently observed in crisis periods. In short, the alpha (constant term) effect is indeed a market risk effect and persistent. The time-varying risk effect is a market risk effect too, but only transient. Our analysis sheds light on resolving the value premium puzzle and thereby contributes to the literature.

We have also shown in Figure 1 that the frequency of rebalancing mattered for the accumulated return on the value strategy. Our analysis suggested that it was probably due to the difference in the long-run risk effect across different rebalancing frequencies. Our analysis arbitrarily chose month, quarter, six months and year, respectively, for the investors' holding period of portfolios before they rebalanced their portfolios. How often should the investors rebalance their portfolios to maximize the value minus growth return? To put it another way, how long should the investors hold their portfolios to be most compensated for their assuming risk before they rebalance their portfolios? To search for such an optimal frequency on a trial and error basis is an insurmountable task. We think that this is what AI and big data can do for investment analysis. Tobek and Hronec (2020) is a related study in the literature. We can crunch the data in countless ways by applying machine learning and deep learning to big data. We suggest that further research along the line be conducted. This is also a challenging issue in the investment community.

8. Concluding Remarks

Reworking the Euler equation, we have shown that the time-varying risk effect on return is reverting: An increase in risk lowers return first before it raises (expected) return. The second effect negates the first effect, and thereby, the temporal effect of the time-varying risk effect on return is negligible when aggregated in time. Growth stocks may outperform value stocks for some periods and vice versa for other periods. However, the long-run risk effect on return exists at a steady state where risk is time-invariant. Indeed, the alpha effect is a market risk effect. The implication is that a riskier asset earns a higher average return over a long horizon.

Based on the Korean stock market data throughout 2000-2020, our preliminary study has shown that the means and standard deviations of value stocks were greater than those of growth stocks, respectively, indicating that value stocks are riskier than growth stocks. These findings are, in fact, consistent with other studies in the literature (e.g., Ai and Kuku, 2013; Ang and Chen, 2007: Bai et al., 2019). Our regression analysis has shown that the time-varying risk effects on the returns of the value, the growth, and the value minus growth stocks were reverting and significant. However, an interesting result is that the long-run risk effect on risk depended on the frequency of rebalancing value stocks and growth stocks. Since the time-varying risk effect on return is negated over time, the long-run risk effect plays a crucial role in determining the accumulated return on value minus growth stocks. Our study has shown that the value strategy's long-term investment performance was better for the bi-annually- and the yearly-rebalanced portfolios than the monthly- and the quarterly-rebalanced portfolios, respectively. It suggested that for the investors to benefit from the long-run risk effect, they needed to hold the value minus growth portfolio for several months before re-balancing the portfolio.

In our empirical analysis, we arbitrarily chose the rebalancing periods i.e., month, quarter, six months, and year. To search for the optimal frequency on a trial and error basis is an insurmountable task. We think that this is what AI and big data can do for investment analysis. We can crunch the data in countless ways by applying machine learning and deep learning to big data. Hopefully, we may search for an optimal frequency to maximize the value minus growth return.

Some authors have recently found that it was premature to call value strategy dead (e.g., Blitz and Hanauer, 2021: Israel et al., 2021). Our results suggest that the value strategy is still valid as long as the long-run risk effect exists.

<Appendix A>

We will show how a steady-state long-run risk could affect return using the Euler equation. We divide (1) by P_t to obtain that

$$E_t(R_{t+1}) = R_f + \delta R_f Cov\,(\frac{u'_{t+1}}{u'_t},\, X_{t+1})$$

Assuming that the investor's utility is quadratic and accounting for heteroscedasticity, we obtain that

$$E_t(R_{t+1}) = R_f + \gamma Var_t(R_{t+1}) \tag{A1}$$

At a steady-state in the long-run, we replace $E_t(R_{t+1})$ and $Var_t(R_t)$ by their unconditional ones to obtain (10).

<Footnote>

1. For example, see Cochrane. The one-period Euler equation is a static model as the Sharpe-Lintner CAPM is so. Since we are studying a time-varying risk and return relation in our paper, we consider (1) a conditional version of the Euler equation as the conditional CAPM is a conditional version of the Sharpe-Lintner CAPM.

2. Even though we use the quadratic utility function for its simplicity in a mean-

variance framework in finance, it has a drawback because of its increasing absolute risk aversion. Generally speaking, if $u'(x)x$ is concave in x, $E_t[u'(x)x]$ decreases in the variance of x due to Jensen's Inequality.

3. Elton (1999) discussed alternative ways to measure expected return in the stock market.

4. We obtained similar results assuming an AR(1) process for x_t.

<Acknowledgement>

We acknowledge that the research is funded by the Ministry of Education of the Republic of Korea and Korea's National Research Foundation (NRF-2017S1A5B5A02023787).

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