Bond holding dynamics by the hedging motive against real exchange rate risk

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Abstract

This study examines bond holding dynamics using a two-country dynamic stochastic general equilibrium (DSGE) model with two bonds. We employ the higher-order approximation method proposed by Devereux and Sutherland (2010) and Till and van Wincoop (2010) and derive the bond holding dynamics equation for fundamental macro variables. Using an equation, we investigate the time-varying movement of bond holding after exogenous shock. The simulation results show that positive productivity shock decreases the home country's holding of home bonds. These results imply that the motivation to hedge against real exchange rate risk is one of the significant reasons for the bond holding dynamics. The empirical test for the robust check strengthens the theoretical results because the covariance between the relative home price and excess returns on home bonds decreases after the home country's output increases. Thus, this study concludes that portfolio investors consider macroeconomic indicators to maximize risk-hedging effects, and the stable movement of the real exchange rate reduces capital flow fluctuation.

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I. Introduction

Understanding economic agents' behavior towards optimal bond holding is important in the literature as bonds are used as main tools for consumption smoothing and risk-sharing. Various factors affect optimal bond holdings, such as interest rate, economic policy, and hedging motive, including macroeconomic factors. For example, many empirical studies have examined the determinants of capital flow, such as GDP growth, exchange rate, and monetary policy.[1] Regarding the hedging motive, Coeurdarcier and Gourinchas (2016) show that investor's hedging motive against real exchange rate risk affects the optimal bond holding. They argue that investors preserve their purchasing power by holding high return bonds when the price of home goods rises. Thus, investors have more home bonds when the covariance between the relative home price and excess return on home bonds (Cov(Px, Rx)) is positive.

In the literature, optimal bond holdings are mostly analyzed in the static framework. For example, Coeurdarcier and Rey (2010) and Amdur (2014) derive the optimal bond holding in the DSGE model. However, their study only explains the steady-state of the bond holding, not the change in bond holding, because they consider non-varying covariance. For it to be more practical, it is necessary to study the bond holding dynamics using the time-varying covariance. The change in the economic condition such as productivity improvement and inflation, causes the change in Cov(Px, Rx). Those changes may weaken or enhance the function of bonds as a hedging tool against real exchange rate risks.

Unlike the previous papers, this paper study the bond holding dynamics in DSGE model. To do so, we derive the equation of bond holding dynamics for state variables using the third-order approximation methods proposed by Devereux and Sutherland (2010) and Till and van Wincoop (2010). And then, using an equation, we confirm the movement of bond holding by the exogenous productivity shock. While Devereux and Sutherland (2010) and Till and van Wincoop (2010) assume the endowment economy, we extend the model to the labor production economy and study bond holding dynamics by hedging motive. In addition, for robustness check, we conduct a simple empirical test on the relationship between bond holding and covariance-variance ratio (Cov(Px, Rx)/Var(Rx))[2].

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[1] For example, Fratzscher (2012), Lane and Milesi-Ferretti (2018), Cerutti, Classens, and Puy (2015), Forbes and Warnock (2012), and Kim et al. (2013) use a fundamental macroeconomic variables to investigate the determinant of capital flow.

[2] Acorrding to Coeurdarcier and Gourinchas (2016), Cov(Px, Rx)/Var(Rx) is used as explanatory variable in empirical test.

According to Amdur (2010), most previous papers focus on predicting the optimal equity portfolio and introducing bond and equity to predict better the optimal equity portfolio rather than the optimal bond portfolio. For instance, Heathcote and Perri (2013), Coeudacier (2009), and Kim and Kim(2021) investigate the equity home bias using the DSGE model with two equity. Amdur (2010) analyze the optimal holding of bonds in zero-order equilibrium using the DSGE model with two bonds, but their research was also related to the non-time-varying of bond holdings.

Regarding portfolio dynamics, Till and van Wincoop (2010) numerically examine the equity holding dynamics in DSGE model with two equity. Devereux and Sutherland (2009) show the movement of the portfolio, such as bond and equity, using the DSGE model under the assumption of one bond and two equity. Devereux and Sutherland (2010) show the characteristics of the time-varying of bond holding in DSGE model with endowment economy. Unlike the previous papers, this study focuses on explaining the bond holding dynamics in DSGE model with labor production economy and two bonds.

Simulation results of this study confirm that when a positive productivity shock occurs in the home country, it decreases the home bonds it holds. This result implies that an increase in output decreases Cov(Px, Rx); thus, home investors reduce their holding of home bonds because the home bond's function as a hedging tool against real exchange rate risk is weakened. The empirical results show that the covariance-variance ratio has a negative relationship with home production and the home country's holdings of home bonds. Thus, we argue that bond holding dynamics can be explained by risk-hedging motivation against real exchange rate risk.

This study examines the link between macroeconomic variable and hedging motive and shows that bond holding dynamics is affected by state variables (also known as fundamental macro variables), such as productivity. Bond investors who want to hedge against real exchange rate risk consider movements in state variables because changes in the state variables affect the Cov(Px, Rx). These results answer why portfolio investors consider the macroeconomic variable.

The structure of this paper is as follows. Section 1 presents the aim of the study, methodological approach, and literature review. Section 2 presents the structure of the DSGE model. Section 3 derives bond holding equilibrium and dynamics in the model. Section 4 presents the simulation results of a theoretical model. Section 5 presents the data analysis and compare it with the empirical results in Section4. Section 6 presents the conclusion and implication.

II. Model

This study constructs a two-country DSGE model with two bonds. We assume that there are two symmetric countries, home and foreign, and each country produces its goods. Each country's households hold both home and foreign bonds and face cash in advance constraint.

Household maximizes the following expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \qquad where \;\; U(C_t, L_t) = \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \kappa \frac{L_t^{1+\mu}}{1+\mu}\right] \;, and \quad \kappa > 0 \tag{1}$$

where L_t is the labor supply of the home country's household and C_t is a home country's aggregate consumption consisted of the home-produced good consumption $c_{H,t}$ and the foreign-produced good consumption $c_{F,t}$ as in the following CES function.

$$C_t = \left[\lambda^{\frac{1}{\theta}} c_{H,t}^{\frac{\theta-1}{\theta}} + (1-\lambda)^{\frac{1}{\theta}} c_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

θ is the elasticity of substitution between home and foreign goods and λ is the share of consumption for home good. The paper assumes that there is a consumption home bias ($\frac{1}{2} < \lambda < 1$).

P_t is the home country's aggregate consumption price as:

$$P_t = \left[\lambda P_{H,t}^{1-\theta} + (1-\lambda) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3}$$

where $P_{H,t}$ is the price of home good and $P_{F,t}$ is the price of foreign good.

The budget constraint of Home country's household is

$$C_t + p_{HB,t}B_{H,t} + Q_t p_{FB,t}B_{F,t} + \frac{M_t}{P_t} = w_t L_t + (p_{HB,t}+1)B_{H,t-1} + Q_t(p_{FB,t}+1)B_{F,t-1} + d_t + \frac{M_{t-1}}{P_t}$$

(4)

where w_t is real wage, and d_t is home firm's real dividends, $Q_t=\frac{P_t^*}{P_t}$ is real exchange rate. M_t is the nominal money holding. $B_{H,t}$ and $B_{F,t}$ are the amount of home and foreign bonds holding by home country respectively, and $p_{HB,t}$ and $p_{FB,t}$ denote real price of home and foreign bonds.

We also assume that each bond is zero net supply as:

$$B_{H,t} + B_{H,t}^* = B_{F,t} + B_{F,t}^* = 0$$

(5)

To obtain bond holding equilibrium and dynamics by using a higher-order approximation method, we transform the budget constraint as:.

$$P_t C_t + NFA_t + M_t = W_t L_t + \alpha_{HB,t-1}(R_t - R_t^*) + R_t^* NFA_{t-1} + D_t + M_{t-1}$$

(6)

NFA_t means net foreign asset which is the sum of holdings of home and foreign bond, $NFA_t = \alpha_{HB,t} + \alpha_{FB,t}$, where $\alpha_{HB,t} = P_t p_{HB,t} B_{H,t}$ and $\alpha_{FB,t} = P_t^* p_{FB,t} B_{F,t}$, which means nominal value of bond. W_t is the nominal wage and D_t is the nominal value of dividend.

Nominal returns of home and foreign bond are

$$R_t = \frac{P_t}{P_{t-1}} r_t, \qquad R_t^* = \frac{P_t^*}{P_{t-1}^*} r_t^*$$

where $r_t = \frac{p_{Hb,t}+1}{p_{Hb,t-1}}$, $r_t^* = \frac{p_{FB,t}+1}{p_{FB,t-1}}$

(7)

where r_t and r_t^* are real returns of home and foreign bond. The nominal bond returns mean that each bond is denominated in the country's aggregate price index and guarantees one unit of aggregate consumption goods in next periods.

Since we also assume that household holds money, M_t, a cash in advance constraint set up as:

$$M_{t-1} \leq P_t C_t + \alpha_{HB,t} + \alpha_{FB,t} - \alpha_{HB,t-1} R_t - \alpha_{FB,t-1} R_t^* \tag{8}$$

Money supply follows an autoregressive process as:

$$lnM_t = \rho_M lnM_{t-1} + \epsilon_{M,t} \tag{9}$$

where exogenous shock ($\epsilon_{M,t}$) follow an i.i.d process with $Var(\epsilon_{M,t}) = \sigma_M^2$

The firm maximizes the nominal value of dividend, D_t. The function of the dividend is

$$D_t = P_{H,t} Y_t - W_t L_t \tag{10}$$

We assume that output, Y_t, is produced by labor only as:

$$Y_t = A_t L_t^{1-\alpha} \tag{11}$$

Productivity, A_t, is an autoregressive process as:

$$lnA_t = \rho_A lnA_{t-1} + \epsilon_{A,t} \tag{12}$$

where exogenous shock ($\epsilon_{A,t}$) follows an i.i.d process with $Var(\epsilon_{A,t}) = \sigma_A^2$. We assume that $Cov(\epsilon_{A,t}, \epsilon_{M,t}) = 0$.

Maximizing household's utility and firm's profit generate the following optimal condition:

$$\frac{C_t^{-\sigma}}{P_t} = \beta E_t\left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} R_{t+1}\right], \quad \frac{C_t^{-\sigma}}{P_t} = \beta E_t\left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} R_{t+1}^*\right] \tag{13}$$

$$\beta \frac{C_{t+1}^{-\sigma}}{P_{t+1}} (1-\alpha) Y_t P_{H,t} = \kappa L_t^{\mu+1} \tag{14}$$

Home and foreign country's optimal conditions yield

$$E_t\left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} R_{t+1}\right] = E_t\left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} R_{t+1}^*\right] \tag{15}$$

$$E_t\left[\frac{C_{t+1}^{*\ -\sigma}}{P_{t+1}^*} R_{t+1}\right] = E_t\left[\frac{C_{t+1}^{*\ -\sigma}}{P_{t+1}^*} R_{t+1}^*\right] \tag{16}$$

According to Devereux and Sutherland (2010), we derive bond holding equilibrium and dynamics by using (15) and (16).

The Market clearing conditions are following equations:

$$M_t = p_{H,t} Y_t \tag{17}$$

$$C_t + C_t^* = Y_t + Y_t^* \tag{18}$$

III. Solution of model

1. Static equilibrium of bond holding

In this section, we derive bond holding equilibrium and dynamics using the method proposed by Devereux and Sutherland (2010) and Till and van Wincoop (2010). According to Devereux and Sutherland (2010) and Till and van Wincoop (2010), portfolio equilibrium and bond holding dynamics can be obtained by taking second-order and third-order approximations to the optimality condition (15) and (16).

First, we define the bond holding equilibrium. the first-order approximation of budget constraint :

$$\frac{\bar{C}}{\bar{Y}}\left(\hat{P}_{t+1}+\hat{C}_{t+1}\right)+N\hat{F}A_{t+1}=\hat{P}_{H,t+1}+\hat{Y}_{H,t+1}+\frac{\bar{\alpha}_{HB}}{\beta\bar{Y}}\left(\hat{R}_{t+1}-\hat{R}^{*}_{t+1}\right)+\frac{1}{\beta}N\hat{F}A_{t}$$

where $N\hat{F}A_t=\frac{NFA_t-N\bar{F}A}{\bar{Y}}$

(19)

where $\bar{Y}=1,\ \bar{P}=\bar{P}_H=1\ ,\bar{R}=\bar{R}^{*}=\frac{1}{\beta}\ \ ,\ N\bar{F}A=0$.

In Equation (19), $\bar{\alpha}_{HB}=\bar{P}\bar{p}_{HB}\bar{B}_H$ is a steady state of nominal bond holding and represents the nominal value of home bond held by the home country, which is only incorporated into the budget constraint.

Thus, the bond holding equilibrium is defined as:

$$\bar{B}_H=\frac{(1-\beta)}{\beta}\bar{\alpha}_{HB}$$

(20)

Second-order approximation of equation in (15) and (16) yields

$$E_t[\{-\sigma(C_{t+1}-C^{*}_{t+1})-(P_{t+1}-P^{*}_{t+1})\}(R_{t+1}-R^{*}_{t+1})]=0$$

(21)

According to Devereux and Sutherland, equation (21) is must hold, in equilibrium, and the bond holding equilibrium can be derived by substituting other first-order approximation equations of the model into equation (21) and replacing $\frac{\bar{\alpha}_{HB}}{\beta\bar{Y}} E_t\left(\hat{R}_{t+1} - \hat{R}^*_{t+1}\right)$ to an exogenous mean-zero i.i.d random variable ξ_t .

By using (17), equation (21) can be transformed as

$$Cov_t\left[\hat{P}_{X,t+1}\hat{R}_{X,t+1}\right] = 0$$

where $\hat{P}_{X,t+1} = \left(\hat{P}_{t+1} - \hat{P}^*_{t+1}\right), \hat{R}_{X,t+1} = (R_{t+1} - R^*_{t+1})$

(22)

Equation (22) means that steady state of bond holding is determined at the *Cov(Px, Rx)* is zero.

2. Bond holding dynamics

$\bar{B}_H$ is a static equilibrium as a steady-state of home bond holding by home country. However, it is necessary to verify the time-varying of bond holding for more realistic explanation about the determinants of bond holdings.

A second-order approximation of budget constraints is

$$\frac{\bar{C}}{\bar{Y}}\left(\hat{P}_{t+1} + \hat{C}_{t+1} + \frac{1}{2}\hat{P}_{t+1} + \frac{1}{2}\hat{C}_{t+1} + \hat{C}_{t+1}\hat{P}_{t+1}\right) + N\hat{F}A_{t+1} - \frac{1}{\beta}\hat{R}^*_{t+1}N\hat{F}A_t$$

$$= \hat{P}_{H,t+1} + \hat{Y}_{H,t+1} + \frac{1}{2}\hat{P}_{H,t+1} + \frac{1}{2}\hat{Y}_{H,t+1} + \hat{P}_{H,t+1}\hat{Y}_{H,t+1}$$

$$+\tilde{\alpha}_{HB}\left(\hat{R}_{t+1} - \hat{R}^*_{t+1}\right) + \frac{1}{2}\tilde{\alpha}_{HB}\left(\hat{R}^2_{t+1} - \hat{R}^{*2}_{t+1}\right) + \frac{1}{\beta}\hat{\alpha}_{HB,t}\left(\hat{R}_{t+1} - \hat{R}^*_{t+1}\right)$$

where $\hat{\alpha}_{HB,t} = \frac{\alpha_{HB,t} - \bar{\alpha}_{HB}}{\bar{Y}}$

(23)

$\hat{\alpha}_{HB,t}$ represents the dynamic of the value of home bond held by home country.

From $\hat{\alpha}_{HB,t}$, we can derive $\hat{B}_{H,t}$ which is a deviation in bond holding from the steady-state denoting a bond holding dynamics.

$$\hat{B}_{H,t} = \frac{1-\beta}{\beta \bar{B}_H} \hat{\alpha}_{HB,t} - \hat{P}_t - \hat{p}_{HB,t} \tag{24}$$

A third order approximation to equations (15) and (16) yields

$$E_t \left[(\hat{c}_{t+1} - \hat{c}^*_{t+1}) + \frac{1}{2} (\hat{c}^2_{t+1} - \hat{c}^{*\,2}_{t+1})(\hat{R}_{t+1} - \hat{R}^*_{t+1}) + \frac{1}{2} (\hat{c}_{t+1} - \hat{c}^*_{t+1})(\hat{R}^2_{t+1} - \hat{R}^{*\,2}_{t+1}) \right] = 0$$

$$\text{where } \hat{c}_{t+1} = -\sigma \hat{C}_{t+1} - \hat{P}_{t+1},\ \hat{c}^2_{t+1} = \left(-\sigma \hat{C}_{t+1} - \hat{P}_{t+1}\right)^2, \hat{c}^*_{t+1} = -\sigma \hat{C}^*_{t+1} - \hat{P}^*_{t+1},$$

$$\hat{c}^{*\,2}_{t+1} = \left(-\sigma \hat{C}^*_{t+1} - \hat{P}^*_{t+1}\right)^2 \tag{25}$$

$\hat{B}_{H,t}$ can be expressed as linear function of the state variables like as:

$$\hat{B}_{H,t} = \gamma_i{}' z_{i,t} \tag{26}$$

where $z_{i,t}$ is a vector of state variable i.[3] This equation means that state variables such as inflation, money supply, and productivity affect bond holding dynamics.

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[3] To derive a equitation, we use a state-space solution provided by Dynare which is similar to the methods proposed by Lombardo and Sutherland(2007). Devereux and Sutherland(2010) argue that any of methods proposed by Judd (1998), Jin and Judd(2002), Sims (2000), Kim et al. (2008), Schmitt-Grohe´ and Uribe (2004), and Lombardo and Sutherland (2007) can be used to derive a second order accurate solution.

We can obtain the equation of bond holding dynamics (26) by substituting other first and second order approximation equations of the model into equation (25) and replacing $\frac{1}{\beta}\hat{\alpha}_{HB,t}\left(\hat{R}_{t+1}-\hat{R}^{*}_{t+1}\right)$ to an exogenous i.i.d variable ξ_t.

The equation (26) can be transformed as:

$$Cov_t\left[\hat{P}_{X,t+1}\hat{R}_{X,t+1}\right]=\frac{1}{2(\sigma-1)}\left\{\left(\hat{\hat{c}}^{2}_{t+1}-\hat{\hat{c}}^{*2}_{t+1}\right)\left(\hat{R}_{t+1}-\hat{R}^{*}_{t+1}\right)-\left(\hat{\hat{c}}_{t+1}-\hat{\hat{c}}^{*}_{t+1}\right)\left(\hat{R}^{2}_{t+1}-\hat{R}^{*2}_{t+1}\right)\right\} \tag{27}$$

In the equation (27), unlikely equation (22), the *Cov(Px,Rx)* moves by the third-order terms. It can be interpreted that the bond holding changes when the *Cov(Px,Rx)* changes due to the effect of state variables.

IV. Simulation results

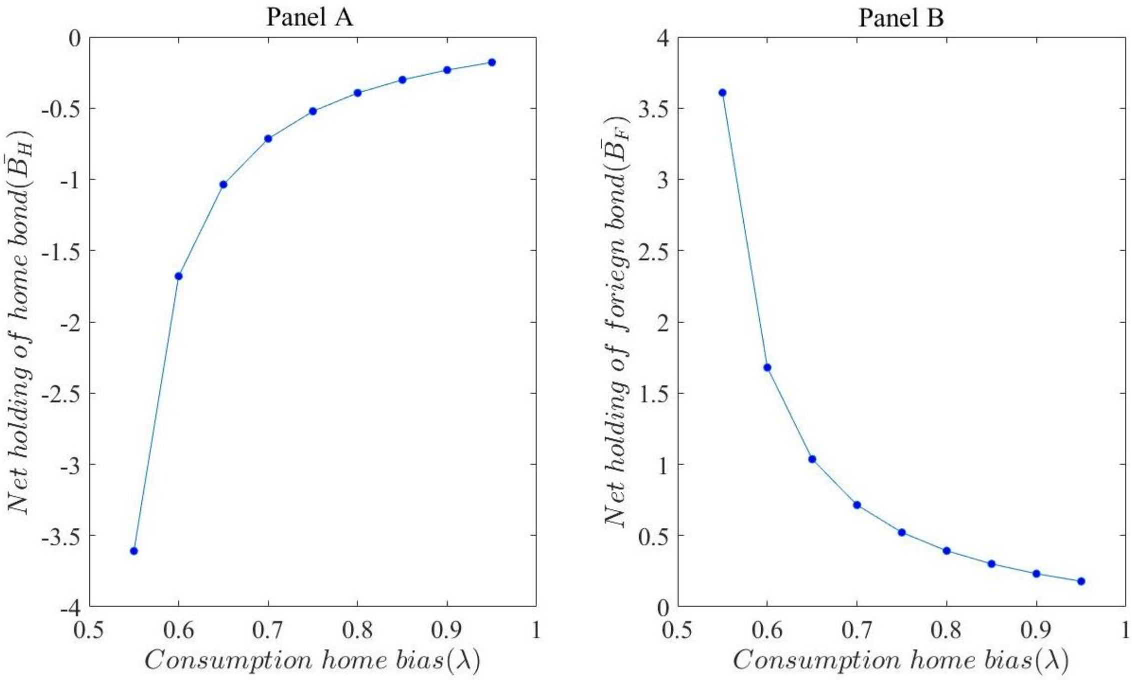
1. Static equilibrium of bond holding

In this section, we investigate the bond holding equilibrium ($\bar{B}_H$,$\bar{B}_F$); as shown in Figure1, we rely on the simulation to confirm the relationship between bond holding equilibrium and consumption home bias (λ).

For calibration, we choose all values of parameters which were typically used in previous literature. The discount factor (β) is set at 0.97 so that the steady-state of the annual interest rate is 3%. The labor share of production function ($1-\alpha$) is set at 0.67. The risk-aversion (σ) is equal to 2. The elasticity of substitution between home and foreign goods (θ) is set at 5 because most previous literature argues that θ is more abundant than unity. The labor supply elasticity of household's utility (μ) is set at 0.5. For exogenous shocks, we normalize productivity shock (σ_A) and monetary supply shocks (σ_M) at both 1%. The persistence parameters on the shock equal to 0.9 ($\rho_A=\rho_M=0.9$) Finally, we assume that there is a consumption home bias ($0.5<\lambda<1$).

Figure 1 confirms that optimal bond holding equilibrium is a short position, which implies that home bonds held by the foreign country (liabilities) are much more than home bonds held by the home country (assets). The results show that a home country's long (short) position in foreign (home) bonds is the portfolio equilibrium; previous studies also had similar results. For instance, Amdur (2010) indicated that major advanced countries have a long position in foreign bonds (short position in home bonds)

<Figure1> Bond holding equilibrium of the home country.



2. Bond holding dynamics

In the previous section, we derived the static equilibrium of bond holding equilibrium which is non-time-varying variables. However, in practice, bond holdings of investors are affected by several factors. Theoretically, bond holding is time-varying variables, so it moves from the equilibrium when exogenous shock occurs. Thus, we derive a bond holding dynamic function using a higher-order approximation. We then employ the bond holding dynamics function to determine how productivity and money supply shocks affect bond holding. To achieve this, based on the literature, we further assume that consumption home bias (λ) is 0.85.

Equation (28) represents the home bond holding dynamics function derived using higher-order approximation. The equation shows that bond holding dynamics are affected by state variables, such as price and money supply, which are usually the fundamental macro variables.

$$\hat{B}_{H,\mathrm{t}} = -0.01N\hat{F}A_t - 0.75\hat{A}_t + 1.41\hat{A}_t^* + 1.39\widehat{M}_t - 1.19\widehat{M}_t^* - 0.07\hat{p}_{HB,t} + 0.07\hat{p}_{FB,t} - 0.07\hat{P}_t + 0.07\hat{P}_t^* \quad (28)$$

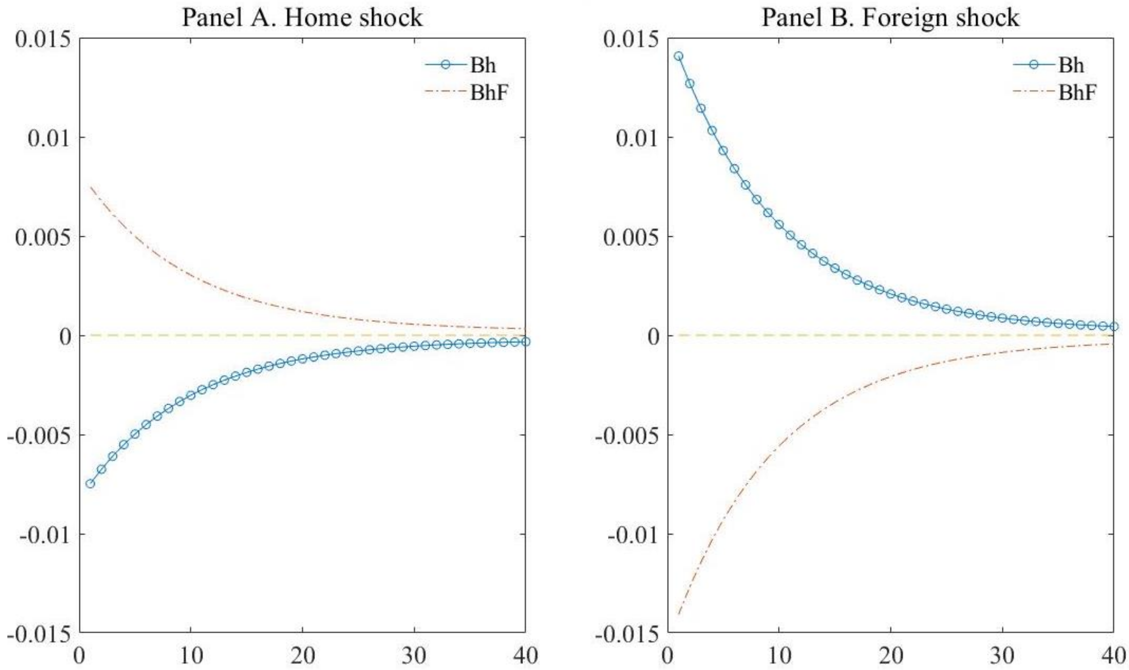
This result is consistent with Devereux and Sutherland (2010) and Till and van Wincoop (2010). They derived the capital flow function of state variables using a higher-order approximation method. For instance, Devereux and Sutherland (2010) derived a bond holding function using the production economy model and showed that an increase in home income decreases home bond holdings. Till and Wincoop (2010) briefly explained that the first-order component is a function of the second-order and third-order components, and state variables drive the changes in the third-order components.

Equation (28) is used to simulate the movement of bond holding after exogenous shock. Panel A of Figure 2 shows how positive productivity shock affects the home country's holding of home bonds. When there is a 1% positive productivity shock in home country, home bonds held by the home country's household decreases by 0.75%. By contrast, Panel B shows that home bonds held by the home country's household increases by 1.5% when there is a 1% positive productivity shock in foreign country.

The results imply a motivation to hedge against real exchange rate risk as proposed by Coeurdacier and Gourinchas (2016), Coeurdacier and Rey (2014), and Amdur (2010). They argued that the home country's household hedges against real exchange rate risk by holding a home bond whose returns have a positive relationship with the relative home price. When home price increases, the purchasing power of households decreases. However, if the return on home bonds increases when home price increases, the purchasing power of households will be stable, so households hold more home bonds. This argument means that when Cov(Px, Rx) decreases, the home country decreases its holding of home bonds. Their results imply a zero-order portfolio equilibrium but do not show the dynamics, so they imply that the relationship between the home price and returns on home bonds is constant.

Unlike the previous studies, the simulation results of this study indicate the dynamics using a higher-order approximation, implying a movement in the relationship between the home price and returns on home bonds. The analysis of the simulation results is as follows. Increasing home output by the home country's positive productivity shock decreases the home price, whereas the excess return on home bonds does not move because based on the study of Devereux and Sutherland(2010), we assume that the excess return on the bonds follows a i.i.d process. Thus, after positive productivity shock occurs, Cov(Px, Rx) becomes negative, so the home country's households decrease their home bond holdings

<Figure2> Bond holding dynamics by productivity shock



Notes 1) **Bh** and **BhF** denotes $\hat{B}_{H,t}$ and $\hat{B}^{*}_{H,t}$ respectively

2) X-axis are periods and Y-axis are change rate from the steady-state of bond holding

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As mentioned earlier, portfolio equilibrium in a steady state is derived under the condition that Co*v(Px, Rx)* is zero. However, the condition can be time-varying after exogenous shock. The simulation result implies that positive productivity shock leads to a negative *Cov(Px*, *Rx)*, and the home bonds held by the home country decreases. Moreover, Panel A shows that the foreign country increases its holding of home bonds after positive productivity shock occurs. A lower home price implies a higher foreign price to the foreign investor; thus, the positive relationship between the foreign price and the returns on home bonds is strengthened when the excess returns on bonds remain positive.

V. Empirical Test

In the previous section, we derive the bond holding equilibrium and bond holding dynamics through the two-country DSGE approach and find that the optimizing agents reduce the domestic bond after the positive home productivity shock. This section implements the empirical analysis to investigate that the actual data shows the same finding as simulation results.

To do so, we use a panel vector-autoregression (panel VAR) model and reports impulse responses of U.S. bonds transaction following a positive production shock. In the analysis, we use net bond transactions between the two countries, U.S(home) bonds denote that U.S. residents purchased from foreign countries minus U.S. bond that foreigners purchased from U.S. residents. These net bond transactions were used as a ratio of GDP (%). The empirical results are important to gain the insight into the behavior of actual investors in response to changes in economic conditions with home and major countries.

1. Data

The dependent variable is the U.S. bond transactions data. We used the monthly data from January 2001 to December 2019 on the Treasury International Capital (TIC) system on the U.S. Treasury Department. We select top-ranking foreign transaction countries of the U.S.,[4] excluding offshore financial centers (tax heaven countries)[5]. Based on the volume of transactions in Dec 2019, eight foreign countries were selected, Canada, China, France, Germany, Japan, Korea, Singapore, and the U.K..

The bond transaction data that we used comprise of treasury bonds, government bond, federal agencies' bonds, and U.S. corporate bonds. The data shows the outward and inward transactions of long-term bonds with partner countries.

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[4] A detailed explanation of the data is in the Appendix.

[5] We excluded several tax heaven countries, such as Mauritius, British Virgin Islands, Bermuda, and Bahamas because we could not trust the price level and industrial price index of these tax haven areas (offshore financial centers), which are significant important macro-economic variables.

We use the macroeconomic variables which are included in the bond holding equation of the previous section as the explanatory variables. We review the relevant literature to choose the appropriate macroeconomic variables and use industrial production index (IPI), consumer price index (CPI), and interest rate as the explanatory variables. Moreover, we calculate the first difference of the logarithm of IPI and CPI. The interest rates are used by calculating the difference between the current and previous period. A detailed description of the variables is included in Appendix B. The global financial crisis is the dummy variable[6].

Based on these data, we derive the hedge ratio (covariance-variance ratio) using the price and interest rates variables of the two countries. According to Couerdacier and Gourinchas (2016), the hedge ratio is related to hedging against real exchange rate fluctuation; an increase in the covariance-variance ratio term implies an increased hedging motive against real exchange rate risk.

2. Model description and statistical test

We employ a reduced-form the panel vector-autoregression (panel VAR) model developed by Love and Zicchino (2006) in Generalized Method of Moments (GMM) environment. This model considers all variables as endogenous variables and allows the individual heteroscedasticity of variables by introducing fixed effects. The model can be simply written as follows:

$$X_{i,t} = \mu_i + \Phi(L)\ X_{i,t} + \varepsilon_{i,t}.$$

$$\text{where } X_{i,t} = \begin{bmatrix} Y_{it}\ (or\ M_{it}) \\ \frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(\ RX_{i,t})} \\ Bh_{i,t} \end{bmatrix} \qquad (29)$$

In the basic model, $X_{i,t}$ represents a macro-variable vector (IPI of the U.S., hedge ratio, and bond transactions between the U.S. and foreign country). We include the hedge ratio, where $PX_{i,t}$ equals

[6] We follow the NBER recession period in selecting the GFC period, from December 2007 to June 2009. However, the empirical result without the GFC period is also the same.

$P_{i,t} - P^{*}{}_{i,t}$, which is the difference between the CPI of the home and foreign countries, and $RX_{i,t}$ equals $R_{i,t} - R^{*}{}_{i,t}$, which is the difference between the bond yield of the home and foreign countries. The bond transaction variable is represented as $Bh_{i,t}$. In addition to the basic model, we extend the model by using M2 variable as the proxy for money supply (M_{it}) shock instead of home production shock. μ_i is the deterministic component, and $\varepsilon_{i,t}$ is the residuals.

For ordering problem, it is general to order output, prices before property prices such as bond like in the previous literature (Belke, Orth, and Setzer (2010)). To choose the lag length, we conduct the lag selection test of the individual countries. We determine the lag length 1 as the optimal selection of home production shock and home monetary supply shock based on the Bayesian Information Criteria (BIC).

In Table 1, we use PVAR Granger Causality test to detect the causal relationship between variables. We find that the bond transactions of home country $(Bh_{i,t})$ can be both explained by home production (Y_{it}) and hedge ratio ($\frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(\ RX_{i,t})}$). In addition, as the first line of Table 1, the lagged variables of home production cause the hedge ratio, but the lagged variables of bond transaction cannot cause the hedge ratio. These results suggest that home production affects the hedge ratio at first, and home production indirectly causes bond transaction using the hedge ratio as the intermediary variable. It also indicates that the motivation for hedging against real exchange rate significantly affects the home country's bond investment; this is consistent with the results presented in the theoretical section.

<Table 1> Granger Causality test

Varables	F-statistic (Prob)	Direction
Y_{it} does not Granger Cause $\frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(\ RX_{i,t})}$	25.890 (0.000)	$Y_{it} \rightarrow \frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(\ RX_{i,t})}$
$Bh_{i,t}$ does not Granger Cause $\frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(\ RX_{i,t})}$	0.035 (0.852)	$Bh_{it} \nrightarrow \frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(\ RX_{i,t})}$
Y_{it} does not Granger Cause $Bh_{i,t}$	31.756 (0.000)	$Y_{it} \rightarrow Bh_{it}$
$\frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(\ RX_{i,t})}$ does not Granger Cause $Bh_{i,t}$	6.058 (0.014)	$\frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(\ RX_{i,t})} \rightarrow Bh_{it}$

Note : $PX_{i,t}$ denotes $P_{i,t} - P^{*}{}_{i,t}$. $RX_{i,t}$ denotes $R_{i,t} - R^{*}{}_{i,t}$

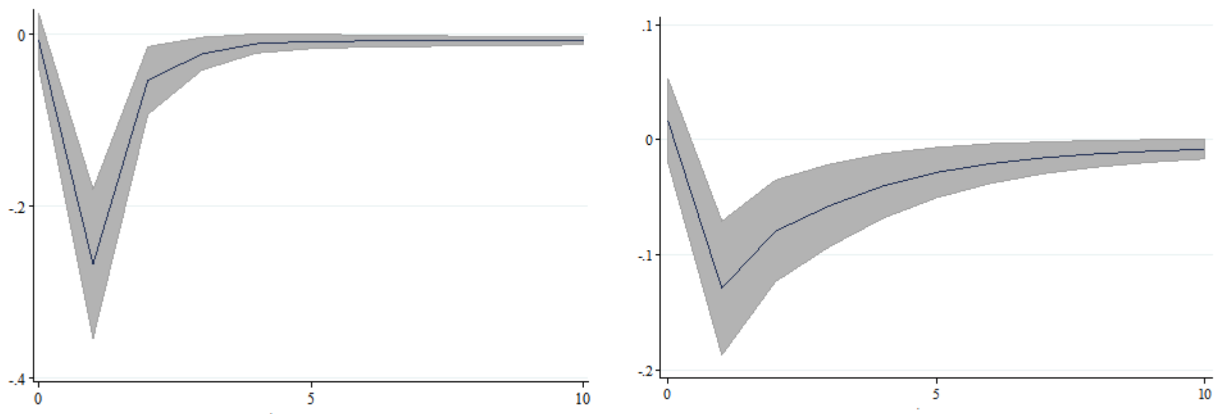
3. Empirical results

Figure 3 shows the impulse response of the positive production shock of the home country (the U.S.). We plot the impulse response functions using the PVAR results. The level of confidence bands is 95% and the confidence band is estimated using the Gaussian approximation based on 200 times of Monte Carlo iteration of the estimated PVAR results.

According to Figure 3, the home country immediately reduces home bond transaction after the positive shock of home production. Contrary to the response for home bond transaction in Panel A, home country immediately increases foreign bond transaction after the home production shock at the first period to home production shock. Immediately after the home production shock, the covariance-variance variable is also negatively affected. They peak in the first one or two periods and then become zero afterward. The results imply that the *Cov(Px, Rx)* is negatively affected by the positive home production shock. The Granger causality and impulse response results are consistent with those presented in the theoretical section. The impulse response of the money supply (M_{it}) shock is also the same as what is presented in the theoretical section, and the impulse response graph is presented in Appendix B. The empirical results indicate that the home country's production shock affects negatively to the covariance with relative prices and bond yields. This shock also affects the volume of bond transaction between home and foreign countries. The empirical results are successfully mapped with the simulation results of the previous section.

<Figure 3> Impulse response to the positive production shock of the U.S. (Y_{it})

Panel A: Response of $Bh_{i,t}$ Panel B: Response of $\frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(RX_{i,t})}$



Notes: X-axisare t period and Y-axis are the impulse response of the variables to the production shock

VI. Conclusion

This study investigates the bond holding dynamics by productivity shock. It shows that portfolio investors change their bond holdings to hedge against real exchange rate risk. These results are consistent with those of previous studies. For instance, Couerdacier and Gourinchas (2016) argued that the home country's households hold more home bonds when *Cov(Px, Rx)* is positive.

Unlike the previous studies that analyzed the constant covariance and steady state portfolio equilibrium, we contribute to the literature by analyzing the relationship between bond holding dynamics and movements in the covariance. The simulation results of this study show that the home country's household decreases its home bond holdings when it experiences positive productivity shock. These results imply that households sell their home bonds to hedge against real exchange rate risk because a reduction in the home price due to the productivity shock makes the covariance negative. The empirical test also shows that an increase in the home country's production decreases both the covariance and home bond transactions.s

The results indicate that state variables, such as production, leads to movements in the covariance. This result can be one reason why portfolio investors consider fundamental macro variables when they are investing in financial assets. Thus, state variables affect the optimal portfolio equilibrium and dynamics in achieving efficient risk-sharing. These results have the following implications. The exchange rate affects various factors, such as the purchasing power of households or the value of financial assets, and investors hold considerable bonds to hedge against real exchange rate risk. Thus, the constant movements of the real exchange rate can reduce capital flow fluctuation.

This study improves the portfolio allocation model using a higher-order approximation method in the DSGE model with two bonds. We adopt a labor production economy, whereas most previous studies used an endowed economy. However, there is still room for developing a model, such as incorporating capital accumulation, investment, and equity assets into the model. Through an empirical test, we verify the relationship between covariance and bond holdings and calculate the covariance using each country's CPI index. In a future study, various covariances, such as the trade-weighted exchange rate or financial exchange rates developed by Lane and Shambaugh (2010), can be used for the robust check.

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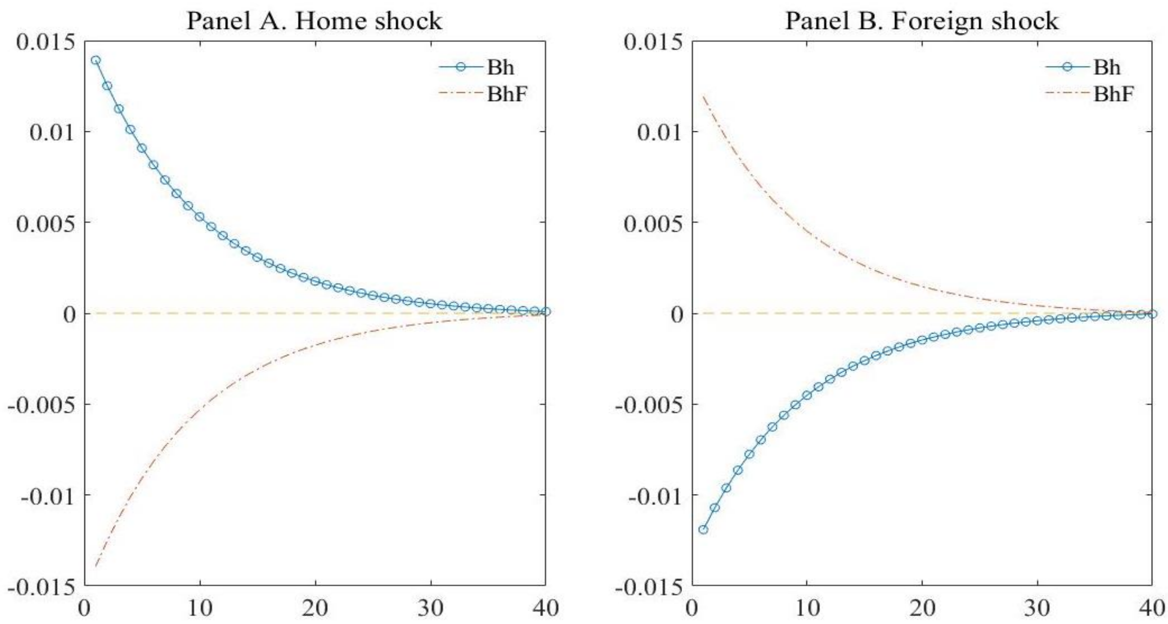
Appendix A

<Figure A.1> Bond holding dynamics by the Money supply shock.



Notes 1) **Bh** and **BhF** denotes $\hat{B}_{H,t}$ and $\hat{B}^{*}_{H,t}$ respectively

2) X-axis are t periods and Y-axis are change rate from the steady-state of bond holding)

Appendix B

<Table B.1> Data Statistics

Variable	Description	Source
Y_{it} $(Y^{*}{}_{it})$	Industrial Production Index (2010=100, Units)	IMF
$P_{i,t}(P^{*}{}_{i,t})$	Consumer Price Index (2010=100, Units)	IMF
$R_{i,t}(R^{*}{}_{i,t})$	Government bond yields (3 Month)	Bloomberg
$M_{i,t}(M^{*}{}_{i,t})$	Monetary base growth rate (percent)	IMF

<Table B.2> Summary statistics of explanatory variables

Variable	US	Canada	China	France	Germ-any	Japan	Korea	Singa-pore	UK
# of Obs	119	119	120	119	120	119	120	120	119
$Y_{it}(Y^{*}{}_{it})$	98.8 (4.4)	112.2 (9.1)	102.3 (4.2)	101.3 (2.1)	98.4 (2.8)	100.3 (6.7)	109.3 (4.6)	88.2 (9.6)	107.7 (4.1)
$P_{i,t}(P^{*}{}_{i,t})$	108.8 (5.0)	108.3 (5.0)	113.6 (7.3)	105.4 (2.8)	102.4 (2.2)	101.7 (1.8)	109.1 (4.6)	110.9 (4.5)	110.6 (5.7)
$R_{i,t}(R^{*}{}_{i,t})$	0.6 (0.8)	0.9 (0.4)	0.9 (0.4)	-0.1 (0.5)	-0.0 (0.1)	0.1 (0.2)	2.3 (0.7)	0.8 (0.6)	0.5 (0.2)
$\frac{Cov(PX_{i,t},\ RX_{i,t})}{Var(RX_{i,t})}$		.0284 (.465)	.1204. (.712)	.0144 (.346)	.0860 (.4093)	.0017 (.190)	.0332 (.884)	.072 (.6994)	-.180 (2.139)

s

<Table B.3> Results of panel unit root tests

Panel	A_{it}	$M_{i,t}$	$Cov(P_{i,t} - P^{*}_{i,t},\ R_{i,t} - R^{*}_{i,t})$	$Bh_{i,t}$	$BhF_{i,t}$
	First Diff	Growth rate	-	First Diff	First Diff
LLC			-	-20.94***	-1.26***
IPS	-8.06***	-8.06***	-12.313***	-35.04***	-4.97***
ADF	-8.81***	-8.81***	-17.058***	-50.01***	-4.77***
PP	-56.87***	-56.87***	-14.158***	-56.87***	-34.69***

<Figure B.1> Impulse response by the money supply ($M_{i,t}$)

Response for the $Bh_{i,t}$