A Credit Rationing Model of Limited Collaterals and Medium-Risk Borrowers

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<Abstract>

This paper analyzes the effects of limited collaterals and explains who and how much will be damaged under the credit rationing. We extend the credit rationing model of Bester(1985) and our equilibrium results show that the most damaged ones are medium-risk borrowers whose collateral valuation is low. We also show that some rejected credit application could be accepted if the bank can narrow down the risk range of the unidentified borrower group, which means that the bank's screening ability is enhanced or information asymmetry is alleviated. The policy implications for the Micro, Small & Medium Enterprises (**MSME**) finance are as follows: 1) alleviate information asymmetry or strengthen the bank's screening ability, 2) support or guarantee the borrowers to increase their collateral valuations, and 3) focus on the medium risk borrowers under the credit rationing.

Key words: Credit rationing, Collateral valuation, Middle range risk borrowers, Microfinance, SME finance, Information asymmetry

JEL code: G21, G28

1. Introduction

Loan contracts between borrowers and lenders could be deterred by information asymmetry, and thus there might be a credit rationing(Freixas and Rochet, 1988). Lenders such as banks may reject some borrowers who are willing to pay high interests because of the adverse selection or the incentive effects(Stiglitz and Weiss, 1981). If the interest burden is too heavy, then only borrowers with riskier investments will apply for the loan ex-ante. Ex-post, investors might choose projects with a higher probability of default or exert less efforts to succeed.

Bester(1985) argues that the contract menu which combines both collaterals and interests can be used as a self-selection mechanism: high risk borrowers prefer the high interest and low collateral contract and vice versa for the low risk ones. The use of collateral seems to be a widespread feature of credit contracts(Coco, 2000; Steijvers and Voordeckers, 2009). This kind of separating equilibriums, however, requires that all the borrowers have collateralizable wealth(Bester, 1987).

In the real world, there must be some borrowers who lack the good collaterals. This paper analyzes the effects of limited collaterals on the loan contracts and explains who will be damaged from it and how much. We propose a credit rationing model in which the borrower's risk types and the lender's collateral valuations are continuous variables. This is a partial extension of Bester(1985), Jin & Zhang(2019), and Kwon(2019), but with some additional restrictions.

Borrowers who lack collaterals need some guarantee or insurance which is expensive and imposes additional costs to the borrowers. Some collateral assets are specific or costly to transact for the lenders and thus these collaterals could be much more discounted in the loan contracts. For example, let's suppose that a firm has only some kinds of assets as collaterals for its bank loan application. If the banks discount the value of the collateral assets by half, then the firm should provide double amount of collaterals to the banks.

Thus the borrowers are characterized by the two parameters: 1) the default risk or the failure probability of the project and 2) the collateral valuation which reflect the transaction costs or the specificity of the borrower's wealth. Our equilibrium results show that, under the credit rationing, the most damaged ones are the medium risk borrowers whose collaterals are valued low. When the collateral valuation is high, the borrowing costs are low and the credit rationing is reduced. Information also matters. If the bank's screening ability is improved and thus the risk characteristics of the unidentified borrowers becomes more similar, then the borrowing costs could drop.

Interestingly, the medium risk borrowers bear more costs relative to the higher or lower risk ones. The high risk borrowers choose to pay high interests and low collaterals, and vice versa for the low risk ones. For the medium risk borrowers, the separating mechanism is ambiguous and it makes the bank loans expensive. Some numerical examples are made and visualized with contour graphs to explain the theoretical results more intuitively. We also briefly talk about the real world policy implications such as the governmental guarantee programs for the SMEs(small-and-medium sized enterprises)(Kwon, 2013). Microfinance policy makers also could gain insights from our arguments.

The remainder of this paper is organized as follows. Section 2 models a self-selection mechanism under credit rationing situation. Section 3 comes up with the equilibrium solutions for the model. Section 4 gives some numerical examples and graphs to explain and visualize the equilibrium results. Section 5 applies the model to the MSME financing support policy analysis. Lastly section 6 summarizes and concludes our arguments.

2. Model

2.1. Investment Projects

All the agents are assumed to be risk neutral and maximize their own expected profits. The banking industry is competitive and thus each bank is expected to earn zero net profits. Each borrower is endowed with an one dollar investment project whose random return is 0 with probability θ or Y with the remaining chance. The failure probability θ of each project is in the continuum support $[0, \theta_H]$. Here the maximum default probability parameter θ_H represents the severity of information asymmetry in the borrower group. The expected return $(1-\theta)Y$ is the same as a constant value, say E, across all the borrowers à la Rothschild and Stiglitz(1976)'s Mean Preserving Spread. Therefore, $Y = \frac{E}{1-\theta}$ and we assume that it is larger than one and thus all the projects have positive NPV(net present value). To remove the possibility of strategic bankruptcies or other complicate opportunistic behaviors, we assume that the default event is verifiable but the value of Y is not.[1] Since, in the real world, the bankruptcy events are more easy to verify than the performance variables such as profits, we could justify that assumption.

[1] This assumption can be formalized by introducing additional equation: $Y = \mu + \epsilon$ where μ is a constant and ϵ is a random error term.

2.2. Collateral Valuations

Now every borrower does not have one dollar and needs a bank loan for the investment. Each borrower has only one kind of assets whose size is sufficiently large for the collateral provision. The borrower's one dollar collateral is valued $k < 1$ to the banks. This kind of discounted valuation is due to the collateral asset specificity(Williamson, 1981) or other transaction costs. We assume that the collateral valuation k is the same for all the banks because the transaction costs and the asset characteristics can usually be observed by the outsiders. Therefore the bank can distinguish each borrower into the groups of collateral valuation k but cannot tell the individual borrower's risk type θ. For example, bank categorize the loans into some groups: real estate collateral loans, special machine collateral loans, loans with no collateral but guarantee insurance, etc. The collateral provision expenses are different for the borrower groups.

2.3. Bank's Zero Profits and Loan Contract Menu

Given the borrower group with the collateral valuation k, banks offer loan contract menu of the ex-post interest payment of $R(\theta)$ and ex-ante collateral provision of $C(\theta)$, considering the failure probability θ. Given the funding cost R_o, the bank's expected profit $\rho(\theta)$ must be zero since we assume the perfect competition in the bank industry. When all the borrowers tell the truth, the bank's zero-profit equation for the collateral valuation k borrowers is:

$$\rho(\theta|k,\theta_H) = \int_0^{\theta_H}\{(1-\theta)R(\theta|k,\theta_H) + k\theta C(\theta|k,\theta_H)\}f(\theta|k)d\theta - R_o = 0 \qquad (1)$$

where $f(\theta|k)$ is the probability density function of the default risk θ of the borrower group whose collateral valuation is k. To deduce tractable results and not to depend on the shape of the borrower risk density functions, we add no-subsidy condition in which the bank earns zero profits from each borrower whose valuation is k:

$$\rho(\theta|k,\theta_H) = (1-\theta)R(\theta|k,\theta_H) + k\theta C(\theta|k,\theta_H) - R_o = 0 \qquad (2)$$

The no collateral contract is to the highest and it is $\{C(\theta_H),\ R(\theta_H)\} = \{0,\ \frac{R_O}{1-\theta_H}\}$. The full collateral contract menu is given to the lowest risk type borrower and it is $\{C(0),\ R(0)\} = \{R_o,\ \frac{R_O}{k}\}$. Note that the collateral provision is costly to the borrowers and the lower risk borrowers pay the informational rents to be separated from the higher risk borrowers. These are qualitatively the same as those of Bester(1985) and will be used as the boundary conditions for the differential equations in the proof of the next <Theorem 1>.

2.4. Incentive Conditions for Truth Telling

When the borrower with risk θ chooses the contract for the risk $\theta_F < \theta$, her expected total cost of borrowing is:

$$\tau(\theta,\ \theta_F) = (1-\theta)R(\theta_F) + \theta C(\theta_F) \qquad (3)$$

The lower risk borrower would not choose the higher risk type contract and thus the truth telling or the separating condition is $\tau(\theta,\ \theta_F) \geq \tau(\theta,\ \theta)$, that is:

$$(1-\theta)R(\theta_F) + \theta C(\theta_F) \geq (1-\theta)R(\theta) + \theta C(\theta) \qquad (4)$$

2.5. Incentive Conditions for Loan Application

For the borrowers to apply for the bank loan, their expected total costs of borrowings $\tau(\theta,\ \theta)$ should be less than the expected project return E. Later, it turns out that there could be some borrowers whose borrowing costs exceed the expected project return and that means the credit rationing could occur. The incentive condition for the loan application is:

$$(1-\theta)R(\theta) + \theta C(\theta) \leq E \qquad (5)$$

3. Continuously Separating Equilibrium

From the above equations, the continuously separating equilibrium is derived. The zero-profit and no-subsidy condition (2) holds for every risk type borrower. By differentiating (2) with the default risk θ and from the equation (2) itself, we get the following condition:

$$\rho'(\theta) = (1-\theta)R'(\theta) + k\theta C'(\theta) - \frac{R(\theta)-R_o}{\theta} = 0 \qquad (6)$$

Next, divide the truth telling condition (4) by the risk difference $(\theta - \theta_F)$ and take the limit such that the difference goes to zero, then we get the following condition:

$$(1-\theta)R'(\theta) + \theta C'(\theta) = 0 \qquad (7)$$

By solving the equation (6) and (7) with the boundary conditions for the highest and the lowest risk borrowers, that is $\{C(\theta_H),\ R(\theta_H)\} = \{0,\ \frac{R_o}{1-\theta_H}\}$ and $\{C(0),\ R(0)\} = \{R_o,\ \frac{R_o}{k}\}$, we can get the next theorems which explain the continuously separating equilibrium.

<Theorem 1> The sufficient conditions of the continuously separating equilibrium for each risk type θ with the collateral valuation k is:

$$R(\theta|k,\theta_H) = R_o\left\{1 + \frac{\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-k}}}{\left(\frac{\theta_H}{1-\theta_H}\right)^{\frac{k}{1-k}}}\right\} = R_o\left\{1 + \left(\frac{\theta}{1-\theta}\right)\left[\frac{\left(\frac{\theta}{1-\theta}\right)}{\left(\frac{\theta_H}{1-\theta_H}\right)}\right]^{\frac{k}{1-k}}\right\} \quad \text{and}$$

$$C(\theta|k,\theta_H) = \frac{R_o}{k}\left\{1 - \left[\frac{\left(\frac{\theta}{1-\theta}\right)}{\left(\frac{\theta_H}{1-\theta_H}\right)}\right]^{\frac{k}{1-k}}\right\}.$$

Therefore, the expected total cost of borrowing is:

$$\varphi(\theta|k,\theta_H) = (1-\theta)R(\theta|k,\theta_H) + \theta C(\theta|k,\theta_H)$$
$$= R_o\left\{1 + \theta\left(\frac{1-k}{k}\right)\left\{1 - \left[\frac{\left(\frac{\theta}{1-\theta}\right)}{\left(\frac{\theta_H}{1-\theta_H}\right)}\right]^{\frac{k}{1-k}}\right\}\right\}$$

<proof> We can verify that those equations satisfy the earlier conditions by some trivial arithmetics and differentiations. QED.

Given collateral valuation k and the highest default probability θ_H in the loan contracting pool, <Theorem 1> tells us the possible equilibrium contract result for the risk type θ. From this and by eliminating the risk variable θ, we can derive the equilibrium relationship between the interest payment and collateral provision in the following <Theorem 2>. The results of <Theorem 2> are depicted by <Figure 1> which is the generalization of the figure 2 in Bester(1985) and it is somewhat similar with the figure 1 in Jin & Zhang(2019). Bester(1985) explains the more general situations but only with the two risk type case. Our results deals with the continuous risk and collateral valuations. The figure 1 in Jin & Zhang(2019) depicts the continuous risk type case, but the different collateral valuations are not considered.

<Theorem 2> The loan contract menu curve for each collateral valuation k is:

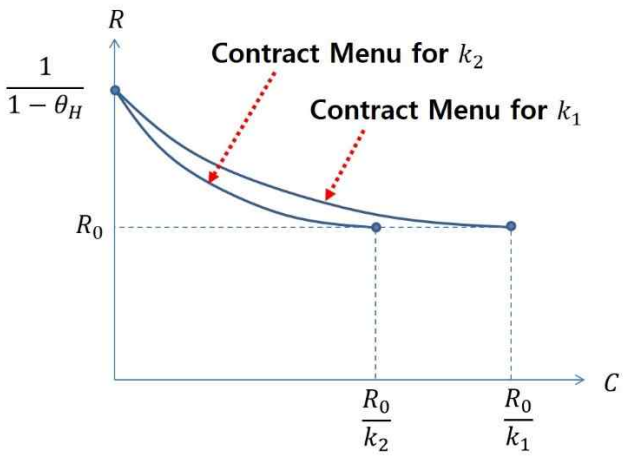
$$R(C|k,\theta_H) = R_o\left\{1 + \left(\frac{\theta_H}{1-\theta_H}\right)\left\{1 - \left(\frac{k}{R_o}\right)C\right\}^{\frac{1}{k}}\right\}$$

Given k, the first and second order differentiations are :

$$\frac{dR}{dC} \propto -\left\{1-\left(\frac{k}{R_o}\right)C\right\}^{\frac{1}{k}-1} < 0 \quad \text{and} \quad \frac{d^2R}{dC^2} \propto \left(\frac{1-k}{R_o}\right)\left\{1-\left(\frac{k}{R_o}\right)C\right\}^{\frac{1}{k}-2} > 0$$

<proof> Note that $C \leq \frac{R_o}{k}$ and thus $\frac{dR}{dC} < 0$. Others are trivially derived from the results of Theorem 1. QED.

<Figure 1> The loan contract menu curves by collateral valuations



<Theorem 2> and <Figure 1> tell us that you should provide higher collaterals if you want to borrow with lower interests and vice versa. And the marginal substitution effects of collaterals are decreasing and that means the more collaterals become much more expensive for the borrowers. Given k and θ_H, the highest risk borrower chooses the no collateral contract and the lowest one chooses the full collateral one. Here we can visually explain two issues: collateral valuations and the severity of information asymmetry matter.

In <Figure 1>, there are two menu curves for different collateral valuations. Higher valuation k and lower severity of information asymmetry θ_H lead to better loan contracts which require the borrowers to pay lower interests and lower collaterals. For example, we may think of k_1 curve as a loan contract for machine collaterals which are more firm specific or difficult to liquidate and k_2 as real estate collaterals which can be easily liquidated. Or we may think of k_1 curve as a loan contract for SMEs whose collaterals are limited or their guarantee premiums are expensive; k_2 curve may be for large corporations which are abundant with good collaterals or able to buy cheap guarantee insurance. Bank's screening is also important. If the banks can lower the highest default probability θ_H in their given borrower group, that is, the severity of information asymmetry is alleviated, then the contract menu can be improved.

Now we analyze the economic meaning of the equilibrium results in the previous section more rigorously. From <Theorem 1>, it is evident that $R'(\theta) > 0$ and $C'(\theta) < 0$. Given k, therefore, higher risk borrowers choose the menu of lower collateral and higher interest payment. On the contrary, with the risk variable θ which moves on a given curve in the <Figure 1>, the collateral valuation k or the maximum default risk θ_H shifts the curve itself. The effects of collateral valuation k on the

interest R is clear: $\frac{\partial R}{\partial k} < 0$. However the effects of k on the collateral amount is ambiguous since there are trade-off between two conflicting effects: substitution and income effects.

An increase in the collateral valuation k shifts the contract line to the left or downwards. Given risk θ, now the cost of using collaterals are lowered and it may lead to more collateral provisions, which is an substitution effect. On the other hand, the borrowers can satisfy the bank's requirement with less collateral provision, which is an income effect. These conflicts are prominent for the higher risk borrowers. For the lower risk borrowers, the income effect dominates. We can visualize these effects in <Figure 3> which depicts some numerical examples. The dotted line means the solution for $\frac{\partial C}{\partial k} = 0$: the substitution effects dominate in the left side and the income effects in the right.

Eventually the most important part is the total expected borrowing costs which is some weighted average of interests and collaterals. It is evident that the higher k and the lower θ_H makes the loan contracts less expensive. The effect of θ, however, is not so simple. The higher risk borrowers want to mimic the lower risk ones if possible, and thus pay less for their loans. Not to be pooled together with the higher risk borrowers, the lower risk ones must pay some separating costs or informational rents which are expensive collaterals in this paper's model. The lower risk borrowers provide much more collaterals but their expected costs are small since their default probabilities are also lower. The most damaged ones are the medium risk borrowers who need to separate from higher risk ones and whose collateral valuations are relatively low and thus their collateral costs are high. <Figure 4> depicts the total expected borrowing costs of some numerical examples and we can confirm these arguments. The dotted line is the solution for $\frac{\partial \tau}{\partial \theta} = 0$ and that means the highest borrowing costs for given collateral valuation k. The <Theorem 3> summarizes the effects of k and θ on the equilibrium results.

<Theorem 3> The effects of risk, collateral valuation, and information asymmetry on the contract results – that is, interests, collaterals, total expected costs – are summarized in the following <Table 1>. Here the rows are the explanatory variables and the columns are the explained. For the not-sign-determined cases of $\frac{\partial C}{\partial k}$ and $\frac{\partial \tau}{\partial \theta}$, there exist solutions for $\frac{\partial C}{\partial k} = 0$ and $\frac{\partial \tau}{\partial \theta} = 0$.

<Table 1> **Comparative Statics Analysis**

The effects of risk, collateral valuation, and information asymmetry on the contract results – that is, interests, collaterals, total expected costs – are represented as the sign of the partial differentials.

Contract results		∂R	∂C	$\partial \tau$
Default risk	$\partial \theta$	+	−	?
Collateral valuation	∂k	−	?	−
Information asymmetry	$\partial \theta_H$	−	+	+

<proof>

The proofs of $\frac{\partial R}{\partial \theta} > 0$, $\frac{\partial C}{\partial \theta} < 0$, $\frac{\partial R}{\partial k} < 0$, $\frac{\partial R}{\partial \theta_H} < 0$, $\frac{\partial C}{\partial \theta_H} > 0$, and $\frac{\partial \tau}{\partial \theta_H} > 0$ are trivial.

For the proof of $\frac{\partial \tau}{\partial k} < 0$, let $x = \frac{k}{1-k}$ and $H = \frac{\left(\frac{\theta}{1-\theta}\right)}{\left(\frac{\theta_H}{1-\theta_H}\right)}$ and $m = \log\left(\frac{1}{H}\right)$.

Note that $x > 0$ and $H \in (0,1)$ and $m > 0$ and $\tau = R_o\left\{1 + \frac{\theta}{x}(1 - H^x)\right\}$.

Since $1 + m\,x < e^{m\,x}$ and $1 + x\log\left(\frac{1}{H}\right) < \left(\frac{1}{H}\right)^x$ and $H^x(1 - x\log H) < 1$,

$$\frac{\partial \tau}{\partial k} \propto \frac{\partial}{\partial x}\left\{\frac{1}{x}(1 - H^x)\right\} = -\frac{1}{x^2} + \frac{H^x}{x^2} - \frac{H^x \log H}{x} < 0.$$

Now we prove that the solution for $\frac{\partial \tau}{\partial \theta} = 0$ exists in the interval $(0,\ \theta_H)$.

Let $p(\theta) = \left(1 + \frac{x}{1-\theta}\right)$ and $q(\theta) = H^{-x} = \left[\frac{\left(\frac{\theta}{1-\theta}\right)}{\left(\frac{\theta_H}{1-\theta_H}\right)}\right]^{-x}$.

Then $\frac{\partial \tau}{\partial \theta} \propto \frac{\partial}{\partial \theta}\{\theta(1 - H^x)\} = 1 - \left(1 + \frac{x}{1-\theta}\right)H^x = 1 - \frac{p(\theta)}{q(\theta)}$.

Note that $p'(\theta) > 0$, $q'(\theta) < 0$, $p(0) < q(0)$, and $p(\theta_H) > q(\theta_H)$.

Therefore $p(\theta)$ and $q(\theta)$ meets at $\theta^* \in (0,\ \theta_H)$.

An example of the solution for $\frac{\partial \tau}{\partial \theta} = 0$ is the dotted line in <Figure 4>.

Lastly, we prove the existence of the solution for $\frac{\partial C}{\partial k} = 0$.

$\frac{\partial C}{\partial k} \propto 1 - \left[\frac{\left(\frac{\theta}{1-\theta}\right)}{\left(\frac{\theta_H}{1-\theta_H}\right)}\right]^{-x} - \frac{x}{1-k}\log\left[\frac{\left(\frac{\theta}{1-\theta}\right)}{\left(\frac{\theta_H}{1-\theta_H}\right)}\right]$ and

$\frac{\partial C}{\partial k} = 0$ means that $\frac{k}{(1-k)^2}\log H = 1 - H^{-\frac{k}{1-k}}$.

With help of some calculations, we can get this equation : $H^{-\frac{k}{1-k}} = \exp\left\{-(1-k)\left(1 - H^{-\frac{k}{1-k}}\right)\right\}$.

Here we use the Lambert W function(**Hayes, 2005**) which is the inverse function $z = f^{-1}(y) = W(y)$ when $y = f(z) = ze^{z}$.

Let $H^{-\frac{k}{1-k}} = w$ and then $w = \exp\{-(1-k)(1-w)\}$.

Note that $-(1-k)w\exp\{-(1-k)w\} = -(1-k)\exp\{-(1-k)\}$.

Let $z = -(1-k)w$ and then

$ze^{z} = -(1-k)\exp\{-(1-k)\}$ and

$-(1-k)w = W[-(1-k)\exp\{-(1-k)\}]$ and

$H^{-\frac{k}{1-k}} = -\frac{W[-(1-k)\exp\{-(1-k)\}]}{(1-k)}$.

Therefore, $H = \exp\left\{\frac{(1-k)^2}{k}\left(1 + \frac{W[-(1-k)\exp\{-(1-k)\}]}{(1-k)}\right)\right\} = \exp\left\{\frac{(1-k)^2}{k} + \frac{1-k}{k}\frac{W[-(1-k)\exp\{-(1-k)\}]}{(1-k)}\right\}$

That makes the default risk θ as an explicit (not an implicit) function of the collateral valuation k, which is the solution for $\frac{\partial C}{\partial k} = 0$.

An examples of this solution is depicted as a dotted line in <Figure 3>. QED.

4. Examples and Visualization of the Equilibrium

We provide some examples and visualize the equilibrium results to understand and explain the economic meanings. <Table 2> shows that credit rationing could happen, that is, some positive NPV(net present value) projects whose expected returns E exceed one dollar investment could be abandoned since the total expected costs of borrowing are higher than the expected returns E.

<Table 2> Examples of the equilibrium results

Here we assume the bank's funding cost $R_o = 1$ without loss of generality.[2]

[2] We can see that the bank's funding cost R_o does not appear in <Theorem 3>, which means that R_o does not have qualitative effects on the contract results.

Case	Bank	Borrowers			Projects	
	Information Asymmetry	Collateral valuation	Default risk	Total expected Costs	$E = 1.15$	$E = 1.2$
1	$\theta_H = 0.9$	$k = 0.5$	$\theta = 0.3$	$\tau \approx 1.28$	Abandon	Abandon
2		$k = 0.7$		$\tau \approx 1.13$	Invest	Invest
3	$\theta_H = 0.5$	$k = 0.5$	$\theta = 0.3$	$\tau \approx 1.17$	Abandon	Invest
4			$\theta = 0.4$	$\tau \approx 1.14$	Invest	Invest
5			$\theta = 0.1$	$\tau \approx 1.09$	Invest	Invest

In <Case 1>, two projects with the expected returns E are 1.15 and 1.2 are all abandoned due to the higher borrowing costs 1.28. If we could raise the valuation k to 0.7 from 0.5 as in <Case 2>, then the borrowing cost goes down to 1.13 and the two projects can be invested. Compared to <Case 1>, in <Case 3>, the more profitable project can be invested due to the alleviated information asymmetry, that is, the maximum possible default risk θ_H went down to 0.5 from 0.9. These examples imply that alleviating information asymmetry or strengthening bank's screening ability and increasing the collateral valuations matters.

There are somewhat counterintuitive results. Now let's compare <Case 3> and <Case 4>. If the borrower's default risk increases, then the borrowing cost might decrease and some abandoned project could be invested now. Meanwhile lowered risk could also reduce the borrowing costs as in <Case 5>. These results make us guess that the medium risk borrowers might pay the highest borrowing costs. We can confirm it again by visualizing these arguments in the following figures.

The following three figures depict the equilibrium results by the borrower types (k, θ). The graphs are numerically drawn using Python codes. Without loss of generality, we assume that $R_o = 1$, that is, the net funding costs are zero. In each figure, the highest failure probability θ_H is assumed to be 0.9 for the left and 0.5 for the right. The collateral valuation k is smaller than 1 and bigger than 0. We cannot find any qualitative differences between the right and the left graphs.

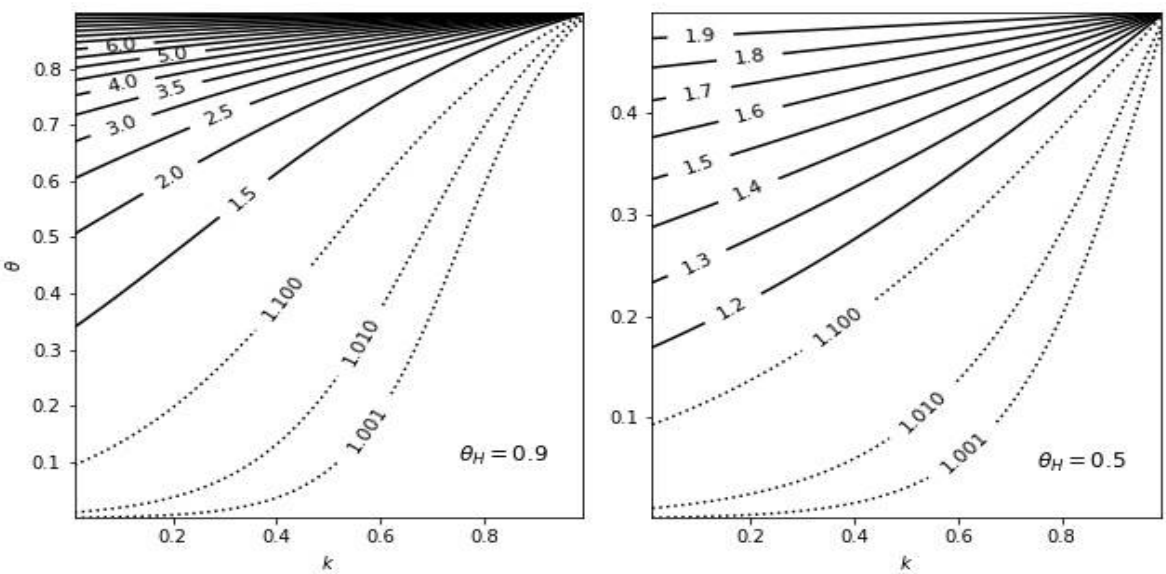
In <Figure 2>, it is evident that the borrowers with higher risks and lower collateral valuations promise to pay more interests. <Figure 3> shows some patterns in which more collateral provisions are required for the borrowers with the lower risk and lower collateral valuations, but it does not always hold true. For the higher risk borrowers, collateral provisions and valuations move in the same direction. That is, lower collateral valuation borrowers provide less collaterals. The lower

collateral valuation means that the banks discount the value of collaterals much deeper, and thus the borrowers prefer to pay more interests rather than to increase the collateral provisions. Those are examples of substitution and income effects which we explained earlier.

<Figure 4> depicts the total expected costs of borrowing by the risk and collateral valuations. The total costs are higher for the lower collateral valuations and the medium risk borrowers. The higher risk borrowers choose the higher interests and lower collateral contract, and vice versa for the lower risk ones. The separating mechanism is ambiguous for the medium risk borrowers. Comparing the left and right figures, the overall costs are higher for the left, that is, the borrower group with the higher maximum default probability $\theta_H = 0.9$ than the right whose $\theta_H = 0.5$. If the bank narrows down the risk range of the given borrower group, the loan contracting costs will shrink.

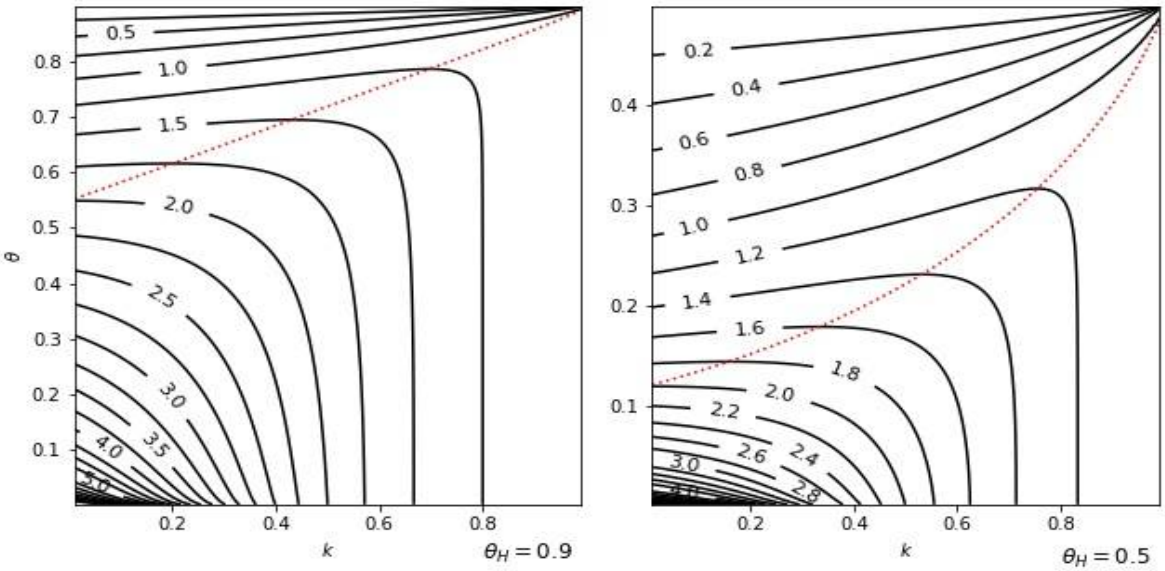
<Figure 2> Repayment levels by borrower types: $R(\theta, k|\theta_H = 0.9)$ and $R(\theta, k|\theta_H = 0.5)$

The dotted contour lines are drawn using finer scales.



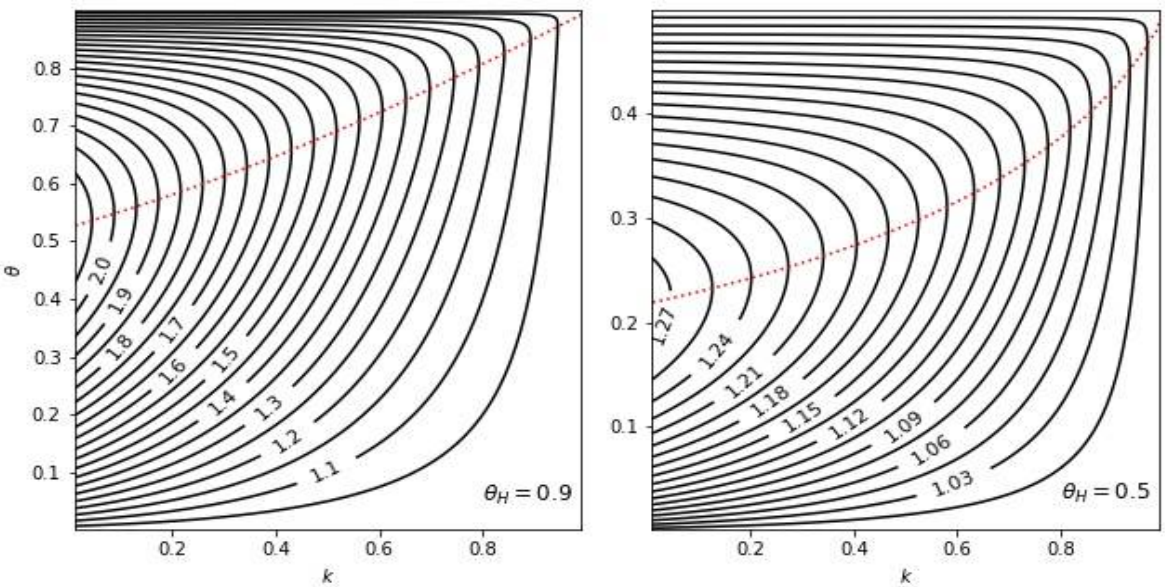
<Figure 3> Collateral levels by borrower types: $C(\theta, k|\theta_H = 0.9)$ and $C(\theta, k|\theta_H = 0.5)$

The dotted line is the solution for $\frac{\partial C}{\partial k} = 0$. It means the highest risk one in the borrower pool with the same collateral provision.



<Figure 4> Total expected cost levels by borrower types: $\tau(\theta, k|\theta_H = 0.9)$ and $\tau(\theta, k|\theta_H = 0.5)$

The dotted line is the solution for $\frac{\partial \tau}{\partial \theta} = 0$. It means the highest collateral valuations in the borrower pool with the same total expected costs.



5. Application: MSME financing support policies

Micro, Small & Medium Enterprises (**MSME**) are often said to be financially constrained due to the market failure. MSMEs also suffer from the inherent lack of suitable collateral(**Ayadi & Gadi, 2013).** Many countries attempt to alleviate these financial constraints with direct loans or guarantee schemes(Cressy, 2002). However there is a lot of interests in the economic efficiencies of these policies(Lerner, 2002; Hahn et al, 2017). In this regard, we discuss here the effects of MSME financing support policies using the results of our model.

Our model explained that some borrowers with limited collaterals could not invest their positive NPV projects, that is, some kinds of credit rationing could happen. Let's assume that some MSME support policies can increase the collateral values and thus make would-be abandoned projects be invested. However, there may be social costs associated with the increased collateral values which is discounted to the banks.

In terms of social interests, there are two kinds of costs in the loan contracts: 1) ex-ante abandonment of positive NPV investments and 2) ex-post foreclosure of collaterals whose value is lower to the banks than to the borrowers. Thus the expected social costs can be represented by the following equation:

$$\text{SoCosts} = \iint_{NoInv} NPV \cdot f(\theta, k)\, d\theta dk \;\; + \;\; \iint_{YesInv} (1-k) \cdot C(\theta, k) \cdot \theta \cdot f(\theta, k)\, d\theta dk \qquad (8)$$

where "NoInv" is the borrowers who abandoned the projects due to their high borrowing costs and "YesInv" is those accepted the projects due to their low borrowing costs. Additionally, we assume that there are no wealth transfer costs which are related with the wealth redistribution between the tax payers and the policy beneficiaries.

The MSME financing support programs will certainly improve the under-investment problems and thus reduce the first term of the Social Costs(cf. <Fighrue 4>). In equation (8), the "NoInv" set will become smaller than before. However, the analysis of the second term is ambiguous due to the conflicts between the income and substitution effects. There could be some possibility of increasing the collaterals, and thus leading to more foreclosures or liquidations. In that case, the second term will increase. The exact summation of these effects are different by the borrower characteristics: the combination of risk and collateral valuation (θ, k). For the higher risk borrowers, increasing the

valuation of collaterals tends to increase collateral provision. However, vice versa for the lower risk ones(cf. <Figure 3>).

Therefore, the policy planners should consider the distributional characteristics of the risk and collateral valuations for the given borrower group eligible for the policy. If there are not so many high-risk and low-valuation borrowers, then collateral support policy such as SME guarantee programs will reduce the social costs associated with the under-investment problems. Comparing the left and the right graphs in <Figure 3>, we can guess that, when the information asymmetry is severe, the collateral support programs might be more effective and more beneficial in the social terms.

We can also consider an indirect support policy for the SMEs' financing. There might be some institutional support for the business credit bureau which can alleviate the information asymmetry between the banks and the SMEs. This can be represented as smaller θ_H in our model and, in equation (8), the shrinked "NoInv" set and the expanded "YesInv" set and the smaller collateral $C(\theta, k)$(cf. <Theorem 1>), and thus the exact total effects are ambiguous. However, we sum these effects all together, we may guess that the total social costs will be reduced because the first term will shrink clearly and the second term change is maybe neutral.

There could be other types of SME support policies such as subordinate loan or equity investment programs in which the government shares the risk of the SMEs and the SME's default risk θ to the bank is reduced. As we saw earlier, the credit rationing is most severe with the middle risk borrowers. Thus the co-investment program should be given to the middle risk ones rather than the higher risk ones. The following <Table 3> summarized the effects of SME financing support policies discussed so far.

<Table 3> The effects of SME financing support policies

Underinvestment	Collateral	Liquidation costs	Total costs
Alleviated	Substitution > Income effects	Could be increasing	Could be ambiguous
	Substitution < Income effects	Clearly decrease	Clearly decrease

6. Summary and Conclusion

This paper extended the result of Bester(1995) by introducing continuous risk types of borrowers. We assume that the bank knows the highest default probability of the borrower pool, even though the individual default risk is known to each borrower only. The collateral valuations are observable to both banks and borrowers. Thus the bank can offer a loan contract menu which consists of the tuple of interests and collaterals for a fixed collateral valuation.

The separating equilibrium results show that the high risk borrowers choose the high interest and low collateral contract, and vice versa for the low risk ones. The higher risk and lower collateral valuation borrowers promise to pay higher interests. However, in terms of collateral, there are some conflicting patterns of substitution and income effects. For the high risk group, the lower collateral valuation borrowers provide less collaterals and vice versa for the low risk group. The total borrowing costs are higher for the lower collateral valuations and the medium risk.

From the equilibrium results, we can deduce some policy implications. First, the bank's ability to narrows down the risk range of the given borrower can decrease the loan contracting costs. This means the importance of information asymmetry. Second, the collateral valuations are important: if we can increase the valuation by some policies such as guarantee programs, the borrowing costs will be reduced. Third, the most damaged group by the credit rationing is the medium risk and low valuation borrowers. If the government consider some support programs, that must be focused on the medium risk borrowers.

That conclusions will be helpful to the policy makers of the microfinance or the governmental support programs for the MSMEs. Small firms and poor people usually lacks collateral and we described it as low collateral valuations in this paper. Banks can help the people and the firms by improving their screening ability and we described it as lowering the highest possible default probability for a given borrower group. If we consider the governmental supports, it should go first to the middle range risk group who suffer from the ambiguous separating mechanism.

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