Volatility Jump using Gumbel Statistic in US Dollar-Euro FX

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I. Introduction

In recent 10 years, one of the most important issues is concerned about discontinuous jump volatility on financial assets such as exchange rates. In 2000's, the jumps of Euro/Dollar exchange rates frequently occurred especially right before and after, or during world financial crisis in years 2007-2008. The US Dollar exchange rate per Euro was 0.9523 on January 1, 2001, 1.3103 on January 10, 2005, 1.4729 on August 15, 2008, 1.4134 on November 26, 2008, 1.3412 on September 24, 2010 and 1.4453 on April 29, 2011, respectively. On March 5, 2015 the US Dollar exchange rate per Euro decreased at large and became 1.0963 due to serious the EU economic situation. On August 3, 2017 the US Dollar exchange rate per Euro was 1.0569, however, the US Dollar exchange rate per Euro increased and was 1.2192 on March 1, 2018 and was 1.1429 on 8 October 2018.

The uncertainty of key exchange rates such as US Dollar/Euro exchange rates may lead decrease world trade seriously and thus may influence the world economy. Since the volatility of Dollar/Euro exchange rates is connected to the EU and U.S economy, the volatility and jumps of Dollar/Euro exchange rates have seriously influenced on the world economy as well as the EU and US economy. Hence, it is so important to estimate the volatility and jumps of Dollar/Euro exchange rates.

Given this backdrop the central questions we seek to answer in our study are as follows: how to correctly estimate the exchange rate volatility and jumps in the Dollar/Euro exchange rates in the recent 2010's? To what extent or how often do jumps occur? Finally, what is probability of jump occurrence? These questions need to be examined by appropriate efficient and robust jump estimation methods and by analyzing the volatility. An important question, often left unaddressed, is whether one should incorporate jumps also in the volatility process or not. In this regard, the effectiveness of estimation of volatility has been tested in different ways, focusing on the volatility of the exchange rate and jumps.

Before 2000's, the parametric approaches such as ARCH models and stochastic volatility models have been mainly used. They rely on explicit functional forms which cannot be inherently exactly specified. The parametric models are almost impossible to explain the discontinuous jump parts of intraday return volatility. Furthermore, as shown in Dewachter et al. (2014), Gaussian quasi-maximum likelihood estimates of GARCH models, subject to the presence of additive jumps, tend to overestimate the volatility for the days following the jumps, and produce also upward-biased estimates of long-term volatility.

To overcome these drawbacks of the parametric approach, in recent years Andersen, Bollerslev, Diebold and Labys (2001, 2003), Andersen, Bollerslev, Diebold (2002, 2004), Barndorff-Nielsen and Shephard (2005a, 2005b, 2006) introduce and develop the nonparametric approaches which used the high frequency daily and intraday asset returns data. Since it is almost impossible to analyze the discontinuous jumps in the volatility of US dollar/Euro exchange rates, we use a modified version of the non-parametric approach like Andersen, Bollerslev, and Diebold (2004, 2007), Huang and Tauchen (2005) and Lee and Mykland (2008).

However, these previous studies do not account for the presence of intraday volatility periodicity. As Boudt, Croux and Laurent (2011b) argued, disregarding this of intraday volatility periodicity leads to seriously influence on the accuracy of the estimated jump statistics and jump detection.

Hence, this paper uses a nonparametric realized volatility model to explain the discrete jumps as well as continuous volatility of Dollar/Euro exchange rates so that this paper introduces and analyzes the realized volatility and relative jump models. Our research topics in this paper are reliable estimates and inferences on the volatility and jump occurred frequently since year 2010, this paper focuses on finding out jumps in Dollar/Euro exchange rates and discontinuous jump probabilities during 2010-2018.

The remainder of the paper is as follows. Section II provides the previous literature survey. Section III introduces the realized volatility and jump statistics which consider the periodicity filter of volatility. Section

IV explains the high frequency data used in this paper and the empirical results of several jump statistics associated with jump probabilities. Section V summarizes and concludes empirical findings.

II. Literature Review

III. Volatility and Jump Model

1. Realized Volatility

Let's consider the following Brownian Semimartingale Process with jumps for the logarithmic price at time t, p(t) in the equation (1). In the equation this paper disposes of T days of M equally spaced intraday returns and denotes the j-th intraday return of day t by $r_{t,j}$. M represents the observed intraday sampling frequency[1].

The daily realized volatility is defined as the summation of realized intraday squared returns following the works of Barndorff-Nielsen and Shephard (2004a), Bollerslev, Kretschmer, Piorsch, and Tauchen (2005) in this paper. Thus, for $\Delta \to 0$, the daily realized volatility or variation of day t(RV_t) is represented the summation of very frequently intraday realized squared variation in equation (1)

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 \,, \quad j=1,\ 2,\ \ldots,\ M. \tag{1}$$

The daily realized volatility converges to the increment of the quadratic variation process as the sampling frequency(M) of the underlying returns goes to the infinity or ((1/M)≡Δ) goes to the zero as Andersen and Bollerslev, and Diebold (2007) pointed out. In reality, however, the jumps in exchange rates occurred occasionally and the occurrence of jumps is generally assumed to follow a Poisson which is a continuous-time discrete process that the realized volatility inherits the continuous sample path process and the discrete jump process. In the presence of jumps, the realized volatility is no longer a consistent estimator of integrated volatility. Thus, for $\Delta \to 0$, the daily realized volatility at the day t converges in probability the sum of continuous integrated variance and the daily summation of discrete N jumps of size of $\kappa_{t,}$ as in equation (2).

$$\lim_{M \to \infty} RV_t = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2 \,. \tag{2}$$

This method is particularly built upon the theories of the realized Bipower variation (BV_t) which Barndorff-Nielsen and Shephard (2004a, 2004b, 2006) developed. The realized Bipower variation(BV_t)

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[1]. Refer Andersen, Bollerslev, and Diebold (2007), Yi(2014) and Huang and Tauchen (2005)

converges in probability to integrated variation, as $\Delta \to 0$ or M goes to be sufficiently large. Bipower variation(BV_t) is robust to jumps because it uses the product between two consecutive returns instead of the squared return.

$$BV_t = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}\left|r_{t,j-1}\right|\left|r_{t,j}\right|. \qquad (3)$$

$$\lim_{M\to\infty} BV_t = \int_{t-1}^{t}\sigma^2(s)ds.$$

2**. Volatility Periodicity Filters**

As Boudt, Croux and Laurent (2011a) proposed, the high frequency return variance $\sigma_{t,i}^2$ has a periodic component $f_{t,i}^2$ due to weekly cycle of opening, lunch and closing times of financial centers. However, the previous studies such as Barndorff-Nielsen and Shephard (2004a), Andersen and Bollerslev and Diebold (2007), did not consider the periodicity of volatility. Thus, this paper adopts the nonparametric estimators in the presence of jumps using periodicity. To identify the periodicity factor $f_{t,i}^2$ for the average variance of day t, the squared periodicity factor has mean one over local window.

We can use this estimator of standard deviation since it is efficient only in the absence of jumps. We, however, cannot use this estimator in the presence of jumps since the observation can be affected by the jump which makes the periodicity estimate arbitrarily large.

Firstly, following a variety of filters proposed by Boudt, Croux and Laurent (2011a, 2011b), this paper adopts the median absolute deviation (MAD). Then, the MAD estimator for the periodicity factor equals

$$\hat{f}_{t,i}^{MAD} = \frac{MAD_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} MAD_{t,j}^2}}. \qquad (4)$$

Second, this paper adopts the Shortest Half scale estimator as efficient as the MAD under normality, proposed by Rousseeuw and Leroy (1988) because the Shortest Half scale estimator is consistent in the presence of infinitesimal contaminations by jumps in the data. Importantly, according to them, it has the property of smallest maximum bias possible which the estimator for which jumps can cause among a wide class of scale estimators. Also it is computationally convenient and does not need any location estimation. Then, the Shortest Half estimator(ShortH) for the periodicity factor equals

$$f_{t,i}^{\widehat{ShortH}} = \frac{ShortH_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} ShortH_{t,j}^{2}}} .$$

$$ShortH_{t,i} = 0.741 \min\left\{ \overline{r_{(h_{t,i}):t,i}} - \overline{r_{(1):t,i}}, \ldots, \overline{r_{(n_{t,i}):t,i}} - \overline{r_{(h_{t,i+1}):t,i}} \right\}. \tag{5}$$

Boudt, Croux, and Laurent (2011a, 2011b) showed that a better trade-off between the efficiency of the standard deviation under normality and the high robustness to jumps of the shortest half dispersion is offered by the standard deviation applied to the returns weighted in function of their outlyingness. Then, the estimator for the periodicity factor under the weighted standard deviation (WSD) estimate equals

$$f_{t,i}^{\widehat{WSD}} = \frac{WSD_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} WSD_{t,j}^{2}}} . \quad WSD_{t,j} = \sqrt{1.081 \frac{\sum_{l=1}^{n_{t,j}} w[(\overline{r_{l:t,j}}/f_{t,j}^{SHortH})^2]\,\overline{r_{l:t,j}^{2}}}{\sum_{l=1}^{n_{t,j}} w[(\overline{r_{l:t,j}}/f_{t,j}^{SHortH})^2]}} \tag{6}$$

where the factor 1.081 is needed to ensure consistency of the estimator under normality.

3. Outlyingness Daily Jump Statistic using Gumbel Distribution

While most previous studies such as Barndorff-Nielsen and Shephard (2005a, 2005b, 2006), Huang and Tauchen (2005) and Andersen, Bollerslev, Diebold (2004, 2007), and Yi(2014) adopted Z-type jump statistics of the standard normal distribution, we use Outlying Weighted Quarticity jump statistic $d_{t,i}$ which measures the local outlyingness of intra-day i-th return of date($r_{t,i}$) like Lee and Mykland (2008),

$$d_{t,i} = \left(\frac{r_{t,i}}{\hat{\sigma}_{t,i}}\right)^2. \tag{7}$$

where $d_{t,i}$ measures the local outlyingness of intra-day i-th return of date($r_{t,i}$) and $\hat{\sigma}$ is the estimator of instantaneous volatility. The outlyingness measure $d_{t,i}$ can be used for a statistic for daily jump detection. Under the null of no jump during day t, $d_{t,i} \sim \chi^2(1), \forall i = 1, 2, \ldots, M$. $Max_{i=1,2,\ldots,M}\sqrt{d_{t,i}}$ follows a Gumbel distribution under the null. More specifically, we reject the null of no jump during day t at the α% critical level if

$$Max_{i=1,2,\ldots,M}\sqrt{d_{t,i}} > G^{-1}(1-\alpha)S_n + C_n , \tag{8}$$

$$C_n = (2\log n)^{0.5} - \frac{\log(\pi)+\log(\log n)}{2(2\log n)^{0.5}}, \qquad S_n = \frac{1}{(2\log n)^{0.5}} .$$

where $G^{-1}(1 - \alpha)$ is the $(1 - \alpha)$ quantile function of the standard Gumbel distribution. When n = M or n = MT, the expected number of spurious (daily) detected jumps respectively equals αT and α.

4. Max Outlyingness Daily Jump Statistics with Periodicity Filters

(1) max outlyingness jump statistic without Periodicity Filters

Firstly, when we do not include the periodicity filter, we can use max outlyingness as the jump statistics.

$$\max DJ_t = Max_{i=1,2,\ldots,M}\sqrt{d_{t,i}} = Max_{i=1,2,\ldots,M}\sqrt{\frac{r_{t,i}}{\hat{\sigma}_{t,i}}} \tag{9}$$

(2) max outlyingness daily jump statistic with Periodicity Filters

Secondly, when we include the periodicity filters such as MAD, ShortH and WSD, we can use max outlyingness as the daily jump statistics as follows.

$$\max DJ_{MAD,t} = Max_{i=1,2,..,M}\sqrt{\frac{r_{t,i}}{\hat{s}_{t,i}\widehat{f^{MAD}}}} \quad , \tag{10}$$

$$\max DJ_{ShortH,t} = Max_{i=1,2,..,M}\sqrt{\frac{r_{t,i}}{\hat{s}_{t,i}\widehat{f^{ShortH}}}} \quad , \tag{11}$$

$$\max DJ_{WSD,t} = Max_{i=1,2,..,M}\sqrt{\frac{r_{t,i}}{\hat{s}_{t,i}\widehat{f^{WSD}}}} \quad . \tag{12}$$

5. Intraday Jump tests with Periodicity Filter

If a return contains a jump component, it should be abnormally big. In times of high volatility, an abnormal return is bigger than an abnormal return in times of low volatility. Hence, they use the ratio of the tested return over a measure of local volatility. The intraday jump statistic $IDJ_{t,i}$ tests whether a jump occurred between intraday time periods i-1 and i of day t. It is defined as the absolute return divided by an estimate of the local standard deviation $\hat{\sigma}_{t,i}$, i.e.

$$IDJ_{t,i} = \frac{|r_{t,i}|}{\hat{\sigma}_{t,i}} \quad . \tag{13}$$

(1) Intraday Jump tests without Periodicity Filters

Under the null of no jump at the testing time, that the process belongs to the family of Brownian Semi-Martingale Jump models, and a suitable choice of the window size for local volatility, asymptotically follows a standard normal distribution. We can replace the local variance $\widehat{\sigma_{t,i}}$ by $\widehat{s_{t,i}} = \sqrt{\frac{1}{M-1}BV_t}$.

$$IDJ_{t,i} = \frac{|r_{t,i}|}{\sqrt{\frac{1}{M-1}BV_t}} \tag{14}$$

(2) Intraday Jump tests with Periodicity Filters using Gumbel Distribution

However, if we ignore the periodic volatility patterns, it leads to spurious jump identification. Boudt, Croux, and Laurent (2011b) propose to account for the strong periodicity in volatility and show that replacing the local variance $\widehat{\sigma_{t,i}}$ by $\widehat{f_{t,i}}\widehat{s_{t,i}}$ (where $s_{t,i}^2 = \frac{\sigma_{t,i}^2}{f_{t,i}^2}$) is more appropriate.

In this paper, to consider periodic volatility patterns we will use the three robust nonparametric estimators such as $\widehat{s_{t,i}}\widehat{f_{t,i}^{MAD}}$, $\widehat{s_{t,i}}\widehat{f_{t,i}^{ShortH}}$ and $\widehat{s_{t,i}}\widehat{f_{t,i}^{WSD}}$ to estimate $\widehat{\sigma_{t,i}}$ and jumps statistics as follows:

$$IDJ_{t,i}^{MAD} = \frac{|r_{t,i}|}{\widehat{s_{t,i}}\widehat{f_{t,i}^{MAD}}}, \tag{15}$$

$$IDJ_{t,i}^{ShortH} = \frac{|r_{t,i}|}{\widehat{s_{t,i}}\widehat{f_{t,i}^{ShortH}}}, \tag{16}$$

$$IDJ_{t,i}^{WSD} = \frac{|r_{t,i}|}{\widehat{s_{t,i}}\widehat{f_{t,i}^{WSD}}}. \tag{17}$$

Under the null of no jump and a consistent estimate $\widehat{\sigma_{t,i}}$, $IDJ_{t,i}$ follows the standard normal distribution which has the absolute value. If the statistic exceeds a plausible maximum, one rejects the null of no jump. When $\Delta \rightarrow 0$, the sample maximum of the absolute value of a standard normal variable (i.e. the jump statistic $IDJ_{t,i}$) follows a Gumbel distribution under the assumption of no jump in the interval i – 1, i of day t. Hence, we reject the null of no jump if

$$IDJ_{t,i} > G^{-1}(1-\alpha)S_n + C_n ,$$

$$C_n = (2\log n)^{0.5} - \frac{\log(\pi)+\log(\log n)}{2(2\log n)^{0.5}}, \qquad S_n = \frac{1}{(2\log n)^{0.5}} . \tag{19}$$

When n = M (i.e. number of observations per day) and n = MT(i.e. total number of observations), this number equals respectively αT and α(i.e. ≈ 0). So if we choose a significance level of α, then we reject the null of no jump at testing time if $IDJ_{t,i} > G^{-1}(1-\alpha)S_n + C_n$.

IV. Empirical Results

1. Data

The empirical analysis is based on data from Olsen and Associates in Zurich, Switzerland. The data set consists of five minute observations on US dollar/Euro exchange rate from February 1, 2010 to February 28, 2018. The all volatility measures are based on the five minute returns as the first difference of the logarithm of US dollar/Euro exchange rate, which results in total of M($=\frac{1}{\Delta}$)=265 high frequency return observations per day. When we have five minute interval per day, we have ($\Delta = 1/265$).

After removing holidays and other inactive trading holidays, we have a total of 2,529 days. The corresponding daily returns of US dollar/Euro for 2,529 days can be represented as $r_{t+1} \equiv r_{t+1,1} \equiv r_{t+\Delta,\Delta} + r_{t+2\Delta,\Delta} + \cdots + r_{t+1,\Delta}$, t= 1, 2, $\cdots$, 2,529. Thus this paper has a total of 670,185(=2,529 X 265) sample observations.

2. Max Outlying $\sqrt{d_{t,i}}$ Daily Jump Statistic with Filters

This section examines the Max outlying $\sqrt{d_{t,i}}$ jump statistic with several periodicity filters instead of Z –type jump statistics in the previous studies such as Barndorff-Nielsen and Shephard (2005a, 2005b, 2006), Huang and Tauchen (2005), Yi(2014), and Andersen, Bollerslev, and Diebold (2004, 2007). Also we classify jump statistics into using the intraday periods(n=M) and using intraday observations(n=MT).

1) Intraday observation case

To obtain the Max outlyingness jump statistics, we use the n=670,185 intraday observations of US dollar/Euro during February 1, 2010 to February 28, 2018. The <Table 1> reports the jumps statistics at α =0.900, α=0.995 and α=0.999 significant levels. It reports the jump detection probability with returns with the no periodicity window and filters using intra-day periods for the maximum outlying jump statistics.

<Table 1> Max Outlying Daily Jump Probability using filters: intraday observations(n=670185)

a.(α=0.999)	$\max DJ_t$	$\max DJ_{MAD,t}$	$\max DJ_{Short H,t}$	$\max DJ_{WSD,t}$
critical value	6.15219	6.15219	6.15219	6.15219
Expected Jumps under Ho=no jump	**0.001**	0.001	0.001	0.001
detected number of jumps	1207	774	784	632

probability of jumps	0.477264	0.30605	0.310004	0.249901
b.(α=0.995)	$\max DJ_t$	$\max DJ_{MAD:t}$	$\max DJ_{Short\,H:t}$	$\max DJ_{WSD:t}$
critical value	5.84109	5.84109	5.84109	5.84109
Expected Jumps under Ho=no jump	0.005	0.005	0.005	0.005
detected number of jumps	1338	886	884	709
probability of jumps	0.529063	0.350336	0.349545	0.280348
c.(α=0.990)	$\max DJ_t$	$\max DJ_{MAD:t}$	$\max DJ_{Short\,H:t}$	$\max DJ_{WSD:t}$
critical value	5.70679	5.70679	5.70679	5.70679
Expected Jumps under Ho=no jump	0.01	0.01	0.01	0.01
detected number of jumps	1413	929	939	739
probability of jumps	0.558719	0.367339	0.371293	0.29221

At the significant level α=0.995(Φ_α=5.84109), according to the maximum outlying jump statistics with the no periodicity window, 1,338 significant jumps occurred during 2010-2018 with the proportion of 52.91%. According to the jump detection probability with filtered returns with the MAD periodicity, 886 significant jumps occurred during 2010-2018 and the proportion of days with significant jumps appeared to be 35.03%. With the Shortest Half Scale periodicity filter, 884 significant jumps occurred during 2010-2018 and the proportion of days with significant jumps appeared to be 34.95%. With the WSD filter, 709 significant jumps occurred during 2010-2018 and the proportion of days with significant jumps appeared to be 28.03%.

2) Intraday period case

Now we obtain the max outlyingness jump statistics of US dollar/Euro exchange rates using 265 intraday periods instead of using intraday observations during February 2010 through February 2018. The <Table 2> reports the jump detection probability with filtered returns with the no periodicity window and several filters using intra-day periods for the maximum outlying jump statistics at α=0.900, α=0.995 and α=0.999 significant levels.

When α=0.999(Φ_α=4.97961), with no periodicity window 1,697 significant jumps occurred with the proportion of 67.10%. According to the jump detection probability with filtered returns with the MAD periodicity, 1,246 significant jumps occurred during 2010-2018 and the proportion of days with significant jumps appeared to be 49.27%. With the Shortest Half Scale periodicity filter, 1,243 significant jumps occurred during 2010-2018 and the proportion of days with significant jumps appeared to be 49.15%. With the WSD filter, 1,000 significant jumps occurred during 2010-2018 and the proportion of days with significant jumps appeared to be 39.54%.

According to the maximum outlying jump statistics, at the significant level α=0.995(Φ_α=4.49723), with the no periodicity window 1,891 significant jumps occurred among 2,529days during 2010-2018 and the proportion of days with significant jumps appeared to be 74.77%. However, as shown in the jump detection probability with filtered returns with the MAD periodicity, 1,517 significant jumps occurred and the proportion of days with significant jumps appeared to be 59.98%. With the Shortest Half Scale periodicity filter, 1,515 significant jumps occurred during 2010-2018 and the proportion of days with significant jumps appeared to be 59.90%. With the WSD filter, 1,197 significant jumps occurred during 2010-2018 and the proportion of days with significant jumps appeared to be 47.33%.

Thus, when we use periodicity filters, the numbers of jumps occurrence and the proportion of days with significant jumps appeared smaller. While one jump of US dollar/Euro exchange rates occurred about per 1.50 days with no periodicity filter, when we use periodicity filters, the jumps occurred about per 2 days at α=0.999

<Table 2> Max Outlying Daily Jump Probability using filters: intraday period

a.(α=0.999)	$\max DJ_t$	$\max DJ_{MAD;t}$	$\max DJ_{Short\,H;t}$	$\max DJ_{WSD;t}$
critical value	4.97961	4.97961	4.97961	4.97961
Expected Jumps under Ho=no jump	2.529	2.529	2.529	2.529
detected number of jumps	1697	1246	1243	1000
probability of jumps	0.671016	0.492685	0.491499	0.395413
b.(α=0.995)	$\max DJ_t$	$\max DJ_{MAD;t}$	$\max DJ_{Short\,H;t}$	$\max DJ_{WSD;t}$
critical value	4.49723	4.49723	4.49723	4.49723
Expected Jumps under Ho=no jump	12.645	12.645	12.645	12.645
detected number of jumps	1891	1517	1515	1197
probability of jumps	0.747726	0.599842	0.599051	0.47331
b.(α=0.990)	$\max DJ_t$	$\max DJ_{MAD;t}$	$\max DJ_{Short\,H;t}$	$\max DJ_{WSD;t}$
critical value	4.28898	4.28898	4.28898	4.28898
Expected Jumps under Ho=no jump	25.29	25.29	25.29	25.29
detected number of jumps	1953	1631	1624	1304
probability of jumps	0.772242	0.644919	0.642151	0.515619

3. Intraday Jump Test

This section examines and compares the intraday jump probability instead of daily jump probability without considering periodicity filter cases and with considering periodicity filter cases**.** Also we classify jump statistics into using the intraday periods(n=M=265) and using intraday observations(n=MT=670,185).

1) Intraday observations case

To obtain the intraday jump statistics, we use the n=670,185 intraday observations of US dollar/Euro during February 2010 through February 2018. <Table 3> reports the jumps statistics at α=0.900, α=0.995 and α =0.999 significant levels. It reports the intraday jump detection probability with returns with the no periodicity window and filters using intra-day observations for the intraday jump statistics.

<Table 3> Intraday Jump Probability using Local Robust Variance: Intraday observations

a. (α=0.999)	$IDJ_{t,i}$	$IDJ^{MAD}_{t,i}$	$IDJ^{ShortH}_{t,i}$	$IDJ^{WSD}_{t,i}$
Critical value, i.e. G(Beta)*Sn+Cn	6.15219	6.15219	6.15219	6.15219
Number of detected jumps	698	683	682	528
Proportion of detected jumps	0.0010415	0.00101912	0.00101763	0.000787842
Number of periods (typically days) with at least one significant jump	579	516	516	403
Proportion of periods with at least one significant jump	0.228944	0.204033	0.204033	0.159352
Expected number of spurious detected jumps (under H0=no jumps)	0.001	0.001	0.001	0.001
b.(α=0.995)	$IDJ_{t,i}$	$IDJ^{MAD}_{t,i}$	$IDJ^{ShortH}_{t,i}$	$IDJ^{WSD}_{t,i}$
Critical value, i.e. G(Beta)*Sn+Cn	5.84109	5.84109	5.84109	5.84109
detected number of jumps	867	796	784	624
Proportion of detected jumps	0.00129367	0.00118773	0.00116983	0.000931086
Number of periods (typically days) with at least one significant jump	694	581	578	458
Proportion of periods with at least one significant jump	0.274417	0.229735	0.228549	0.181099
Expected number of spurious detected jumps (under H0=no jumps)	0.005	0.005	0.005	0.005
c. (α=0.990)	$IDJ_{t,i}$	$IDJ^{MAD}_{t,i}$	$IDJ^{ShortH}_{t,i}$	$IDJ^{WSD}_{t,i}$
Critical value, i.e. G(Beta)*Sn+Cn	5.70679	5.70679	5.70679	5.70679
Number of detected jumps	957	860	855	672
Proportion of detected jumps	0.00142796	0.00128323	0.00127577	0.00100271

Number of periods (typically days) with at least one significant jump	751	619	619	484
Proportion of periods with at least one significant jump	0.296955	0.244761	0.244761	0.19138
Expected number of spurious detected jumps (under H0=no jumps)	0.01	0.01	0.01	0.01

Firstly, when α=0.999(Φ_α=6.15219), the number of detected jumps with the no periodicity window was 698. Among 698 jumps, the number of days with at least one significant jump was 579 among 2,529 days. Thus, significant intraday jumps occurred with the proportion of 22.89%.

The number of detected jumps was 683 as shown in the jump detection probability with filtered returns with the MAD periodicity. At least one significant jump occurred in 516 days during 2010-2018 and the proportion of days with significant jumps appeared to be 20.40%.

With the Shortest Half Scale periodicity filter, the number of detected jumps was 682. The number of days with at least one significant jump was 516. The proportion of days with significant jumps appeared to be 20.40% as the MAD filter case. With the WSD filter, the number of detected jumps was 528. At least one significant jump occurred in 403 days during 2010-2018 and the proportion of days with significant jumps appeared to be 15.94%.

Secondly, at the significant level α=0.995(Φ_α=5.84109), the number of detected jumps with the no periodicity window was 867. The number of days with at least one significant jump was 694. The significant intraday jumps occurred with the proportion of 27.44%. According to the jump detection probability with filtered returns with the MAD periodicity, the number of detected jumps was 796. At least one significant jump occurred in 581 days during 2010-2018 and the proportion of days with significant jumps appeared to be 22.97%.

With the Shortest Half Scale periodicity filter, the number of detected jumps was 784. The number of periods (typically days) with at least one significant jump was 578. The proportion of days with significant jumps appeared to be 22.85%. With the WSD filter, the number of detected jumps was 624. At least one significant jump occurred in 458 days during 2010-2018 and the proportion of days with significant jumps appeared to be 18.11%.

Thirdly, at the significant level α=0.990(Φ_α=5.70679), the number of detected jumps with the no periodicity window was 957. The number of days with at least one significant jump was 751. The significant intraday jumps occurred with the proportion of 29.70%.

As shown in the jump detection probability with filtered returns with the MAD periodicity, the number of detected jumps was 860. At least one significant jump occurred in 619 days during 2010-2018 and the proportion of days with significant jumps appeared to be 24.48%. With the Shortest Half Scale periodicity filter, the number of detected jumps was 855. The number of periods (typically days) with at least one significant jump was 619. The proportion of days with significant jumps appeared to be 24.48%.

With the WSD filter, the number of detected jumps was 672. At least one significant jump occurred in 484 days during 2010-2018 and the proportion of days with significant jumps appeared to be 19.13%. Thus, if we do not consider periodicity filters, intraday jump detection probability is much higher than when we consider periodicity filters such as MAD, Shortest Half Scale and WSD filters.

2) Intraday period case

To obtain the intraday jump statistics, we use the n=265 intraday periods of US dollar/Euro during February 2010 through February 2018. The <Table 4> reports the intraday jump detection probability with returns with the no periodicity window and filters using intra-day periods for the intraday jump statistics at α=0.900, α=0.995 and α=0.999 significant levels.

When α=0.999(Φ_α=4.97961), the number of detected jumps with the no periodicity window was 1,586. The number of days with at least one significant jump was 1,091. Thus, significant intraday jumps occurred with the proportion of 43.13%. As shown in the jump detection probability with filtered returns with the MAD periodicity, the number of detected jumps was 1,395. At least one significant jump occurred in 922 days during 2010-2018 and the proportion of days with significant jumps appeared to be 36.46%. With the Shortest Half Scale periodicity filter, the number of detected jumps was 1,395. The number of days with at least one significant jump was 928. The proportion of days with significant jumps appeared to be 36.70%. With the WSD filter, the number of detected jumps was 1,089. At least one significant jump occurred in 709 days during 2010-2018 and the proportion of days with significant jumps appeared to be 28.03%.

At the significant level α=0.995(Φ_α=4.49723), the number of detected jumps with the no periodicity window was 2,259. The number of days with at least one significant jump was 1,383. The significant intraday jumps occurred with the proportion of 54.69%. As shown in the jump detection probability with filtered returns with the MAD periodicity, the number of detected jumps was 1,974. At least one significant jump occurred in 1,184 days during 2010-2018 and the proportion of days with significant jumps appeared to be 46.81%.

With the Shortest Half Scale periodicity filter, the number of detected jumps was 1,982. The number of days with at least one significant jump was 1,189. The proportion of days with significant jumps appeared to be 47.01%. With the WSD filter, the number of detected jumps was 1,557. At least one significant jump occurred in 939 days during 2010-2018 and the proportion of days with significant jumps appeared to be 37.13%.

At the significant level α=0.990(Φ_α=4.28898), the number of detected jumps with the no periodicity window was 2,688. The number of days with at least one significant jump was 1,522. The significant intraday jumps occurred with the proportion of 60.18%. According to the jump detection probability with filtered returns with the MAD periodicity, the number of detected jumps was 2,334. At least one significant jump occurred in 1,335 days during 2010-2018 and the proportion of days with significant jumps appeared to be 52.79%.

With the Shortest Half Scale periodicity filter, the number of detected jumps was 2,341. The number of days with at least one significant jump was 2,341. The proportion of days with significant jumps appeared to be 53.14%. With the WSD filter, the number of detected jumps was 1,823. At least one significant jump occurred in 1,047 days during 2010-2018 and the proportion of days with significant jumps appeared to be 41.40%.

Thus, if we do not consider periodicity filters, intraday jump detection probability in using intraday period also is much higher than when we consider periodicity filters such as MAD, Shortest Half Scale and WSD filters. We need to use periodicity filters to get more robust and consistent estimators of volatility jumps and jump probabilities of Euro exchange rates unlike previous Barndorff-Nielsen and Shephard (2004a, 2004b, 2005a, 2005b, 2006), Andersen, Bollerslev and Diebold(2004, 2007) who did not consider the periodicity window factors of volatility nor more efficient outlying weighted variances.

<Table 4> Intraday Jump Probability using Local Robust Variance: Intraday period

a. (α=0.999)	$IDJ_{t,i}$	$IDJ_{t,i}^{MAD}$	$IDJ_{t,i}^{ShortH}$	$IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	4.97961	4.97961	4.97961	4.97961
Number of detected jumps	1586	1395	1395	1089
probability of jumps	0.00236651	0.00208151	0.00208151	0.00162492
Number of periods (typically days) with at least one significant jump	1091	922	928	709
Proportion of periods with at least one significant jump	0.431396	0.364571	0.366943	0.280348
Expected number of spurious detected jumps (under H0=no jumps)	2.529	2.529	2.529	2.529
b.(α=0.995)	$IDJ_{t,i}$	$IDJ_{t,i}^{MAD}$	$IDJ_{t,i}^{ShortH}$	$IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	4.49723	4.49723	4.49723	4.49723
detected number of jumps	2259	1974	1982	1557
Proportion of detected jumps	0.00337071	0.00294546	0.00295739	0.00232324
Number of periods (typically days) with at least one significant jump	1383	1184	1189	939
Proportion of periods with at least one significant jump	0.546856	0.468169	0.470146	0.371293
Expected number of spurious detected jumps (under H0=no jumps)	12.645	12.645	4.49723	4.49723
c. (α=0.990)	$IDJ_{t,i}$	$IDJ_{t,i}^{MAD}$	$IDJ_{t,i}^{ShortH}$	$IDJ_{t,i}^{WSD}$
Critical value, i.e. G(Beta)*Sn+Cn	4.28898	4.28898	4.28898	4.28898
Number of detected jumps	2688	2334	2341	1823
Proportion of detected jumps	0.00401083	0.00348262	0.00349307	0.00272014

Number of periods (typically days) with at least one significant jump	1522	1335	1344	1047
Proportion of periods with at least one significant jump	0.601819	0.527877	0.531435	0.413998
Expected number of spurious detected jumps (under H0=no jumps)	25.29	25.29	25.29	25.29

V. Conclusion

This paper analyzes the recent realized continuous volatility and discrete jump volatility of US dollar/Euro returns using the ultra-high frequency five minute returns spanning the period from February 2010 through February 2018. In particular, this paper considers the several periodicity filters such as MAD, Short Half Scale, and WSD to obtain more efficient and robust jump estimators. These estimators have the advantage that they are little affected by volatility periodicity in exchange rate returns. This paper can find out the followings.

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Thus, when we consider the periodicity filters of volatility such as MAD, Short Half Scale and WSD, the five minute returns of US dollar/Euro have considerably smaller daily and intraday jump probabilities. Therefore, if we do not consider periodicity filters of volatility, we can have overestimated jump probabilities so that we have to consider periodicity filters of volatility to get the more robust estimation of jumps and jump probabilities five minute returns of US dollar/Euro.

However, if we use the longer period but very expensive exchange rate data, we will get more interesting rigid analysis. Of course, if we can identify the factors such as economic events and psychology aspects, we will get more interesting results in volatility and jumps in Dollar-Euro exchange rates in 2010's, it goes beyond the scope of the current paper and is thus left for future work.

<References>