Pricing Default Risk of a Convertible Bond by Simulation[+]

Abstract

In this paper, we offer a simple way to price a defaultable, convertible and callable bond by applying the Longstaff-Schwartz Least Squares simulation method. In our model, the stock price is a driving force for valuing the security. A key idea is to terminate the simulated sample path immediately when the issuer defaults on the bond at time t, the same as when the investor and the issuer optimally exercise their option, and to discount back the resulting cash flows at a risk-free rate. In turn, the defaulted group of the sample paths belongs to a bottom x percentile of the realized stock prices at each time, which is exogenously given by the cumulative or marginal default probability of a firm equally rated as the issuer. We apply our simulation model to a LYON-like security and show that the price depends on its default probability and recovery ratio.

JEL classification: G12, G13, G17

KEYWORDS: Least Squares simulation, optimal decision rule, marginal default probability, recovery ratio, default-triggering stock price level

1. Introduction

Pricing convertible bond has been an important issue both in the academics and industry. Convertible bond is a hybrid security of equity and bond with complicated features, and it is known to be difficult to price it. Analytic solution for convertible bond valuation can be obtained in simpler cases (e.g., Ingersoll, 1977; Lewis, 1991). Because of the complexity of the security, however, it can be solved only numerically in most other practical cases.

In the literature, convertible bond is numerically valued in two different ways. In the structural approach (e.g., Brennan and Schwartz, 1977 and 1980), the driving force of the valuation is the issuing firm value which is assumed to follow a stochastic process. This approach allows one to model default risk of the convertible bond in a straightforward manner by setting a certain level of the firm value (which equals the book value of the firm liability) to triggering a default event. In the reduced form approach (e.g., McConnell and Schwartz, 1986; Ho and Pfeffer, 1996; Tsiverietis and Fernandes, 1998; Ayache, Forsyth and Vetzal, 2003), the driving force of the valuation is the issuing firm stock price which is assumed to follow a stochastic process. In this approach, however, it still remains an important consideration how to model default risk of the convertible bond since there is no clear-cut way to link the stock price to a default event.

In this paper, we offer a simple way to price default risk of a convertible bond by simulation in the framework of the reduced form approach. Two related studies to our paper in the literature are Tsiverietis and Fernandes (1998), and Ayache, Forsyth and Vetzal (2003). The former prices default risk of cash-only part of the convertible bond by applying a higher risk-adjusted discount rate. The payment in equity of the convertible bond is, on the other hand, discounted at a risk-free rate. The latter decomposes the higher discount rate into the parts due to exogenously determined default probability and recovery rate of the convertible bond.

Our paper differs from these studies in the way which the numerical method is applied. Both of them derived the Black-Scholes type of partial differential equations and solved them numerically by applying traditional finite difference methods. In this paper, we price the convertible bond by applying the Least-Squares Monte Carlo simulation method developed by Longstaff and Schwartz (2001). We think that our Monte Carlo method (compared with traditional finite-difference methods) is a more efficient computational tool for valuing a defaultable convertible bond with complicated features or with multiple state variables, which is in line with those assertions by Broadie and Glasserman (1997) and Ammann, Kind and Wilde (2008).

Modeling default risk of a convertible bond in terms of the stock price hinges centrally on how to relate its default risk to the stock price. In a recent empirical paper, as a matter of fact, Campbell, Hilscher and Szilagyi (2008) reported that market capitalization was a reliable long-run default indicator for the firm.

Merton (1974) showed a monotonic increasing relationship between the equity value on the right-hand side of the balance sheet and the firm asset value on the left-hand side of the balance sheet. It is because equity is considered to be a call option on the firm value struck at the book value of the liability. Given an empirical default probability of the firm, there should be a certain level of the firm value which triggers a default event as for the KMV's Expected Default Frequency (EDF) indicator. Henceforth, the one-to-one correspondence between the equity value and the firm value means that there should be a corresponding level of the stock price which triggers a default event to the firm. Once the stock price hits the default event triggering level, it may jump down to zero (absolute priority rule) or close to zero (restructuring). After a default occurs, we assume that the recovery ratio of the convertible bond is constant and lies between zero and one. In these simplified assumptions, only the empirical default probability and the recovery ratio do essentially matter for pricing the default risk. It does not actually matter whether the stock price would jump down to zero or close to zero after the default event.

In this paper, we consider a zero-coupon, convertible, callable and defaultable bond. The bond is same as LYON valued by McConnell and Schwartz (1986) except for its puttability.[1] We may call it a Lyon-like security. Investor can opt to convert it into a specified number of shares of equity any time prior to or at the maturity date. The issuer may call the bond after a pre-specified call protection period and the call price escalates through time. The issuer may default any time prior to or at the maturity date.

In our simulation, the market value of the convertible bond at each node is the expected cash flows from continuing, which is estimated by the Longstaff-Schwartz (LS) Least Squares Monte Carlo simulation method. The optimal stopping rule can be then determined by the simultaneous optimal exercise strategies of both the investor and the issuer. At each node, the issuer would not call the bond unless the market value of the convertible bond exceeds the call price. If the market value exceeds the call price and thereby the issuer calls the bond, the investor can choose to cash it in at the call price or to convert into a fixed number of shares (forced conversion) to his or her advantage. If the bond is not called, the investor can opt to carry it to the next period or to immediately convert it into a fixed number of shares (voluntary conversion), depending on whether the market value of the bond is greater or lesser than the conversion value.

At this junction, an important consideration for our simulation is how to identify a default

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[1] Practically, the put option could be exercised only when the issuing firm becomes delisted or when the firm is restructured due to an M&A event. Besides, the portion of puttable convertible comprises of only 5.2% of the total issues in the U.S. (Grimwood and Hodges, 2002).

event at each node prior to or at maturity. For that purpose, we use the transition probability matrix for rated firms, showing the probability of rating migration between year one and year two, from which marginal default probability in year n (conditional on a particular starting rating and not having defaulted prior to year n) can be obtained. That was actually done in Elton, Gruber, Agrawal and Mann (2001). If the marginal default probability of the bond is x percent in year n (conditional on not having defaulted prior to year n), we then assign a default event to the nodes in year n belonging to the bottom x percentile of the distribution of the random stock prices in year n.

We think that our paper contributes to the literature in important ways. The earlier works in the literature (e.g., McConell and Schwaltz, 1986; Tsiverietis and Fernandes, 1988; Ayache, Forsyth and Vetzal, 2003) have priced default risk of the convertible bond by discounting at a higher risk-adjusted rate. However, as pointed out by Batten, Khaw and Young (2013), this approach is subject to criticism since credit risk spreads are neither constant over time, nor constant along the yield curve. In our approach, we price default risk by reducing the cash flows to the investor to a fraction of the face value of the bond immediately when a default occurs, and hence discounting the resulting cash flows at a (lower) risk-free rate. In doing so, we adhere to the general pricing rule under martingale measure. Besides, we can extend our Monte Carlo simulation model to the valuation of default risk of a convertible bond with multiple state variables (such as stochastic interest rate and volatility of stock return) in a more computationally efficient manner.

The paper proceeds as follows. Section 2 discusses our valuation of convertible bond which focuses on dealing with the default risk. Simulation is designed in Section 3. We apply our model to pricing a LYON-like security in Section 4. In Section 5, we further discuss extensions of our simulation model including the cases of stochastic interest rate and stochastic (stock price) volatility. Finally, we offer our concluding remarks in Section 6.

2. The Model

Our model prices a zero-coupon, convertible, callable and defaultable bond in terms of the stock price by a simulation method. Hence the valuation depends crucially on how the stock price evolves over time and the optimal stopping rules of both the investor and the issuer.

We assume that the issuer's stock price follows a diffusion process with a constant volatility :

$$dS_t/S_t = (\mu - d_y)dt + \sigma\, dZ_t\,, \tag{1}$$

where S_t is the stock price at time t ; μ is its instantaneous expected return ; σ is the standard deviation of the rate of return ; d_y is the continuous dividend yield. Z_t denotes the usual Wiener process. The stochastic process for S is related to the corresponding stochastic process for the firm value, V thru Ito's Lemma because S is a function of V.

The optimal stopping rules are determined by the simultaneous exercise strategies of the investor and the issuer. The investor would exercise to maximize the value of the convertible bond, while the issuer would exercise to minimize the value of the convertible bond. Hence, at any point in time, the value of the convertible bond must be no less than its conversion value :

$$C_t(S_t, t) \geq mS_t\,, \tag{2}$$

where $C_t(S_t, t)$ is the market value of the convertible bond at time t, and mS_t is its immediate conversion value. If (2) does not hold, the investor would voluntarily convert to take an immediate profit.

At the same time, the market value of the convertible bond must be no greater than the maximum of the call price and the conversion value :

$$C_t(S_t, t) \leq Max\,\{K_t, mS_t\} \qquad (3)$$

where K_t is the call price at time t. If $C_t(S_t, t) > K_t$, the issuer would call the bond to take an immediate profit. Once the bond is called, the investor chooses to cash it in at the call price or to convert it into a fixed number of shares, whichever is greater. Hence (3) should hold under these optimal strategies.

In order to consider default risk in pricing the convertible bond, we relate the default risk to the stock price at time t. We posit that there exist a certain level of the stock price at which a default event is triggered, given the evolution of marginal default probability of the issuer over time. Once the stock price hits this triggering level, the stock price may jump down to zero under absolute priority rule or close to zero due to a departure from the absolute priority rule under a debt reorganization program at time t. Assuming a constant recovery rate in the event of default, the investor will receive a fraction, θ of the (discounted) face value of the bond when the issuer defaults at time t :

$$C_t(S_t, t) = e^{-r\,(T-t)}\,(\theta F), \quad \text{for } 0 \leq S_t \leq \underline{S_t} \qquad (4)$$

where r is a constant risk-free (discount) rate ; $T - t$ is the remaining time to maturity, F is the face value of the bond, and $\underline{S_t}$ is the lowest stock price level at time t below which the issuer defaults on the bond.[2]

The existence of $\underline{S_t}$ may be shown using one-to-one correspondence between S_t and V_t, the firm asset value at time t as exhibited in Figure 1. If we set $\underline{V_t}$ be the lowest level below which the issuer defaults on the bonds,[3] we find $\underline{S_t}$ which corresponds to $\underline{V_t}$ at time t because equity is a call option on the firm value with the strike price being the liability book value. Hence, we must have that for $t \in [\,1,2,\cdots\cdots,T]$.

[2] Since we are considering a zero-coupon bond, there would not actually occur a default on coupon payments before maturity. However, when the issuer defaults on other payments, a cross default clause may be applied to this bond, too

[3] In the structural approach to pricing credit risk, $\underline{V_t}$ is directly linked to the liability book value of the issuer (e.g., Merton, 1974)

$$\Pr(S_t \le \underline{S}_t/(S_{t-1} > \underline{S}_{t-1})) = \Pr\big(V_t \le \underline{V}_t/(V_{t-1} > \underline{V}_{t-1})\big) \tag{5}$$

= conditional marginal default probability at time t.

Equation (5) means that there exists $\underline{S}_t$ at time t, given the marginal default probability at time t conditional on not having defaulted prior to time t.

Finally, the pay-off to the investor at maturity is :

$$C_{\mathrm{T}}(S_T, T) = mS_T \quad for\ mS_T \ge F \ , \tag{6}$$

$$= F \quad for\ m\underline{S}_T < mS_T \le F$$

$$= \theta F \quad for\ 0 \le mS_T \le m\underline{S}_T \ .$$

where the recovery ratio is assumed that $0 \le \theta \le 1$.

3. Simulation Design

For our simulation, we assume discrete time over the interval with a finite time set $t \in [0, 1, \cdots\cdots, T]$, where $t = 0$ for today, and $t = T$ for the day of maturity. We generate random numbers for Z_t in (1). For that purpose, we employ the inverse transform method such that

$$Z_t = F^{-1}\ (U_t)\ , \quad U_t \sim Unif\ [0\ , 1]\ , \tag{7}$$

where F is a standard normal cumulative distribution function, F^{-1} is the inverse of F, and $Unif\ [0\ , 1]$ denotes the uniform distribution on [0 , 1].

We simulate and compute the current value of the convertible bond as an average of discounted (at a risk free rate) payoffs that the (risk-neutral) investor would receive along the simulated paths at the time set $t \in [0, 1, \cdots, T]$ under martingale measure. In doing so, we duly account for that both the investor and the issuer would make their exercise decisions optimally during the course as explained in Section 2. In addition to these optimal exercise decisions, we include a default decision by the issuer.[4] As a matter of fact, there are six decisions that can be made at each node in our simulation work : (cash) call, forced conversion, voluntary conversion, continuation, default, and redemption at maturity as shown in Table 1. Eventually every sample path is going to be terminated by one out of these six decisions over the time course, $t \in [0, 1, \cdots, T]$.

One important consideration for setting the optimal decision rule in Table 1 is how to figure out the market value of the convertible bond, $C_t(S_t, t)$ at time t along the simulated paths. By applying the Longstaff-Schwartz Least Squares simulation method, we estimate the conditional expected cash flows from continuing for $C_t(S_t, t)$. This is done by regressing the subsequent realized cash flows from continuation on a basis function of a constant, S and S^2.

A striking feature of our simulation work is that we consider that the issuer may default on the bond any time before or at maturity and we directly take into account this default risk for our valuation. This is done by reducing the resulting cash flows immediately to a fraction, θ of (discounted) face value of the bond when a default occurs along the simulated paths and

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[4] In our paper, we simply assume that a default occurs at time t when the stock price, S_t hits the lowest level, $\underline{S}_t$ and the investor receives a fraction, θ of (discounted) face value of the bond. In reality, however, a default decision is more likely to be made thru a lengthy renegotiation process between the investor and the issuer (e.g., Mella-Barrel, 1999).

discounting back the resulting cash flows at a risk-free rate. By doing so, we adhere to the general pricing rule under matingale measure. We think that this would make our work differentiated in an important way from others where a higher risk-adjusted discount rate was applied to account for the default risk (e.g., Tsiverietis and Fernandes, 1988 ; Ho and Pfeffer, 1996; Ayache, Forsyth and Vetzal, 2003 ; Ammann, Kind and Wilde, 2008).

4. An Application

In Section 4, we consider a specific convertible bond and compute the theoretical price using our simulation model. It is a zero coupon, convertible, callable, and defaultable bond issued by a BBB-rated firm. As a matter of fact, it is exactly same as LYON issued by Waste Management, Inc. on April 12, 1985 except for its puttability. Since LYON was actually priced by McConnell and Schwartz using a finite difference method, we can also compare our prices to theirs.

We briefly describe the characteristics of the bond : It has a face value of $1,000 and matures on January 21, 2001. Any time prior to or at the maturity date, the investor can opt to convert it into 4.36 shares of the issuer's stock. The issuer can also elect to call the bond at cash call prices that escalate through time as shown in Table 2[5] :

We assume that the stock return volatility is 30% per year and the dividend yield is a constant 1.6% per year the same as in McConnell and Schwartz (1986). The risk-free rate is constant 9.86% per year.[6] Furthermore, assuming that the issuer is BBB-rated, we use the

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[5] If the bond is called between the dates shown in Table 2, we linearly interpolate the call prices. Because of some call protection, the issuer may not call the bond prior to June 30, 1987 unless the issuer's stock price rises above $86.01.

[6] On the other hand, McConell and Schwartz (1986) used a higher (risk-adjusted) interest rate of 11.21%, to account for the default risk of LYON. Our rate of 9.86% was obtained by substracting a credit spread of 1.35% on a 10-year BBB-rated corporate bond from 11.21%.

marginal default probability in year n which was computed by Elton, Gruber, Agrawal and Mann (2001) as in Table 3. It is a sum of the product of the rating transition probability and the default probability between year $\mathrm{n}-1$ and year n. Lastly, if a default occurs over the time set $\mathrm{t} \in [0, 1, \cdots, \mathrm{T}]$, we choose a constant recovery ratio of 28.8% which was empirically observed for subordinated bonds (e.g., Altman, 2008).

We simulate 100,000 (50,000 plus 50,000 antithetic) paths with 50 time steps per year before maturity to compute theoretical prices of the bond. The simulated paths are generated by the risk-neutral dynamics of the stock return instead of (1) :

$$dS_t/_{S_t} = (\mathrm{r} - d_y)dt + \sigma\, dZ_t \ . \tag{8}$$

We also regard that default probabilities are same under physical measure and martingale measure by assuming that default risks are all diversifiable.[7] Hence we can discount the cash flows that the (risk-averse) investor would receive over the time course $\mathrm{t} \in [0, 1, \cdots, \mathrm{T}]$ at a risk-free rate, r for valuing the convertible bond under martingale measure.

Tables 4 and 5 report our theoretical price for a LYON-like security with and without default risk. We consider default risk in two different ways. Table 4 is for when a default may occur only at the day of maturity of the security, while Table 5 for when a default may occur any time prior to or at the day of maturity. To make the comparison between the theoretical prices in Tables 4 and 5 on a level ground, we set the condition such that

$$(1 - CDP_n) = \prod_{i=1}^{n}(1 - MDP_i) \ , \tag{9}$$

[7] If default risks are not all diversifiable, one may want to use CDS premium on a BBB-rated bond to price the security. It is because CDS premium represents a (risk-neutral) default probability under martingale measure.

where CDP_n is the cumulative default probability for n years, and MDP_i is the marginal default probability in year i. Given the marginal default probabilities in year i for a BBB-rated firm in Table 3, we then obtain a 14.77% cumulative default probability for 15.7 years, which is the time to maturity of our LYON-like security.

From Tables 4 and 5, we find three things : First, the callability of the security reduces its price as expected, and the reduction ranges between two and three percent of the price. Second, the security's default risk clearly lowers its price, which is a main result of our paper. Last, with default risk, the reduction in the price is more severe in Table 5 than in Table 4, as visually seen in Figure 2, and it deserves a further explanation.

As above-mentioned, we set the cumulative default probability of 14.77% for the 15.7 years of time to maturity, using (9). This means that the issuer would default over the time set $t \in [0, 1, \cdots, T]$ with that probability of 14.77%, given the evolution of the marginal default probability of a BBB-rated firm in Table 3. Why is then the price in Table 5 lower than in Table 4, even though we apply an equal default probability to the both cases in Tables 4 and 5? We think that it is because an otherwise defaulted firm in earlier years may continue to survive and eventually pay out a higher sum to the investor at maturity for the case in Table 4. However, for the case in Table 5, the investor loses the chance of a higher sum being paid out at maturity when a default is allowed to occur in earlier years. Because of this non-trivial difference in the theoretical price of the security, one should allow an earlier default to occur before maturity in order to accurately price default risk of any convertible bond.[8]

When we benchmark our theoretical price to the LYON market price, we do for

[8] This point is further clarified by considering a non-callable, and non-convertible zero-coupon bond. We compute its theoretical prices using the cumulative default probability of 14.77% and the corresponding marginal default probability in Table 3. The prices are \$190.42, and \$188.89, respectively. The difference may be due to an approximation and trivial.

convertible only since the effects of callability and puttability tend to be netted out, and LYON is puttable, while ours is not. In that convertible only case, the LYON market price is quite close to our theoretical price computed using the marginal default probability (panel A in Table 5).

5. Further Discussions

If and when the issuer defaults on the convertible bond over the time course $\mathrm{t} \in [0, 1, \cdots, \mathrm{t}]$, the stock price may not follow the sample paths governed by (8). Rather in that event, the stock price would sharply fall down to zero or close to zero. In order to get around such a problem, one may want to posit that the diffusion process, (1) is conditional on that the issuer has not defaulted till time, t. In doing so, we can value the convertible value at $\mathrm{t} = 0$ as

$$\mathrm{C} = e^{-(T-t)\,r} \{ \sum_{t=0}^{T} [\, H(t)p(t) + \theta F \left(1 - p(t)\right)] \}$$

$$\text{for} \quad \mathrm{t} \in [0, 1, \cdots, \mathrm{N}], \tag{10}$$

where $H(t)$ are the cash flows that the investor would receive at time t conditional on that the issuer has survived till time t. On the other hand, $\theta F \left(1 - p(t)\right)$ is the cash flow that the investor would receive conditional on that the issuer has not defaulted till time $\mathrm{t} - 1$, but the issuer defaults at time t. The stock return dynamics, (8) will now apply only to the sample paths for $H(t)$, given $p(t)$ and the number of those sample paths would decrease as the marginal default probability cumulates as time passes .

One possible extension of our simulation model is to consider a stochastic volatility of the stock return dynamics. As discussed in McConnell and Schwaltz (1986), and Batten, Khaw and Young (2013), even though one may assume a constant volatility for the firm value return dynamics, it is not appropriate to assume a constant volatility for the stock return dynamics. It is well known that since equity is a call option on the firm value, stock price and asset value volatility are related by the following expression :

$$\sigma_S = \left(\frac{A}{E}\right)(\Delta)\sigma_A \quad , \tag{11}$$

where $\sigma_S(\sigma_A)$ is the stock (asset) volatility, $A(E)$ is the firm asset (equity) value, and Δ denotes the call option delta. Here even though σ_A is constant, σ_S is not because leverage (A/E) and delta Δ are both stochastic. Furthermore, Anderson, Benzoni and Lund (2002) empirically reported that the stock return dynamics with a stochastic volatility is a better fit than other candidate return dynamics.

Another extension is to introduce a stochastic interest rate. Particularly if interest rate falls sharply in the future, the issuer would call the convertible bond and refinance at a lower cost. In that event, callable (and convertible) bond becomes more valuable.

As we point out earlier in our paper, pricing by simulation becomes more computationally efficient and flexible with multiple state variables as compared with traditional finite difference methods. In that regard, our default pricing simulation model can be extended with a greater flexibility when we consider interest rate uncertainty and stock return volatility uncertainty simultaneously.

6. Concluding Remarks

In this paper, we have presented a simple way to price default risk of a convertible bond by simulation. Indeed, our simulation application has shown that the convertible bond price was quite sensitive to default risk. Particularly, it has depended on how likely the issuer would default on the bond in the future course, and the recovery ratio in the event of default. We have also shown that allowing an earlier default decision (rather than only at maturity) lowered the convertible bond price because the investor would lose the chance of a higher sum being paid out later in time. In modeling default risk of a convertible bond, hence, it is important to allow a default to occur in earlier years when and if the stock price hits the lowest default-triggering level before the maturity of the debt.

We have priced default risk of the convertible bond by taking an average of (contingent) cash flows discounted at a risk-free rate. This is contrasted with other studies in the literature where cash flows were discounted at a higher risk-adjusted interest rate to account for the default risk. In this regard, our approach is more consistent with the general pricing rule under martingale measure.

In industry, convertible bond is mostly issued by a firm with a greater uncertainty in the future, and hence with a large downside risk as well as upside potential. Therefore, pricing accurately its downside risk may well be as important as pricing accurately its upside potential. Besides, our simulation method becomes more efficient and flexible with an addition of new state variables such as interest rate and stock return volatility than traditional finite difference methods in the literature. In this respect, we think that our default risk pricing model by simulation would contribute both to the literature and to the industry.

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Table 1. Optimal Decision Rule

This table presents the optimal decision rule of both the investor and the issuer. The first column of the table contains the payoffs resulting from each optimal decision. The condition for such an optimal decision is listed in the second column, respectively. There are, as a matter of fact, six decisions that can be made at each node in our simulation work : call, forced conversion, voluntary conversion, continuation, default, and redemption at maturity.

Payoff	Condition	Decision
$\boldsymbol{K_t}$	$C_t > K_t$ and $\mathrm{K(t)} > mS_t$	(cash) call
$\boldsymbol{mS_t}$	$C_t > K_t$ and $\mathrm{K(t)} < mS_t$	forced conversion
$\boldsymbol{mS_t}$	$C_t < mS_t$	voluntary conversion
0	$C_t < K_t$ and $C_t > mS_t$	continuation (not exercising immediately)
F	$mS_T < F$	redemption at maturity
$\boldsymbol{e^{-(T-t)}}(\boldsymbol{\theta}\mathbf{F})$	$mS_t < m\underline{S}_t$	default

Table 2. The LYON Call Prices

The call prices below are quoted from McConnell and Schwartz (1986).

Date	Call Price	Date	Call Price
At Issuance	$ 272.50	June 30, 1994	563.63
June 30, 1986	297.83	June 30, 1995	613.04
June 30, 1987	321.13	June 30, 1996	669.45
June 30, 1988	346.77	June 30, 1997	731.04
June 30, 1989	374.99	June 30, 1998	798.34
June 30, 1990	406.00	June 30, 1999	871.80
June 30, 1991	440.08	June 30, 2000	952.03
June 30, 1992	477.50	At Maturity	1000.00
June 30, 1993	518.57		

Table 3. Marginal Default Probability

The marginal default probability in year n can be computed by a sum of the product of the rating transition probability and the default probability between year n-1 and year n. This table is taken from Table Ⅴ in Elton, Gruber, Agrawal and Mann (2001).

Year	Aaa	Aa	A	Baa	Ba	B	Caa
1	0.000	0.000	0.000	0.103	1.594	8.903	22.052
2	0.000	0.004	0.034	0.274	2.143	8.664	19.906
3	0.001	0.011	0.074	0.441	2.548	8.355	17.683
4	0.002	0.022	0.121	0.598	2.842	8.003	15.489
5	0.004	0.036	0.172	0.743	3.051	7.628	13.421
6	0.008	0.053	0.225	0.874	3.193	7.246	11.554
7	0.013	0.073	0.280	0.991	3.283	6.867	9.927
8	0.019	0.095	0.336	1.095	3.331	6.498	8.553
9	0.027	0.120	0.391	1.185	3.348	6.145	7.416
10	0.036	0.146	0.445	1.264	3.340	5.810	6.491
11	0.047	0.174	0.499	1.331	3.312	5.496	5.743
12	0.060	0.204	0.550	1.387	3.271	5.203	5.141
13	0.074	0.234	0.599	1.435	3.218	4.930	4.654
14	0.089	0.265	0.646	1.474	3.157	4.678	4.258
15	0.106	0.297	0.691	1.506	3.092	4.444	3.932
16	0.124	0.329	0.733	1.532	3.022	4.229	3.662
17	0.143	0.362	0.773	1.552	2.951	4.030	3.435
18	0.163	0.394	0.810	1.567	2.878	3.846	3.241
19	0.184	0.426	0.845	1.578	2.806	3.676	3.074
20	0.206	0.457	0.877	1.585	2.735	3.519	2.928

Table 4. Simulated Prices with and without Default Risk

This table reports our theoretical prices for a LYON-like security which can be defaulted only at the day of maturity of the security. Here, we assume 14.77% for the 15.7 year cumulative default probability of a BBB-rated firm. MS price refers to the theoretical price of LYON by McConnell and Schwartz (1986).

Panel A : Convertible Only

Date	Stock Price (closing)	Lyon Market Price (closing)	MS Price	Our Simulated Price w/o Default Risk	Our Simulated Price w/t Default Risk
April 12, 1985	$52 1/4	$258.75	$262.7	290.96	271.37
15	53	258.75	264.6	293.94	272.85
16	52 5/8	257.5	263.7	292.15	272.72
17	52	—	262.1	291.33	269.96
18	52 3/8	257.5	263.0	292.02	271.97
19	52 3/4	257.5	264.0	293.43	272.70
22	52 1/2	257.5	263.3	292.47	272.42
23	53 1/4	260.0	265.3	294.82	274.03
24	54 1/4	265.0	267.9	296.76	277.93
25	54 1/4	265.0	267.9	296.77	277.16
26	54	265.0	267.2	296.36	275.04
29	54 3/4	260.0	266.6	298.98	279.15
30	52 1/8	260.0	262.4	291.65	272.14
May 1, 1985	49 3/4	252.5	256.7	285.05	266.21
2	50 1/2	250.0	258.4	287.57	267.45
3	50 3/4	252.5	259.0	287.78	269.18
6	50 1/2	252.5	258.4	287.43	267.73
7	50 7/8	255.0	259.3	288.84	269.69
8	50 3/4	253.75	259.0	288.37	268.25

9	51 1/4	255.0	260.3	290.03	269.45
10	53 1/8	260.0	265.0	294.06	273.86

Panel B : Convertible and Callable

Date	Stock Price (closing)	Lyon Market Price (closing)	MS Price	Our Simulated Price w/o Default Risk	Our Simulated Price w/t Default Risk
April 12, 1985	$52 1/4	$258.75	$262.7	271.54	254.80
15	53	258.75	264.6	273.64	257.02
16	52 5/8	257.5	263.7	272.41	255.46
17	52	—	262.1	270.93	254.47
18	52 3/8	257.5	263.0	271.69	255.18
19	52 3/4	257.5	264.0	272.51	256.29
22	52 1/2	257.5	263.3	272.11	255.62
23	53 1/4	260.0	265.3	274.44	257.29
24	54 1/4	265.0	267.9	276.47	260.87
25	54 1/4	265.0	267.9	276.95	260.64
26	54	265.0	267.2	275.60	259.32
29	54 3/4	260.0	266.6	278.21	262.00
30	52 1/8	260.0	262.4	271.49	260.43
May 1, 1985	49 3/4	252.5	256.7	265.66	248.31
2	50 1/2	250.0	258.4	267.27	250.32
3	50 3/4	252.5	259.0	268.20	251.30
6	50 1/2	252.5	258.4	267.44	250.59
7	50 7/8	255.0	259.3	268.20	251.48
8	50 3/4	253.75	259.0	268.03	251.21
9	51 1/4	255.0	260.3	269.13	252.63
10	53 1/8	260.0	265.0	273.81	257.65

Table 5. Simulated Prices with and without Default Risk

This table reports our theoretical prices for a LYON-like security which can be defaulted any time prior to at the day of maturity of the security. Here, we assume the marginal default probability of a BBB-rated firm in Table 3 for the security. MS price refers to the theoretical price of LYON by McConnell and Schwartz (1986).

Panel A : Convertible Only

Date	Stock Price (closing)	Lyon Market Price (closing)	MS Price	Our Simulated Price w/o Default Risk	Our Simulated Price w/t Default Risk
April 12, 1985	$52 1/4	$258.75	$262.7	290.96	257.54
15	53	258.75	264.6	293.94	260.33
16	52 5/8	257.5	263.7	292.15	258.80
17	52	—	262.1	291.33	258.27
18	52 3/8	257.5	263.0	292.02	258.53
19	52 3/4	257.5	264.0	293.43	259.99
22	52 1/2	257.5	263.3	292.47	259.13
23	53 1/4	260.0	265.3	294.82	261.68
24	54 1/4	265.0	267.9	296.76	263.84
25	54 1/4	265.0	267.9	296.77	264.45
26	54	265.0	267.2	296.36	262.86
29	54 3/4	260.0	266.6	298.98	265.46
30	52 1/8	260.0	262.4	291.65	257.30
May 1, 1985	49 3/4	252.5	256.7	285.05	250.60
2	50 1/2	250.0	258.4	287.57	253.69
3	50 3/4	252.5	259.0	287.78	254.92
6	50 1/2	252.5	258.4	287.43	252.76
7	50 7/8	255.0	259.3	288.84	254.49
8	50 3/4	253.75	259.0	288.37	254.16
9	51 1/4	255.0	260.3	290.03	255.56

10	53 1/8	260.0	265.0	294.06	260.84

Panel B : Convertible and Callable

Date	Stock Price (closing)	Lyon Market Price (closing)	MS Price	Our Simulated Price w/o Default Risk	Our Simulated Price w/t Default Risk
April 12, 1985	$52 1/4	$258.75	$262.7	271.54	243.95
15	53	258.75	264.6	273.64	245.86
16	52 5/8	257.5	263.7	272.41	244.21
17	52	—	262.1	270.93	242.35
18	52 3/8	257.5	263.0	271.69	244.31
19	52 3/4	257.5	264.0	272.51	244.73
22	52 1/2	257.5	263.3	272.11	244.83
23	53 1/4	260.0	265.3	274.44	246.93
24	54 1/4	265.0	267.9	276.47	249.14
25	54 1/4	265.0	267.9	276.95	249.42
26	54	265.0	267.2	275.60	249.29
29	54 3/4	260.0	266.6	278.21	251.43
30	52 1/8	260.0	262.4	271.49	243.11
May 1, 1985	49 3/4	252.5	256.7	265.66	236.88
2	50 1/2	250.0	258.4	267.27	238.66
3	50 3/4	252.5	259.0	268.20	239.71
6	50 1/2	252.5	258.4	267.44	238.42
7	50 7/8	255.0	259.3	268.20	240.02
8	50 3/4	253.75	259.0	268.03	239.32
9	51 1/4	255.0	260.3	269.13	240.83
10	53 1/8	260.0	265.0	273.81	246.50

Figure 1. One-to-one correspondence between S_t and V_t

$\underline{V}_t(S_t)$ denotes the lowest level of $V_t(S_t)$ below which the issuer defaults on the bond at time t. The convex curve exhibits that equity is a call option on the firm value.

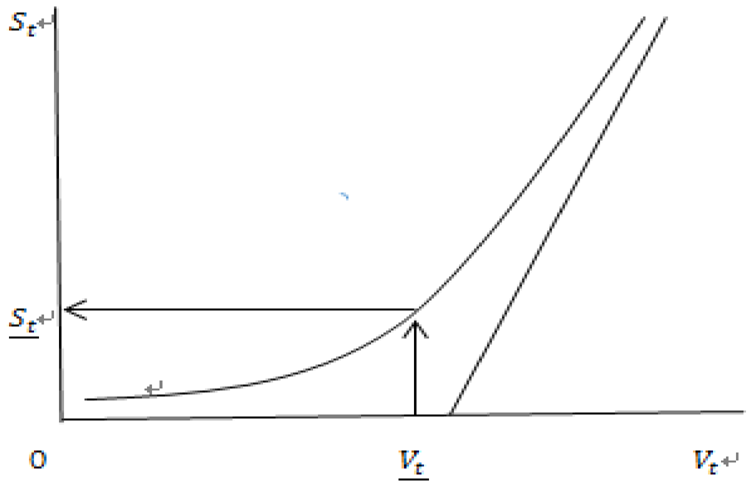
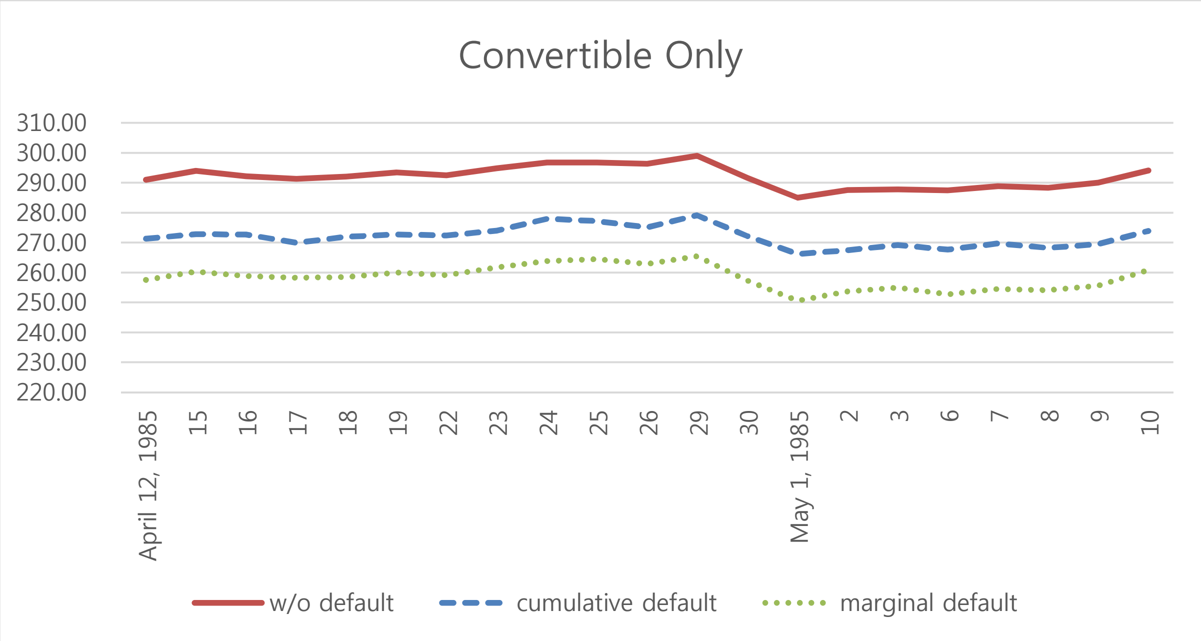


Figure 2. Theoretical Prices with and without Default Risk

Panel A : Convertible Only



Panel B : Convertible and Callable