# Asset Correlation and Bank Capital Regulation: A Macroprudential Perspective*

Sangwon Suh†

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**Abstract**

A strong asset correlation across financial institutions may pose a high systemic risk if a common shock negatively affects asset values. In this paper, we present a simple model with multiple banks in which bank defaults are correlated each other and elicit macroprudential implications of asset correlation on bank capital regulation. We analytically show that if bank failure exhibits an increasing social-cost to scale property, optimal bank capital level gets higher as asset correlations across banks become stronger. Moreover, we quantitatively show that an optimal bank capital level is not constant but varies with economic situations. A strong asset correlation across banks may lead to the so-called "too-many-to-fail" problem under regulation forbearance. Our findings suggest that analogously to the bank capital surcharges for the systemically-important financial institutions to prevent the "too-big-to-fail" problem in the Basel III framework, another bank capital surcharge could preemptively respond to the "too-many-to-fail" problem.

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†School of Economics, Chung-Ang University; Email: ssuh@cau.ac.kr.

# 1 Introduction

Financial stability has been one of top priorities for international policy makers since the recent global financial crisis (GFC) occurred in 2007/2008. At the same time, much academic research has been devoted on the relationship between financial instability and systemic risks and also on desirable macroprudential policies to maintain financial stability.

Systemic risk roughly refers to an undesirable situation where many financial institutions simultaneously fail. This systemic risk may arise as a result of a common shock, contagion, or other causes. Contagion may occur when the failure of one financial institution leads to the default of others through a domino effect in the interbank market, the payment system, or asset markets (see, for example, Allen, Babus, and Carletti, 2009). Asset commonality may also cause systemic risk, as shown by Allen, Babus, and Carletti (2012).

When financial institutions hold similar asset portfolios, and if a common shock negatively affects asset values, then many financial institutions simultaneously get into financial distress, leading to systemic risk. A collapse of residential or commercial real estate values during the GFC period is such an example (see, for example, Reinhart and Rogoff, 2009). Therefore, a strong asset correlation across financial institutions may pose a high potential systemic risk.

This notion of asset correlation effect has been well recognized when quantitatively measuring systemic risk level (see, for example, Huang et al. 2009; Acharya et al., 2010; Adrian and Brunnermeier, 2010; Billio et al., 2010; Brownlees and Engle, 2012; Suh, 2012). Surprisingly, however, financial regulations have not formally taken into account the systemic risk associated with asset correlation yet. For example, Basel II (BCBS, 2004) introduced a risk-based bank capital regulation framework which accounts only for asset correlation across individual loans held by the same bank but not for asset correlation across banks, being lack of a macroprudential perspective. Basel III framework developed Basel II in order to fulfill macroprudential objectives. The countercyclical buffer (BCBS, 2010) was introduced into Basel III in order to ameliorate procyclicality problem. BCBS (2013) also suggested to identify systemically-important financial institutions (SIFIs) and to levy systemic capital surcharges (from 1% to 3.5%) for the SIFIs in order to reduce systemic risk of a cross-sectional nature. Even with these added regulation tools and others, however, Basel III framework fails to explicitly account for systemic risks arising from asset correlation across banks. Beside formal financial regulatory frameworks, even though many regulatory proposals have been made recently (see, for example, Brunnermeier et al., 2009; Geanakoplos, 2009; Perotti and Suarez, 2009; Gorton and Metrick, 2010a,b; FSB, 2013; Calomiris et al., 2014), no one has addressed the issue of systemic risks related with asset correlation across banks yet.

In this paper, we try to fill the gap between the systemic risks related with asset correlation and financial regulatory framework. In particular, we try to elicit macroprudential implications of asset correlation on bank capital regulation. To this end, we present a simple model with multiple banks in which bank defaults are correlated each other.

We analytically show that when bank failure exhibits an increasing social-cost to scale property, optimal bank capital level would be higher with higher asset correlation across banks. Moreover, we quantitatively show that the gap between the two optimal bank capital levels (i.e., the one with a consideration of asset correlation effect and the other without that consideration) is not constant but varies with economic situations: for example, not only overall riskiness but also risk profile of bank loan portfolios, loss given default, cost of bank capital, and bank loan interest rates.

Our findings have profound implications on financial regulation policy. Both the so-called "too-big-to-fail" and the "too-many-to-fail" problems commonly have a cross-sectional nature.[1] Bank capital surcharges for the SIFIs can be preemptively used to prevent the "too-big-to-fail" problem. In contrast, the "too-many-to-fail" problem does not have such an explicit policy tool within the current regulatory frameworks, including Basel III. Our findings suggest that another type of bank capital surcharges would be needed to account for the asset correlation effect. Furthermore, the capital surcharges would differ across countries, depending upon their economic situations.

The rest of this paper is organized as follows. Section 2 presents the model. In Section 3, we analyze banks' decision on bank capital level and its welfare implications and provide some analytic results on optimal bank capital level. In Section 4, we illustrate our previous

[1] Refer to, for example, Sorkin (2009) for the too-big-to-fail problem. Regarding the too-many-to-fail problem, refer to Acharya and Yorulmazer (2007) for the theoretical analysis with a regulation forbearance view, and refer to Brown and Dinc (2011) for its empirical evidence.

results with numerical examples. In addition, we numerically illustrate comparative statics on various parameters in the model. Section 5 summarizes the results and concludes. The Appendix A provides the proofs of the analytic results. The Appendix B presents a factor model to explain asset correlations across banks. The Appendix C provides some concrete examples to motivate the increasing social-cost to scale property of bank failures.

# 2 Model

In this section, we set up a model to be used for our analysis. We simplify the model of Repullo and Suarez (2013) to obtain analytical results but extend it to incorporate multiple banks whose defaults are correlated each other. The economy has an infinite horizon of repetitive one-period with two dates $(t = 0, 1)$. It is populated with four classes of risk-neutral agents: entrepreneurs, investors, banks, and a government. Entrepreneurs finance their projects by borrowing from banks. Investors provide their investment funds to the banks in the form of deposits and equity capital. Banks intermediate funds from investors to entrepreneurs. The government insures bank deposits.

We consider a continuum of ex ante identical entrepreneurs who have a one-period investment project but do not have their own wealth to invest so that they need to borrow funds from banks to finance projects. Each project requires a unit investment and is expected to yield a return $1 + \mu$ if it is successful and $1 - \lambda$ if it fails, where $\mu > 0$ and $\lambda \in (0, 1]$.

Each project has an identical probability of failure $p$ at time 0. Since the outcomes of the projects which are financed by borrowings from a certain bank will be stochastic and positively but imperfectly correlated each other, the aggregate failure rate $x$, defined by the ratio of default bank loans to the total bank loans lent by the same bank, will be a continuous random variable with support $[0, 1]$ whose cumulative distribution function (cdf) is denoted by $F(x)$. Therefore, we have the following relation:

$$p = E[x] = \int_0^1 x dF(x). \tag{1}$$

At time 0, a large number of investors willingly supply banks with their investment funds in the form of deposits and equity capital in a perfectly elastic way at some required rate of return. Bank deposits are assumed to be insured by the government and their required rate is normalized to zero. On the other hand, bank capital requires a positive excess return $\delta$, following the related literature (see, for example, Holmström and Tirole, 1997; Diamond and Rajan, 2000; Repullo and Suarez, 2012).

Banks are the only financial intermediaries in a competitive market. In particular, banks are funded with insured deposits at a (normalized) zero cost and equity capital at costs of $\delta$ and provide entrepreneurs with funds at a prevailing market gross rate $R$. To avoid uninteresting cases, we impose the following parameter restriction:

$$1 + \mu > R, \tag{2}$$

with which entrepreneurs have positive expected profits from projects. Similarly, to guarantee positive expected profits for banks (with zero bank capital), we also impose the restriction that

$$(1 - p) R + p (1 - \lambda) > 1. \tag{3}$$

Banks are managed in a way to maximize their shareholders' wealth. The shareholders are protected by limited liability. Since the economy infinitely evolves, bank failure would cost to the shareholders a certain amount of continuation (or charter) value $V$.[2]

A government insures bank deposits, and therefore the government needs to levy taxes to cover the cost of repaying deposits in case of bank default. In this analysis, we take a normative perspective by taking into account potential negative externalities arising from bank failures. In particular, we measure social welfare by including not only bank shareholders' wealth but also government deposit insurance costs. In addition, we introduce multiple banks in the economy and assess the welfare implications of asset correlations across banks.

[2] This continuation value may be related with a present value of future cash flows to bank shareholders if banks do not default. However, other factors may deviate the continuation value from its fundermental one. Considering various deviating factors, we simply assume a certain amount of continuation value, instead of explicitly modelling it. Notably, with zero continuation value, banks have no reason to hold bank capital in absence of capital regulation. Therefore, a positive continuation value may play a role to deter this zero bank capital result.

Even though we do not explicitly analyze minimum bank capital regulation, our normative analysis results will be directly related with bank capital regulation.

# 3 Analytic Results

In this section, we analyze banks' decision on bank capital level and its welfare implications and provide some analytic results on optimal bank capital level. We first analyze the case of a representative bank and then the case of multiple banks.

## 3.1 A representative bank

Consider a representative bank that lends a unit-size loan to the measure one continuum of entrepreneurs at time 0. Then, bank asset value at time 1 will be

$$a' = (1 - x) R + x (1 - \lambda) . \tag{4}$$

The unit-size bank loan is partially financed through bank capital $k$ which the bank will optimally choose, and the remaining amount $1 - k$ will be financed in the form of bank deposits. Bank default is assumed to occur if bank asset value is less than bank deposits, i.e, $a' < 1 - k$. Then, the bank default probability is associated with the riskiness of bank loan (i.e., aggregate failure rate). Specifically, we obtain the bank default probability as follows:

$$\Pr [a' < 1 - k] = 1 - F (\widehat{x}) , \tag{5}$$

where

$$\widehat{x} \equiv \frac{R - 1 + k}{R - 1 + \lambda} \equiv A + Bk; A \in (0, 1) , B > 0. \tag{6}$$

That is, bank default occurs if the aggregate failure rate $x$ exceeds the threshold level $\widehat{x}$.

At time 1, the bank capital will be

$$k' = \begin{cases} a' - (1 - k) , & \text{if } x < \widehat{x} \\ 0, & \text{if } x \geq \widehat{x} \end{cases} , \tag{7}$$

and the bank (firm) value will be

$$v' = \begin{cases} k', & \text{if } x < \widehat{x} \\ -V, & \text{if } x \geq \widehat{x} \end{cases} . \tag{8}$$

The bank is supposed to be managed so as to maximize bank shareholders' net wealth by optimally choosing bank capital level, i.e.,

$$\max_k \pi_b \equiv E\left[v'\right] - k\left(1+\delta\right), \tag{9}$$

where required excess rate of return on bank capital is taken into account as an opportunity cost.

Given the linear relationship between bank capital $k$ and the bank default threshold level $\widehat{x}$ as shown in (6), we will state decision problems in terms of $\widehat{x}$ instead of $k$ for analytical convenience. By introducing a notation

$$\widehat{p}\left(\widehat{x}\right) \equiv \int_0^{\widehat{x}} x dF\left(x\right), \tag{10}$$

the bank shareholders' net wealth is written as

$$\begin{aligned} \pi_b &= \int_0^{\widehat{x}} \left[\left(1-x\right)R + x\left(1-\lambda\right) - \left(1-k\right)\right] dF\left(x\right) \\ &\quad -V \int_{\widehat{x}}^1 dF\left(x\right) - \left(\widehat{x} - A\right)\left(1+\delta\right)B^{-1} \\ &= \left(\widehat{x}F\left(\widehat{x}\right) - \widehat{p}\left(\widehat{x}\right)\right)B^{-1} - V\left(1 - F\left(\widehat{x}\right)\right) - \left(\widehat{x} - A\right)\left(1+\delta\right)B^{-1}. \end{aligned} \tag{11}$$

The optimal bank capital level $k_b$ and the associated threshold level $x_b$ should satisfy the following first order condition for an optimal bank capital decision:

$$\frac{\partial \pi_b}{\partial \widehat{x}} = F\left(x_b\right)B^{-1} + Vf\left(x_b\right) - \left(1+\delta\right)B^{-1} = 0, \tag{12}$$

and also the second order condition:

$$\frac{\partial^2 \pi_b}{\partial \widehat{x}^2} = f(x_b) B^{-1} + V f'(x_b) < 0, \tag{13}$$

where $f(x)$ is the pdf of the aggregate failure rate, and $f'(x)$ denotes its first derivative. Note that the second order condition implies that

$$f'(x_b) < 0. \tag{14}$$

The following assumption formally restricts the set of distributions of the aggregate failure rate $F(x)$ to guarantee the existence of optimal bank capital level and its uniqueness.

**Assumption 1**: *Aggregate failure rate distribution function* $F(x)$ *satisfies both conditions of* $(12)$ *and* $(13)$. *In addition,* $F(x)$ *satisfies that*

$$f'(x) < 0 \text{ for } x > x_b, \tag{15}$$

*and that both the optimal threshold level* $x_b$ *and the associated optimal bank capital level* $k_b$ *lie in the region of* $(0, 1)$.

To elicit implications for bank capital regulation, we take into account the fact that bank failures entail a social cost. In particular, banks do not internalize bank deposit insurance costs, hence their bank capital choice will be socially inefficient. Government, a risk-neutral regulator, deals with this externality by choosing a socially-optimal bank capital level so as to maximize social welfare. This social welfare includes not only bank shareholders' net wealth but also bank deposit insurance costs. The socially-optimal bank capital level is determined by maximizing the social welfare; i.e.,

$$\max_k W = \pi_b - \pi_g,$$

where social welfare $W$ accounts for the expected bank deposit insurance costs $\pi_g$.

Note that the expected bank deposit insurance costs $\pi_g$ is written as

$$\begin{aligned}\pi_g &= \int_0^1 \max\left[(1-k)-a',0\right]\cdot dF(x) \\ &= B^{-1}\left[p-\widehat{p}(\widehat{x})-\widehat{x}(1-F(\widehat{x}))\right]. \end{aligned} \tag{16}$$

We can analytically derive the fact that the bank capital level chosen by banks which do not internalize social costs is less than its socially-optimal level, which is formally stated in the following proposition.

**Proposition 1** *Socially optimal bank capital level $k_s$ is higher than the level $k_b$ chosen by the bank; that is,*

$$k_s > k_b. \tag{17}$$

The proof of Proposition 1 is provided in the Appendix A.

## 3.2 Multiple banks

In this subsection, we consider $n$ symmetric banks instead of a representative bank in order to elicit implications of asset correlation on bank capital regulation. Each symmetric bank has an identical bank default probability $1-F(\widehat{x})$. Bank defaults are, however, dependent each other. For analytical simplicity, we assume the same pairwise correlation coefficient $\rho$ for bank default dependence.[3] Denote by $X_j$ bank $j$'s default indicator variable which takes value one for default event at time 1 and zero otherwise. Then, $X_j$ is a random variable following Bernoulli distribution with success (bank default) probability of $1-F(\widehat{x})$. Due to bank default dependence, the number of default banks $Y$ does not follow binomial distribution;

[3] In Appendix B, we present a factor model to explicitly relate individual loans with an identical asset correlation coefficient $\rho$ across banks.

however, we can express its first and second moments as follows:

$$\begin{aligned}
Y &\equiv \sum_{j=1}^{n} X_j, &(18)\\
E[Y] &= n[1-F(\widehat{x})], &(19)\\
Var[Y] &= n[1+2(n-1)\rho]F(\widehat{x})[1-F(\widehat{x})], &(20)\\
E\left[Y^2\right] &= n[1-F(\widehat{x})][F(\widehat{x})+2(n-1)\rho F(\widehat{x})+n\{1-F(\widehat{x})\}]. &(21)
\end{aligned}$$

In this framework of multiple banks, we take a macroprudential perspective. In particular, we assume that bank defaults entail not only bank deposit insurance costs but also other extra costs which may be justified by prevailing perceptions that bank failures entail social costs via various externality channels (see, for example, De Nicoló et al., 2012; Claessens, 2014). To capture this macroprudential notion, we assume that social cost per default bank is not constant but increasing in the number of bank defaults. Therefore, as more banks default, the associated social costs grow at an accelerating rate, exhibiting an *increasing social-cost to scale* property. Specifically, the (expected) social costs consist of not only bank deposit insurance costs but also other extra costs:

$$C \equiv \pi_g^{def} E(Y) + \eta \pi_s E\left(Y^2\right), \tag{22}$$

where $\pi_g^{def}$ denotes bank deposit insurance costs conditional on bank default, and $\pi_s$ denotes social costs other than the deposit insurance cost conditional on bank default.[4] Note that social cost per default bank $\pi_g^{def} + \eta \pi_s E(Y)$ is proportionate to the expected number of default banks $E(Y)$ with sensitivity parameter $\eta$. In contrast, we name as a *constant social-cost to scale* property a situation where social cost per default bank is constant and the same with $\pi_g^{def}$ (i.e., $\eta = 0$). The social welfare in the multiple-bank framework is defined as:

$$W \equiv n\pi_b - C. \tag{23}$$

[4]To motivate the increasing social-cost to scale property, we provide some concrete examples in Appendix C.

The conditional bank deposit insurance cost is written as:

$$\begin{aligned}\pi_g^{def} &\equiv E\left[(1-k) - a' \middle| x \geq \widehat{x}\right] \\ &= \frac{1}{1-F(\widehat{x})}\pi_g. \end{aligned} \tag{24}$$

The other social costs $\pi_s$ would be specified according to social costs under consideration. We impose the following restrictions on the social costs $\pi_s$ for analytical tractability.

**Assumption 2**: *The social cost other than the deposit insurance cost, $\pi_s$, is a function of bank capital level $k$ (equivalently the threshold level $\widehat{x}$) and satisfies both conditions:*

$$\frac{\partial \pi_s}{\partial \widehat{x}} < 0, \tag{25}$$

$$\frac{\partial^2 \pi_s}{\partial \widehat{x}^2} > 0. \tag{26}$$

We can derive several implications of macroprudential perspective and asset correlation across banks on optimal bank capital level.[5] First, if we do not take a macroprudential notion on bank defaults, then an analysis with a representative bank would be equivalent to the analysis with multiple banks. Proposition 2 formally states this result.

**Proposition 2** *If bank failure exhibits a constant social-cost to scale property (i.e., $\eta = 0$ in* (22)*), then socially-optimal bank capital for the case of multiple banks is the same with that of a representative bank.*

The proof of Proposition 2 is provided in the Appendix A.

Second, Proposition 3 shows that even though bank default are independent each other, if we take a macroprudential notion on bank defaults in the framework of multiple banks, then socially-optimal bank capital would be higher than that of the case where the macroprudential notion is ignored.

[5] Both 'asset correlation' and 'bank default correlation' are interchangeably used in this paper.

**Proposition 3** *If bank failure exhibits an increasing social-cost to scale property, and bank defaults are independent each other (i.e., $\eta > 0$ but $\rho = 0$ in (22)), then the socially-optimal bank capital is higher for $\eta > 0$ but $\rho = 0$ than that for $\eta = 0$.*

The proof of Proposition 3 is provided in the Appendix A.

Furthermore, if bank defaults are dependent, socially-optimal bank capital level would be higher than that in case of independent bank defaults, which is provided in Proposition 4.

**Proposition 4** *If bank failure exhibits an increasing social-cost to scale property, and bank defaults are dependent (i.e., $\eta > 0$ and $\rho > 0$ in (22)), and if aggregate failure rate distribution function $F(x)$ satisfies that socially-optimal bank capital lies in the region satisfying $F(x) > \frac{1}{2} - \frac{f(x)^2}{f'(x)}$, then the socially-optimal bank capital is higher for $\eta > 0$ and $\rho > 0$ than for $\eta > 0$ but $\rho = 0$.*

The proof of Proposition 4 is provided in the Appendix A.

# 4 Numerical Illustrations

In this section, we illustrate our previous results with numerical examples. In addition, we numerically illustrate comparative statics on various parameters in the model.

We assume the aggregate failure rate to follow beta distribution. Beta distribution is a flexible probability distribution family with support $(0, 1)$ and two parameters $\alpha$ and $\beta$. Under the beta distribution assumption, the pdf and the cdf of the aggregate failure rate are specified as follows:

$$f(x) = \frac{\Gamma(\alpha, \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}; \alpha, \beta > 0, \tag{27}$$

$$F(x) = I_x(\alpha, \beta), \tag{28}$$

where $\Gamma(\cdot)$ is the gamma function, and $I_x(\alpha, \beta)$ is the regularized incomplete beta function.

Its mean will be matched with the probability of failure $p$; that is,

$$E[X] = \frac{\alpha}{\alpha + \beta} = p. \tag{29}$$

Therefore, we have only one free parameter $\alpha$ regarding the cdf of the aggregate failure rate $F(x)$.

Following Repullo and Suarez (2013), we calibrate $p$ as 3.6%. Figure 1 illustrates the pdfs and the cdfs of the aggregate failure rate, and Figure 2 shows its variance, skewness, and excess kurtosis with various levels of $\alpha$. With greater $\alpha$, the aggregate failure rate tends to have lower variance and kurtosis but higher skewness.

We calibrate $\lambda$ as 0.45 and $\delta$ as 8%, following Repullo and Suarez (2013). We employ an average intermediation margin of 3.97% from Repullo and Suarez (2013) to calibrate $R$ by setting

$$(1 - p) R + p (1 - \lambda) = 1.0397,$$

which implies $R$ of 5.8%. The number of banks in the economy $n$ is assumed as 20. For other parameters such as $V$, $\rho$, and $\eta$, we vary their levels.

Figure 3 illustrates how the optimal bank capital level is determined in case of a representative bank by showing bank shareholders' net wealth $\pi_b$, expected deposit insurance costs $\pi_g$, and social welfare $W$ along with various threshold levels. Intuitively, higher threshold level (with higher bank capital level) would reduce bank default probability, which in turn reduces deposit insurance costs. Too low threshold level (equivalently, too low bank capital level) would not be optimal because of too high probability of bank default which entails a large amount of losses to bank shareholders. On the other hand, too high bank capital level would not be optimal either because high opportunity costs of bank capital relative to deposits will reduce bank profitability. Therefore, optimal bank capital level from bank shareholders' perspective should not be either too low or too high, which is illustrated in Figure 3. Moreover, the optimal capital levels are uniquely found.

Table 1 compares optimal bank capital levels from bank shareholders' perspective with socially-optimal bank capital levels along with various levels of $V$ in case of a representative bank. Due to negative externalities (deposit insurance costs), socially-optimal bank capital level turns out to be higher than the optimal level from the perspective of bank shareholders. Understandably, bank manager would have an incentive to hold sufficient bank capital to guard against bank default if bankruptcy cost is large. This incentive implies that as additional bankruptcy cost $V$ becomes greater, not only optimal bank capital level tends to be higher, but also the difference between the two decreases. In addition, we observe that optimal bank capital level tends to be higher with smaller $\alpha$ which corresponds to larger variance and kurtosis as shown in Figure 2. Indeed, larger variance and kurtosis imply greater bank default risks which are to be contained by a higher bank capital.

Table 2 shows the results on optimal bank capital levels when there exist multiple banks in the economy and we consider social costs $\pi_s$ associated with deposit insurance funding costs which are explained in Appendix C. This table shows the effects of $\eta$, $V$, and $\alpha$ on optimal bank capital levels from bank shareholders' perspective and socially-optimal bank capital levels, and the difference between the two along with various levels of $\rho$. As asset correlation across banks becomes greater, social costs from bank defaults also become greater, which in turn requires a higher socially-optimal bank capital level. On the other hand, bank managers do not internalize this asset correlation across banks; therefore, the optimal bank capital level from bank shareholders' perspective remains the same even though the correlation varies.

Panel A of Table 2 illustrates the effect of $\eta$ on the optimal bank capital levels. As social costs per default bank become more sensitive to the number of bank failures, socially-optimal bank capital levels should be higher. On the other hand, Panel B of Table 2 illustrates the effect of $V$ on the optimal bank capital levels. Greater bankruptcy cost $V$ would provide more incentive to avoid bank default; thus, socially-optimal bank capital levels should be higher, which is consistent with the case of a representative bank. Panel C of Table 2 shows the effect of $\alpha$ on the optimal bank capital levels. Larger variance and kurtosis (smaller $\alpha$) imply greater bank default risks which in turn requires higher socially-optimal bank capital levels, consistent with Table 1.

For the robustness of the results, we vary the calibrated parameter values and investigate the effects of those parameters on the optimal bank capital levels. Table 3 compares the three optimal bank capital levels: optimal bank capital levels from bank shareholders' perspective ($\eta = 0$), socially-optimal bank capital levels with zero asset correlation ($\eta = 5n, \rho = 0$), and socially-optimal bank capital levels with positive asset correlation ($\eta = 5n, \rho = 0.3$). Panel A illustrates the effect of aggregate failure rate $p$ on the optimal bank capital levels. Intuitively, a higher riskiness of bank loan portfolios would require a higher bank capital level in order to contain a heightened default risks. Consistent with our analytical results, socially-optimal bank capital level would be higher than that of optimal bank capital levels from bank shareholders' perspective, and it would be higher if asset correlations across banks exist and are positive than otherwise. Optimal bank capital level would be lower if the opportunity cost of bank capital $\delta$ becomes higher, which is demonstrated in Panel B. In contrast, a greater loss given default $\lambda$ would yield a higher probability of bank default which in turn requires a higher level of optimal bank capital. Panel C numerically confirms this intuition. Higher gross loan rate $R$ could contribute to a higher bank capital level in the next period and thus would require a lower optimal bank capital level, which is numerically shown in Panel D.

We conduct similar numerical exercises for the case where we consider social costs $\pi_s$ associated with liquidation costs which are explained in Appendix C. Tables 4 and 5 provide the results which are similar with the case of deposit insurance funding costs (as shown in Tables 2 and 3).

# 5 Conclusion

Financial institutions typically hold similar asset portfolios, which may lead to systemic risk, if a common shock negatively affects asset values, and thereby many financial institutions simultaneously get into financial distress. Therefore, a strong asset correlation across financial institutions may pose a high potential systemic risk. Even though this asset correlation effect has been well recognized, surprisingly, however, it has not been formally taken into account within financial regulations but also has rarely been studied from an optimal regulation view yet.

In this paper, we present a simple model with multiple banks in which bank defaults are correlated each other and elicit macroprudential implications of asset correlation on bank capital regulation. In particular, we analytically show that when bank failure exhibits an increasing social-cost to scale property, an optimal bank capital level would be higher with a higher asset correlation across banks. Moreover, we quantitatively show that an optimal bank capital level is not constant but varies with economic situations such as riskiness of bank loan portfolios, loss given default, cost of bank capital, and bank loan interest rates.

Our findings have profound implications on financial regulation policy. A strong asset correlation across banks may lead to the so-called "too-many-to-fail" problem under regulation forbearance. Our findings suggest that to preemptively respond to the "too-many-to-fail" problem, we may need an additional bank capital surcharge which is analogous to the bank capital surcharges for the systemically-important financial institutions to prevent the "too-big-to-fail" problem in the Basel III framework.

To derive analytical results and to highlight the effect of asset correlation across banks on an optimal bank capital level, we present a simplified model, abstract from many realistic features. Therefore, to obtain more realistic quantitative results, it would be necessary to develop a richer model which encompasses asset correlation across banks as well as other realistic features. There would be many related empirical issues of importance to examine how much asset correlations differ across countries, how much asset correlation changes over time, how much changes in asset correlation are related with economic situations, and whether and how closely asset correlation is related with systemic risk.

In our model, the increasing social-cost to scale property is a prerequisite for the effect of asset correlation across banks to affect an optimal bank capital level. Alternatively, even though bank failure exhibits the constant social-cost to scale property, if a government (or a regulator) is not risk-neutral but risk-averse or subject to a VaR-type risk constraint, then asset correlation across banks may affect an optimal bank capital level. A combination of the increasing social-cost to scale property with the risk-averseness of the regulator would reinforce the effect of asset correlation. This issue is also explorable in a future research.

# Appendix A. Proofs

In the Appendix, we provide the proofs of Propositions 1 to 4. For the proofs, we begin with the following lemma.

**Lemma:** *Suppose that a continuous and twice-differentiable function $h(x)$ attains its maximum at $x^*$ (i.e., $h'(x^*) = 0$ and $h''(x^*) < 0$) and satisfies both $h'(x) < 0$ and $h''(x) < 0$ for $x > x^*$ and that another continuous and twice-differentiable function $\widetilde{h}(x)$ satisfies both $\widetilde{h}'(x) < 0$ and $\widetilde{h}''(x) > 0$ for $x \geq x^*$. Then, the difference between the two functions $H(x) \left( \equiv h(x) - \widetilde{h}(x) \right)$ attains its maximum at $x^{**}$ which is greater than $x^*$ (i.e., $x^{**} > x^*$).*

**Proof:** Since

$$\begin{aligned} H'(x^*) &= h'(x^*) - \widetilde{h}'(x^*) \\ &= -\widetilde{h}'(x^*) > 0, \end{aligned} \tag{A-1}$$

the first derivative of $H(x)$ evaluated at $x^*$ is positive. From the fact that

$$H''(x) = h''(x) - \widetilde{h}''(x) < 0, \tag{A-2}$$

the first derivative of $H(x)$ is uniformly decreasing for $x \geq x^*$. Therefore, we have the following relation:

$$H'(x^*) > 0 = H'(x^{**}), \tag{A-3}$$

which implies that

$$x^* < x^{**}.$$

Moreover, the fact that

$$H''(x^{**}) = h''(x^{**}) - \widetilde{h}''(x^{**}) < 0 \tag{A-4}$$

guarantees that $H(x)$ attains its maximum at $x^{**}$. ■

**Proof of Proposition 1:** From the lemma, it is enough to show that

$$\begin{aligned} \frac{\partial \pi_g}{\partial \widehat{x}} &< 0, & \text{(A-5)} \\ \frac{\partial^2 \pi_g}{\partial \widehat{x}^2} &> 0. & \text{(A-6)} \end{aligned}$$

Notice that

$$\begin{aligned} \frac{\partial \pi_g}{\partial \widehat{x}} &= B^{-1}\left[-\widehat{x} f(\widehat{x}) - 1 + F(\widehat{x}) + \widehat{x} f(\widehat{x})\right] \\ &= B^{-1}\left[-1 + F(\widehat{x})\right] < 0, \qquad \text{(A-7)} \end{aligned}$$

and

$$\frac{\partial^2 \pi_g}{\partial \widehat{x}^2} = B^{-1} f(\widehat{x}) > 0. \qquad \text{(A-8)}$$

Therefore, we have

$$\begin{aligned} x_s &> x_b, & \text{(A-9)} \\ k_s &> k_b. & \text{(A-10)} \end{aligned}$$

■

**Proof of Proposition 2:** Notice that if $\eta = 0$,

$$\begin{aligned} W &= n\pi_b - \pi_g^{def} E[Y] \\ &= n[\pi_b - \pi_g], \qquad \text{(A-11)} \end{aligned}$$

which is simply proportionate to the social welfare for the case of a single bank. Therefore, the optimal bank capital for multiple banks would be the same with that of a single bank.■

**Proof of Proposition 3:** With $\eta > 0$ but $\rho = 0$, the social welfare function is rewritten as

$$
\begin{aligned}
W &= n\pi_b - \pi_g^{def} E[Y] - \eta \pi_s E\left[Y^2\right] \\
&= n[\pi_b - \pi_g] - \eta \pi_s E\left[Y^2\right] \\
&\equiv n[\pi_b - \pi_g] - \eta M, \qquad \text{(A-12)}
\end{aligned}
$$

where

$$
M \equiv \pi_s E\left[Y^2\right].
$$

From the lemma, it is enough to show that

$$
\begin{aligned}
\frac{\partial M}{\partial \widehat{x}} &< 0, \qquad \text{(A-13)} \\
\frac{\partial^2 M}{\partial \widehat{x}^2} &> 0. \qquad \text{(A-14)}
\end{aligned}
$$

Now, we obtain the following expression:

$$
\frac{\partial M}{\partial \widehat{x}} = \pi_s'(\widehat{x}) E\left[Y^2\right] + \pi_s(\widehat{x}) \frac{\partial E[Y^2]}{\partial \widehat{x}}. \qquad \text{(A-15)}
$$

The first term is negative from (25). Since

$$
\begin{aligned}
\frac{\partial E[Y^2]}{\partial \widehat{x}} &= -nf(\widehat{x})[F(\widehat{x}) + n\{1 - F(\widehat{x})\}] \\
&\quad -n(n-1) f(\widehat{x})[1 - F(\widehat{x})] \\
&< 0, \qquad \text{(A-16)}
\end{aligned}
$$

the second term is also negative; therefore, $(A-13)$ is established.

Next, we obtain the following expression:

$$
\frac{\partial^2 M}{\partial \widehat{x}^2} = \pi_s''(\widehat{x}) E\left[Y^2\right] + 2\pi_s'(\widehat{x}) \frac{\partial E[Y^2]}{\partial \widehat{x}} + \pi_s(\widehat{x}) \frac{\partial^2 E[Y^2]}{\partial \widehat{x}^2}, \qquad \text{(A-17)}
$$

where the first term is positive from (26). The second term is also positive from both (25) and $(A-16)$. Lastly, the third term is positive from the fact that

$$\begin{aligned}\frac{\partial^2 E\left[Y^2\right]}{\partial \widehat{x}^2} &= -nf'(\widehat{x})\left[F(\widehat{x})+n\left\{1-F(\widehat{x})\right\}\right] \\ &\quad +2n(n-1)f(\widehat{x})^2 \\ &\quad -n(n-1)f'(\widehat{x})\left[1-F(\widehat{x})\right] \\ &> 0. \end{aligned} \tag{A-18}$$

Since the three terms are positive, $(A-14)$ holds. ■

**Proof of Proposition 4:** With $\eta > 0$ and $\rho > 0$, the social welfare function is rewritten as

$$\begin{aligned} W &= n\pi_b - \pi_g^{def} E[Y] - \eta\pi_s(\widehat{x}) E\left[Y^2\right] \\ &\equiv n\left[\pi_b - \pi_g\right] - \eta M - \eta Q, \end{aligned} \tag{A-19}$$

where $Q$ is defined as

$$Q \equiv 2n(n-1)\rho\pi_s(\widehat{x})\left[1-F(\widehat{x})\right]F(\widehat{x}). \tag{A-20}$$

From the lemma, it is enough to show that

$$\frac{\partial Q}{\partial \widehat{x}} < 0, \tag{A-21}$$

$$\frac{\partial^2 Q}{\partial \widehat{x}^2} > 0. \tag{A-22}$$

Now, we can show the following inequality relation:

$$\begin{aligned}\frac{\partial Q}{\partial \widehat{x}} &= 2n(n-1)\rho\pi_s'(\widehat{x})\left[1-F(\widehat{x})\right]F(\widehat{x}) \\ &\quad +2n(n-1)\rho\pi_s(\widehat{x})f(\widehat{x})\left[1-2F(\widehat{x})\right] \\ &< 0 \end{aligned} \tag{A-23}$$

if the following condition holds:

$$\frac{1}{2} < F(\widehat{x}). \tag{A-24}$$

Note that this condition implies that bank default probability should be less than a half at the optimal bank capital level, i.e., $1 - F(\widehat{x}) < 0.5$.

Next, we obtain the following inequality relation:

$$\begin{aligned} \frac{\partial^2 Q}{\partial \widehat{x}^2} &= 2n(n-1)\rho\pi_s''(\widehat{x})[1 - F(\widehat{x})]F(\widehat{x}) \\ &\quad +4n(n-1)\rho\pi_s'(\widehat{x})f(\widehat{x})[1 - 2F(\widehat{x})] \\ &\quad +2n(n-1)\rho\pi_s(\widehat{x})\left[f'(\widehat{x}) - 2f'(\widehat{x})F(\widehat{x}) - 2f(\widehat{x})^2\right] \\ &> 0 \end{aligned} \tag{A-25}$$

if

$$f'(\widehat{x}) - 2f'(\widehat{x})F(\widehat{x}) - 2f(\widehat{x})^2 > 0,$$

or, equivalently,

$$F(\widehat{x}) > \frac{1}{2} - \frac{f(\widehat{x})^2}{f'(\widehat{x})}. \tag{A-26}$$

Note that the first term in $(A-25)$ is positive from (26), and the second term is also positive from (25) and $(A-24)$. The condition $(A-26)$ encompasses $(A-24)$.■

# Appendix B. Factor model for asset correlation

In Subsection 3.2, asset correlation is exogenously given. In this appendix, we present a factor model to explicitly relate individual loans with an identical asset correlation coefficient $\rho$ across banks. Bank $j$ contains a continuum of measure one loans with the same size. An individual loan (indexed by $i$) default is defined by the event that a latent variable takes a negative value. The latent variable is modeled with three factors: economy-wide factor $u$, bank-specific factor $w_j$, and idiosyncratic factor $\varepsilon_{j,i}$; that is, the latent variable is specified as:

$$z_{j,i} = p + \sqrt{\omega} u + \sqrt{\xi} w_j + \sqrt{1 - \omega - \xi} \varepsilon_{j,i}, \tag{A-27}$$

where

$$\begin{aligned} u &\sim G_u; w_j \sim i.i.d.G_w; \varepsilon_{j,i} \sim i.i.d.G_\varepsilon, \\ E(u) &= E(w_j) = E(\varepsilon_{j,i}) = 0, \\ \omega,\ \xi,\ \omega + \xi &\in (0,1), \end{aligned}$$

and the three factors are independent each other.

With a continuum of individual loans, bank $j$'s aggregate failure rate $x_j$ is a function of of the realizations of the common factor $u$ and the bank-specific factor $w_j$. By the law of large numbers, the aggregate failure rate becomes

$$\begin{aligned} x_j &= \Pr\left[ p + \sqrt{\omega} u + \sqrt{\xi} w_j + \sqrt{1 - \omega - \xi} \varepsilon_{j,i} < 0 \middle| u, w_j \right] \\ &= \Pr\left[ \varepsilon_{j,i} < -\frac{p + \sqrt{\omega} u + \sqrt{\xi} w_j}{\sqrt{1 - \omega - \xi}} \middle| u, w_j \right] \\ &= G_\varepsilon\left[ -\frac{p + \sqrt{\omega} u + \sqrt{\xi} w_j}{\sqrt{1 - \omega - \xi}} \right]. \end{aligned} \tag{A-28}$$

Hence, bank $j$'s default probability is written as

$$\begin{aligned}
\Pr\left[x_j > \widehat{x}\right] &= \Pr\left[G_\varepsilon\left[-\frac{p + \sqrt{\omega}u + \sqrt{\xi}w_j}{\sqrt{1-\omega-\xi}}\right] > \widehat{x}\right] \\
&= \Pr\left[w_j < -\frac{p + \sqrt{\omega}u + \sqrt{1-\omega-\xi}G_\varepsilon^{-1}\left(\widehat{x}\right)}{\sqrt{\xi}}\right] \\
&= \int G_w\left[-\frac{p + \sqrt{\omega}u + \sqrt{1-\omega-\xi}G_\varepsilon^{-1}\left(\widehat{x}\right)}{\sqrt{\xi}}\right] \cdot dG_u\left(u\right). \qquad \text{(A-29)}
\end{aligned}$$

Lastly, the pairwise correlation coefficient between defaults by banks $j$ and $l$ is defined by

$$\rho \equiv \frac{E\left[X_j X_l\right] - E\left[X_j\right]E\left[X_l\right]}{\sqrt{E\left[X_j\right] - E\left[X_j\right]^2}\sqrt{E\left[X_l\right] - E\left[X_l\right]^2}}. \qquad \text{(A-30)}$$

Note that the mean of bank $j$'s default indicator variable is given by

$$E\left[X_j\right] = \Pr\left[x_j > \widehat{x}\right] = 1 - F\left(\widehat{x}\right), \qquad \text{(A-31)}$$

and the expected value of a product of two default indicator variables can be calculated via the following joint cdf of aggregate failure rates:

$$\begin{aligned}
E\left[X_j X_l\right] &= \Pr\left[x_j > \widehat{x}, x_l > \widehat{x}\right] \\
&= \int \left\{G_w\left[-\frac{p + \sqrt{\omega}u + \sqrt{1-\omega-\xi}G_\varepsilon^{-1}\left(\widehat{x}\right)}{\sqrt{\xi}}\right]\right\}^2 \cdot dG_u\left(u\right). \qquad \text{(A-32)}
\end{aligned}$$

This factor model explicitly relates bank asset correlation with risk factors. As an extreme case, bank defaults would be perfectly correlated without bank-specific factor (i.e., $\xi = 0$). As another extreme, bank defaults would be perfectly uncorrelated without economy-wide factor (i.e., $\omega = 0$). Between the two extremes, asset correlation across banks will be governed by a relative strength of the two factors.

Analogously, individual loans within the same bank loan portfolios are correlated each other in this factor model, and its identical correlation coefficient will be determined by a relative strength of the three risk factors. Obviously, the asset correlation across individual loans within the same bank loan portfolios differs from that across banks.

# Appendix C. Increasing social-cost to scale property of bank failures

In Subsection 3.2, we simply assume an increasing social-cost to scale property for bank failures. To motivate the increasing social-cost to scale property, we consider some concrete examples in this appendix.

**Deposit insurance funding costs.**

Suppose that deposit insurance costs should be first financed by government (real) bond issuance in the bond market, and then taxes will be collected later for repaying the government debt. Demand for government bond issuance amounts to the conditional bank deposit insurance cost per default bank times the number of default bank, i.e.,

$$D_g = \pi_g^{def} Y. \tag{A-33}$$

Supply for funds in the bond market is assumed to be positively related with a unit borrowing cost $p_g$. In particular, we specify an upward-sloping supply function as

$$S_g = c_g + b_g p_g, \;\; b_g > 0. \tag{A-34}$$

In equilibrium, unit borrowing cost becomes

$$p_g^* = \frac{\pi_g^{def} Y - c_g}{b_g}, \tag{A-35}$$

which is rewritten as

$$p_g^* = 1 + \widetilde{b}_g \pi_g^{def} Y. \tag{A-36}$$

This equation implies that the unit borrowing cost $p_g$ is normalized to one with an infinitely elastic supply for funds (i.e., $b_g = \infty$ and $\widetilde{b}_g = 0$) but increases as demand for funds increases.

The total deposit insurance costs amount to the government bond issuance amount times the unit borrowing cost; that is,

$$\pi_g^{def} Y p_g^* = \pi_g^{def} Y \left( 1 + \widetilde{b}_g \pi_g^{def} Y \right),$$

therefore, the total social costs are

$$C = \pi_g^{def} E(Y) + \widetilde{b}_g \left(\pi_g^{def}\right)^2 E\left(Y^2\right). \tag{A-37}$$

Note that the social cost per default bank is not constant but increases in the expected number of bank defaults $E(Y)$. In this example, $\pi_s = \left(\pi_g^{def}\right)^2$ and $\eta = \widetilde{b}_g$. Since $(A-7)$ satisfies $(25)$, and $(A-8)$ implies $(26)$, Assumption 2 holds for this example.

**Liquidation costs.**

Suppose that assets held by default banks will be liquidated by the government. The assets to be liquidated are traded in the special market for liquidation.[6] In addition to the deposit insurance costs per default bank $\pi_g^{def}$, additional costs may arise from this liquidation process.

The assets to be liquidated per default bank is equal to $a'$, and the supply of assets in the market for asset liquidation is

$$S_a = Ya', \quad \text{if } x \geq \widehat{x}. \tag{A-38}$$

The demand for liquidated assets is assumed to be negatively related with asset price $p_a$. Specifically, we assume a downward-sloping demand function as

$$D_a = c_a - b_a p_a, \ b_a > 0. \tag{A-39}$$

In equilibrium, the liquidated asset price is determined as

$$p_a^* = \frac{c_a - Ya'}{b_a}, \quad \text{if } x \geq \widehat{x},$$

which is rewritten as

$$p_a^* = 1 - \widetilde{b}_a a' Y. \tag{A-40}$$

[6] This market is special in the sense that it is segmented from other markets, and asset prices in this market do not affect other market prices. Obviously, this assumption is unreal but made for analytical simplicity.

This expression implies that the liquidated asset price $p_a$ is normalized to one with an infinitely elastic demand for funds (i.e., $b_a = \infty$ and $\widetilde{b}_a = 0$) but decreases as supply for funds increases.

The liquidation costs are

$$S_a \left(1 - p_a^*\right) = a' \widetilde{b}_a Y^2 = \frac{a'^2}{b_a} Y^2.$$

In this example, $\eta = b_a^{-1}$, and

$$\pi_s = \frac{1}{1 - F\left(\widehat{x}\right)} \int_{\widehat{x}}^{1} a'^2 dF\left(x\right) = \frac{1}{1 - F\left(\widehat{x}\right)} \int_{\widehat{x}}^{1} \left(R - B^{-1}x\right)^2 dF\left(x\right). \tag{A-41}$$

Note that the following inequality holds

$$\begin{aligned} \frac{\partial \pi_s}{\partial \widehat{x}} &= \frac{-f\left(\widehat{x}\right)}{\left[1 - F\left(\widehat{x}\right)\right]^2} \int_{\widehat{x}}^{1} \left(R - B^{-1}x\right)^2 dF\left(x\right) - \frac{f\left(\widehat{x}\right)}{1 - F\left(\widehat{x}\right)} \left(R - B^{-1}\widehat{x}\right)^2 \\ &< 0, \end{aligned} \tag{A-42}$$

because each of the two terms is clearly negative. We can also establish the following inequality:

$$\begin{aligned} \frac{\partial^2 \pi_s}{\partial \widehat{x}^2} &= \left[\frac{-f'\left(\widehat{x}\right)}{\left[1 - F\left(\widehat{x}\right)\right]^2} + \frac{2f\left(\widehat{x}\right)^2}{\left[1 - F\left(\widehat{x}\right)\right]^3}\right] \int_{\widehat{x}}^{1} \left(R - B^{-1}x\right)^2 dF\left(x\right) \\ &\quad + \frac{f\left(\widehat{x}\right)^2}{\left[1 - F\left(\widehat{x}\right)\right]^2} \left(R - B^{-1}\widehat{x}\right)^2 + \frac{f\left(\widehat{x}\right)}{\left[1 - F\left(\widehat{x}\right)\right]^2} \left(R - B^{-1}\widehat{x}\right)^2 f\left(\widehat{x}\right) \\ &\quad + \frac{1}{1 - F\left(\widehat{x}\right)} 2B^{-1} \left(R - B^{-1}\widehat{x}\right) f\left(\widehat{x}\right) - \frac{1}{1 - F\left(\widehat{x}\right)} \left(R - B^{-1}\widehat{x}\right)^2 f'\left(\widehat{x}\right) \\ &> 0, \end{aligned} \tag{A-43}$$

because all terms are positive. Here, (15) is utilized for the positiveness of the first and the last terms. The above two inequalities imply that Assumption 2 holds for this example.

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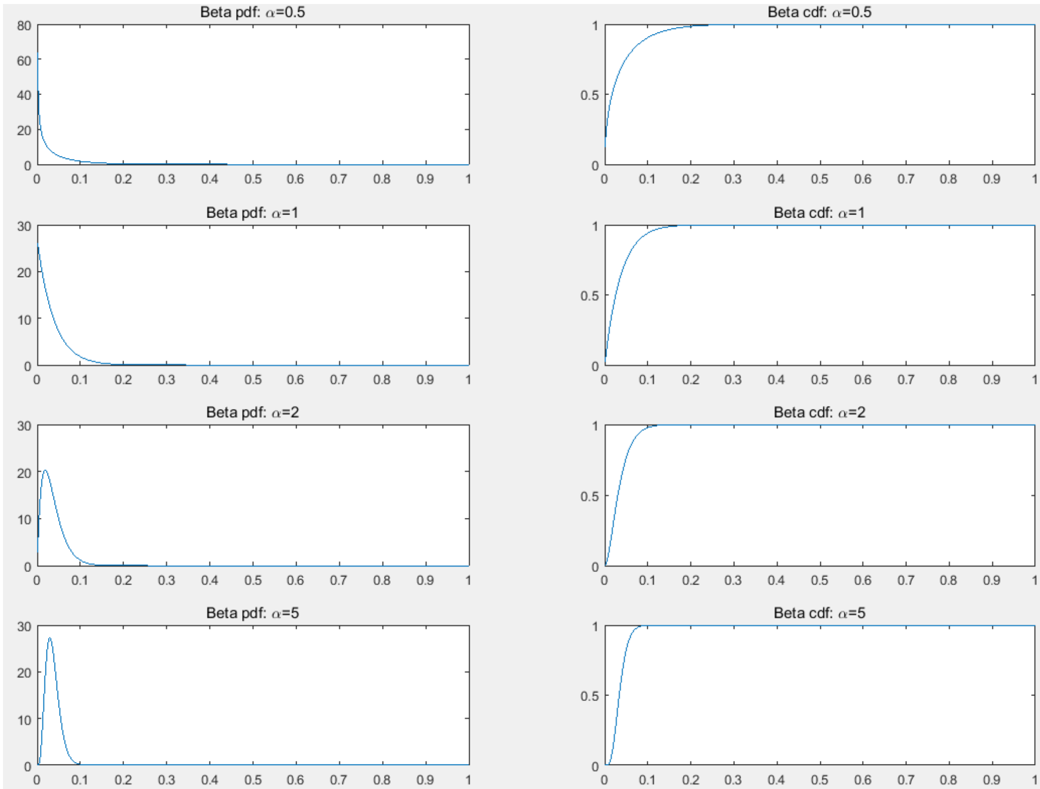


Figure 1. Pdfs and cdfs of the aggregate failure rate. This figure illustrates the pdfs and the cdfs of the aggregate failure rate which is assumed to follow beta distribution. Parameter $\alpha$ varies while parameter $\beta$ is determined so as to match the mean with the calibrated level of the probability of failure $p$.

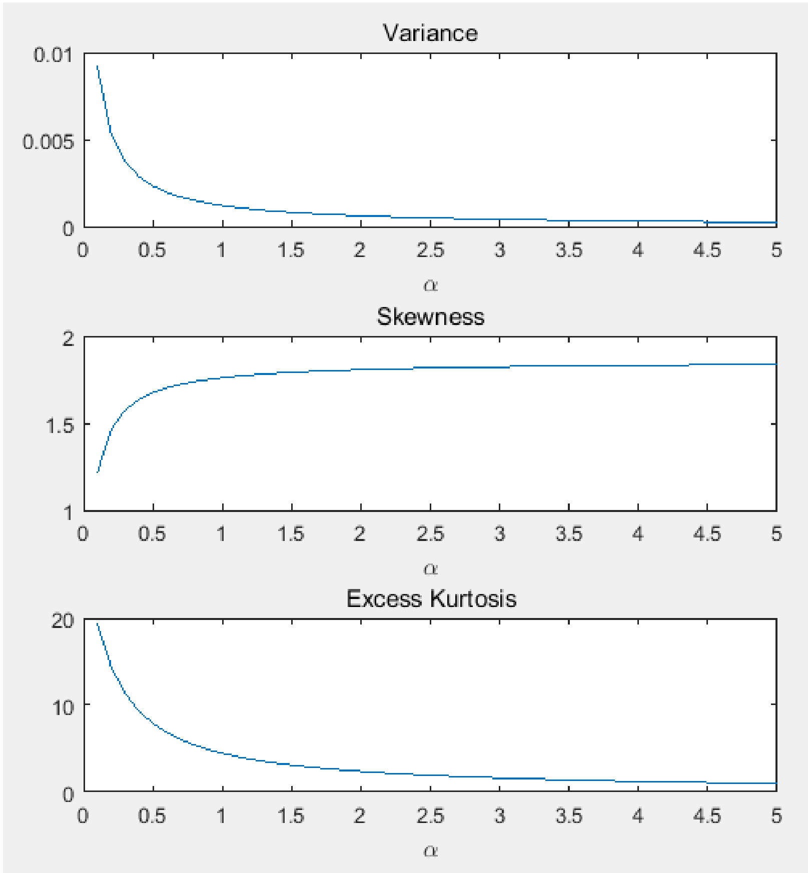


Figure 2. Variance, skewness, and excess kurtosis of the aggregate failure rate. This figure illustrates the variance, skewness, and excess kurtosis of the aggregate failure rate. Refer to Figure 1 for other explanations.

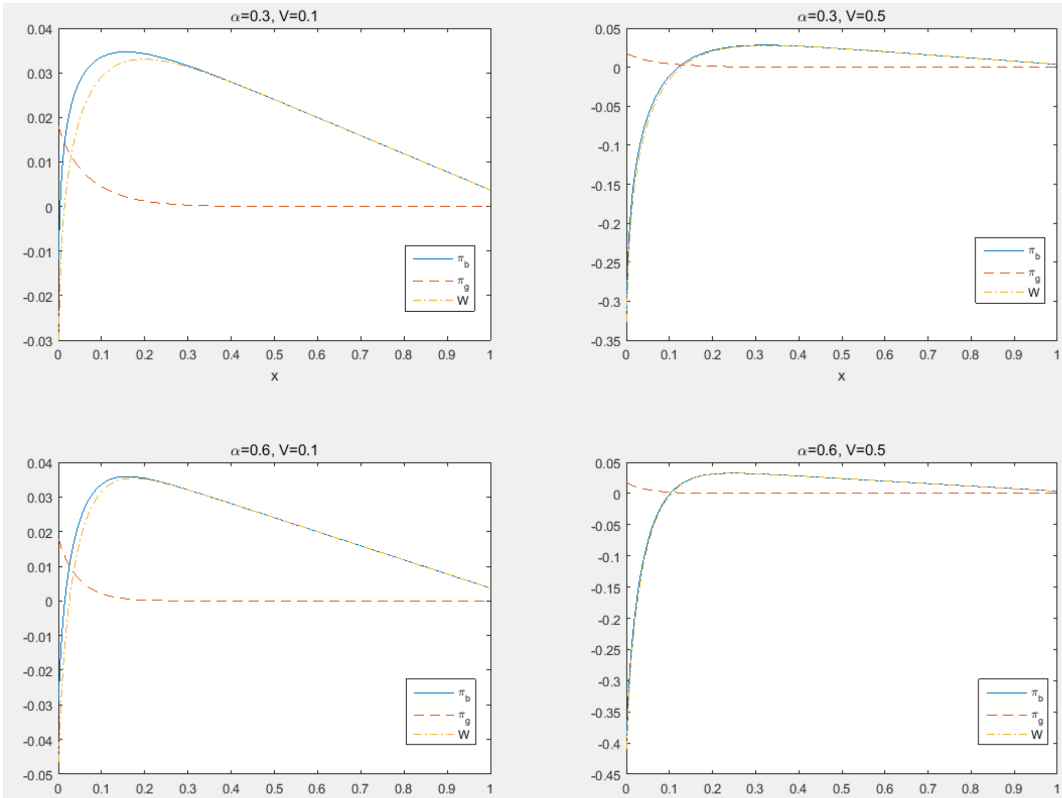


Figure 3. Bank shareholders' net wealth, expected deposit insurance costs, and social welfare: A representative bank case. This figure illustrates bank shareholders' net wealth $\pi_b$, expected deposit insurance costs $\pi_g$, and social welfare $W$ along with various threshold levels in case of a representative bank.

Table 1. Optimal bank capital levels: A representative bank case.

This table shows optimal bank capital levels from bank shareholders' perspective ($k_b$) and socially-optimal bank capital levels ($k_s$), and the difference between the two along with various levels of $V$ in case of a representative bank.

| $V$ | $\alpha = 0.3$ | | | $\alpha = 0.6$ | | |
|---|---|---|---|---|---|---|
| | $k_s$ | $k_b$ | $k_s - k_b$ | $k_s$ | $k_b$ | $k_s - k_b$ |
| 0.10 | 4.26 | 2.13 | 2.13 | 2.99 | 2.33 | 0.66 |
| 0.20 | 7.15 | 6.19 | 0.97 | 4.72 | 4.41 | 0.30 |
| 0.30 | 8.83 | 8.22 | 0.61 | 5.68 | 5.48 | 0.20 |
| 0.40 | 10.00 | 9.59 | 0.41 | 6.34 | 6.19 | 0.15 |
| 0.50 | 10.91 | 10.61 | 0.30 | 6.90 | 6.75 | 0.15 |
| 0.60 | 11.63 | 11.37 | 0.25 | 7.31 | 7.21 | 0.10 |
| 0.70 | 12.23 | 12.03 | 0.20 | 7.66 | 7.61 | 0.05 |
| 0.80 | 12.79 | 12.59 | 0.20 | 7.97 | 7.92 | 0.05 |
| 0.90 | 13.25 | 13.10 | 0.15 | 8.22 | 8.17 | 0.05 |
| 1.00 | 13.66 | 13.50 | 0.15 | 8.48 | 8.42 | 0.05 |

Table 2. Optimal bank capital levels for multiple banks: Deposit insurance funding costs case.

This table shows the effects of $\eta$ (panel A), $V$ (panel B), and $\alpha$ (panel C) on optimal bank capital levels from bank shareholders' perspective ($k_b$) and socially-optimal bank capital levels ($k_s$), and the difference between the two along with various levels of $\rho$ in case of multiple banks. Refer to Appendix C for the deposit insurance funding costs.

| A. Effect of $\eta : \alpha = 0.45, V = 0.5$ | | | | | | |
|---|---|---|---|---|---|---|
| $\rho$ | $\eta = 5n$ | | | $\eta = 10n$ | | |
| | $k_s$ | $k_b$ | $k_s - k_b$ | $k_s$ | $k_b$ | $k_s - k_b$ |
| 0.10 | 8.58 | 8.27 | 0.30 | 8.68 | 8.27 | 0.41 |
| 0.20 | 8.63 | 8.27 | 0.36 | 8.83 | 8.27 | 0.56 |
| 0.30 | 8.73 | 8.27 | 0.46 | 8.98 | 8.27 | 0.71 |
| 0.40 | 8.78 | 8.27 | 0.51 | 9.09 | 8.27 | 0.81 |
| 0.50 | 8.88 | 8.27 | 0.61 | 9.24 | 8.27 | 0.97 |
| 0.60 | 8.93 | 8.27 | 0.66 | 9.34 | 8.27 | 1.07 |
| 0.70 | 9.03 | 8.27 | 0.76 | 9.44 | 8.27 | 1.17 |
| 0.80 | 9.09 | 8.27 | 0.81 | 9.54 | 8.27 | 1.27 |
| 0.90 | 9.14 | 8.27 | 0.86 | 9.64 | 8.27 | 1.37 |
| B. Effect of $V : \alpha = 0.45, \eta = 5n$ | | | | | | |
| $\rho$ | $V = 0.3$ | | | $V = 0.6$ | | |
| | $k_s$ | $k_b$ | $k_s - k_b$ | $k_s$ | $k_b$ | $k_s - k_b$ |
| 0.10 | 7.15 | 6.65 | 0.51 | 9.09 | 8.88 | 0.20 |
| 0.20 | 7.26 | 6.65 | 0.61 | 9.14 | 8.88 | 0.25 |
| 0.30 | 7.41 | 6.65 | 0.76 | 9.19 | 8.88 | 0.30 |
| 0.40 | 7.51 | 6.65 | 0.86 | 9.29 | 8.88 | 0.41 |
| 0.50 | 7.66 | 6.65 | 1.02 | 9.34 | 8.88 | 0.46 |
| 0.60 | 7.76 | 6.65 | 1.12 | 9.39 | 8.88 | 0.51 |
| 0.70 | 7.87 | 6.65 | 1.22 | 9.44 | 8.88 | 0.56 |
| 0.80 | 7.97 | 6.65 | 1.32 | 9.49 | 8.88 | 0.61 |
| 0.90 | 8.07 | 6.65 | 1.42 | 9.54 | 8.88 | 0.66 |
| C. Effect of $\alpha : V = 0.5, \eta = 5n$ | | | | | | |
| $\rho$ | $\alpha = 0.3$ | | | $\alpha = 0.6$ | | |
| | $k_s$ | $k_b$ | $k_s - k_b$ | $k_s$ | $k_b$ | $k_s - k_b$ |
| 0.10 | 11.17 | 10.61 | 0.56 | 6.95 | 6.75 | 0.20 |
| 0.20 | 11.37 | 10.61 | 0.76 | 7.00 | 6.75 | 0.25 |
| 0.30 | 11.52 | 10.61 | 0.91 | 7.00 | 6.75 | 0.25 |
| 0.40 | 11.68 | 10.61 | 1.07 | 7.05 | 6.75 | 0.30 |
| 0.50 | 11.83 | 10.61 | 1.22 | 7.10 | 6.75 | 0.36 |
| 0.60 | 11.98 | 10.61 | 1.37 | 7.15 | 6.75 | 0.41 |
| 0.70 | 12.13 | 10.61 | 1.52 | 7.15 | 6.75 | 0.41 |
| 0.80 | 12.29 | 10.61 | 1.68 | 7.21 | 6.75 | 0.46 |
| 0.90 | 12.39 | 10.61 | 1.78 | 7.26 | 6.75 | 0.51 |

Table 3. Optimal bank capital levels for multiple banks with various parameter values: Deposit insurance funding costs case.

This table shows optimal bank capital levels from bank shareholders' perspective ($k_b$; $\eta = 0$), socially-optimal bank capital levels with zero asset correlation ($k_{s1}$; $\eta = 5n, \rho = 0$), and socially-optimal bank capital levels with positive asset correlation ($k_{s2}$; $\eta = 5n, \rho = 0.3$). Panel A illustrates the effect of aggregate failure rate $p$ on the optimal bank capital levels, Panel B the effect of excess required rate of return on bank capital $\delta$, Panel C the effect of loss given default $\lambda$, and Panel D the effect of gross loan rate $R$. Other parameters are set as follows: $V = 0.5$, $\alpha = 0.3$. Refer to Appendix C for the deposit insurance funding costs.

| A. Effect of $p$ | | | | | |
|---|---|---|---|---|---|
| $p$ | $k_{s2}$ | $k_{s1}$ | $k_b$ | $k_{s2} - k_{s1}$ | $k_{s2} - k_b$ |
| 0.01 | 1.87 | 1.82 | 1.77 | 0.05 | 0.10 |
| 0.02 | 5.92 | 5.72 | 5.62 | 0.20 | 0.30 |
| 0.03 | 9.51 | 9.06 | 8.85 | 0.45 | 0.66 |
| 0.04 | 12.86 | 12.05 | 11.69 | 0.82 | 1.17 |
| 0.05 | 16.03 | 14.84 | 14.33 | 1.19 | 1.70 |
| 0.06 | 19.06 | 17.44 | 16.76 | 1.61 | 2.29 |
| 0.07 | 21.88 | 19.94 | 19.09 | 1.95 | 2.79 |
| 0.08 | 24.61 | 22.38 | 21.37 | 2.24 | 3.25 |
| 0.09 | 27.13 | 24.71 | 23.58 | 2.42 | 3.55 |
| 0.10 | 29.49 | 27.04 | 25.79 | 2.45 | 3.70 |
| B. Effect of $\delta$ | | | | | |
| $\delta$ | $k_{s2}$ | $k_{s1}$ | $k_b$ | $k_{s2} - k_{s1}$ | $k_{s2} - k_b$ |
| 0.01 | 18.99 | 18.64 | 18.43 | 0.36 | 0.56 |
| 0.02 | 16.65 | 16.25 | 15.99 | 0.41 | 0.66 |
| 0.03 | 15.23 | 14.72 | 14.47 | 0.51 | 0.76 |
| 0.04 | 14.17 | 13.66 | 13.35 | 0.51 | 0.81 |
| 0.05 | 13.35 | 12.79 | 12.49 | 0.56 | 0.86 |
| 0.06 | 12.64 | 12.03 | 11.78 | 0.61 | 0.86 |
| 0.07 | 12.03 | 11.42 | 11.12 | 0.61 | 0.91 |
| 0.08 | 11.52 | 10.91 | 10.61 | 0.61 | 0.91 |
| 0.09 | 11.07 | 10.41 | 10.10 | 0.66 | 0.97 |
| 0.10 | 10.66 | 10.00 | 9.69 | 0.66 | 0.97 |
| 0.11 | 10.25 | 9.59 | 9.29 | 0.66 | 0.97 |
| 0.12 | 9.95 | 9.24 | 8.93 | 0.71 | 1.02 |
| 0.13 | 9.59 | 8.93 | 8.58 | 0.66 | 1.02 |
| 0.14 | 9.34 | 8.63 | 8.27 | 0.71 | 1.07 |
| 0.15 | 9.03 | 8.32 | 8.02 | 0.71 | 1.02 |
| 0.16 | 8.78 | 8.07 | 7.71 | 0.71 | 1.07 |

Table 3. continued.

| C. Effect of $\lambda$ | | | | | |
|---|---|---|---|---|---|
| $\lambda$ | $k_{s2}$ | $k_{s1}$ | $k_b$ | $k_{s2}-k_{s1}$ | $k_{s2}-k_b$ |
| 0.10 | 1.67 | 1.65 | 1.64 | 0.01 | 0.03 |
| 0.15 | 3.25 | 3.23 | 3.19 | 0.02 | 0.06 |
| 0.20 | 4.76 | 4.68 | 4.63 | 0.07 | 0.12 |
| 0.25 | 6.19 | 6.07 | 5.95 | 0.12 | 0.24 |
| 0.30 | 7.55 | 7.38 | 7.24 | 0.18 | 0.32 |
| 0.35 | 8.93 | 8.60 | 8.40 | 0.32 | 0.53 |
| 0.40 | 10.22 | 9.76 | 9.53 | 0.46 | 0.68 |
| 0.45 | 11.52 | 10.91 | 10.61 | 0.61 | 0.91 |
| 0.50 | 12.83 | 11.99 | 11.59 | 0.84 | 1.23 |
| 0.55 | 14.14 | 13.04 | 12.55 | 1.10 | 1.59 |
| 0.60 | 15.41 | 14.01 | 13.48 | 1.39 | 1.92 |
| 0.65 | 16.71 | 14.99 | 14.35 | 1.72 | 2.36 |
| 0.70 | 18.05 | 15.98 | 15.21 | 2.07 | 2.84 |
| 0.75 | 19.38 | 16.84 | 16.02 | 2.54 | 3.36 |
| 0.80 | 20.68 | 17.72 | 16.76 | 2.96 | 3.92 |
| 0.85 | 22.06 | 18.64 | 17.44 | 3.41 | 4.61 |
| 0.90 | 23.42 | 19.43 | 18.16 | 4.00 | 5.26 |
| D. Effect of $R$ | | | | | |
| $R$ | $k_{s2}$ | $k_{s1}$ | $k_b$ | $k_{s2}-k_{s1}$ | $k_{s2}-k_b$ |
| 1.027 | 13.74 | 13.22 | 12.93 | 0.52 | 0.81 |
| 1.038 | 12.97 | 12.43 | 12.14 | 0.54 | 0.83 |
| 1.048 | 12.24 | 11.64 | 11.34 | 0.60 | 0.90 |
| 1.058 | 11.50 | 10.89 | 10.59 | 0.61 | 0.91 |
| 1.069 | 10.77 | 10.09 | 9.78 | 0.67 | 0.99 |
| 1.079 | 10.03 | 9.34 | 8.97 | 0.69 | 1.06 |
| 1.089 | 9.29 | 8.54 | 8.16 | 0.76 | 1.13 |
| 1.100 | 8.55 | 7.78 | 7.39 | 0.77 | 1.15 |
| 1.110 | 7.80 | 6.96 | 6.57 | 0.84 | 1.23 |
| 1.121 | 7.06 | 6.20 | 5.75 | 0.86 | 1.31 |

Table 4. Optimal bank capital levels for multiple banks: Liquidation costs case.

This table shows the effects of $\eta$ (panel A), $V$ (panel B), and $\alpha$ (panel C) on optimal bank capital levels from bank shareholders' perspective ($k_b$) and socially-optimal bank capital levels ($k_s$), and the difference between the two along with various levels of $\rho$ in case of multiple banks. Refer to Appendix C for the liquidation costs.

| A. Effect of $\eta : \alpha = 0.45, V = 0.5$ | | | | | | |
|---|---|---|---|---|---|---|
| $\rho$ | $\eta = 0.05n$ | | | $\eta = 0.1n$ | | |
| | $k_s$ | $k_b$ | $k_s - k_b$ | $k_s$ | $k_b$ | $k_s - k_b$ |
| 0.10 | 9.44 | 8.27 | 1.17 | 10.10 | 8.27 | 1.83 |
| 0.20 | 9.95 | 8.27 | 1.68 | 10.86 | 8.27 | 2.59 |
| 0.30 | 10.41 | 8.27 | 2.13 | 11.47 | 8.27 | 3.20 |
| 0.40 | 10.76 | 8.27 | 2.49 | 11.93 | 8.27 | 3.66 |
| 0.50 | 11.12 | 8.27 | 2.84 | 12.34 | 8.27 | 4.06 |
| 0.60 | 11.42 | 8.27 | 3.15 | 12.69 | 8.27 | 4.42 |
| 0.70 | 11.68 | 8.27 | 3.40 | 12.95 | 8.27 | 4.67 |
| 0.80 | 11.88 | 8.27 | 3.61 | 13.25 | 8.27 | 4.98 |
| 0.90 | 12.08 | 8.27 | 3.81 | 13.45 | 8.27 | 5.18 |
| B. Effect of $V : \alpha = 0.45, \eta = 0.05n$ | | | | | | |
| $\rho$ | $V = 0.3$ | | | $V = 0.6$ | | |
| | $k_s$ | $k_b$ | $k_s - k_b$ | $k_s$ | $k_b$ | $k_s - k_b$ |
| 0.10 | 8.48 | 6.65 | 1.83 | 9.80 | 8.88 | 0.91 |
| 0.20 | 9.24 | 6.65 | 2.59 | 10.25 | 8.88 | 1.37 |
| 0.30 | 9.80 | 6.65 | 3.15 | 10.66 | 8.88 | 1.78 |
| 0.40 | 10.25 | 6.65 | 3.61 | 11.02 | 8.88 | 2.13 |
| 0.50 | 10.66 | 6.65 | 4.01 | 11.32 | 8.88 | 2.44 |
| 0.60 | 11.02 | 6.65 | 4.37 | 11.57 | 8.88 | 2.69 |
| 0.70 | 11.27 | 6.65 | 4.62 | 11.83 | 8.88 | 2.95 |
| 0.80 | 11.57 | 6.65 | 4.93 | 12.03 | 8.88 | 3.15 |
| 0.90 | 11.78 | 6.65 | 5.13 | 12.23 | 8.88 | 3.35 |
| C. Effect of $\alpha : V = 0.5, \eta = 0.05n$ | | | | | | |
| $\rho$ | $\alpha = 0.3$ | | | $\alpha = 0.6$ | | |
| | $k_s$ | $k_b$ | $k_s - k_b$ | $k_s$ | $k_b$ | $k_s - k_b$ |
| 0.10 | 12.13 | 10.61 | 1.52 | 7.66 | 6.75 | 0.91 |
| 0.20 | 12.84 | 10.61 | 2.24 | 8.12 | 6.75 | 1.37 |
| 0.30 | 13.40 | 10.61 | 2.79 | 8.48 | 6.75 | 1.73 |
| 0.40 | 13.91 | 10.61 | 3.30 | 8.83 | 6.75 | 2.08 |
| 0.50 | 14.32 | 10.61 | 3.71 | 9.09 | 6.75 | 2.34 |
| 0.60 | 14.67 | 10.61 | 4.06 | 9.34 | 6.75 | 2.59 |
| 0.70 | 15.03 | 10.61 | 4.42 | 9.54 | 6.75 | 2.79 |
| 0.80 | 15.28 | 10.61 | 4.67 | 9.69 | 6.75 | 2.95 |
| 0.90 | 15.59 | 10.61 | 4.98 | 9.90 | 6.75 | 3.15 |

Table 5. Optimal bank capital levels for multiple banks with various parameter values: Liquidation costs case.

This table shows optimal bank capital levels from bank shareholders' perspective ($k_b$; $\eta = 0$), socially-optimal bank capital levels with zero asset correlation ($k_{s1}$; $\eta = 0.05n, \rho = 0$), and socially-optimal bank capital levels with positive asset correlation ($k_{s2}$; $\eta = 0.05n, \rho = 0.3$). Panel A illustrates the effect of aggregate failure rate $p$ on the optimal bank capital levels, Panel B the effect of excess required rate of return on bank capital $\delta$, Panel C the effect of loss given default $\lambda$, and Panel D the effect of gross loan rate $R$. Other parameters are set as follows: $V = 0.5$, $\alpha = 0.3$. Refer to Appendix C for the liquidation costs.

| A. Effect of $p$ | | | | | |
|---|---|---|---|---|---|
| $p$ | $k_{s2}$ | $k_{s1}$ | $k_b$ | $k_{s2} - k_{s1}$ | $k_{s2} - k_b$ |
| 0.01 | 2.81 | 1.82 | 1.77 | 0.99 | 1.04 |
| 0.02 | 7.47 | 5.72 | 5.62 | 1.75 | 1.85 |
| 0.03 | 11.33 | 9.06 | 8.85 | 2.27 | 2.47 |
| 0.04 | 14.70 | 12.05 | 11.69 | 2.65 | 3.01 |
| 0.05 | 17.78 | 14.84 | 14.33 | 2.94 | 3.45 |
| 0.06 | 20.57 | 17.44 | 16.76 | 3.13 | 3.80 |
| 0.07 | 23.15 | 19.94 | 19.09 | 3.21 | 4.05 |
| 0.08 | 25.52 | 22.38 | 21.37 | 3.14 | 4.15 |
| 0.09 | 27.78 | 24.71 | 23.58 | 3.07 | 4.20 |
| 0.10 | 29.93 | 27.04 | 25.79 | 2.88 | 4.14 |
| B. Effect of $\delta$ | | | | | |
| $\delta$ | $k_{s2}$ | $k_{s1}$ | $k_b$ | $k_{s2} - k_{s1}$ | $k_{s2} - k_b$ |
| 0.01 | 20.41 | 18.64 | 18.43 | 1.78 | 1.98 |
| 0.02 | 18.23 | 16.25 | 15.99 | 1.98 | 2.24 |
| 0.03 | 16.86 | 14.72 | 14.47 | 2.13 | 2.39 |
| 0.04 | 15.89 | 13.66 | 13.35 | 2.24 | 2.54 |
| 0.05 | 15.08 | 12.79 | 12.49 | 2.29 | 2.59 |
| 0.06 | 14.47 | 12.03 | 11.78 | 2.44 | 2.69 |
| 0.07 | 13.91 | 11.42 | 11.12 | 2.49 | 2.79 |
| 0.08 | 13.40 | 10.91 | 10.61 | 2.49 | 2.79 |
| 0.09 | 13.00 | 10.41 | 10.10 | 2.59 | 2.90 |
| 0.10 | 12.59 | 10.00 | 9.69 | 2.59 | 2.90 |
| 0.11 | 12.23 | 9.59 | 9.29 | 2.64 | 2.95 |
| 0.12 | 11.93 | 9.24 | 8.93 | 2.69 | 3.00 |
| 0.13 | 11.63 | 8.93 | 8.58 | 2.69 | 3.05 |
| 0.14 | 11.32 | 8.63 | 8.27 | 2.69 | 3.05 |
| 0.15 | 11.07 | 8.32 | 8.02 | 2.74 | 3.05 |
| 0.16 | 10.81 | 8.07 | 7.71 | 2.74 | 3.10 |

Table 5. continued.

| C. Effect of $\lambda$ | | | | | |
|---|---|---|---|---|---|
| $\lambda$ | $k_{s2}$ | $k_{s1}$ | $k_b$ | $k_{s2}-k_{s1}$ | $k_{s2}-k_b$ |
| 0.10 | 2.44 | 1.65 | 1.64 | 0.78 | 0.80 |
| 0.15 | 4.29 | 3.23 | 3.19 | 1.06 | 1.10 |
| 0.20 | 6.03 | 4.68 | 4.63 | 1.34 | 1.39 |
| 0.25 | 7.66 | 6.07 | 5.95 | 1.59 | 1.71 |
| 0.30 | 9.21 | 7.38 | 7.24 | 1.83 | 1.97 |
| 0.35 | 10.66 | 8.60 | 8.40 | 2.06 | 2.26 |
| 0.40 | 12.09 | 9.76 | 9.53 | 2.33 | 2.55 |
| 0.45 | 13.40 | 10.91 | 10.61 | 2.49 | 2.79 |
| 0.50 | 14.67 | 11.99 | 11.59 | 2.69 | 3.08 |
| 0.55 | 15.91 | 13.04 | 12.55 | 2.88 | 3.36 |
| 0.60 | 17.07 | 14.01 | 13.48 | 3.05 | 3.58 |
| 0.65 | 18.21 | 14.99 | 14.35 | 3.22 | 3.86 |
| 0.70 | 19.28 | 15.98 | 15.21 | 3.30 | 4.07 |
| 0.75 | 20.36 | 16.84 | 16.02 | 3.52 | 4.34 |
| 0.80 | 21.38 | 17.72 | 16.76 | 3.66 | 4.62 |
| 0.85 | 22.33 | 18.64 | 17.44 | 3.69 | 4.89 |
| 0.90 | 23.23 | 19.43 | 18.16 | 3.80 | 5.07 |
| D. Effect of $R$ | | | | | |
| $R$ | $k_{s2}$ | $k_{s1}$ | $k_b$ | $k_{s2}-k_{s1}$ | $k_{s2}-k_b$ |
| 1.027 | 15.46 | 13.22 | 12.93 | 2.24 | 2.53 |
| 1.038 | 14.77 | 12.43 | 12.14 | 2.34 | 2.63 |
| 1.048 | 14.08 | 11.64 | 11.34 | 2.44 | 2.74 |
| 1.058 | 13.38 | 10.89 | 10.59 | 2.49 | 2.80 |
| 1.069 | 12.69 | 10.09 | 9.78 | 2.59 | 2.90 |
| 1.079 | 11.99 | 9.34 | 8.97 | 2.65 | 3.02 |
| 1.089 | 11.29 | 8.54 | 8.16 | 2.75 | 3.13 |
| 1.100 | 10.58 | 7.78 | 7.39 | 2.80 | 3.19 |
| 1.110 | 9.88 | 6.96 | 6.57 | 2.91 | 3.30 |
| 1.121 | 9.23 | 6.20 | 5.75 | 3.02 | 3.48 |