

# Entrepreneurship and Income Distribution Dynamics: Why Are Top Income Earners Unaffected by Business Cycles?\*

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## Abstract

Income shares of the bottom three income quintiles are procyclical; while those of the other quintiles are countercyclical. However, the very top five percent income group is unaffected by the business cycle. This study attempts to explain the cyclical behavior of the income distribution over the business cycle by incorporating an entrepreneurial choice to a heterogeneous agent model with indivisible labor. We find that the model economy successfully reproduces the acyclical behavior of the income share of the top five percent and sufficiently replicates the income transition matrices over occupational choices obtained from the U.S. data.

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# 1 Introduction

The “Occupy Wall Street” movement attracted much attention to the inequality of income distribution, in particular, to the concentration of wealth and income on the top one percent. The long-run trend that income or wealth distribution becomes more unequal motivated many researchers to figure out the causes and effects of the concentration of wealth and income. It may be surprising that Thomas Piketty’s (2014) 700-page book, *Capital in the Twenty-First Century*, was listed as a best-seller on Amazon, but it reflects a lot of interest in the inequality of income and wealth. In addition to the level and long-run trend of inequality in wealth and income, the issue of income distribution dynamics over the business cycle is also central to the discussion of economic policies, such as redistributive as well as stabilization policies. In spite of its importance, there are only a few studies developed for this issue and the conventional models trying to explain the business cycle dynamics of income distribution have had limited success. In other words, there has been some success in reproducing the steady state distributions of income with heterogeneous agent models, but replicating the cyclical behavior of income distribution is in its infancy. This study is another attempt to explain the business cycle behaviors of income distribution with an emphasis on the top income earners.

In the U.S. economy, there is a significant difference in the cyclical properties between the low and high-income groups. As Castañeda et al. (1998) found, the income shares of the first three lowest income quintiles are strongly procyclical over the business cycle, strongly countercyclical for the fourth quintile and the 80-95 percentile, and, interestingly, acyclical for the top five percent (see Table 1).<sup>1</sup> That is, during expansion, the income shares of the individuals in the low-income groups tend to increase, while those of the individuals in the high-income groups tend to decrease. Particularly, the share of the top five percent group is not affected by the business cycle at all. These empirical findings are quite surprising and interesting in that individuals in the top income groups not only earn a higher portion of the total income, but their income share is not affected by the business cycle.

This study attempts to replicate the cyclical behavior of the income distribution over the business cycle, focusing on the share of the top five percent income earners, by incorporating an entrepreneurial choice to a heterogeneous agent model with indivisible labor. We extend the model economies of entrepreneurial choice by Quadrini (2000), Cagetti and De Nardi (2006) and Terajima (2006), as well as those of Chang and Kim (2006, 2007) that feature extensive margins of labor supply. The key features of our model economy are as follows. First, there are three types of occupations (workers, entrepreneurs, and non-employed workers) in our model economy while entrepreneurial-choice models have two occupational choices between workers and entrepreneurs and the conventional indivisible labor choice models have two choices between

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<sup>1</sup>We use the Current Population Survey (CPS) data for the period of 1968-2006 based on Heathcote et al. (2010), while Castañeda et al. (1998) use the CPS samples from 1948-86.

workers and non-employed workers. As Castañeda et al. (1998) find, the indivisible labor supply decision, which makes it possible for households to become non-employed workers, may play a role in the overall cyclicity of income shares over income groups.<sup>2</sup> In particular, incorporating a decision to become entrepreneurs may be important in replicating the acyclicity of the income share of the top income earners. Second, our model explicitly introduces aggregate shocks to a model of occupational choice, while the conventional studies that model occupational choices only focus on the steady state equilibrium. As far as we know, this study is the first quantitative work studying a heterogeneous agent model with occupational choices in a dynamic stochastic general equilibrium (DSGE) framework under aggregate shocks. Through combining aggregate shocks and occupational choice, we can analyze the cyclical property of the income shares over occupational choices as well as the income mobility of occupational choices over the income distribution. In summary, using Castañeda et al. (1998) as a motivational point, we endogenize a household’s indivisible labor decision following Chang and Kim (2006, 2007), and incorporate an entrepreneurial choice (Quadrini, 2000; Cagetti and De Nardi, 2006; Terajima, 2006).

The main findings of this study are summarized as follows. First, our model economy with an entrepreneurial choice successfully reproduces the cyclical behavior of income distribution dynamics including the acyclical behavior of the income share of the top five percent income earners. During expansions, there are more opportunities for entrepreneurship, especially for the top income earners, which induces more individuals to become entrepreneurs and earn high income. This offsets a decline in the workers’ income share in the top income group, which is caused by an increase in the employment of workers in the low-income quintiles. Second, the model economy replicates reasonably well the transition matrices across the income groups over occupational choices in the U.S. data. Based on the analysis of the transition probabilities of occupational choices over the income distribution, entrepreneurial opportunities play a significant role in accounting for an upward movement across income groups. Moreover, we find that entrepreneurs in the top five percent income group may be important in explaining the acyclical behavior of the income share of the top income earners, since entrepreneurs in the top five percent income group tend to stay in that group. Third, an indivisible labor choice model with heterogeneous capital returns is able to reproduce the acyclical behavior of the income share of the top five percent income group.<sup>3</sup> We propose that heterogeneous capital returns can be interpreted as a reflection of entrepreneurial activities in the form of higher

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<sup>2</sup>While Castañeda et al. (1998) introduce exogenous individual labor shocks perfectly correlated with aggregate shocks, and employment shocks, we allow households to optimally choose indivisible hours of work, following Chang and Kim (2006, 2007), which may reduce the overall cyclicity of the income shares of the income groups.

<sup>3</sup>The idea of heterogeneity in the rate of return to capital across individuals may not be attractive unless the source of this heterogeneity is explicitly constructed in a theoretical model. The theoretical background for this idea can be found in Lusardi and Mitchell (2014) that survey the models with heterogeneity arising from human capital on financial knowledge.

capital returns.

This paper is related to two strands of the quantitative literature: the cyclical behavior of income distribution and entrepreneurial activities. The work of Castañeda et al. (1998) is the first comprehensive study attempting to account for the income distribution dynamics over the business cycle using a dynamic general equilibrium model with infinitely-lived agents and unemployment risk. They document the income distribution dynamics and introduce unemployment spells and a cyclically moving labor share to account for the cyclical behavior of the income distribution. While they find that unemployment spells may play a significant role in the income distribution dynamics, which is supportive for the indivisibility of labor, they do not see the role of a cyclically moving labor share in explaining the cyclical properties of such dynamics. In fact, they only manage to account for the contemporaneous correlations between the income shares and output in signs for each income group. However, the correlations between income shares and output in their model are very high in absolute terms, and the income share earned by the top five percent income group shows a strong negative correlation with output, though acyclical in the data. Heer (2013) also tries to explain the cyclical behavior of the income distribution using a model with overlapping generations and the savings motive for retirement. He finds that the model economy does not help to understand the cyclical behavior of the income distribution, including the acyclical property of the income shares of the top five percent. Further, his other specification with rigid wages produces only a small improvement.

The second strand of the previous quantitative studies closely related to our work is the entrepreneurial choice models by Quadrini (2000), Cagetti and De Nardi (2006), and Terajima (2006). Quadrini (2000) constructs a dynamic general equilibrium model that incorporates an entrepreneurial choice and financial frictions under incomplete markets. He shows that this model economy can generate a high inequality in wealth distribution and replicate the empirical patterns of wealth mobility.<sup>4</sup> Cagetti and De Nardi (2006) also study a model economy with an occupational choice allowing for entrepreneurial activities, similar to Quadrini (2000), to analyze the effects of borrowing constraints as a source of wealth concentration. In contrast to Quadrini (2000), who introduces exogenous financial frictions, Cagetti and De Nardi (2006) construct endogenously determined borrowing constraints in equilibrium. They find that the model economy sufficiently reproduces the observed wealth distribution for entrepreneurs and workers, and more restrictive borrowing constraints generate less wealth concentration. Terajima (2006) also studies a model considering relations between occupational and educational choices to analyze the interaction between the changes in earnings and wealth inequality over time. He finds that the model can explain about 33 percent of the change in the relative average wealth between different education-occupation groups.

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<sup>4</sup>A standard model with idiosyncratic risks under incomplete markets, initiated by Huggett (1993) and Aiyagari (1994), has not been successful in accounting for the tails of the distributions (see Quadrini (2000)).

This paper is organized as follows. Section 2 summarizes some empirical facts on the cyclical behavior of the U.S. economy, focusing on that of the income shares of the quintiles as well as the transition probabilities of occupational choices over the income distribution. Section 3 introduces the heterogeneous agent model economies with three occupational choices. The simulation results are discussed in Section 4, with regard to steady state distributions and cyclical properties, focusing on the cyclical behavior of income distribution. Section 5 summarizes the findings and concludes.

## 2 Empirical Facts

In this section, we summarize some empirical evidence on the relationship between entrepreneurship and the cyclical behavior of the income shares earned by the top income earners; then, we investigate the role of entrepreneurship in income mobility. We mainly use two sets of survey data: the March Current Population Survey (CPS) and the Panel Study of Income Dynamics (PSID).<sup>5</sup> The basic unit of observations is a household for the CPS and a family for the PSID. The sample survey periods of the CPS data span from 1968 to 2005, or 39 years. Since the CPS data do not have any information on wealth, we use the 1984 survey of the PSID for both wealth distribution and income distribution.<sup>6</sup> For calculating the average income transition matrices across occupations, the PSID surveys from 1983-1989 are used.

We use the following question in the PSID survey to classify a household by occupational status among three categories—entrepreneurs, workers, and non-employed workers: *Do you (HEAD) work for someone else, yourself, or what?* We define the occupations of households in the PSID as follows. Entrepreneurs are families in which the head of household is self-employed;<sup>7</sup> workers are those in which the head of household is working for someone else;<sup>8</sup> and non-employed workers are those in which the head of household is unemployed and looking for work, retired, permanently disabled, keeping house, in school, or working 10 hours or less per week. With regard to the CPS, since we are using the data set constructed by Heathcote et al. (2010), it is limited to pulling out all variables of interest. Fortunately, there it contains information on self-employment income so that we can define entrepreneurs. However, there is no variable in the CPS data to identify workers and non-employed workers. We use 1,800 working hours for the head of households per year as an ad hoc criterion to separate workers

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<sup>5</sup>The CPS data are downloaded from Heathcote et al. (2010).

<sup>6</sup>We choose the PSID data from 1984 because the year 1984 is approximately the median of the sample period of the CPS, and the PSID 1984 data contains information on family wealth.

<sup>7</sup>Cagetti and De Nardi (2006) find that entrepreneurs tend to take a large fraction of the total net wealth and a relatively small fraction of the total population from the Survey of Consumer Finances (SCF), regardless of their various definitions. Quadrini (2000) also finds the same tendency in the PSID.

<sup>8</sup>In case that the head of a family is working for someone else and self-employed at the same time, we regard the family as an entrepreneur. The portion of this type of families is small.

from those that are non-employed. In summary, our definition of occupational status for the CPS is that entrepreneurs are heads of household that earn self-employed income; workers are those that do not have self-employed income and work 1,800 hours or more per year;<sup>9</sup> and non-employed workers are those that do not have self-employed income and work less than 1,800 hours per year.<sup>10</sup> Under this classification, most statistics of interest of the CPS are comparable to those of the PSID. For example, the shares of workers, entrepreneurs, and non-employed workers from the two data sets following our definitions are around 60, 10, and 30 percent, respectively.

## 2.1 Entrepreneurship and income inequality

The U.S. economy shows a contrasting difference in cyclical properties between low- and high-income groups. The first row of Table 1 reports the contemporaneous correlations of the income shares with output for the first four quintiles, next 15 percent (80-95 percentile), and top five percent. As Castañeda et al. (1998) find, it is clear that the income shares of the three lowest quintiles tend to be procyclical, while those of the fourth quintile and 80-95 percentile are countercyclical. The contemporaneous correlations of the income shares in the three lowest quintiles are 0.72, 0.59, and 0.12, respectively, while those in the fourth quintile and 80-95 percentile are -0.35 and -0.53, respectively. Therefore, we can easily conclude that, in a relative sense, business expansions benefit low-income earners but hurt high-income earners, resulting in an improvement in inequality in general.<sup>11</sup>

The top five percent group, however, does not show any cyclicity with output showing the contemporaneous correlation of approximately zero with output fluctuations (-0.06), which is a contrasting behavior to the fourth quintile and 80-95 percentile. To investigate the role of entrepreneurship in the cyclical property of the income shares earned by the top income earners, we separate entrepreneurs from the original sample, keeping the income quintile groups based on the total sample. The second row of Table 1 indicates the cross correlations of the income shares in the six categories with output without entrepreneurs. Interestingly, the correlation between the income share of the top five percent and output is higher, in absolute terms, when we drop entrepreneurs, while the other groups' correlations remain almost unchanged. This reflects that entrepreneurs may play a crucial role in reducing the cyclicity of the income share of the top five percent group. Thus, one of the objectives of this study is to attempt to

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<sup>9</sup>The average working hours of households for three quarters is approximately 1,800.

<sup>10</sup>The definition of non-employed workers we examine for the CPS is slightly different from that of the model economy. In the model economy, they are defined as heads of household that do not work in the fourth quarter, which is when we assume the survey is conducted, since we convert the simulated data at a quarterly frequency into an annual frequency for comparison.

<sup>11</sup>As observed in many studies, income distribution tends to improve during expansions and worsen during recessions, which is confirmed by the negative cross correlations of the Gini coefficient with output in the last column of the lower panel of Table 8.

explain this phenomenon and reproduce it in a heterogeneous agent DSGE model.

Inequality across individuals may be analyzed in various ways. We start with income distribution since it is the focus of our study. The U.S. income distribution is shown in the upper panel of Table 2. The income share of the first quintile is very small and the top 20 percent earns about 50 percent of the total income. Thus, the income distribution shows a large inequality, in particular, the top five percent accounts for about 20 percent of the total income. This unequal distribution makes the income Gini index about 0.5 in the 1984 PSID and 0.4, on average, in the CPS 1968-2006. As for wealth distribution, inequality increases. The bottom panel of Table 2 summarizes the wealth shares of the four quintiles, 80-95 percent group, and top five percent earners. As already documented in many empirical studies, the wealth distribution is more unequal than is the income distribution. The wealth share of the first quintile implying the lowest 20 percent group ranked by wealth, is negative, and the top 20 percent accounts for about 80 percent of the total wealth. Further, the top five percent accounts for more than half of the total wealth and the Gini index for this wealth distribution is 0.75 or 0.77, depending on the data. We argue that reproducing the unequal wealth distribution may be critical for, or at least related to, the acyclical behavior of the income share of the top five percent group, which will be discussed in detail later.

To analyze the relationship between entrepreneurship and income inequality focusing on the top income earners, we first examine the income share, population share, and relative average income of entrepreneurs over the income groups. Figure 1 shows the shares of entrepreneur income within each of the income groups. The importance of entrepreneurs, in terms of the income share, increases in the higher income groups, especially in the top five percent income group. According to the PSID 1984, entrepreneurs account for approximately 40 percent of the total income of the 95-100 percentile income group, but approximately 10-15 percent of the total income of the fourth quintile and 90-95 percentile groups and only 2-9 percent of the total income of the three lowest quintile groups. The CPS survey also shows a similar pattern in which entrepreneurs in the top five percent earn a large portion of the income within the group, when compared to those in the other high-income groups.

The income share of entrepreneurs may be decomposed into two factors: the population share of and relative average income for entrepreneurs.<sup>12</sup> The relative average income for entrepreneurs is defined as the average income for the entrepreneurs relative to that of each income group. We first examine the population share of entrepreneurs. Figure 2 exhibits the population share of entrepreneurs across income groups. Similar to the findings from the income share for entrepreneurs, the population share of entrepreneurs in the top five percent is much higher than in the other income groups, even though the share increases in the higher income

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<sup>12</sup>We think of the population share of entrepreneurs as an extensive margin, and relative average income of entrepreneurs as an intensive margin, for the income share of entrepreneurs.

groups. In the PSID 1984, the fraction of entrepreneurs is 31.2 percent in the top five percent income group, while the corresponding figures in the fourth quintile and 80-95 percentile are 11.0 and 15.2 percent, respectively.

The income share of entrepreneurs in the top five percent is larger than the population share of entrepreneurs in the group, while the income share for entrepreneurs in the fourth quintile and 80-95 percentile are similar to the population shares of entrepreneurs in the corresponding income groups. This implies that the average income of entrepreneurs relative to the average income of total households in the top five percent is greater than 1, while the relative average incomes of entrepreneurs in the fourth quintile and 80-95 percentile are approximately 1. Figure 3 confirms this conjecture, showing that the relative average income of entrepreneurs in the top five percent is 1.18 in the PSID 1984, or 1.17 in the CPS, while the relative average incomes of entrepreneurs in the fourth quintile and 80-95 percentile are 1.00 and 1.01, respectively. In summary, the income share of entrepreneurs in the top five percent is much higher, when compared to those in the fourth quintile or 80-95 percentile, due to the higher population share and relative average income of entrepreneurs. As such, we think that entrepreneurial activities play an important role in the cyclical behavior of the income shares over the business cycle, and incorporating entrepreneurs in a model may be critical to replicating the acyclical behavior of the income shares of the top income earners. Therefore, we will explicitly introduce entrepreneurial activities in the model and focus on the effect of incorporating them in the cyclical behaviors of the income groups.

## 2.2 Entrepreneurship and income mobility

Quadrini (2000) shows that entrepreneurs experience a higher upward mobility in wealth distribution than workers. Since our primary interest is on the behavior of the income distribution, we examine the role of entrepreneurship in income mobility across the income groups. We classify households into four categories, according to changes in the occupational status between the previous and current years, as follows: staying entrepreneurs, switching entrepreneurs, staying non-entrepreneurs, and switching non-entrepreneurs. Moreover, we divide households into six categories according to the family income in the previous and current years: the first four quintiles, 80-95 percentile, and top five percent. Table 3 reports the average one-year income transition matrices among each of the occupational statuses over the period 1983-1989 in the PSID data.

The upper panel of Table 3 reports data for households that were entrepreneurs in the previous year. In the first five income groups (from the first quintile to 80-95 percentile), the percentage of moving to a higher income group is greater for staying entrepreneurs than for households switching from entrepreneur to non-entrepreneur in the current year. In the first quintile, for example, about 60 percent (the sum of 34.4, 11.5, 8.3, 1.7, and 3.2 percent)

of households that were entrepreneurs in the previous year move to a higher income group when they remain entrepreneurs, while only 30 percent (the sum of 27.5, 1.3, 1.9, 0.0, and 0.0 percent) of households move to a higher income group when they switch from entrepreneur to non-entrepreneur. On the contrary, the likelihood of falling to a lower income group is much less for staying entrepreneurs than for switching entrepreneurs in the current year in the second quintile through the top five percent group. In addition, the probability of staying in the top five percent for staying entrepreneurs is much higher than for switching entrepreneurs. Therefore, entrepreneurial activities are more likely to provide an opportunity to earn a higher income.

The transition probabilities across the income groups for households that were not entrepreneurs in the previous year are reported in the bottom panel of Table 3. In the same token, it is evident that the households that switch to entrepreneurs are more likely to move to a higher income group, when compared to staying non-entrepreneurs in the lower five income groups. The households switching from non-entrepreneur to entrepreneur in the top five percent income group in the previous year have a slightly higher probability (74.5 percent) of staying in the same income group than do staying non-entrepreneurs (73.3 percent).

In summary, the statistics from the PSID 1983-1989 show that there is a noticeable difference in the income mobility of entrepreneurs and non-entrepreneurs across the income groups: entrepreneurship provides a higher probability of upward mobility to a higher income group than does non-entrepreneurship. This finding is in line with Quadrini (2000), who focuses on the role of entrepreneurs in wealth mobility. Thus, we hypothesize that entrepreneurial activities play an important role in explaining the acyclical behavior of the income share of the top income earners.

### 3 The Model Economy

Our motivational starting point of constructing a model is Castañeda et al.'s (1998) heterogeneous agent indivisible labor model. However, their model is a limited version of an indivisible labor choice model in the sense that the individual productivity types of their model are finite in five cases, and the agents' employment is determined randomly, rather than by optimal choices. We build a simple dynamic stochastic general equilibrium model with a large population of heterogeneous households in labor efficiency under an incomplete capital market, and add an additional feature of entrepreneurship with heterogeneous entrepreneurial productivity. This model is an extended version of the traditional heterogeneous agent models by Huggett (1993) and Aiyagari (1994), and is also in line with Chang and Kim (2006, 2007), featuring extensive margins of labor supply with aggregate shocks as well as heterogeneous labor productivity shocks across households. Moreover, it develops entrepreneurship emphasized by Quadrini (2000), Cagetti and De Nardi (2002), and Terajima (2006), which introduce an occu-

pational decision for a household to become an entrepreneur with idiosyncratic entrepreneurial technology shocks as well as heterogeneous labor ability.<sup>13</sup>

The key features of our model economy can be summarized as follows. First, a household can make an occupational decision among the three occupational choices—employment, entrepreneurship, and no work—after the realization of idiosyncratic shocks in labor efficiency and entrepreneurial productivity shocks. A household may decide to supply an indivisible unit of labor to the market as a *worker*, to participate in its own business by investing its labor and capital as an *entrepreneur*, or not to work due to unfavorable individual labor productivity shocks and/or low entrepreneurial productivity shock, which we call a *non-employed worker*. Second, we introduce aggregate productivity shocks to analyze business cycle properties focusing on the dynamics of income distribution with occupational mobility. Most of the previous studies on entrepreneurship only focus on the steady state properties, not on the business cycle properties. Incorporating aggregate productivity shocks in addition to idiosyncratic labor productivity shocks, as well as heterogeneous entrepreneurial shocks, allows for the simulation of income distribution over the business cycle. The indivisible labor supply choice, which enables households to be non-employed, is important in determining the overall cyclicity of income shares over income groups, as Castañeda et al. (1998) find. Therefore, introducing a decision to become entrepreneurs may play a crucial role in reproducing the acyclical behavior of the income share of the top income earners.

### 3.1 Preferences

Each household maximizes the expected lifetime utility over consumption  $c_t$  and hours of work  $h_t$ :

$$U = \max_{\{c_t, h_t\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right] \quad (1)$$

with

$$u(c, h) = \log c - \lambda \frac{h^{1+\theta}}{1+\theta}, \quad (2)$$

where  $E_0[\cdot]$  denotes the expectation operator conditional on the initial period information, and  $0 < \beta < 1$  is the time discount factor. The functional form of the monetary utility is assumed additively separable between consumption and time devoted to work.  $\lambda > 0$  is the disutility parameter on work, and  $1/\theta > 0$  represents (compensated) labor-supply elasticity or

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<sup>13</sup>The labor supply in our model is different from that in the previous studies in that we explicitly consider an employment decision, while employment is either randomly assigned in Castañeda et al. (1998) or a fixed unit of labor supply is exogenously supplied by all households in Quadrini(2000), Cagetti and De Nardi (2002), and Terajima (2006).

the intertemporal elasticity of substitution for hours.<sup>14</sup>

## 3.2 Heterogeneity

There are two types of heterogeneity in the model economy: labor efficiency,  $e$ , and entrepreneurial productivity,  $\eta$ ; each household faces idiosyncratic risks in  $e$  and  $\eta$ . Each shock follows a stochastic process with transition probabilities  $Q_s(s'|s) = \Pr(s_{t+1} = s' | s_t = s)$ , where  $s, s' \in \mathcal{M}_s \equiv \{1, 2, \dots, n_s\}$  and  $s \in \{e, \eta\}$ . These idiosyncratic shocks are assumed to be independently distributed across households and identically distributed within each worker type. In addition, we assume that the two shocks,  $e$  and  $\eta$  are uncorrelated with each other and follow an AR(1) process:

$$\ln s' = \rho_s \ln s + \varepsilon_s, \quad \varepsilon_s \sim N(0, \sigma_s^2), \quad (3)$$

where  $s \in \{e, \eta\}$ . The capital market is incomplete following Huggett (1993) and Aiyagari (1994): the physical capital,  $a$ , is the only asset available to insure against idiosyncratic risks in  $e$  and  $\eta$ .

## 3.3 Technology

There are two production sectors in the model economy: corporate and entrepreneurial. The corporate production sector employs labor and capital, and pays wages and interest rates to the production factors as in a conventional aggregate production in the DSGE literature. The market wage and rental rates are determined by the supply of and demand for the production factors in the corporate sector. On the contrary, the entrepreneurial production sector uses a part or all of the entrepreneur's capital and all of its own labor. Therefore, entrepreneurial production activities can be considered self-employment, small business firms using their own production factors, or independent business projects, which is in line with Quadrini (2000). The entrepreneurs are not allowed to borrow from outside or combine other entrepreneurial business activities to insure their own entrepreneurial individual risk. However, they are allowed to optimally allocate capital between their own entrepreneurial activity and the corporate production sector.

**Corporate production** The production technology in the corporate sector is represented by a constant-returns-to-scale Cobb-Douglas production function:

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<sup>14</sup>We assume that the disutility parameter,  $\lambda$ , is different across the household's two occupational choices:  $\lambda = \lambda_W$  when the household decides to work in the market and  $\lambda = \lambda_E (\neq \lambda_W)$  when it chooses to become an entrepreneur.

$$F(K, L, z) = zK^\alpha L^{1-\alpha}, \quad (4)$$

where  $K$  and  $L$  denote capital and effective labor used in the corporate production, respectively, and  $z$  is an aggregate productivity shock, which follows a stochastic process with transition probabilities,  $Q_z(z'|z) = \Pr(z_{t+1} = z'|z_t = z)$ . The aggregate productivity  $z$  follows an AR(1) process in logs:

$$\ln z' = \rho_z \ln z + \varepsilon_z, \quad \varepsilon_z \sim N(0, \sigma_z^2). \quad (5)$$

It is assumed that the aggregate shocks and the two idiosyncratic shocks for each individual are independent of one another.

**Entrepreneurial production** The entrepreneurial production technology is given by:

$$f(k, l, \eta, z) = z\chi\eta k^v l^{1-v}, \quad (6)$$

where  $\chi$ ,  $k$ ,  $l$ , and  $v$  denote the entrepreneurial production parameter, capital invested in the entrepreneurial production, effective labor ( $l = eh$ ) used for the entrepreneurial firm, and entrepreneur's capital income share, respectively. We assume that the depreciation rate for physical capital,  $\delta$ , is common in both of the production sectors.

### 3.4 Household's problem

**Worker's problem** The budget constraint of a worker is:

$$c = weh + (1 + r)a - a', \quad (7)$$

and

$$c \geq 0 \text{ and } a' \geq 0, \quad (8)$$

where  $w$  and  $r$  are the wage rate and interest rates on asset  $a$ , respectively.

A worker can purchase an asset, implying a claim to physical capital,  $a'$ , as a means of saving. We impose a restriction that workers face a borrowing constraint: the level of asset holding,  $a$ , cannot be negative.<sup>15</sup> An important feature introduced in the model is that a labor

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<sup>15</sup>Relaxing this assumption (i.e., allowing negative asset holding to a certain level, such as in Chang and Kim (2006)) does not produce any qualitatively different simulation results, but makes the simulation process more complicated and calculation take longer. Castañeda et al. (1998) also assume that workers cannot borrow at any time.

choice by each worker is indivisible, following Hansen (1985) and Rogerson (1988). That is, a worker decides to work for a certain amount of hours ( $h = \bar{h}$ ) or none ( $h = 0$ ). It is well-known from the previous studies that a decision to work for a fixed amount of hours or none is the main source of variation in total hours worked (Heckman, 1984), emphasizing the role of extensive margins of labor. In particular, Castañeda et al. (1998) emphasize the role of unemployment spells in determining the cyclical behavior of income distribution, following Clark and Summers (1981), Kydland (1984), and Ríos-Rull (1993).

The state variables for each household are the vector  $\omega \equiv (a, e, \eta)$  and the economywide state is the vector  $\Omega \equiv (\mu, z)$ , where  $\mu$  is a distribution of assets, idiosyncratic labor productivity, and entrepreneurial productivity across individual agents. The value function for a worker, denoted by  $V^W$ , is

$$V^W(\omega, \Omega) = \max_{c, a'} \left\{ \ln c - \lambda_W \frac{\bar{h}^{1+\theta}}{1+\theta} + \beta E[V(\omega', \Omega')] \right\}, \quad (9)$$

subject to

$$c = w e \bar{h} + (1+r)a - a', \quad c \geq 0, \quad a' \geq 0 \quad \text{and} \quad \mu' = \phi(\Omega), \quad (10)$$

where  $\phi$  denotes a transition operator for  $\mu$ .

**Entrepreneur's problem** We assume that an entrepreneur uses capital in its own entrepreneurial business up to the level of asset that it holds ( $k \leq a$ ). It is also assumed that the labor supply decision for the entrepreneur is indivisible. If a household decides to be an entrepreneur, its value function  $V^E$  is:

$$V^E(\omega, \Omega) = \max_{c, a', k} \left\{ \ln c - \lambda_E \frac{\bar{h}^{1+\theta}}{1+\theta} + \beta E[V(\omega', \Omega')] \right\}, \quad (11)$$

subject to

$$c = z \chi \eta k^v t^{1-v} + (1-\delta)k + (1+r)(a-k) - a', \quad c \geq 0, \quad a' \geq 0, \quad k \leq a, \quad \text{and} \quad \mu' = \phi(\Omega). \quad (12)$$

**Non-employed worker's problem** The value function for a non-employed worker who decides not to work, denoted by  $V^N$ , is:

$$V^N(\omega, \Omega) = \max_{c, a'} \{ \ln c + \beta E[V(\omega', \Omega')] \}, \quad (13)$$

subject to

$$c = (1 + r(\Omega))a - a' , \quad c \geq 0, \quad a' \geq 0, \quad \text{and } \mu' = \phi(\Omega). \quad (14)$$

**Household's occupational decision** The household's value function,  $V$ , is defined as:

$$V(\omega, \Omega) = \max \{V^W(\omega, \Omega), V^E(\omega, \Omega), V^N(\omega, \Omega)\} , \quad (15)$$

and its occupational decision is  $V(\omega, \Omega) = V^i(\omega, \Omega)$  where  $i \in \{W, E, N\}$ .

### 3.5 Definition of equilibrium

A recursive competitive equilibrium consists of a forecasting function  $\phi(\Omega)$  for  $\mu$ , a set of input functions in the corporate production sector  $\{K(\Omega), L(\Omega)\}$ , pricing functions  $\{r(\Omega), w(\Omega)\}$ , value functions  $\{V^W(\omega; \Omega), V^E(\omega; \Omega), V^N(\omega; \Omega), V(\omega; \Omega)\}$ , and optimal decision rules  $\{c(\omega, \Omega), a'(\omega, \Omega), k(\omega, \Omega), h(\omega, \Omega)\}$ , such that :

1. A household's optimization: The optimal decision rules  $c(\omega, \Omega), a'(\omega, \Omega), k(\omega, \Omega), h(\omega, \Omega)$  solve the value functions  $V^W(\omega, \Omega), V^E(\omega, \Omega), V^N(\omega, \Omega), V(\omega, \Omega)$ , given  $r(\Omega), w(\Omega)$  and  $\phi(\Omega)$ .
2. The corporate firm's optimization: For all  $\Omega$ ,

$$r(\Omega) = F_K(K(\Omega), L(\Omega), z) - \delta, \quad (16)$$

$$w(\Omega) = F_L(K(\Omega), L(\Omega), z). \quad (17)$$

3. Market clearing: For all  $\Omega$ ,

$$L(\Omega) + \int l(\omega, \Omega) d\mu = \int e h(\omega, \Omega) d\mu, \quad (18)$$

$$K(\Omega) + \int k(\omega, \Omega) d\mu = \int a d\mu. \quad (19)$$

4. Consistency of individual and aggregate behavior: The law of motion for the distribution  $\phi(\Omega)$  is consistent with that implied by the optimal decision rule  $a'(\omega, \Omega)$ .

### 3.6 Calibration

Simulation results often depend on or may sometimes be sensitive to the parameter values used in the models, implying that the choice of parameters must be appropriately justified. Table 4 summarizes the parameter values. As is standard in the business cycle literature and empirical labor supply literature, we use the conventional parameter values adopted in many previous studies. Even though our model is simulated at a quarterly frequency, we add up the values of the aggregate and individual variables, on an annual basis, to compare the simulation results with the data and previous studies. When the income distribution at an annual frequency is calculated from the simulated data, the individual data, such as income, are added up agent by agent over the four quarters.

The parameter  $1/\theta$  which corresponds to the elasticity of the labor supply, is set to 0.4 ( $\theta = 2.5$ ), considering that many micro data estimates of the elasticity of the labor supply at an individual level are small.<sup>16</sup> The extensive margin of the labor supply,  $\bar{h}$ , is chosen to be one-third, since it is well-known that a typical worker allocates about that much time for work. There are three more parameters to be calibrated in a household's preferences,  $\beta$ ,  $\lambda_W$ , and  $\lambda_E$ . We jointly choose the discount factor and the disutility parameters of labor to be consistent with a one percent quarterly return to capital, the population share of workers, 60%, and the population share of entrepreneurs, 10%.<sup>17</sup>

As for individual labor productivity shock, which we believe may be an important part of calibration, many studies attempt to estimate the individual labor productivity parameters,  $\rho_e$  and  $\sigma_e$  (Floden and Lindé, 2001; French, 2005; Chang and Kim, 2006, 2007). It is common that individual labor productivity shocks have a large variance and high persistence. We use  $\rho_e = 0.929$  and  $\sigma_e = 0.227$ , following Chang and Kim's (2007) estimates from the PSID of the years 1979-1992. Chang and Kim (2007) estimate an AR(1) wage process using a maximum-likelihood estimation method, following Heckman (1979) in order to remedy the problem of the wages of non-employed workers not being observed in the data. For aggregate productivity shocks, we simply choose  $\rho_z = 0.95$  and  $\sigma_z = 0.007$ , following Kydland and Prescott (1982). The capital income share,  $\alpha$  is 0.36, and the depreciation rate,  $\delta$  is 2.5 percent per quarter.

Unfortunately, there is little empirical evidence concerning the persistence and variance of the idiosyncratic entrepreneurial productivity shock process. We simply assume that the persistence of the entrepreneurial productivity shock process is the same as that of aggregate productivity shocks, by choosing  $\rho_\eta = \rho_z = 0.95$  as in Khan and Thomas (2008), since there is no agreement on the persistence of plant-specific idiosyncratic shocks in the literature on firm dynamics (see Khan and Thomas (2008) and Siemer (2014) for discussion). For the standard

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<sup>16</sup>In fact, since the labor supply decision for the household is indivisible, choosing any values of  $\theta$  does not affect the simulated results, but only changes the parameter values of  $\lambda_W$  and  $\lambda_E$  in the model.

<sup>17</sup>The US data such as the CPS 1968-2006, PSID 1984, and SCF 2007, consistently report that the population shares of workers and entrepreneurs are around 60% and 10%, respectively.

deviation of idiosyncratic entrepreneurial productivity,  $\sigma_\eta$ , we choose 0.50, which dictates the income Gini coefficient for entrepreneurs to be 0.60, as in the SCF 1992 (Díaz-Giménez et al., 1997). Entrepreneurial production parameter,  $\chi$ , is chosen to target the income share of the self-employed, which is around 20 percent in the U.S. data, such as that of the PSID 1984 and SCF 1992. The entrepreneur’s capital income share,  $v$ , is 0.33, as in Heathcote et al. (2010), which calculate the capital income share of self-employment from the data.

### 3.7 Alternative model with heterogeneous capital returns

When most previous studies do not consider explicitly entrepreneurial activities in the model, they usually count two-thirds of the total income of the entrepreneurs or self-employed as labor income, and the remaining one-third as their capital income. In our model, households become entrepreneurs only when the entrepreneurial opportunity is better than the employment opportunity. Therefore, it is reasonable to think that the rate of return to entrepreneurs’ capital is always higher than the market rate of return to capital. We simply introduce exogenously heterogeneous capital returns across agents when we do not explicitly consider entrepreneurial choices. We propose a heterogeneous rate of return to capital, proportional to the level of individual productivity. This feature may increase the inequality of wealth distribution, which we think plays a role in explaining the cyclical behavior of income distribution over the business cycle, in particular, offsetting the cyclical behavior of labor income across the income quintiles. Even though the heterogeneous rate of return on capital across individuals is assumed to be a linear function of the productivity level of each agent, it is not very counterfactual. Figure 4 plots the relationship between the rate of return to capital and wage rate from the PSID 1994.<sup>18</sup> The rate of return and wage rate are calculated as averages in each income decile.<sup>19</sup> Surprisingly, they are linearly related with a cross correlation coefficient, 0.88.

All other specifications are the same as in the models described above, except for two features: no entrepreneurial choice and heterogeneity in the rate of return to capital. The rate of return on assets,  $r$ , is assumed to depend on the agent’s labor productivity level,  $e$ :  $\tilde{r} = \kappa e r$ , where  $\tilde{r}$  is the rate of return on assets each individual faces and  $r$  is the rate of return on the aggregate capital. Regarding calibration, we determine the value of  $\kappa$  to satisfy the following equilibrium condition in the aggregate economy:

$$rK = \int \tilde{r} a d\mu. \tag{20}$$

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<sup>18</sup>The wage rate is defined as the annual wage income divided by annual working hours of head of household, and capital return is defined as the annual capital income over total wealth for each decile. We use the PSID 1994 survey instead of PSID 1984 because the former contains more information on capital incomes and family wealth.

<sup>19</sup>We take the average of the deciles after deleting the observations with zero assets or unemployment (typically with annual working hours under 100).

## 4 Findings

### 4.1 Entrepreneurship and income inequality

We first investigate the steady state distributions of the model economies where aggregate uncertainty is eliminated, by calculating the average of the long time series of the simulated data for income and wealth. We compare the steady state distributions of wealth and income in the model economies with those of the U.S. economy in Figures 5 and 6. Figure 5 shows the Lorenz curves for the wealth distribution of the model economies and U.S. data. The model economy with entrepreneurs replicates the steady state wealth distribution significantly better than does the model economy without entrepreneurs, even though the inequality of wealth distribution is not generated enough to match the data.<sup>20</sup> Introducing entrepreneurs seems to generate higher inequality in wealth: the model economy makes the wealth Gini index about 0.62, while the corresponding value for the model without entrepreneurs is about 0.53. Figure 6 plots the Lorenz curves for the income distributions of the model economies and U.S. data. The model economies sufficiently replicate the income distribution of the U.S., regardless of entrepreneurial activities, making the income Gini index of the simulated model 0.48, which is in the range of what the U.S. data show (0.41–0.51).

To analyze the relationship between entrepreneurship and income inequality focusing on the top income earners, we compare the income shares of each occupation over the income groups generated by the model economy with those of the U.S. data. Table 5 shows that the model economy successfully replicates the income shares of the income groups over the three occupational statuses: workers, entrepreneurs, and non-employed workers. The income shares of entrepreneurs are higher in the higher income groups. In particular, the income share of entrepreneurs is much higher in the top five percent income group than in the other income groups, which is consistent with the U.S. data. In the model economy, the entrepreneurs in the top five percent earn around 55 percent of the income of the 95-100 percentile, while the entrepreneurs in the fourth quintile and 80-95 percentile earn approximately 10 percent of the income in each of the income groups. Even though the income share of entrepreneurs for the top five percent in the model economy is larger, when compared to the PSID 1984 and CPS averages, the model economy sufficiently reproduces the pattern found in the U.S. data of entrepreneurs being more important in the high-income groups than in the low.

Table 6 shows the population shares of each occupation across the income groups for the U.S. data and the model economy. Similar to the data, the model economy shows the largest population share of entrepreneurs in the top five percent, around 38 percent, of all income

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<sup>20</sup>We can generate the wealth inequality close to that of the data by increasing the standard deviation of the individual heterogeneous shocks. However, the higher heterogeneity across individuals makes some other features (e.g., earnings inequality) inconsistent with the data.

groups. The corresponding numbers for the fourth quintile and the 80-95 percentile are 10.3 and 9.8 percent, respectively. As can be seen in Table 7 and Figure 3, the relative average income of entrepreneurs is much higher than that of workers in the top five percent income group, 43 percent higher in the model, and 17–18 percent higher in the data. On the contrary, there is no significant difference in the average income of entrepreneurs and that of workers in all other income groups in the data, as well as in the model. Therefore, the model economy successfully replicates the fact that entrepreneurs are very important in terms of the income share, population share, and average income in particular, in the top five percent income group, which might give an important implication for the business cycle behavior of the income share.

## 4.2 Entrepreneurship and income distribution dynamics

The main focus of this study is the role of entrepreneurship in the business cycle behavior of income distribution. We first examine the aggregate business cycle properties of the model economy in the presence of exogenous shifts in aggregate productivity. The conventional set of business cycle statistics of the model economy are reported in Table 8, which shows the volatility of output, relative volatilities, and cross correlations with output of the key aggregate variables for the model economy. Although the volatility of output in the model is small compared to the actual output volatility, most statistics are similar to those found in the standard models: consumption is about 50 percent as volatile as output, investment is about three times as volatile as output, and so forth.

However, we are more interested in the income distribution dynamics summarized in Table 9 and Figure 7. Table 9 shows the contemporaneous cross correlations of the income shares with output for the six income groups in the data, as well as in the model economy and the other previously simulated economies. The model economy successfully matches the cyclical behavior of the income shares with the U.S. data. The contemporaneous correlations of the income shares of the first, second, and third quintiles with output are 0.82, 0.68, and 0.31, respectively, in the model economy, and 0.72, 0.59, and 0.12, respectively, in the data. Therefore, the model economy successfully reproduces the procyclicality of the income shares of the bottom 60 percent of households. The intuitive explanation is as follows. Since the labor decision of the household in the model economy is indivisible, when a positive aggregate shock occurs, more workers participate in the labor market (an increase in the extensive margin). Most of the new employment occurs in the low-income groups, since the households in the high-income groups are already almost fully employed, which results in an increase in the income share of the low-income quintiles.

In the lowest income group, there may be a compositional change due to aggregate shocks. With a favorable aggregate technology shock and individual idiosyncratic shocks, there are movements across quintiles. With a favorable aggregate shock and a good idiosyncratic shock,

some workers that were in the low quintiles move to the upper, while some workers that were in the high quintiles move to the low, due to a favorable aggregate shock but a bad idiosyncratic shock. The workers moving from the high quintiles to the low may have a higher income than the average of the incumbents of the low quintiles, while the workers moving from the low quintiles to the high may have a lower income than the average of the incumbents of the high quintile. This compositional change creates an increase in the income shares of the low-income quintiles during expansion and a decrease in the income shares of the high-income quintiles. This intuition is confirmed as the negative contemporaneous correlations of the income shares with output, -0.41 for the fourth quintile and -0.67 for the 80-95 percentile. This has already been pointed out by previous studies such as Castañeda et al. (1998), emphasizing that the indivisible labor choice for the households is important in accounting for the overall cyclicity of the income shares. However, these studies were not successful in explaining the acyclical behavior of the income share of the top five percent earners’.

One of the main contributions of our work is that the model economy successfully reproduces the acyclical behavior of the top five percent. The contemporaneous correlations of the income shares of the top five percent is -0.12, which is approximately acyclical, while the corresponding value in the previous studies are strongly negative, reflecting a decrease in the income shares of the high-income quintiles during expansions, as explained above. We think that introducing the occupational choice that includes entrepreneurs seems important in generating the acyclical behavior. To investigate the relationship between entrepreneurship and the cyclical property of the income shares earned by the top income earners, we also calculate the correlations after dropping entrepreneurs in the same income groups. The second row of the second panel in Table 9 reports the cross correlations of the income shares in the six income groups with output, after excluding entrepreneurs. It is very interesting that the correlations of the income share of the top five percent income group, after excluding entrepreneurs, with output becomes strongly negative, which is in line with the countercyclical behavior of the income shares of the fourth quintile and 80-95 income percentile in the model economy, as well as in the data. Therefore, we think that entrepreneurial activities play an important role in generating the acyclical behavior of the income share of the top five percent income group. In addition, we report the simulation results of the model economy without entrepreneurial choice to see the role of entrepreneurship in the cyclical property of the income shares. The model without entrepreneurial choice (an indivisible heterogeneous agent model with only work choice) shows a contrasting result to the model economy with entrepreneurial choice.<sup>21</sup> Without entrepreneurial choice, the contemporaneous correlation of the income shares of the top five percent becomes -0.73. These two exercises confirm that entrepreneurial activities play a crucial role in generating

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<sup>21</sup>Comparing the results of Castañeda et al. (1998) and those of the model without entrepreneurial choice, the endogenously determined indivisible labor supply model reduces, to some extent, the strong cyclicity of the income shares over the income groups.

an acyclical behavior of the income share of the top five percent income group.

We also examine the cyclical behavior of income share, population share, and relative average income of each occupation in the top five percent to obtain a better understanding, which is summarized in Table 10. According to the CPS data, the contemporaneous correlations with output of the income share for workers and entrepreneurs in top five percent are -0.19 and 0.39, respectively.<sup>22</sup> In other words, during expansions, workers in the top five percent lose their relative income share, while the income share of entrepreneurs increases. The opposite signs of the two correlations imply that the income share of the top five percent is likely to be acyclical. A decrease in workers' income share of the total income, caused by an increase in employment in the lower income groups, may be cancelled out by an increase in entrepreneurs' income share of the total income.<sup>23</sup> Surprisingly, the model economy reproduces exactly the same behavior regarding the opposite signs of the correlations of the income shares with output, -0.32 for workers and 0.24 for entrepreneurs.

The income share of entrepreneurs can be decomposed into two factors: the population share of entrepreneurs and relative average income of entrepreneurs. In the U.S. data, the contemporaneous correlations of the population share of entrepreneurs in the top five percent with output is positive, 0.37, which implies that the number of entrepreneurs in this group increases with a favorable aggregate shock. This behavior is roughly reproduced in the model with a correlation of 0.17. The relative average income for entrepreneurs in the top income group also shows a procyclical behavior in the CPS data with a correlation coefficient of 0.20, which is also roughly generated in the model economy.<sup>24</sup> In summary, the model economy with entrepreneurial choice successfully replicates the acyclical behavior of the income share of the top five percent by procyclical entrepreneurial activities.

### 4.3 Entrepreneurship and income mobility

Another important feature of this study is the role of entrepreneurship in income mobility. From Table 3, we observe that entrepreneurs tend to experience higher upward mobility than do non-entrepreneurs. Table 11 reports the one-year transition probabilities across the income groups of staying entrepreneurs, switching entrepreneurs, staying non-entrepreneurs, and switching non-entrepreneurs for the model economy. The transition probability matrices of the model are very similar to those of the data in Table 3.

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<sup>22</sup>We focus our analysis on the two occupational groups, workers, and entrepreneurs, since non-employed workers are relatively negligible in the top five percent income group, in terms of both population and income.

<sup>23</sup>These opposite signs of the two correlations for the two occupations within the income group are not observed in the other high-income groups: the contemporaneous correlations of the income shares of workers and entrepreneurs with output show the same positive signs in the fourth quintile and the 80-95 percentile (not reported in Table 10).

<sup>24</sup>We think that the increase in the number of entrepreneurs at extensive margins is more important in explaining the cyclical behavior of the income share than is the change in the relative average income.

The model economy reproduces the feature that the probability of moving to higher income quintiles for staying entrepreneurs in each income group is higher than that of the households switching from entrepreneurs to non-entrepreneurs, which is consistent with the data shown in Table 3. For example, a household staying as an entrepreneur of the first quintile in the current period has a probability of 56.5 percent of moving to a higher income group, while the probability is 47.1 percent for a household in the first quintile that switches from entrepreneur to non-entrepreneur. The model economy also shows the behavior that the likelihood of falling to a lower income group is less for staying entrepreneurs than for the households switching to non-entrepreneurs in each of the income groups. As for the top five percent, the model economy replicates the fact that staying entrepreneurs are more likely to stay in the top five percent than are switching entrepreneurs. Among staying entrepreneurs, 72.8 percent of the households were successful in remaining in the top five percent, while only 28.5 percent of the households switching to non-entrepreneurs, in the top five percent, managed to stay in the same income group. The higher probability to move upward to a higher income group for switching households from non-entrepreneurs to entrepreneurs than for staying non-entrepreneurs is also successfully replicated in the model economy. The bottom panel of Table 11 reports the transition probabilities for non-entrepreneurs in the previous year.

Therefore, the model economy shows that involvement in entrepreneurial activities is related to upward movement across the income groups, which is particularly important for the cyclical behavior of the top five percent, since the fraction of entrepreneurs in this group is the largest of all income groups. This is consistent with the idea that entrepreneurial activities are a way to explain the acyclical behavior of the income share of the top income earners.

#### **4.4 Heterogeneous capital returns and income distribution dynamics**

Table 12 shows the simulation results of the indivisible labor choice model with no entrepreneurial choice, but with heterogeneous capital returns, as described in Section 3.7. The model economy successfully reproduces the acyclical behavior of the income distribution for the top five percent income earners, with the correlation coefficient of their income share with output of -0.09. Since two-third of the entrepreneurs' income is treated as capital income, and entrepreneurial opportunities are the most active in the top income group, the heterogeneous capital returns can be interpreted as a reflection of entrepreneurial activities.

## 5 Concluding Remarks

This study attempts to explain the cyclical behavior of the income distribution dynamics by developing and analyzing a heterogeneous-agent general equilibrium model with indivisible labor and entrepreneurial choices. We construct an indivisible labor choice heterogeneous agent model and incorporate entrepreneurial activities, making the three occupational choices: workers, entrepreneurs, and non-employed workers.

From the simulation of the model economy, we summarize the findings as follows. First, we find the significant role of entrepreneurs in explaining the cyclical behavior of the income share in the income groups. The model economy successfully reproduces the acyclical behavior of the income share of the top five percent income earners. During expansions, there are more entrepreneurial opportunities, which make more people become entrepreneurs, particularly in the top five percent, offsetting the decline in the income share of the high-income earners from the workers' side. In turn, this generates an acyclical behavior of the income share of the top five percent. Second, the model economy replicates reasonably well the transition matrices across the income groups over the occupational choices in the U.S. data, which shows that entrepreneurial activities are related to upward movement across income groups. The feature that staying households as entrepreneurs of the top five percent tend to remain in the same group is a key to understanding the role of entrepreneurs in explaining the acyclicity of the income share of the top income earners. Third, from an additional exercise in an indivisible labor choice model with heterogeneous capital returns, we are able to reproduce the acyclical behavior of the income share of the top five percent income group, which is a reflection of entrepreneurial activities in the form of heterogeneous capital returns.

# Appendix

## A.1 Data

### A.1.1 Micro data

**CPS** Heathcote et al. (2010) compile various data for inequality from the CPS. The survey period of the CPS is 39 years, 1968-2006, and the number of households in the CPS is around 60,000 per year. We define household income as the sum of labor income, self-employment income, and net asset income, excluding private transfers, public transfers, and taxes, to be consistent with the definition of income in the model economy.<sup>25</sup>

**PSID** We use PSID samples for 1984 to obtain the income and wealth distribution because the year is around the median of the sample periods of CPS, and the 1984 survey contains information on family wealth. We also use the PSID surveys from 1983-1989 to calculate the average one-year transition probabilities for income across occupations for Table 3. The number of families in the PSID is around 6,000 per year. We summarize the PSID variables that we use for income, wealth, and the definitions of the three occupations.

- Income: total taxable income of head and wife (PSID 1984: V10277)
- Wealth: sum of values of six asset types, net of debt value, and value of home equity (PSID 1984: S117)
- Occupation: work for someone else, yourself, or what (PSID 1984: V10456)

### A.1.2 Aggregate data

This subsection explains the data sources used for the moments provided in Table 8. All data, except for the employment rate for workers and entrepreneurs, are drawn from the St. Louis Fred economic database.

- Output ( $Y$ ): real gross value added: nonfarm business (Fred ID: A358RX1Q020SBEA)
- Consumption ( $C$ ): sum of real consumption of nondurable goods and services (Fred ID: DNDGRA3A086NBEA+DSERRA3A086NBEA)
- Investment ( $I$ ): sum of real consumption of durable goods and real fixed private investment (Fred ID: DDURRA3A086NBEA+B007RA3A086NBEA)
- Total hours ( $N$ ): hours worked by full-time and part-time employees (Fred ID: B4701C0A222NBEA)

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<sup>25</sup>See the appendix of Heathcote et al. (2010) for details.

- Average hours ( $H$ ): weekly hours worked: manufacturing for the United States (Fred ID: HOHWMN02USM065S)
- Population share for workers ( $PW$ ): CPS 1968-2006
- Population share for entrepreneurs ( $PE$ ): CPS 1968-2006
- Gini: income Gini ratio of families by race of householder, all races (Fred ID: GINIALLRF)

## A.2 Computational Algorithm

In this subsection, we summarize the computational methods and procedures for the model economy. The computational algorithm is similar to that of Chang and Kim (2014).

### A.2.1 Steady state

We use the algorithm suggested by Ríos-Rull (1999) to find the stationary measure,  $\mu_s(a, e, \eta)$ . The steps are as follows.

1. *Setting guess for endogenous parameters*: We start with the initial guess for endogenous parameters.<sup>26</sup>
2. *Constructing grids for  $a$ ,  $e$  and  $\eta$* : The numbers of  $a$ ,  $e$ , and  $\eta$  grids are denoted by  $n_a$ ,  $n_e$ , and  $n_\eta$ , respectively. We choose  $n_a = 201$ ,  $n_e = 11$ , and  $n_\eta = 7$ . The range of the asset holding is  $[0, 350]$  and asset grids are not equally spaced. Since  $\ln s$  follows an AR(1) process, we set up the range of  $\ln s$ ,  $[-3\sigma_{\ln s}, 3\sigma_{\ln s}]$  where  $\sigma_{\ln s} = \sigma_s / \sqrt{1 - \rho_s^2}$ , where  $s \in \{e, \eta\}$ . In addition, the grid points for  $\ln s$  are constructed equally, where  $s \in \{e, \eta\}$ .
3. *Approximating the transition probability matrices for  $e$  and  $\eta$* : We use the method developed by Tauchen (1986) to approximate the transition probability matrices for  $e$  and  $\eta$ ,  $Q_e(e'|e)$  and  $Q_\eta(\eta'|\eta)$ , respectively.
4. *Solving the individual value functions*: Given the parameters, we solve a set of value functions,  $\{V^W, V^E, V^N, V\}$ , on each grid point of the individual states. In this step, we also obtain the optimal decision rules for asset holding,  $a'(a, e, \eta)$ ; consumption,  $c(a, e, \eta)$ ; investment on entrepreneurial production,  $k(a, e, \eta)$ ; and hours worked,  $h(a, e, \eta)$ .
5. *Obtaining time-invariant measures*: We obtain the time-invariant measures,  $\mu_s(a, e, \eta)$  using  $a'(a, e, \eta)$ ,  $Q_e(e'|e)$ , and  $Q_\eta(\eta'|\eta)$  which were obtained from the previous steps.

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<sup>26</sup>The endogenous parameters are  $\beta$ ,  $\lambda_W$ ,  $\lambda_E$ ,  $\chi$ , and  $\sigma_\eta$ .

6. *Updating the parameters* : If the obtained values are close enough to the targeted values in the steady state equilibrium, then we are done. Otherwise, we update the parameters and go back to Step 1.

### A.2.2 Dynamics

For an equilibrium with aggregate fluctuations, we follow Krusell and Smith (1998), who suggest that a very high precision can be obtained by approximating the type distribution of individuals using a mean asset, the first moments of the distribution. In this model, however, it is a mean asset invested in the corporate production sector,  $K$ , that affects the market prices. The steps are as follows.

1. *Constructing  $K$ ,  $w$ , and  $z$  grids*: The grids for individual state variables are the same with those used in the steady state economy, except for the range of the asset holding, which is  $[0, 450]$ . We choose seven equally-spaced grid points ( $n_K = 7$ ) for  $K$  in the range of  $[0.85K_s, 1.15K_s]$ , where  $K_s$  denotes steady state mean capital used for corporate production. We also choose seven equally-spaced grid points ( $n_w = 7$ ) for  $w$  in the range of  $[0.85w_s, 1.15w_s]$ , where  $w_s$  denotes the steady state wage rate. Since  $\ln z$  follows an AR(1) process, we set up the range of  $\ln z$ ,  $[-3\sigma_{\ln z}, 3\sigma_{\ln z}]$  where  $\sigma_{\ln z} = \sigma_z / \sqrt{1 - \rho_z^2}$  and the number of  $z$  grids is seven ( $n_z = 7$ ).
2. *Parameterizing the forecasting function*: We use the following log-linear functional form for forecasting functions of  $K'$  and  $w$ :

$$\ln K' = b_0 + b_1 \ln K + b_2 \ln z,$$

$$\ln w = d_0 + d_1 \ln K + d_2 \ln z.$$

We set the initial guess for the coefficients in the forecasting functions. The conservative and reasonable guesses are  $b_0 = \ln K_s$ ,  $b_1 = 0$ ,  $b_2 = 0$ ,  $d_0 = w_s$ ,  $d_1 = 0$ , and  $d_2 = 0$ , respectively. In this step, we obtain the forecasting functions for each  $K$  and  $w$  grid.

3. *Solving the optimization problem for the individual agents*: Using the forecasting functions obtained in the previous step, we solve the individual optimization problem to obtain a set of value functions and optimal decision rules.
4. *Implementing the simulation*: Using the forecasting functions, value functions, and optimal decision rules obtained in the previous step, we simulate the decision rules of 50,000 individuals ( $N = 50,000$ ) for 2,500 periods ( $T = 2,500$ ). We then generate a set of time

series data  $\{K_{t+1}, w_t\}_{t=1}^T$ .<sup>27</sup> We drop the first 500 periods ( $T_0 = 500$ ) to eliminate the influence of the arbitrary choice of initial  $K$  and  $z$ .

5. *Obtaining new values for coefficients:* We obtain the new set of coefficients in the forecasting functions,  $b'$  and  $d'$ , from an OLS regression with  $\{K_{t+1}, w_t, z_t\}_{t=T_0+1}^T$ . If the new coefficients obtained are close enough to the previous coefficients, we are done. Otherwise, we update the set of coefficients and go back to Step 3.
6. *Checking the goodness of fit:* We check the goodness of fit for the forecasting functions with  $R^2$ . The forecasting functions for  $K'$  and  $w$  provide high accuracy:<sup>28</sup>

$$\ln K' = 0.10264 + 0.95447 \ln K + 0.08916 \ln z, \quad R^2 = 0.996988,$$

$$\ln w = -0.07264 + 0.41522 \ln K + 0.87332 \ln z, \quad R^2 = 0.996143.$$

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<sup>27</sup> $K_{t+1}$  is obtained as follows. We use the forecasting function for  $K'$  to obtain a set of value functions in period  $t + 1$ . Comparing the value functions, we are able to obtain the occupational status for each household  $i$  in period  $t + 1$ , and household  $i$ 's investment in corporate firms in period  $t + 1$ ,  $\tilde{a}_{t+1,i}$ . Finally, we can compute  $K_{t+1} = \sum_i^N \tilde{a}_{t+1,i}/N$ .

<sup>28</sup>All coefficients in each forecasting function are statistically significant at the five percent level.

## References

1. Aiyagari, S. Rao, 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* 109(3), 659-84.
2. Cagetti, Marco and De Nardi, Mariacristina, 2006. "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy* 114(5), 835-70.
3. Castañeda, Ana, Díaz-Giménez, Javier, and Ríos-Rull, José-Victor, 1998. "Exploring the Income Distribution Business Cycle Dynamics," *Journal of Monetary Economics* 42(1), 93-130.
4. Chang, Yongsung and Kim, Sun-Bin, 2006. "From Individual To Aggregate Labor Supply: A Quantitative Analysis Based On A Heterogeneous Agent Macroeconomy," *International Economic Review* 47(1), 1-27.
5. Chang, Yongsung and Kim, Sun-Bin, 2007. "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations," *American Economic Review* 97(5), 1939-56.
6. Chang, Yongsung and Kim, Sun-Bin, 2014. "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations: Reply," *American Economic Review* 104(4), 1461-66.
7. Clark, Kim B. and Summers, Lawrence H., 1981. "Demographic Differences in Cyclical Employment Variation," *Journal of Human Resources* 16(1), 61-79.
8. Díaz-Giménez, Javier, Glover, Andrew, and Ríos-Rull, José-Victor, 2011. "Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update," *Quarterly Review* (Federal Reserve Bank of Minneapolis) 34(1), 2-31.
9. Díaz-Giménez, Javier, Quadrini, Vincenzo, and Ríos-Rull, José-Victor, 1997. "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth," *Quarterly Review* (Federal Reserve Bank of Minneapolis) 21, 3-21.
10. Floden, Martin and Lindé, Jesper, 2001. "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?," *Review of Economic Dynamics* 4(2), 406-437.
11. French, Eric, 2005. "The Effects of Health, Wealth, and Wages on Labor Supply and Retirement Behavior," *Review of Economic Studies*, 72(2), 395-427.
12. Hansen, Gary D., 1985. "Indivisible labor and the business cycle," *Journal of Monetary Economics* 16(3), 309-27.
13. Heathcote, Jonathan; Perri, Fabrizio and Violante, Giovanni L., 2010. "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967-2006," *Review of Economic Dynamics* 13(1), 15-51.
14. Heckman, James, 1979. "Sample Selection Bias as a Specification Error," *Econometrica* 47, 153-62.
15. Heckman, James, 1984. "Comments on Ashenfelter and Kydland Papers," *Carnegie-Rochester Conference Series on Public Policy* 21, 209-24.
16. Heer, Burkhard, 2013. "A Note on the Cyclical Behaviour of the Income Distribution," *OECD Journal: Journal of Business Cycle Measurement and Analysis*, vol. 2013(1), 1-7.
17. Huggett, Mark, 1993. "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control* 17(5-6), 953-69.

18. Khan, Aubhik and Thomas, Julia K., 2008. "Idiosyncratic Shocks and the Role of Non-convexities in Plant and Aggregate Investment Dynamics," *Econometrica* 76(2), 395-436.
19. Krusell, Per and Smith, Jr., Anthony A., 1998. "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy* 106(5): 867-96.
20. Kydland, Finn E. and Prescott, Edward C., 1982. "Time to Build and Aggregate Fluctuations," *Econometrica* 50(6), 1345-70.
21. Kydland, Finn E., 1984. "Labor-Force Heterogeneity and the Business Cycle," *Carnegie-Rochester Conference Series on Public Policy* 21, 173-208.
22. Lusardi, Annamaria and Mitchell, Olivia S., 2014, "The Economic Importance of Financial Literacy: Theory and Evidence," *Journal of Economic Literature* 52(1), 5-44.
23. Piketty, Thomas, 2014. *Capital in the Twenty-First Century*, translated by Arthur Goldhammer, Belknap Press.
24. Quadrini, Vincenzo, 2000, "Entrepreneurship, Saving, and Social Mobility," *Review of Economic Dynamics* 3, 1-40.
25. Ríos-Rull, José-Victor, 1993. "Working in the Market, Working at Home and the Acquisition of Skills: A General Equilibrium Approach," *American Economic Review* 83, 893-907.
26. Ríos-Rull, José-Victor, 1999. "Computation of Equilibria in Heterogeneous-Agent Models," in *Computational Methods for the Study of Dynamic Economies*, edited by Ramon Marimon and Andrew Scott, 238–64. New York: Oxford University Press.
27. Rogerson, Richard, 1988. "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 21(1), 3-16.
28. Siemer, Michael, 2014. "Firm Entry and Employment Dynamics in the Great Recession," Finance and Economics Discussion Series 2014-56, Board of Governors of the Federal Reserve System.
29. Tauchen, George. 1986. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20(2): 177–81.
30. Terajima, Yaz, 2006. "Education and Self-Employment: Changes in Earnings and Wealth Inequality," Bank of Canada Working Paper 2006-40.

Table 1: Contemporaneous Correlations of Income Shares with Output

	Quintiles				Top (%)	
	1st	2nd	3rd	4th	80-95	95-100
Total Sample	0.72	0.59	0.12	-0.35	-0.53	-0.06
Excluding Entrepreneurs	0.67	0.39	0.07	-0.44	-0.62	-0.35

Notes: The data are from the CPS for the period of 1968-2006 based on Heathcote et al. (2010)

Table 2: Distributions of Income and Wealth

	Quintiles				Top (%)		Gini Index
	1st	2nd	3rd	4th	80-95	95-100	
Income Distribution							
PSID 1984	0.3	6.2	15.3	26.3	31.6	20.4	0.51
CPS 1968-2006	2.9	10.6	16.8	24.1	26.8	18.7	0.41
Wealth Distribution							
PSID 1984	-0.7	0.7	4.5	13.9	28.2	53.4	0.77
SCF 1992	-0.4	1.7	5.7	13.4	26.0	53.5	0.75

Notes: SCF 1992 data are from Díaz-Giménez et al. (1997)

The Gini index is calculated by the Lorenz curve based on the income and wealth distribution of the six groups in the table.

Table 3: Average One-Year Transition Probabilities for Income: PSID 1983-1989.

	1st	2nd	3rd	4th	80-95	95-100	1st	2nd	3rd	4th	80-95	95-100
	Staying Entrepreneurs						Switching Entrepreneurs					
1st	41.0	34.4	11.5	8.3	1.7	3.2	69.3	27.5	1.3	1.9	0.0	0.0
2nd	11.1	55.9	22.0	6.8	3.2	1.0	23.1	54.1	19.6	2.3	0.9	0.0
3rd	3.4	14.7	45.8	25.5	8.2	2.4	5.2	24.4	49.6	15.9	4.9	0.0
4th	2.2	4.4	20.2	47.3	22.6	3.3	5.1	11.6	16.4	51.3	14.3	0.0
80-95	1.5	1.2	5.7	20.3	54.2	17.1	0.6	2.2	4.0	24.9	58.5	6.7
95-100	0.3	0.6	1.9	3.8	19.4	74.0	0.0	3.2	4.4	9.0	22.0	61.3
	Staying Non-Entrepreneurs						Switching Non-Entrepreneurs					
1st	85.2	13.0	1.3	0.3	0.1	0.0	53.3	38.8	7.5	0.4	0.0	0.0
2nd	13.8	65.5	17.9	2.4	0.3	0.1	12.9	59.7	21.3	4.1	1.9	0.0
3rd	2.2	15.8	63.5	16.9	1.4	0.1	3.4	20.3	52.2	21.1	3.1	0.0
4th	0.6	2.7	14.7	67.9	13.5	0.6	0.6	8.8	20.8	49.1	19.2	1.6
80-95	0.2	0.8	2.0	18.0	73.1	6.0	0.0	2.1	6.1	21.1	54.5	16.2
95-100	0.2	0.7	0.8	2.7	22.3	73.3	1.5	0.0	0.0	3.8	20.3	74.5

Table 4: Parameters of the Model Economies

Parameter	Description
$\beta$	0.97751 Time discount factor
$\lambda_W$	139.4 Disutility from working for workers
$\lambda_E$	20.4 Disutility from working for entrepreneurs
$\bar{h}$	1/3 Fixed hours for full-time workers
$1/\theta$	0.4 Labor supply elasticity
$\alpha$	0.36 Capital income share for corporate production
$\chi$	0.14 Entrepreneurial production parameter
$\nu$	0.33 Capital income share for entrepreneurial production
$\delta$	0.025 Quarterly depreciation rate
$\rho_e$	0.929 Persistence of labor productivity shocks
$\sigma_e$	0.227 Standard deviation of labor productivity shocks
$\rho_\eta$	0.95 Persistence of entrepreneurial productivity shocks
$\sigma_\eta$	0.50 Standard deviation of entrepreneurial productivity shocks
$\rho_z$	0.95 Persistence of aggregate shocks
$\sigma_z$	0.007 Standard deviation of aggregate shocks
$\kappa$	0.724 Capital return coefficient on labor productivity

Table 5: Income Share of Each Occupation over Income Groups

	Quintiles				Top (%)		Total
	1st	2nd	3rd	4th	80-95	95-100	
PSID 1984							
Workers	26.1	62.4	78.3	82.2	81.5	59.6	75.3
Entrepreneurs	2.6	7.4	8.4	10.9	15.6	37.1	17.1
Non-Employed	71.3	30.3	13.4	6.9	2.9	3.4	7.5
CPS 1968-2006							
Workers	33.5	58.0	67.5	72.3	73.4	62.8	67.3
Entrepreneurs	7.6	8.7	8.8	9.1	11.0	22.2	11.9
Non-Employed	58.8	33.4	23.7	18.6	15.7	14.9	20.7
Model Economy							
Workers	32.7	60.3	71.1	78.6	83.6	43.0	67.7
Entrepreneurs	4.4	9.5	10.6	9.7	10.4	54.9	20.6
Non-Employed	62.9	30.2	18.3	11.7	6.0	2.1	11.7

Table 6: Population Share of Each Occupation over Income Groups

	Quintiles				Top (%)		Total
	1st	2nd	3rd	4th	80-95	95-100	
PSID 1984							
Workers	9.3	57.1	78.0	81.8	81.8	65.6	60.0
Entrepreneurs	0.9	7.2	8.0	11.0	15.2	31.2	9.5
Non-Employed	89.8	35.6	14.0	7.2	3.0	3.2	30.5
CPS 1968-2006							
Workers	16.8	57.6	67.8	72.9	74.3	67.0	57.5
Entrepreneurs	5.0	8.7	8.8	9.2	10.9	19.4	8.9
Non-Employed	78.2	33.7	23.3	17.9	14.8	13.6	33.5
Model Economy							
Workers	23.7	59.6	70.7	78.3	83.5	59.2	61.9
Entrepreneurs	3.1	9.3	10.6	9.8	10.3	38.3	10.0
Non-Employed	73.1	31.0	18.7	12.0	6.3	2.5	28.0

Table 7: Relative Average Income of Each Occupation over Income Groups

	Quintiles				Top (%)		Total
	1st	2nd	3rd	4th	80-95	95-100	
PSID 1984							
Workers	2.78	1.09	1.00	1.00	1.00	0.91	1.24
Entrepreneurs	2.73	1.01	1.04	0.99	1.03	1.18	1.84
Non-Employed	0.80	0.85	0.96	0.96	0.96	1.05	0.25
CPS 1968-2006							
Workers	1.97	1.01	1.00	1.00	1.00	0.94	1.17
Entrepreneurs	1.56	1.00	1.00	1.00	1.01	1.17	1.34
Non-Employed	0.76	0.98	0.99	0.99	0.99	1.02	0.62
Model Economy							
Workers	1.38	1.01	1.01	1.00	1.00	0.73	1.09
Entrepreneurs	1.40	1.02	0.99	1.00	1.02	1.43	2.06
Non-employed	0.86	0.97	0.98	0.98	0.96	0.83	0.42

Note : Average income for each income group is normalized to 1.

Table 8: Volatilities and Comovements of Aggregate Variables

Volatilities	$\sigma_Y$	$\sigma_C/\sigma_Y$	$\sigma_I/\sigma_Y$	$\sigma_H/\sigma_Y$	$\sigma_{\frac{Y}{H}}/\sigma_Y$	$\sigma_{PW}/\sigma_Y$	$\sigma_{PE}/\sigma_Y$	$\sigma_{Gini}/\sigma_Y$
US Data	2.52	0.45	2.39	0.50	0.77	0.47	0.14	0.17
Model Economy	1.38	0.49	2.97	0.39	0.72	0.32	0.06	0.19
Comovements		$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{H,Y}$	$\rho_{\frac{Y}{H},Y}$	$\rho_{PW,Y}$	$\rho_{PE,Y}$	$\rho_{Gini,Y}$
US Data		0.85	0.82	0.66	0.98	0.83	0.35	-0.13
Model Economy		0.86	0.96	0.82	0.95	0.54	0.18	-0.63

Notes:  $\sigma_i$  and  $\rho_{j,Y}$  are standard deviation of  $i$  and cross correlation of  $j$  with output ( $Y$ ). Subscripts  $C$ ,  $I$ ,  $H$ ,  $PW$ ,  $PE$ , and  $Gini$  denote consumption, investment, total hours, population share for workers, population share for entrepreneurs, and Gini coefficients, respectively. All variables other than  $PW$ ,  $PE$  and  $Gini$  are logged and all variables are detrended by the HP filter.

Table 9: Contemporaneous Correlations of Income Shares with Output

	Quintiles				Top (%)	
	1st	2nd	3rd	4th	80-95	95-100
CPS 1968-2006						
All Households	0.72	0.59	0.12	-0.35	-0.53	-0.06
Excluding Entrepreneurs	0.67	0.39	0.07	-0.44	-0.62	-0.35
Model with Entrepreneurial Choice						
All Agents	0.82	0.68	0.31	-0.41	-0.67	-0.12
Excluding Entrepreneurs	0.75	0.71	0.31	-0.32	-0.64	-0.30
Model without Entrepreneurial Choice						
	0.86	0.85	0.75	-0.48	-0.78	-0.73
Castañeda et al. (1998)						
	0.97	0.97	0.85	-0.98	-0.96	-0.98
Heer (2013)						
	0.85	0.89	0.90	-0.32	-0.88	-0.88

Table 10: Contemporaneous Correlations of Income Share, Population Share, and Relative Average Income of Each Occupation with Output for Top Five Percent Income Group

CPS 1968-2006			
Occupation	Income Share	Population Share	Relative Average Income
Workers	-0.19	-0.15	-0.18
Entrepreneurs	0.39	0.37	0.20
Non-Employed	0.46	-0.36	-0.27
Model Economy			
Occupation	Income Share	Population Share	Relative Average Income
Workers	-0.32	-0.28	-0.25
Entrepreneurs	0.24	0.17	0.04
Non-Employed	0.21	0.31	-0.16

Table 11: One-Year Transition Probabilities for Income: Model Economy

	1st	2nd	3rd	4th	80-95	95-100	1st	2nd	3rd	4th	80-95	95-100
	Staying Entrepreneurs						Switching Entrepreneurs					
1st	43.5	37.4	9.7	4.9	3.6	0.9	52.9	34.5	10.6	1.9	0.1	0.0
2nd	10.6	51.3	22.2	7.0	5.9	3.1	29.3	33.6	26.4	9.5	1.2	0.0
3rd	1.0	22.5	43.1	18.8	7.5	7.0	14.0	25.8	29.3	24.4	6.4	0.2
4th	0.3	4.8	25.8	40.6	16.3	12.2	5.7	15.1	24.8	32.5	20.3	1.6
80-95	0.2	2.9	7.7	24.1	38.5	26.7	3.7	8.4	13.5	27.1	38.3	9.0
95-100	0.0	0.6	3.4	7.5	15.7	72.8	1.3	7.7	12.7	18.3	31.5	28.5
	Staying Non-entrepreneurs						Switching Non-entrepreneurs					
1st	58.0	24.1	11.6	5.4	0.8	0.0	35.1	39.1	17.1	6.2	2.0	0.5
2nd	20.8	42.4	24.8	9.7	2.3	0.1	14.8	40.4	27.4	11.9	4.3	1.1
3rd	13.2	17.8	36.8	25.8	6.3	0.2	5.5	27.1	38.3	20.9	6.2	2.0
4th	9.1	9.2	18.2	40.8	21.7	1.0	1.8	9.4	29.1	39.9	16.2	3.7
80-95	3.4	5.9	8.2	22.7	50.0	9.8	0.3	2.7	9.0	32.1	45.4	10.5
95-100	0.3	2.0	3.6	7.7	37.1	49.2	0.0	0.3	2.0	8.2	45.7	43.7

Table 12: Contemporaneous Correlations of Income Shares with Output: Model with Heterogeneous Capital Returns

	Quintiles				Top (%)	
	1st	2nd	3rd	4th	80-95	95-100
CPS 1968-2006	0.72	0.59	0.12	-0.35	-0.53	-0.06
Model with Entrepreneurs	0.82	0.68	0.31	-0.41	-0.67	-0.12
Model with Heterogeneous Capital Returns	0.87	0.81	0.45	-0.52	-0.71	-0.09

Figure 1: Income Share of Entrepreneurs across Income Groups

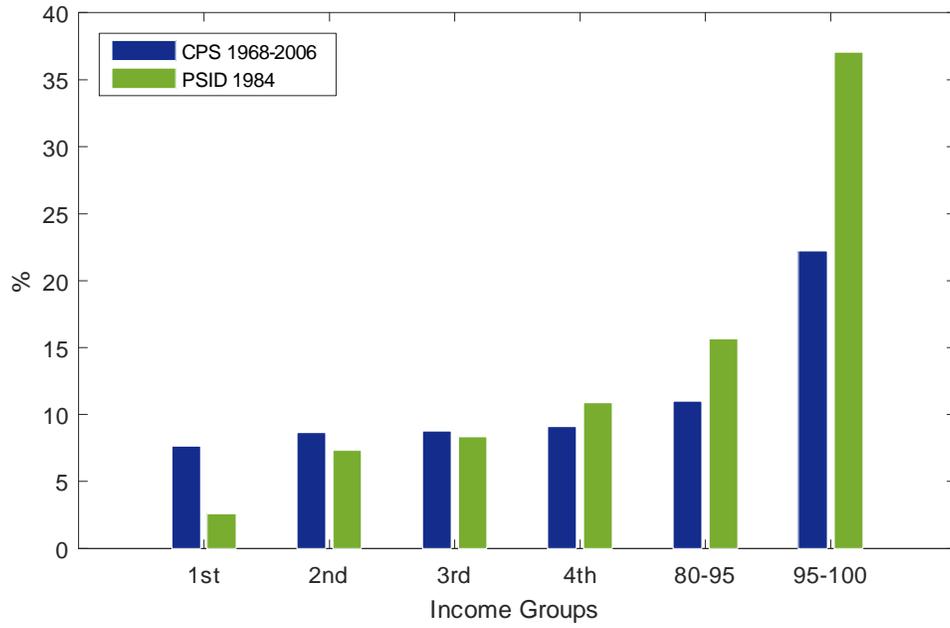


Figure 2: Population Share of Entrepreneurs across Income Groups

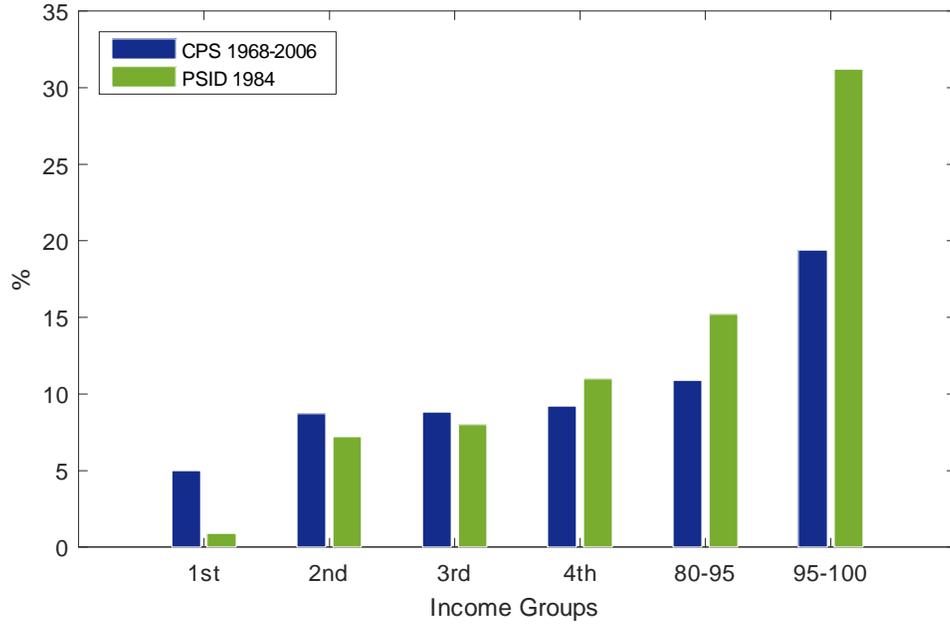


Figure 3: Relative Average Income of Entrepreneurs across Income Groups

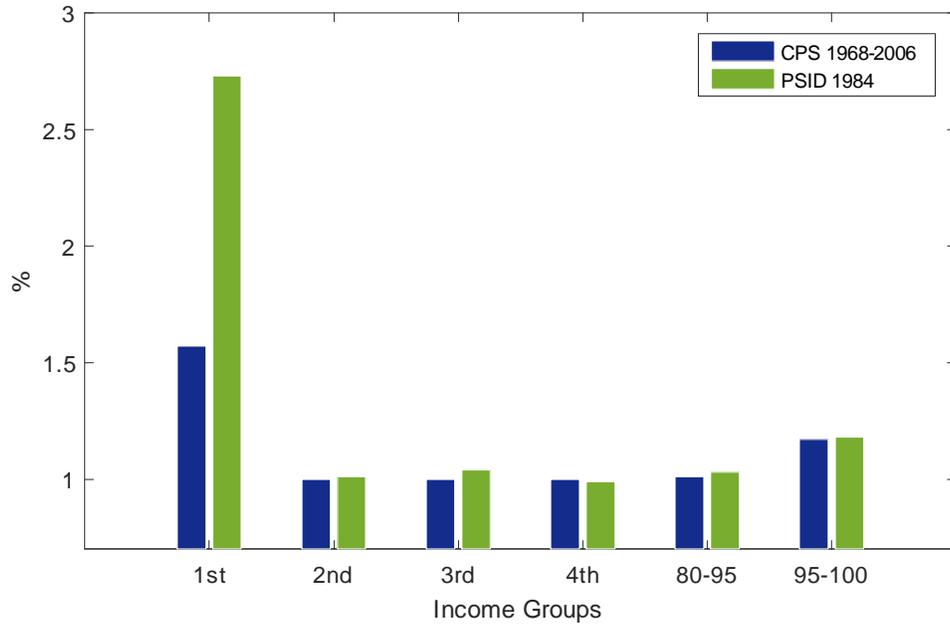


Figure 4: Wage Rates and Capital Returns

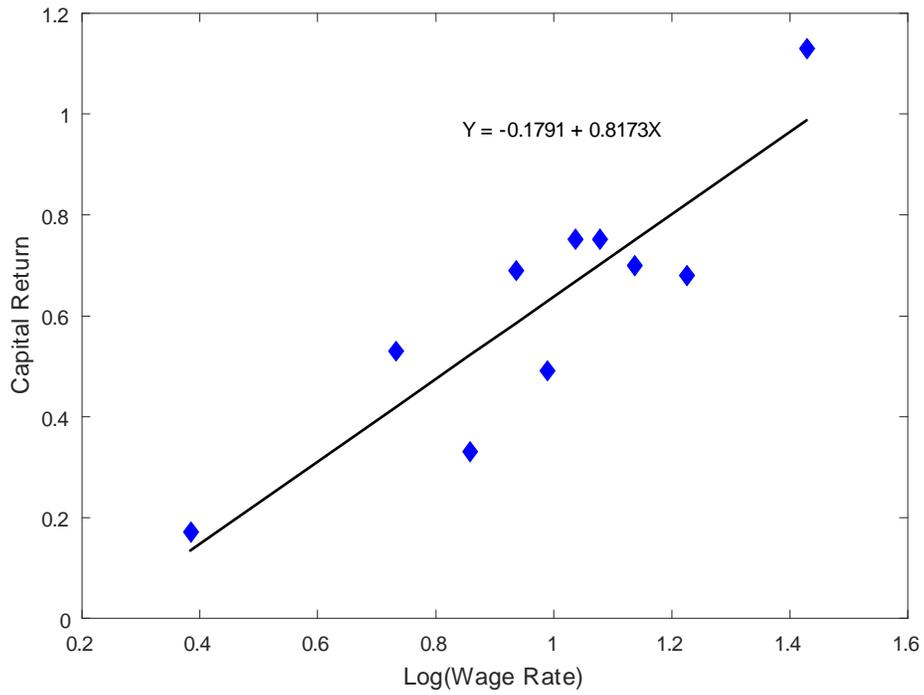


Figure 5: Lorenz Curves for Wealth

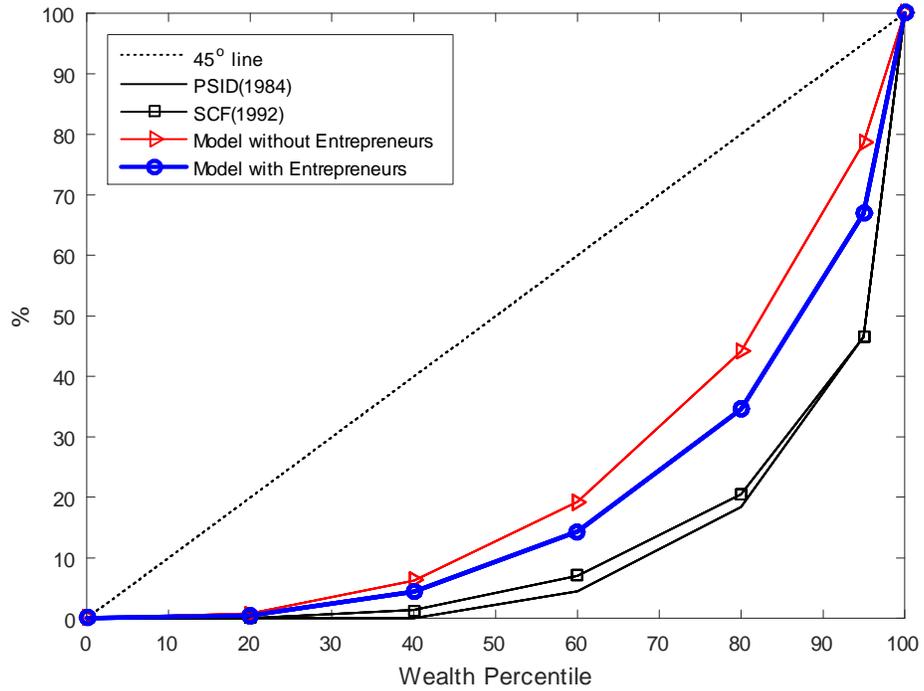


Figure 6: Lorenz Curves for Income

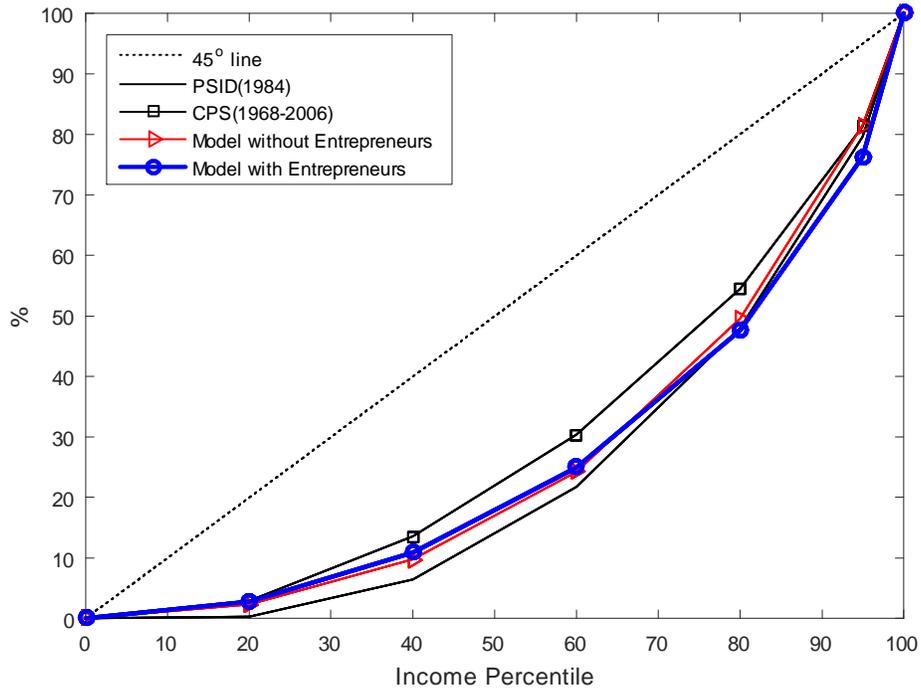


Figure 7: Contemporaneous Correlations of Income Shares with Output

