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Loan Rate Differences across Financial Firm Types: A Mechanism Design Approach

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The views herein are those of the authors and not necessarily those of the Bank.

Introduction

- Loan interest rates offered across various financial firms are hugely different.
- In Korea, you can get a loan at 5% from banks, 10% from mutual savings banks, 20% from credit card companies, and 30% from loan business companies.
- We explore the cross-sectional structure of loan rates across financial firm types.
  - Why do they offer so different loan interest rates?
  - Is it too wide or the other way around?
  - How many financial firm types can exist in an economy?
  - Possible link to the monetary policy?

- There are many factors that affect the characteristics of loans: Differences in credit rating, for example.
- You can argue that loan rate differences mainly reflect different levels of credit rating
- But in the theoretical point of view, this is a tautology.
- We assume that agents are all the same from the ability perspective. That is, each agent has an ex-ante indistinguishable project with the same probabilistic outcome on the same support. Agents differ only in the history of paying back their debt.

(Contribution of This Paper)

- There are two well-known mechanisms in costly state verification model
  - Verification in case of default (Townsend 1979, Gale and Hellwig 1985): Standard debt contract is optimal in the sense that it minimizes monitoring costs. Verification takes place when agent says that he cannot pay back the loan (due to bad realization of ex ante profitable production plan, for example).
  - Stochastic verification (Mookherjee and Png 1989): Standard debt contract is not optimal once random auditing is allowed in standard CSV models (no threshold bankruptcy point). All audits must be random. If borrower's report is audited and verified to be truthful then the borrower must be rewarded so that the borrower strictly prefers to be audited.
- This paper adds no verification mechanism: Financial firms never verify even if agents are not paying back their debt. (But still, of course, truth-telling incentive compatibility conditions are satisfied in the equilibrium as will be shown).

- The main idea in this paper is that the loan rate differences are representing a mechanism that makes agents (borrowers) tell the truth.
  - Spse, after failing to pay back the debt to the bank, you cannot borrow from banks again and must contact higher-interest charging mutual savings banks to get a new loan.
  - Now, there exist a well-defined incentive: pay back and remain in the banks' loan market vs. don't pay back and move to mutual savings banks' loan market (higher interest rate).
- Based on this idea, this paper will derive how many financial sectors can co-exist in the economy.
- Also discusses possible interesting extensions: relief fund, some business cycle aspects (in a boom it's easier to get loans at lower rates)
- Note that better understanding of financial market is a starting point of more efficient monetary policy.

Model

(Basic Environment)

- Time is discrete and indefinite. Each period consists of two sub-periods, period 0 and period 1.
- There are N financial sectors occupied by corresponding financial firm type: B_1, B_2, $\cdots$, B_N. WLOG $r_1 \leq r_2 \leq \cdots \leq r_N$, where r_j is the loan rate charged by B_j.
- For convenience, there is $(N+1)$th sector in which an imaginary financial firm charges loan rate r_{N+1}: In this sector no loans can be made because IR condition of either financial firm or agents are violated. Agents falling into $(N+1)$th sector leave the economy.
- Continuum of agents $a_i \in [0,1]$. $\sum_j K_j = 1, K_j \equiv \int_j a_i di$.
- Agent i borrows α and invest in his/her project at period 0. The project returns $y_i \sim U[0,\bar{y}]$ in period 1.

- Discount rate between sub-period 1 at time t and sub-period 1 at time $t+1 = \beta$.
- Agent i in sector B_j, $j \in \{1, 2, \cdots, N\}$ reports $\tilde{y}_i \in \{Yes, No\}$ to the financial firm. If $\tilde{y}_i =$ 'Yes' then debt (principal and interest) is cleared and agent i remains in sector B_j.
- If $\tilde{y}_i =$ 'No' then two cases. First, an audit can take place, and all the ownership of the corresponding project is transferred from agent i to the financial firm. An audit by B_j can fully recover true y_i at a cost of c. Second, an audit does not take place and the financial firm cannot recover any and agent i keeps y_i in his/her pocket. In either case, agent i moves to sector B_{j+1} with probability m.

- Note following the convention by letting $\tilde{y}_i \in [0, \bar{y}]$ does not make any difference in this paper's context.
  If $\tilde{y}_i \geq \alpha(1+r_j)$ then debt (principal and interest) is cleared and agent i remains in sector B_j.
  Otherwise, two cases can happen: (i) an audit takes place and B_j recover true y_i at a cost of c. (ii) an audit does not take place and agent i keeps y_i in his/her pocket. That is, B_j cannot recover anything without an audit. In either case, agent i moves to sector B_{j+1} with probability m.
- But letting $\tilde{y}_i \in \{Yes, No\}$ makes the consequence of an audit clearer in this paper's context, we guess.
- No capital. No collateral. No aggregate uncertainty.
- Consider flow of agents across financial sectors: In equilibrium, inflow into B_j = outflow from B_j, $j \in \{1, 2, \cdots, N\}$. Therefore, assume inflow into B_1 = outflow from B_N.

(Value Functions of Agents)

- For a moment, let audit prob $q \in \{0, 1\}$.
- Consider agent i in sector $j \in \{1, 2, \cdots, N\}$ with realization y_i:

$$\begin{aligned} V_j(y_i) = \max\Big\{ & y_i - \alpha(1+r_j) + \beta E[V_j(\cdot)], \\ & q\big[\beta(1-m)E[V_j] + \beta m E[V_{j+1}]\big] \\ & \quad + (1-q)\big[y_i + \beta(1-m)E[V_j] + \beta m E[V_{j+1}]\big]\Big\} \end{aligned} \tag{1}$$

- Revelation principle allows us to focus on truth-telling mechanism: Green color represents payoff when $y_i \geq \alpha(1+r_j)$ and red color represents payoff when $y_i < \alpha(1+r_j)$.
- If $y_i \geq \alpha(1+r_j)$ then $\tilde{y}_i =$ 'Yes.' If $y_i < \alpha(1+r_j)$ then $\tilde{y}_i =$ 'No.' In the latter case, it is physically impossible to cheat.

- Under truth-telling mechanism:

$$
\begin{aligned}
E[V_j(\cdot)] = & \frac{1}{\bar{y}} \int_{\alpha(1+r_j)}^{\bar{y}} y_i - \alpha(1+r_j) + \beta E[V_j(\cdot)] dy_i \\
& + \frac{1}{\bar{y}} \int_{0}^{\alpha(1+r_j)} q\big[\beta(1-m)E[V_j] + \beta m E[V_{j+1}]\big] \\
& \quad + (1-q)\big[y_i + \beta(1-m)E[V_j] + \beta m E[V_{j+1}]\big] dy_i \\
= & \frac{1}{\bar{y}} \int_{\alpha(1+r_j)}^{\bar{y}} y_i - \alpha(1+r_j) + \beta E[V_j(\cdot)] dy_i \\
& + \frac{1}{\bar{y}} \int_{0}^{\alpha(1+r_j)} (1-q) y_i + \beta(1-m)E[V_j] + \beta m E[V_{j+1}] dy_i
\end{aligned}
\tag{2}
$$

- Rearranging yields:

$$2(1-\beta)\bar{y}E[V_j(\cdot)] = [\bar{y}-\alpha(1+r_j)]^2 + (1-q)\alpha^2(1+r_j)^2 - 2\beta m\{E[V_j(\cdot)] - E[V_{j+1}(\cdot)]\}\alpha(1+r_j) \quad (3)$$

(Incentive Compatibility Conditions for Agents)

- Note that agent with realization $y_i < \alpha(1+r_j)$ cannot repay the debt not because of IC but because of physical impossibility. In other words, IC condition can hold with strong inequality for all realization y_i in the support.
- Since we are interested in maximum numbers of financial sectors that can co-exist, we will let IC bind with equality at $y_i = \alpha(1+r_j)$ for $j \in \{1, 2, \cdots, N\}$.

- Then, from Eq. (1),

$$\beta E[V_j(\cdot)] = (1-q)\alpha(1+r_j) + \beta(1-m)E[V_j(\cdot)] + \beta m E[V_{j+1}(\cdot)]$$
$$\Rightarrow \beta m \{E[V_j(\cdot)] - E[V_{j+1}(\cdot)]\} = (1-q)\alpha(1+r_j),$$
$$for\ j = \{1, 2, \cdots, N-2\} \quad \text{(reason below)}. \tag{4}$$

- Plug Eq. (4) into Eq. (3) to get

$$E[V_j(\cdot)] = \frac{1}{2(1-\beta)\bar{y}} \left\{ [\bar{y} - \alpha(1+r_j)]^2 - (1-q)\alpha^2(1+r_j)^2 \right\},$$
$$for\ j = \{1, 2, \cdots, N-1\} \tag{5}$$

- **Boundary Condition for No-verification Case** IR condition makes $V_{N+1} = 0$.
  Claim: When $q = 0$, Nth sector cannot exist.
  Spse $E[V_N] \geq (1-q)\alpha(1+r_N)$ (From Eq. (4)). Then Nth sector cannot be the last sector and there could exist one more sector. A contradiction. On the contrary, spse $E[V_N] < (1-q)\alpha(1+r_N)$. Then Nth sector can't exist since Eq. (4) is violated. When $q = 1$, this does not happen.

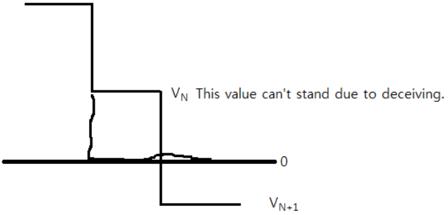

Figure: Similar to limited participation problem: You cannot make an incentive scheme that punishes agents strong enough due to IR condition.

(Individual Rationality Conditions for Agents)

- $E[V_1(\cdot)] \geq \cdots \geq E[V_N(\cdot)] \geq 0 = E[V_{N+1}(\cdot)]$ implies:

$$E[V_N(\cdot)] = \frac{1}{2(1-\beta)\bar{y}} \left\{ [\bar{y} - \alpha(1+r_N)]^2 - (1-q)\alpha^2(1+r_N)^2 \right\} \geq 0 \tag{6}$$

- When $q = 1$, IR condition is automatically satisfied.
- When $q = 0$, need the following condition to be satisfied:

$$\bar{y} \geq 2\alpha(1+r_N) \Rightarrow 1 + r_N \leq \frac{\bar{y}}{2\alpha}. \tag{7}$$

- Note that the value of the terms inside $\{\}$ is strictly increasing in q.

(Financial Firms)

- Profit of financial firm in sector j:

$$
\begin{aligned}
& K_j\Bigg\{\frac{1}{\bar{y}}[\bar{y}-\alpha(1+r_j)][\alpha(1+r_j)-\alpha] \\
& \quad \frac{1}{\bar{y}}\int_0^{\alpha(1+r_j)} q(y_i-\alpha-c)+(1-q)(-\alpha)\Bigg\} \\
& \quad = \frac{\alpha B_j}{2\bar{y}}\Bigg\{-\alpha(2-q)(1+r_j)^2+2(\bar{y}-qc)(1+r_j)-2\bar{y}\Bigg\}
\end{aligned} \tag{8}
$$

- Rearrange the profit Eq. (8) wrt r_j to get:

$$
\begin{aligned}
& \frac{\alpha K_j}{2\bar{y}}\Bigg\{-\alpha(2-q)r_j^2+2[(\bar{y}-qc)-\alpha(2-q)]r_j \\
& \qquad -\alpha(2-q)-2qc\Bigg\}
\end{aligned} \tag{9}
$$

- Focus on terms inside $\{\}$. Condition to have two different real roots:

$$(\bar{y} - qc)^2 - 2\alpha(2-q)\bar{y} > 0 \tag{10}$$

Assumption 1

We assume that $\bar{y} \gg \alpha > c$. In particular,

$$\bar{y} > 4\alpha, \alpha > c \tag{11}$$

- Under Assumption 1, two real roots (denoted by $\underline{r}$, $\bar{r}$) for Eq (9) $= 0$ exists and for $r \in [\underline{r}, \bar{r}]$, profit for a financial firm is weakly positive. (Figure in the next page)
- Note under $\alpha > c$, LHS of Eq. (10) is strictly increasing in q.

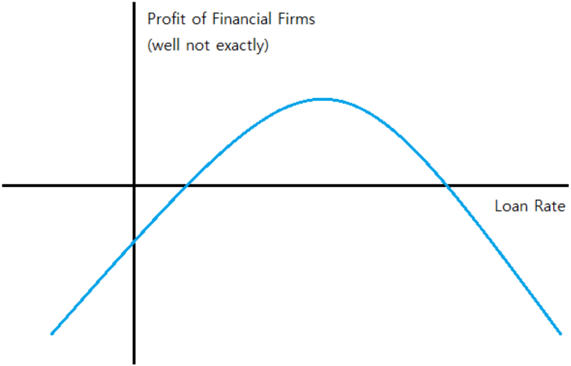

Figure: Loan Rates Satisfying IRC for Financial Firms

(Behavior of Financial Firms)

- Financial firms can be assumed to operate as long as profit is weakly greater than zero. In this case, WLOG we can assume that financial firms take K_j as given.
- Alternatively, financial firms are assumed to maximize profits. To highlight this, consider the case in which one financial firm exists in each sector (monopoly) and loan rate decisions are made sequentially: $B_1 \rightarrow B_2 \rightarrow \cdots \rightarrow B_N$.
- Will cover both cases.

Co-existence of Financial Sectors

(Flow of Agents across Financial Sectors)

- In a stationary state, agent population of a specific financial sector must remain the same across periods. That is, inflow into a specific sector must be equal to outflow from the sector.
- Will deal with N-1 sectors in both $q = 0$ and $q = 1$ cases. (Turns out that $q = 1$ case does not matter.)

Proposition 1

Let K_j represents the population in financial sector B_j ($\sum_j K_j = 1$). Then,

$$K_1(1 + r_1) = K_2(1 + r_2) = \cdots = K_{N-1}(1 + r_{N-1}). \qquad (12)$$

(Existence of Equilibrium)

- A stationary equilibrium: Agents max their value functions while IC for agents, IR for agents and IR for financial firms are all satisfied. Of course, K_j remains the same across periods.

Proposition 2

Suppose that financial firms operate if profit is weakly positive. Stationary equilibrium for $q = 1$ (full verification mechanism) exists under Assumption 1. Stationary equilibrium for $q = 0$ (no verification mechanism) exists under Assumption 1 provided that $\bar{r} \geq r_2$.

Proof.

(SKETCH) Let $q = 1$. IR for agents is always satisfied. IR for financial firms states that $\underline{r} = \frac{\bar{y}-\alpha-c-\sqrt{(\bar{y}-\alpha-c)^2-\alpha(\alpha-c)}}{\alpha}$ and $\bar{r} = \frac{\bar{y}-\alpha-c+\sqrt{(\bar{y}-\alpha-c)^2-\alpha(\alpha-c)}}{\alpha}$, which are both greater than zero under Assumption 1. Note that intersection of IR for agents and IR for financial firms ($r \in [\underline{r}, \bar{r}]$) is not empty under Assumption 1. Note that any loan rate $r \in [\underline{r}, \bar{r}]$ can be an equilibrium loan rate. Now let $q = 0$. IR for agents can be simplified to $1 + r_N \leq \bar{y}/(2\alpha)$. Under Assumption 1, $\underline{r} = \frac{\bar{y}-2\alpha-\sqrt{\bar{y}^2-4\bar{y}\alpha}}{2\alpha} > 0$ and $\bar{r} = \frac{\bar{y}-2\alpha+\sqrt{\bar{y}^2-4\bar{y}\alpha}}{2\alpha} > 0$, and intersection of IR conditions for agents and financial firms is not empty. Any r in this intersection can be r_1. Due to the boundary condition, need $\bar{r} - \underline{r} \geq r_2 - r_1$ to sustain sector B_1. WLOG we can let $r_1 = \underline{r}$, which leads to $\bar{r} \geq r_2$. (Note if only one sector can exist, agents who fail to pay back must leave the economy.) $\square$

(Characteristics of the Equilibrium)

Proposition 3

Suppose that financial firms operate if profit is weakly positive and that financial firms verify whenever an agent fails to pay back ($q = 1$). Then, two or more financial sectors cannot co-exist. That is, only one financial sector can exist in the economy.

Proof.

(SKETCH) Eq. (4) implies $E[V_j] = E[V_{j+1}]$, which is achieved only if $r_j = r_{j+1}$ for any $j = 1, 2, \cdots, N-1$. In other words, $r_1 = r_2 = \cdots = r_N$. Hence the result follows. □

Proposition 4

Suppose that financial firms operate if profit is weakly positive and that financial firms never verify even if an agent fails to pay back $(q = 0)$. Then, multiple financial sectors can co-exist depending on parameter values. In particular, maximum number of sectors that can co-exist is determined by the largest integer $\bar{N}$ that satisfies:

$$\bar{N} \leq \min\left\{ \frac{log\left[\frac{\sqrt{\bar{y}(\bar{y}-4\alpha)}}{\alpha(1+\underline{r})}+1\right]}{log\left[1+\frac{1-\beta}{\beta m}\right]}, \frac{log\left[\frac{\bar{y}}{2\alpha(1+\underline{r})}\right]}{log\left[1+\frac{1-\beta}{\beta m}\right]} \right\}. \tag{13}$$

Proof.

(SKETCH) Since financial firms operates as long as profit is weakly positive, WLOG, let $r_1 = \underline{r}$. From Eq. (4) and Eq. (5)

$$1 + r_{j+1} = (1 + r_j)\left(1 + \frac{1-\beta}{\beta m}\right), \quad j = 1, 2, \cdots, N-1. \tag{14}$$

This leads to:

$$1 + r_j = (1 + r_1)\left(1 + \frac{1-\beta}{\beta m}\right)^{j-1}, \quad j = 1, 2, \cdots, N. \tag{15}$$

Now the first term in Inequality (13) comes from $(1 + r_N) - (1 + r_1) = (1 + \underline{r})\left\{\left(1 + \frac{1-\beta}{\beta m}\right)^{N-1} - 1\right\} \leq \bar{r} - \underline{r}$ and the second term from $r_N = (1 + \underline{r})\left(1 + \frac{1-\beta}{\beta m}\right)^{N-1} - 1 \leq \frac{\bar{y}}{2\alpha}$. The boundary condition leads to the final result. □

(Co-existence of Financial Sectors)

- Some simulation results
  - Parameters: $\bar{y} = 100, \alpha = 10, \beta = 0.9, m = 0.5$

No. of Sectors	r_1	r_2	r_3	r_4	$r_{\bar{N}}$
7	0.127	0.377	0.684	1.058	2.757

  - Parameters: $\bar{y} = 100, \alpha = 2, \beta = 0.95, m = 1$

No. of Sectors	r_1	r_2	r_3	r_4	$r_{\bar{N}}$
62	0.021	0.075	0.131	0.191	22.325

  - Parameters: $\bar{y} = 100, \alpha = 24.5, \beta = 0.9, m = 0.8$

No. of Sectors	$r_1 = r_{\bar{N}}$	r_2	r_3	r_4	r_5
1	0.752	-	-	-	-

(Optimization of Financial Firms)

- What if financial firms sequentially maximize their profits? $(B_1 \to B_2 \to \cdots \to B_N)$

Proposition 5

Suppose that financial firms optimize their profits. Then, two or more financial sectors cannot co-exist regardless of the verification scheme ($q \in \{0,1\}$). That is, only one financial sector can exist in the economy.

Proof.

(SKETCH) When $q = 1$, Proposition 3 still applies. When $q = 0$, it is easy to show that K_j is not dependent on loan rates. Then from Eq. (9), $r_1^* = \frac{\bar{y}}{2\alpha} - 1$, which means that the financial firm charges loan rate such that IR condition for agents, Inequality (7), binds with equality taking all the ex-ante surplus generated by loans. Therefore, another financial sector cannot exist. □

- This result may make a room for gov't intervention: setting upper bound of loan rates charged by financial firms, especially r_1.
- Note, in this model, once r_1 is set, $r_j, j \in \{2,3,\cdots,N-1\}$ is mainly determined by IC condition for agents.
- In addition, the gov't may want to restrict upper bound of r_N.
- Formal treatments call for some welfare analysis.
- And this discussion extends for the case in which financial firms operates as long as profit is weakly positive.
- Below, will focus on the case in which financial firms operate as long as profit is weakly positive.

(Efficiency)

- From social efficiency perspective, no verification equilibrium brings higher efficiency.
- The same production and less cost.

Proposition 6

No verification equilibrium is more efficient than verification equilibrium.

(Comparative Statics and Policy Implications)

- α, m, $\bar{y}$
  - $\frac{\partial \bar{N}}{\partial \alpha} < 0, \frac{\partial \bar{N}}{\partial m} > 0, \frac{\partial \bar{N}}{\partial \bar{y}} > 0$
- Boom can be interpreted as an increase in $\bar{y}$.
- Relief fund: Govt saves proportion k of agents moving down to next sector.

(Stochastic Verification in This Context)

- Verification also occurs under stochastic verification scheme, which incurs social costs.
- Total production does not change.
- Therefore, less efficient than no verification equilibrium.

Conclusion

- There is a reason that loan rates across financial sectors are different and somewhat discrete.
- The loan rate differences are representing a mechanism that makes agents (borrowers) to tell the truth.
- This mechanism is efficient in the sense that verification cost is nil.
- That said, optimal differences of loan rate across financial firm types are a matter of empirical analysis: they can be wide or narrow depending on the environment.
    - Example: As verification probability q increases more sectors can co-exist, which means that loan rate differences across financial firm types should get smaller.

(End of Presentation)