Stock Based CEO Compensation under Ambiguity

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Abstract

The paper investigates how the long-term value of a publicly traded firm and its stock price affect executive compensation when some traders have ambiguous information about the firm's fundamental. It incorporates a standard principal-agent model into Grossman and Stiglitz's (1980) stock pricing model when uninformed traders face ambiguity. We analyze comparative statics of weights on the long-term value and stock price in managerial incentive contract. We find that an increase of the proportion of uninformed traders with ambiguity makes the contract more sensitive to the long-term value of the firm and less sensitive to the stock prices. On the other hand, if the degree of ambiguity increases, the weight on firms long-term value decreases but that on stock prices increases.

Keywords: principal-agent problem; executive compensation; asymmetric information; information acquisition; price informativeness

JEL Classification: G30, D86

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1 Introduction

It is well known that the owners of publicly trading firms attempt to monitor the performance of managers by observing stock markets. In a few decades, various studies the effects of stock markets on the incentive contracts between owners and managers. The purspose of this paper is to investigate how the long-term value of a publicly trading firm and stock prices affect managerial compensation under ambiguity in the stock market. To do this, we incorporate a standard principal-agent model (e.g., Holmström, 1979) into Grossman and Stiglitz's (1980) stock pricing model when uninformed traders face ambiguous information. In our model, before the stocks are traded, the owner of the firm offers contract to a manager (CEO). The firm's long-term value depends on unobservable effort of the manager. The incentive contract between the owner and the manager is assumed to be a linear in the firm's long-term value and the stock price. Our model contains two ambiguity factors: the (individual) degree of ambiguity and the population of uninformed traders.[1] Each uninformed trader has multiple beliefs about the variance of risky stock's true value but exact information about its mean. This assumption allows us to simplify the analysis of stock market equilibrium.[2]

We analyze comparative statics of weights on the firm's long-term value and the stock price in managerial incentive contract. Each weight is affected by ambiguity factors in different ways. We find that an increase of the proportion of uninformed traders makes the contract more sensitive to the long-term value of the firm and less sensitive to the stock prices. On the other hand, if the degree of ambiguity increases, the weight on firms long-term value decreases but that on stock prices increases.

Our model is different from the previous literature on market-based compensation in three aspects. First, we consider all rational traders participate in trading whether they are informed or uninformed. In Holmström and Tirole (1993), Kang and Liu (2010), and Calcagno and Heider (2014), all rational traders who participate in trading are informed and the others are liquidity traders. Second, we allow changes of the proportion of informed (uninformed) traders. Epstein and Schneider (2008) and Illeditsch (2011) assume that there is a representative risk-averse trader with ambiguous information, while Ozsoylev and Werner (2011) consider an economy where there are a single risk-averse informed trader and a single risk-neutral uninformed trader with ambiguity.[3] As a result, the literature do not take into account effects induced by changes of the population of uninformed traders with ambiguity. Third, based on Grossman and Stiglitz (1980), we can explain how ambiguity has influence on the managerial incentive contract. To the best of our knowledge, this study is the first report which examines the relationship between ambiguous information in stock market and managerial contract.

[1] The latter determines the aggregate level of ambiguity in asset markets given a degree of ambiguity.

[2] In general, uninformed traders may have exact information about neither mean nor variance as assumed in Easley and O'Hara (2009, 2010) and Ozsoylev and Werner (2011).

[3] Note that Epstein and Schneider (2008) and Illeditsch (2011) do not consider asymmetric information between rational traders.

The rest of the paper is organized as follows. In Section 2, we introduce the model of principal agent problem while ambiguous information is present in the stock market. The stock market equilibrium is derived in Section 3. In Section 4, we analyze managerial contract between the owner and the manager. Concluding remarks are given in Section 5. All the proofs are relegated to Appendix.

2 The Model

We consider three periods, indexed $t = 0, 1, 2$. At the initial period ($t = 0$), a publicly traded firm is established and the firm's owner hires a manager. The owner offers him an management contract. The true value θ of the firm consists of managerial effort level e and a factor η outside the manager's control, which is normally distributed with mean zero and variance σ^2. Thus the true value θ has normal distribution with mean e and variance σ^2. The z shares of the stock are issued and traded at time 1. To ensure partial revealing of stock prices, we assume the randomness of stock supply z, which is normally distributed with mean zero and variance σ_z^2. At the final period, i.e., $t = 2$, the terminal value v of the firm is realized and the manager is paid.

2.1 Managerial Contract

As in Baiman and Verrecchia (1995) and Holmström and Tirole (1993), we assume that there are two performance measures of the manager: the stock price p and the firm's terminal payoff v. The manager's income is given by

$$I = a_0 + a_1 v + a_2 p,$$

where a_0 represents a fixed wage, and a_1 and a_2 means the weights of the managerial compensation to v and p, respectively. Note that $a_1 v$ means compensation for the firm's long-term value and $a_2 p$ means that for stock prices. The manager is paid $a_0 + a_2 p$ in cash and $a_1 v$ is paid in the stock. We assume that the manager chooses his effort level e at time 0 and has CARA utility function with absolute risk aversion coefficient $\tau : u_m(w) = -e^{-\tau w}$. It is also assumed that the manager is barred from trading. This assumption reflects real world where managers are subject to laws and restriction on stock trading.

2.2 Stock Market and Traders

There are two stocks: a risky stock and a risk-free bond. At period 1, the price of the risky stock and the bond are given by p and 1, respectively. A trader t invests his initial wealth w_t between x_t shares of the risky stock and b_t shares of the bond with the budget constraint $b_t + px_t = w_t$. At the end of the period, i.e., $t = 2$, the risky stock gives random payoff $\tilde{v} - (a_0 + a_1\tilde{v})$. The bond gives deterministic payoff 1. Thus his portfolio (x_t, b_t) yields wealth $w_t' = w_t + (\tilde{v} - (a_0 + a_1\tilde{v}) - p)x_t$.

There is a continuum of traders denoted by interval $[0,1]$. Traders are divided into two groups: informed traders and uninformed traders. All the traders in each group are identical. They have rational expectations so that they understand the functional relationship $\tilde{p}$ between p and (θ, z). Informed traders observe realization (p, θ) of $(\tilde{p}, \tilde{\theta})$, while uninformed traders only observe p. Uninformed traders have ambiguous information about σ^2 with knowing that it belongs to $[\underline{\sigma}^2, \bar{\sigma}^2]$ but have exact information about μ.[4] The length $\Delta\sigma^2$ of interval $[\underline{\sigma}^2, \bar{\sigma}^2]$ is called the *(individual) degree of ambiguity*. Let λ and λ_u denote the fractions of informed and uninformed traders respectively, where $\lambda + \lambda_u = 1$ and $\lambda \in (0,1]$. We assume that λ and λ_u are exogenously given.[5] Note that our model excludes the case where all the traders are uninformed.

The whole process is illustrated in Figure 1 below.

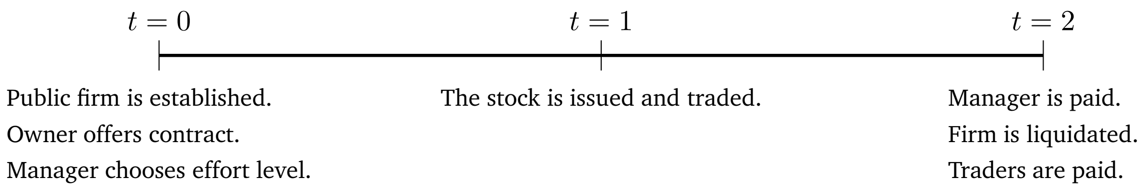

Figure 1. Sequence of Process

3 Stock Market Equilibrium

3.1 Portfolio Choice of Trader

A trader t invests his initial wealth w_t between x_t shares of the risky stock and b_t shares of the bond with the budget constraint $b_t + px_t = w_t$. Thus his portfolio (x_t, b_t) results in wealth $w_t' = w_t + (\tilde{v} - p)x_t$. All the traders have CARA utility with the constant degree of risk aversion $\gamma > 0$: $u(w) = -\exp(-\gamma w)$. For the convenience of analysis, we assume that traders are sufficiently averse to risk such that $\gamma > \frac{1}{\sqrt{\sigma_\varepsilon \sigma_z}}$.

For the optimal portfolio choice, informed trader i with initial wealth w_i solves

$$\max_{x_i} \mathbb{E}[-\exp\left(-\gamma[w_{0i} + (\tilde{v} - p)x_i]\right) |(\tilde{p}, \tilde{\theta}) = (p, \theta)]$$

and his demand for the stock is given by

$$x_i(p, \theta) = \frac{\theta - p}{\gamma \sigma_\varepsilon^2}. \tag{3.1}$$

[4]Cao et al. (2005), Ui (2011), and Mele and Sangiorgi (2013) assume that traders have ambiguous information about the mean while have exact information about the variance.

[5]When $\Delta\sigma^2 = 0$, our model reduces to that of Grossman and Stiglitz (1980) without endogenous information acquisition.

Let $\mathcal{P}$ be the set of normal distributions with mean e and variance $(\sigma')^2 \in [\underline{\sigma}^2, \bar{\sigma}^2]$. The mean and variance under belief $\pi \in \mathcal{P}$ are denoted by $\mathbb{E}_\pi[\cdot]$ and $\mathrm{Var}_\pi[\cdot]$, respectively. Equipped with the maxmin expected utility of Gilboa and Schmeidler (1989), uninformed trader u with initial wealth w_u solves

$$\max_{x_u} \min_{\pi \in \mathcal{P}} \mathbb{E}_\pi \left[-\exp\left(-\gamma[w_u + (\tilde{v} - p)x_u]\right) \middle| \, \tilde{p} = p \right].$$

Proposition 3.1. *The demand of uninformed trader u for the (risky) stock is given by*

$$x_u(p,\tilde{p}) = \begin{cases} \dfrac{\mathbb{E}_{\hat{\pi}}[\tilde{v}|\tilde{p}=p] - p}{\gamma Var_{\hat{\pi}}[\tilde{v}|\tilde{p}=p]}, & \text{if} \quad p < \mathbb{E}_{\hat{\pi}}[\tilde{v}|\tilde{p}=p], \\ 0, & \text{if} \quad \mathbb{E}_{\hat{\pi}}[\tilde{v}|\tilde{p}=p] \le p \le \mathbb{E}_{\bar{\pi}}[\tilde{v}|\tilde{p}=p], \\ \dfrac{\mathbb{E}_{\bar{\pi}}[\tilde{v}|\tilde{p}=p] - p}{\gamma Var_{\bar{\pi}}[\tilde{v}|\tilde{p}=p]}, & \text{if} \quad p > \mathbb{E}_{\bar{\pi}}[\tilde{v}|\tilde{p}=p], \end{cases} \tag{3.2}$$

where $\hat{\pi} \in \mathcal{P}$ minimizes $\mathbb{E}_\pi[v|\tilde{p} = p]$ and maximizes $Var_\pi[v|\tilde{p} = p]$, and $\bar{\pi} \in \mathcal{P}$ maximizes both $\mathbb{E}_\pi[v|\tilde{p} = p]$ and $Var_\pi[v|\tilde{p} = p]$.

From Proposition 3.1, one may expect that uninformed traders do not participate in trading when stock price falls in some intermediate region. Other literature which assume that ambiguous information of mean such as in Cao, Wang, and Zhang (2005), Ozsoyleve and Werner (2011), and Mele and Sangiorgi (2013) show that uninformed traders with ambiguity refuse to take positions unless stock prices are sufficiently high or low. However, in our model, non-participation region disappears in equilibrium.[6]

3.2 Equilibrium Stock Price

We consider a rational expectations equilibrium as in Grossman and Stigltiz (1980). A rational expectations equilibrium stock price function $\tilde{p} : (\theta, z) \mapsto p$ of (θ, z) satisfies the market clearing condition: for every $p = \tilde{p}(\theta, z)$,

$$\lambda x_i(p,\theta) + \lambda_u x_u(p,\tilde{p}) = z, \tag{3.3}$$

where $\lambda + \lambda_u = 1$. Following Grossman and Stigltiz (1980),[7] we define a compound signal function $\tilde{s} : (\theta, z) \mapsto s$, which encapsulates θ and z:

$$\tilde{s}(\theta, z) := \theta - \frac{\gamma\sigma_\varepsilon^2}{\lambda} z.$$

Clearly, $\tilde{s}$ is normally distributed with mean μ and variance $\sigma_s^2 = \sigma^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2/\lambda^2$. We define equilibrium stock price function $P : s \mapsto p$ of signal s by $\tilde{p}(\theta, z) := P(\tilde{s}(\theta, z))$ and conjecture that P strictly increases in signal s, which is verified by Proposition 3.1 below.

[6]We verify this in Section 3.2

[7]Mele and Sangiorgi (2013) use a slightly different signal function from ours and Grossman and Stigltiz (1980).

Proposition 3.2. *There exists a unique rational expectations equilibrium stock price function given by*

$$P(s) = (1-\alpha)\mu + \alpha s$$

where

$$\alpha = \frac{\lambda(\lambda\bar{\sigma}^2 + \gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda^2\bar{\sigma}^2 + \lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2} \in (0,1].$$

The equilibrium price function strictly increases in $s \in \mathbb{R}$. Thus information obtained from p is equivalent to that from s. In other words, for uninformed traders, observing p and observing s are indifferent. Since uninformed traders choose $\bar{\sigma}^2$ both when they buy the stock and sell it, non-participation region disappears and the equilibrium price function $P(s)$ becomes linear.[8] Thus $\bar{\sigma}^2$ can be considered as the degree of ambiguity.

Our model reduces to that of Grossman and Stiglitz (1980) if uninformed traders observe exact value of σ^2 and then the stock price function becomes

$$P_0(s) = (1-\alpha_0)\mu + \alpha_0 s$$

where

$$\alpha_0 = \frac{\lambda(\lambda\sigma^2 + \gamma^2\sigma^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda^2\sigma^2 + \lambda\gamma^2\sigma^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2} \in (0,1].$$

Since

$$\frac{\partial\alpha_0}{\partial\sigma^2} = \frac{(1-\lambda)\lambda(\lambda^2\sigma^2 + \gamma^2\sigma^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{(\lambda^2\sigma^2 + \lambda\gamma^2\sigma^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)^2} > 0,$$

α is greater than α_0. This implies that the stock price responds to s more sensitively when ambiguity is present than when it is absent.

Following Grossman and Stiglitz (1980), the informativeness of the stock price can be defined as squared correlation coefficient ρ^2 between P and θ, which is given by

$$\rho^2 = \frac{1}{1+m}$$

where $m = \gamma^2\sigma_\varepsilon^4\sigma_z^2/(\lambda^2\bar{\sigma}^2)$. Since ρ^2 decreases in m, the stock price become more informative as the proportion λ of informed traders or the degree of ambiguity increases.

4 The Manager's Contract

This section is devoted to analyze the incentive contract between the owner and the manager. It is assumed that the reservation value of the manager equals 1. The owner's problem is given by

$$\begin{aligned} &\max_{a_0,a_1,a_2,e} \mathbb{E}[v - I] \\ &s.t.\ \ \mathbb{E}[I] - \frac{\tau}{2}\text{Var}[I] - \frac{1}{2}ke^2 \geq 0, \\ &\qquad e = \arg\max_{\hat{e}} \mathbb{E}[I] - \frac{\tau}{2}\text{Var}[I] - \frac{1}{2}k\hat{e}^2. \end{aligned}$$

[8]In fact, literature which assume ambiguity of mean such as Ozsoylev and Werner (2011) and Mele and Sangiorgi (2013) exhibit kinks in stock price functions.

Proposition 4.1. *In equilibrium, compensation contract is given by*

$$\begin{aligned} a_0 &= \frac{(\sigma_\eta^2+\sigma_\varepsilon^2+\alpha^2\sigma_s^2)(\alpha^2\sigma_s^2 k\tau(\sigma_\eta^2+\sigma_\varepsilon^2)-(\sigma_\eta^2+\sigma_\varepsilon^2+\alpha^2\sigma_s^2))}{2\alpha^2\sigma_s^2 k(1+k\tau(\sigma_\eta^2+\sigma_\varepsilon^2))(\sigma_\eta^2+\sigma_\varepsilon^2+\alpha^2\sigma_s^2+\alpha^2\sigma_s^2 k\tau(\sigma_\eta^2+\sigma_\varepsilon^2))}, \\ a_1 &= \frac{\alpha^2(1+m)}{\sigma_\eta^2+\sigma_\varepsilon^2+\alpha^2(1+m)(1+k\tau(\sigma_\eta^2+\sigma_\varepsilon^2))}, \\ a_2 &= \frac{\sigma_\eta^2+\sigma_\varepsilon^2}{\alpha^2(1+m)(1+k\tau(\sigma_\eta^2+\sigma_\varepsilon^2))}. \end{aligned}$$

As the manager is more averse to risk, the owner does not have incentive to offer a higher compensation for the manager's performance. Furthermore, he knows that the manager will accept the contract with lower compensation for firm's long-term value and stock prices. Thus, in the equilibrium, the owner offers contract with lower a_1 and a_2 as τ increases. Note that a_1 decreases in price informativeness while a_2 increases in that. Containing more information about θ, stock price reflect more about the effort of the manager. Then, in the contract, stock prices are treated as the important performance measure of the manager relative to long-term value.

In Kang and Liu (2010), coefficient of firm's long-term value v is negative. In their model, the owner believes that high v is due to high exogenous factors and the effort levels of managers are low. However, our model shows that the managerial compensation increases in both v and p. It is intuitive since the expected values of v and p increase due to increases of effort level e of the manager.

Ambiguity affects the contract through price informativeness and price volatility. As the degree of ambiguity increases, the stock price contains more information about θ and responds more sensitively to compound signal s. From Proposition 4.1, we see that a_1 increases in price sensitivity α and decreases in informativeness ρ^2, and a_2 displays reverse relationship with α and ρ^2. Since the proportion λ_u of uninformed traders decreases in both both price sensitivity and informativeness but the degree $\bar{\sigma}^2$ of ambiguity increases in them, ambiguity effects on the contract rely on relative changes of α and ρ^2.

Proposition 4.2. *The following hold.*

1. *As the proportion λ_u of uninformed traders increases, a_1 increases and a_2 decreases.*
2. *Suppose $\lambda \in [1/3, 1]$. As the degree $\bar{\sigma}^2$ ambiguity increases, a_1 decreases and a_2 increases.*

Proposition 4.2 shows that each ambiguity factor affects the contract in different ways. As the proportion λ_u of uninformed traders with ambiguity increases, the sensitivity of the stock price to s decreases since

$$\frac{\partial \alpha}{\partial \lambda} = \frac{\left[(2-\lambda)\lambda\sigma^2+\gamma^2\sigma^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2\right]\gamma^2\sigma_\varepsilon^4\sigma_z^2}{(\lambda^2\sigma^2+\lambda\gamma^2\sigma^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2)^2} > 0,$$

while the reciprocal $(1+m)$ of price informativeness increases. However, since the former effect is dominated by the latter one, the weight to firm's long-term value increases but that to to stock prices decreases in the contract. On the other hand, as the degree of ambiguity increases, the stock prices respond to s more sensitively, while the informativeness of stock price increases. In this case, the latter effect dominates the former one, and thus the weight to firm's long-term value decreases but that to stock prices increases.

5 Concluding Remarks

The paper investigates the effects of market-based compensation in managerial contract by incorporating a standard principal-agent problem into Grossman and Stiglitz's (1980) stock pricing model when uninformed traders face ambiguous informaiton. We analyze comparative statics of weights on the long-term value of a publicly trading firm and stock prices in managerial contract. Each weight is affected by ambiguity factors. An increase of the proportion of uninformed traders makes the contract more sensitive to the long-term value of the firm and less sensitive to the stock prices. If the degree of ambiguity increases, the weight on firms long-term value decreases but that on stock prices increases. Topics of future research may include ambiguous information to examine how the manager manipulates stock market information.

Appendix

PROOF OF PROPOSITION 3.1: The optimization problem of the uninformed trader u with initial wealth w_u is equivalent to

$$\max_{x_u} \min_{\pi \in \mathcal{P}} \left(w_u + (\mathbb{E}_\pi[v|\tilde{p} = p] - p)x_u - \frac{1}{2}\gamma x_u^2 \text{Var}_\pi[v|\tilde{p} = p] \right).$$

Consider three cases: (i) $x_u > 0$, (ii) $x_u < 0$, and (iii) $x_u = 0$.

(i) Since the uninformed trader u considers probability distribution $\hat{\pi}$ under worst case scenario, the demand of u is given by

$$x_u = \frac{\mathbb{E}_{\hat{\pi}}[v|\tilde{p} = p] - p}{\gamma \text{Var}_{\hat{\pi}}[v|\tilde{p} = p]}$$

if and only if $\mathbb{E}_{\hat{\pi}}[v|\tilde{p} = p] - p > 0$.

(ii) Similar to the case (i), the uninformed trader u considers $\bar{\pi}$ under worst case scenario. Then we obtain

$$x_u = \frac{\mathbb{E}_{\bar{\pi}}[v|\tilde{p} = p] - p}{\gamma \text{Var}_{\bar{\pi}}[v|\tilde{p} = p]}$$

if and only if $\mathbb{E}_{\bar{\pi}}[v|\tilde{p} = p] - p < 0$.

(iii) The demand x_u does not depend on any $\pi \in \mathcal{P}$. Thus $x_u = 0$ if and only if $\mathbb{E}_{\hat{\pi}}[\tilde{v}|\tilde{p} = p] \leq p \leq \mathbb{E}_{\bar{\pi}}[\tilde{v}|\tilde{p} = p]$. ■

Proof of Proposition 3.2: We conjecture that P is a strictly increasing function of s. Then information generated by the equilibrium price p is equivalent to that by s. Since $\tilde{s}$ and $\tilde{v}$ are normally distributed, we have

$$\mathbb{E}_{\pi}[\tilde{v}|\tilde{p}=p] = \mathbb{E}_{\pi}[\tilde{v}|\tilde{s}=s] = \frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2\mu + \lambda^2\sigma_\pi^2 s}{\lambda^2\sigma_\pi^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2},$$
$$\mathrm{Var}_{\pi}[\tilde{v}|\tilde{p}=p] = \mathrm{Var}_{\pi}[\tilde{v}|\tilde{s}=s] = \frac{\sigma_\varepsilon^2(\lambda^2\sigma_\pi^2 + \gamma^2\sigma_\pi^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda^2\sigma_\pi^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2},$$

where σ_π^2 is the variance of probability distribution π. Note that $\mathbb{E}_{\pi}[\tilde{v}|\tilde{s}=s]$ increases in σ^2 if $s > \mu$ and decreases in $s < \mu$, and $\mathrm{Var}_{\pi}[\tilde{v}|\tilde{s}=s]$ increases in σ^2.

Consider the three cases: (i) $p < \mathbb{E}_{\hat{\pi}}[v|\tilde{s}=s]$, (ii) $p > \mathbb{E}_{\bar{\pi}}[v|\tilde{s}=s]$, and (iii) $\mathbb{E}_{\hat{\pi}}[v|\tilde{s}=s] \le p \le \mathbb{E}_{\bar{\pi}}[v|\tilde{s}=s]$.

(i) We conjecture that $s < \mu$. Since $\mathbb{E}_{\pi}[\tilde{v}|\tilde{s}=s]$ decreases in σ^2, uninformed traders choose $\bar{\sigma}^2$ from their belief set $[\underline{\sigma}^2, \bar{\sigma}^2]$. Then demand of uninformed traders becomes

$$x_u = \left(\frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2\mu + \lambda^2\bar{\sigma}^2 s}{\lambda^2\bar{\sigma}^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2} - p\right) \Big/ \left(\frac{\sigma_\varepsilon^2(\lambda^2\bar{\sigma}^2 + \gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda^2\bar{\sigma}^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}\right)$$

From the market clearing condition, we obtain

$$\begin{aligned} P(s) &= \frac{(1-\lambda)\gamma^2\sigma_\varepsilon^4\sigma_z^2\mu}{\lambda^2\bar{\sigma}^2 + \lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2} + \frac{\lambda(\lambda\bar{\sigma}^2 + \gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda^2\bar{\sigma}^2 + \lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2} s \\ &= (1-\alpha)\mu + \alpha s \end{aligned}$$

where

$$\alpha = \frac{\lambda(\lambda\bar{\sigma}^2 + \gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda^2\bar{\sigma}^2 + \lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}.$$

Since

$$\mathbb{E}_{\hat{\pi}}[v|\tilde{s}=s] - P(s) = \frac{\lambda\gamma^2\sigma_\varepsilon^4\sigma_z^2(\lambda^2\bar{\sigma}^2 + \gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{(\lambda^2\bar{\sigma}^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)(\lambda^2\bar{\sigma}^2 + \lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}(\mu - s),$$

$\mathbb{E}_{\hat{\pi}}[v|\tilde{s}=s] - P(s)$ is greater than zero if and only if $s < \mu$.

(ii) We conjecture that $s > \mu$. Since $\mathbb{E}_{\pi}[\tilde{v}|\tilde{s}=s]$ increases in σ^2, uninformed traders choose $\bar{\sigma}^2$. Similar with case (i), we have

$$P(s)(1-\alpha)\mu + \alpha s.$$

Since

$$\mathbb{E}_{\bar{\pi}}[v|\tilde{s}=s] - P(s) = \frac{\lambda\gamma^2\sigma_\varepsilon^4\sigma_z^2(\lambda^2\bar{\sigma}^2 + \gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{(\lambda^2\bar{\sigma}^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)(\lambda^2\bar{\sigma}^2 + \lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}(\mu - s),$$

$\mathbb{E}_{\hat{\pi}}[v|\tilde{s}=s] - P(s)$ is less than zero if and only if $s > \mu$.

(iii) From (i) and (ii), if $x_u = 0$, then $\mu = s$ holds. Thus $P(s) = \mu$.

As a consequence, for every $s \in \mathbb{R}$, the equilibrium stock price is given by

$$P(s)(1-\alpha)\mu + \alpha s.$$

■

PROOF OF PROPOSITION 4.1. At the equilibrium of the stock market, it holds that

$$\begin{aligned}
\mathbb{E}[I] &= a_0 + a_1 e + \frac{a_2}{1+a_2}(e - a_0 - \delta), \\
\text{Var}[I] &= a_1^2(\sigma_\eta^2 + \sigma_\varepsilon^2) + \frac{a_2^2}{(1+a_2)^2}\alpha^2 \left(\sigma_\eta^2 + \frac{\gamma^2\sigma_\varepsilon^4}{\lambda^2}\sigma_z^2\right).
\end{aligned}$$

The first order condition for the manager's incentive compatibility implies

$$e^* = \frac{1}{k}\left(a_1 + \frac{a_2}{1+a_2}\right).$$

Then we obtain

$$\mathbb{E}[v - I] = \frac{1}{k}\left(a_1 + \frac{a_2}{1+a_2}\right) - \frac{\tau}{2}\left(a_1^2(\bar{\sigma}^2 + \sigma_\varepsilon^2) + \frac{a_2^2}{(1+a_2)^2}\alpha^2\sigma_s^2\right) - \frac{1}{2k}\left(a_1 + \frac{a_2}{1+a_2}\right)^2.$$

The first order conditions for owner's profit are given by

$$\begin{aligned}
\frac{1-(1+a_2)a_1(1+k\tau(\bar{\sigma}^2+\sigma_\varepsilon^2))}{k(1+a_2)} &= 0, \\
\frac{1-k\tau a_2\alpha^2\sigma_s^2 - a_1(1+a_2)}{k(1+a_2)^3} &= 0,
\end{aligned}$$

which implies

$$\begin{aligned}
a_1 &= \frac{\alpha^2\sigma_s^2}{\bar{\sigma}^2 + \sigma_\varepsilon^2 + \alpha^2\sigma_s^2(1+k\tau(\bar{\sigma}^2+\sigma_\varepsilon^2))}, \\
a_2 &= \frac{\bar{\sigma}^2 + \sigma_\varepsilon^2}{\alpha^2\sigma_s^2(1+k\tau(\sigma_\varepsilon^2+\bar{\sigma}^2))}.
\end{aligned}$$

The individual rationality condition for the manger holds as equality in equilibrium, we obtain

$$a_0 = \frac{(\bar{\sigma}^2+\sigma_\varepsilon^2+\alpha^2\sigma_s^2)(\alpha^2\sigma_s^2 k\tau(\bar{\sigma}^2+\sigma_\varepsilon^2) - (\bar{\sigma}^2+\sigma_\varepsilon^2+\alpha^2\sigma_s^2))}{2\alpha^2\sigma_s^2 k(1+k\tau(\bar{\sigma}^2+\sigma_\varepsilon^2))(\bar{\sigma}^2+\sigma_\varepsilon^2+\alpha^2\sigma_s^2+\alpha^2\sigma_s^2 k\tau(\bar{\sigma}^2+\sigma_\varepsilon^2))}.$$

■

PROOF OF PROPOSITION 4.2.

1. Note that

$$\frac{\partial\alpha^2(1+m)}{\partial\lambda} = \frac{(1-\lambda)\lambda^2\bar{\sigma}^2 - \lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2(1-\lambda-\gamma^2\bar{\sigma}^2\sigma_z^2 - \gamma^2\sigma_z^2)}{(2\gamma^2\sigma_\varepsilon^4\sigma_z^2(\lambda\bar{\sigma}^2+\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2))^{-1}(\lambda^2\bar{\sigma}^2+\lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2)^3},$$

   which decreases in λ. Therefore as λ_u increases, α_1 increases and α_2 decreases.

2. We have
$$\frac{\partial \alpha^2(1+m)}{\partial \bar{\sigma}^2} = -\frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2(\lambda\bar{\sigma}^2+\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2)}{\bar{\sigma}^4(\lambda^2\bar{\sigma}^2+\lambda\gamma^2\bar{\sigma}^2\sigma_\varepsilon^2\sigma_z^2)^3}\kappa$$

   where

$$\kappa = (1-3\lambda)\gamma^2\bar{\sigma}^2\sigma_\varepsilon^4\sigma_z^2(\lambda+\gamma^2\sigma_\varepsilon^2\sigma_z^2)+\lambda\bar{\sigma}^4((1-2\lambda)\lambda^2+2\lambda^2\gamma^2\sigma_\varepsilon^2\sigma_z^2+\gamma^4\sigma_\varepsilon^4\sigma_z^4).$$

   Since κ increases in $\bar{\sigma}^2$, $\alpha^2(1+m)$ decreases in $\bar{\sigma}^2$. Therefore as $\bar{\sigma}^2$ increases, α_1 decreases and α_2 increases.

■

References

Baiman, S. and R. Verrecchia (1995): "Earnings and Price-Based Compensation Contracts in the Presence of Discretionary Trading and Incomplete Contracting," *Journal of Accounting and Economics*, 20, 93–121.

Calcagno, R. and F. Heider (2014): "Market-Based Pay for Long-Term CEO Performance," Mimeo.

Cao, H., T. Wang and H. Zhang (2005): "Model Uncertainty, Limited Market Participation, and Asset Prices," *Review of Financial Studies*, 18, 1219-1251.

Easley, D. and M. O'Hara (2009): "Ambiguity and Nonparticipation: The Role of Regulation," *Review of Financial Studies*, 22, 1817-1843.

Easley, D. and M. O'Hara (2010): "Microstructure and Ambiguity," *Journal of Finance*, 65, 1817-1846.

Grossman, S. and J. Stiglitz (1980): "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70, 393–408.

Holmström, B. (1979): "Moral Hazard and Observability," *Bell Journal of Economics*, 10, 74–91.

Holmström, B. and P. Milgrom (1987): "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 55, 303–328.

Holmström, B. and J. Tirole (1993): "Market Liquidity and Performance Monitoring," *Journal of Political Economy*, 101, 678–709.

Gilboa, I. and D. Schmeidler (1989): "Maxmin Expected Utility with Non-Unique Prior," *Journal of Mathematical Economics*, 18, 141-153.

Kang, Q. and Q. Liu (2010): "Information-Based Stock Trading, Executive Incentives, and Principal-Agent Problem," *Management Science*, 56, 682–698.

Kim, O. and Suh. Y (1993): "Incentive Efficiency of Compensation Based on Accounting and Market Performance," *Journal of Accounting and Economics*, 16, 25–53.

Kyle, A. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53, 1315–1336.

Ozsoylev, H. and J. Werner (2011): "Liquidity and Asset Prices in Rational Expectations Equilibrium with Ambiguous Information," *Economic Theory*, 48, 469-491.

Mele, A. and F. Sangiorgi (2013): "Uncertainty, Information Acquisition and Price Swings in Asset Markets," Mimeo.

Vives, X. (1995): "Short-Term Investment and the Informational Efficiency of the Market," *Review of Financial Studies*, 8, 125–160.

Ui, T. (2011): "The Ambiguity Premium vs. the Risk Premium under Limited Market Participation," *Review of Finance*, 15, 245-275.