# A Monetary Approach to the Gravity Model

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This study derives a new gravity model from a monetary general equilibrium. It offers a theoretical framework for identifying a monetary factor as a determinant of bilateral trade flows. To apply the micro-founded monetary model of Lagos and Wright (2005) to open macroeconomics, this study endogenously determines the optimal currency for international transactions between agents from exporting and importing countries. The empirical results confirm that a monetary factor has a significant effect on bilateral trade flows.

*Keywords:* Monetary general equilibrium; gravity model; optimal currency

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## 1. Introduction

The gravity model plays a significant role in identifying the determinants of bilateral trade flows. For example, by using the gravity model with trade barriers such as the bilateral distance and the border between two countries, McCallum (1995) shows that the amount of trade between Canadian provinces in 1988 was 22 times greater than trade between US states and Canadian provinces. This finding is referred to as the "border puzzle," because the border between the US and Canada unexpectedly affects the ratio of intranational to international trade. Anderson and van Wincoop (2003; henceforth, AvW) resolve the border puzzle by developing a new gravity model derived from a general equilibrium. McCallum's model lacks theoretical foundations, mainly due to the absence of multilateral resistance factors, which represent the barriers to trade that each country faces with all of its trading partners.

These gravity equations have been applied to investigate how a monetary factor (i.e., a real exchange rate) affects bilateral trade flows. However, those studies (e.g., Bergstrand, 1985, 1989, Soloaga and Winters, 2001, and Brun et al., 2005) suffer from the absence of a micro-founded monetary theory. Studies that use the AvW model cannot offer a theoretical foundation for the monetary approach to the gravity model because they do not explicitly consider the role of money. Furthermore, the real exchange rate can be just a variable derived from a partial equilibrium of the current model, in the sense that the optimal condition, except for the endogenous choice of international transaction currency, addresses the real exchange rate. Thus, the aim of this study is to develop a new gravity model that includes a monetary general equilibrium. To achieve this, the benchmark gravity model is vital. The AvW model offers a good framework for the extension because it is a micro-founded trade model that controls for the important multilateral resistance factors that the traditional gravity model omits. Furthermore, because the AvW gravity model is a theoretical framework for identifying the determinants of bilateral trade flows between international agents in circumstances where trade frictions such as trade costs exist, the model environment is similar to that of the endogenous monetary model.[1] Rauch (1999) demonstrates that trade costs accounted for by distance/contiguity and language/colonial ties act as search frictions in bilateral trade between international buyers and sellers, in the sense that remoteness and information costs correspond to the proximity and network factors, respectively.

It is appropriate to use the model of Lagos and Wright (2005; henceforth, LW) as the monetary model because it not only has sufficient micro-foundations for the essentiality of money, but also solves the money indivisibility problem, as explained below. The overlapping generations models (e.g., Wallace, 1980) and cash-in-advance models (e.g., Lucas, 1980) are frequently used to address monetary economics. These models lack micro-foundations for the role of money. Thus, search-theoretical models (e.g., Kiyotaki and Wright, 1989) have been developed to formalize the role of money as a medium of exchange (MoE) in the presence of trade frictions, and thereby provide a theoretical foundation for monetary economics. The first generation models, however, impose the restriction that money and goods are indivisible. Due to this limitation, the models cannot determine prices. Subsequently, Shi (1995, 1996) and Trejos and Wright (1995) partially relax the indivisibility assumption while keeping the indivisibility of money. Zhu (2005) and Molico (2006) make some progress with the divisibility of money, but demonstrate that this assumption significantly complicates the analysis. This

[1] Anderson and van Wincoop (2004) define trade costs as all of the costs incurred in getting a good to a final user, other than the marginal cost of producing the good itself.

complication comes from the fact that a distribution of money holdings can affect the equilibrium. LW then offers the monetary model that functions well in controlling for the heterogeneity of money holdings across agents. The LW model gives an agent periodic access to a decentralized market (DM) and a competitive market (CM), and excludes wealth effects in the demand for money by assuming quasi-linear preferences in the CM.

Hence, the objective of this study is to derive a new gravity model by introducing the monetary factor into the AvW gravity model through the LW model. To achieve this, the LW model needs to be revised because a unit of account (UoA) of DM money (or DM transaction currency) can be different from that of CM money (or CM transaction currency). The LW model does not take into account an endogenous transaction currency and simply imposes the restriction that the DM transaction currency is the same as the CM transaction currency. Goldberg and Tille (2008) and Goldberg (2010) show that rather than using their own currencies, many countries choose to use the US dollar as an international transaction currency.

The use of money as a UoA for future payments has received little attention, although it is effectively widespread. A number of studies examine the choice of money as the UoA for international transactions.[2] Matsuyama et al. (1993) adopt a two-country, two-currency version of the Kiyotaki and Wright model to investigate the conditions under which a currency would emerge as an international currency. As in the first

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[2] There is also a stream of literature on domestic transactions. Doepke and Schneider (2013) develop a theory that rationalizes the use of a dominant UoA in an economy, but neglect the role of money as an MoE and focus only on the CM transactions. Kim and Lee (2013) provide a theoretical account for the separation of a UoA from an MoE by explicitly considering a seller's choice of UoA in terms of either an MoE or a unit of metal weight in the commodity–money system.

version of the search-theoretical models, the obvious shortcoming of the Matsuyama et al. (1993) model is that it cannot consider the determination of prices and exchange rates because it follows the Kiyotaki and Wright model. Thus, Trejos and Wright (1996) extend the model by allowing for divisible goods under the specific bargaining protocol (i.e., take-it-or-leave-it offers). They, however, make the assumption that money is indivisible, as in the second version of the search-theoretic models. Based on the general equilibrium open economy model with the cash-in-advance assumption, Bacchetta and van Wincoop (2005) show that market share of an exporting country and product differentiation are important factors when choosing the invoicing currency. Goldberg and Tille (2008) use a partial equilibrium approach to demonstrate that the coalescing effect, relative country size, hedging consideration, and transaction costs are causal factors for the choice of invoicing currency in international trade. These studies use the old versions of the models with either extreme assumptions or insufficient micro-foundations for the role of money. In contrast, Zhang (2014) develops an open-economy search model to provide the micro-foundations for the internationalization of currencies based on the formal framework addressing the essential role of money. The model, however, is not appropriate for extending the AvW gravity model that deals with only foreign goods, because it introduces local goods and foreign goods simultaneously in the DM. Furthermore, the study focuses on a seller's acceptance of a currency as a means of payment based on the imperfect recognizability of currencies rather than an optimal choice of DM transaction currency, which is the focus of interest in the current study.

The critical differences between this study and those in the literature are summarized as follows. First, this study theoretically develops a new gravity model to identify the monetary factor as a determinant of international trade. Thus, it is likely that the new gravity model renovated by the driving force of bilateral trade can act as a useful empirical workhorse for various analyses related to international trade. Second, the model endogenously derives the optimal UoA for the DM transaction money. Although the LW model focuses solely on the role of money as an MoE, it systemically considers two roles of money: as an MoE and as a UoA. As this model builds on the LW model, which has adequate micro-foundations for the role of money, this study complements the literature.

The remainder of this paper is organized as follows. Section 2 establishes the model for generating the link between monetary theory and the AvW gravity model. Section 3 provides empirical evidence regarding the validity of the new gravity model. Section 4 concludes the paper.

## 2. The Model

This section builds on the LW framework to set up the model for generating the link between monetary theory and the AvW gravity model. A monetary equilibrium condition is derived first, followed by an introduction to the new gravity model extended by monetary theory, along with a comparison of the monetary equilibrium condition and the AvW gravity model.

### *2.1. Monetary Economy*

The model economy consists of the DM and CM for each period. Neither market permits trades by credit. All objects are perfectly divisible. All goods are non-storable, while money is storable. Each country has a continuum of agents who live forever with the discount factor $\beta \in (0, 1)$. The mass of agents in each country in period t is denoted as $A_t^j$ and the total mass of agents as $A_t^1 + A_t^2 \cdots + A_t^N = 1$. Each agent can transform one unit of labor into one unit of output in the CM and DM. In the CM, general goods are traded, which are homogeneous goods that are traded in organized exchanges with a well-defined international price (e.g., wheat).[3] All agents from each country share symmetric features, except that they use their own local currency when consuming general goods. In the DM, search goods are traded, which are differentiated goods that do not have well-defined product standards and are not traded on organized exchanges (e.g., footwear). Agents of each country bilaterally trade with those of other countries but their identities are anonymous. All bilateral pairs are symmetric and transactions among agents from the same country are not permitted. It is assumed that each country is specialized in the production of only one type of search goods and thus agents from the same country deal with homogeneous search goods.[4] Note that the DM transaction currency is endogenously determined while the CM transaction currency is given.

The timing of events in a typical period is illustrated in Figure 1. At the beginning of the DM, buyers and sellers are randomly matched. In each match, a seller offers one of the search goods to a buyer and the buyer transfers the corresponding amount of DM currency to the seller, according to the Nash bargaining protocol.[5] In the CM, the seller exchanges the DM currency received from the buyer for his local currency and purchases general goods from the buyer with the local currency. The buyer receives the amount of money denominated in the seller's currency from the seller, in

[3] This model follows the classification of Rauch (1999) in the sense that general goods and search goods correspond to homogeneous goods and differentiated goods, respectively.
[4] This corresponds to the assumption of the AvW model that products are differentiated by place of origin.
[5] Although barter exchange can occur, the timing of events is focused on the single-coincidence matching.

exchange for the general goods produced with his labor. Note that unmatched agents do not have an incentive to produce general goods in the CM because they can meet the financial demand using the money that they do not spend in the DM. Furthermore, there is no distinction between domestic and international transactions in the CM as general goods are homogenous, agents are symmetric, and trade frictions do not exist in this market. In turn, the buyer exchanges the seller's currency for his local currency and then determines the amount of general goods he consumes and prepares the amount of local currency he needs to exchange for DM currency in the next period. At the end of the CM, the buyer exchanges the local currency for the DM currency. In sum, the DM offers the opportunity for search goods trades and the CM acts as a place for general goods trades and currency exchanges.

<Insert Figure 1>

The value function for an agent of country i holding $m \in \Re_{+}$ units of money, evaluated at the beginning of the CM, satisfies

$$W_t(m) = \max_{X_t^i,\, H_t^i,\, m',\, u_t^e,\, u_t^i,\, u_t^v} \left\{ U(X_t^i) - H_t^i + \beta E_t V_{t+1}(m') \right\} \tag{1}$$

subject to

$$X_t^i + (\phi_t^e \;\; \phi_t^i \;\; \phi_t^v) m' (u_t^e \;\; u_t^i \;\; u_t^v)^T \leq H_t^i + \phi_t^i m u_t^i \,, \tag{2}$$

$$u_t^e + u_t^i + u_t^v = 1 \,, \tag{3}$$

$$0 \leq u_t^a \leq 1 \text{, where } a = e,\, i,\, v \,. \tag{4}$$

$W_t$ and $V_{t+1}$ are value functions of the CM in period t and the DM in period t+1, respectively; $m$ and $m'$ represent a quantity of money at the beginning of the CM and the succeeding DM, respectively; $X_t^i \in \Re_+$ and $H_t^i \in \Re_+$ are consumption and labor during the CM; $E_t$ is the expectation operator conditional on information at time t; $u_t^e$, $u_t^i$, and $u_t^v$ on the left-hand side (LHS) of the agent's budget constraint denote currency e, currency i, and the vehicle currency, respectively, as candidate UoAs for $m'$; $\phi_t^e$, $\phi_t^i$, and $\phi_t^v$ are the real values of the corresponding currencies expressed in units of the general good;[6] and $\phi_t^i m u_t^i$ on the right-hand side (RHS) of constraint (2) indicates the money balance, expressed in units of currency i, as one of the financial sources for the agent to consume in the CM and to exchange for the DM transaction currency.

Typical preference assumptions are applied such that U(·) are twice continuously differentiable with $U' > 0$, $U'' \leq 0$.[7] Constraint (2) suggests that the quantity of money itself has no identity as a currency of a specific country when its role as a UoA is separated from that of an MoE. The reason is that variations across countries' currencies arise from a relative gap in the value of each currency generated by different UoAs. Thus, the quantity variable $m$ can serve as an endogenous state variable in this model. Uncertainty in this model comes from the exogenous variables $\phi_t^e$ and $\phi_t^v$, both of which follow a stochastic AR(1) process. Their realizations are

[6] The second term on the LHS of the agent's budget constraint is consistent with the spirit of Doepke and Schneider (2013).

[7] In Nosal and Rocheteau's (2011) similar model, it is simply assumed that $U(X_t^i) = X_t^i$, so there is no gain from producing the general good for oneself.

learned at the beginning of period t. Unlike the LW framework, this model characterizes the agent's choices of money UoA and also its holdings in the CM for the DM international transactions.

For a better understanding of the budget constraint, constraint (2) needs to be transformed into

$$X_t^i + \phi_t^i \left( E_{t(i/e)} \quad 1 \quad E_{t(i/v)} \right) m' (u_t^e \quad u_t^i \quad u_t^v)^T \leq H_t^i + \phi_t^i m u_t^i , \tag{5}$$

where $E_{t(i/e)}$ is the exchange rate between currencies i and e, with an increase corresponding to a depreciation in currency i. The second term on the LHS of constraint (5) refers to the money balances expressed in currency i, based on the exchange rates, for the amount of money used for the DM transactions. After determining the DM transaction currency, the agent exchanges his local currency for that currency at the end of period t.

Substituting $H_t^i$ from constraint (2) and $u_t^i$ from constraint (3), and inserting the optimal value of $X_t^i$ into Equation (1) yields

$$W_t(m) = \phi_t^i m + U(X_t^{i*}) - X_t^{i*} + \max_{m', u_t^e, u_t^v} \begin{Bmatrix} -[\phi_t^i + (\phi_t^e - \phi_t^i) u_t^e + (\phi_t^v - \phi_t^i) u_t^v ] m' \\ + \beta E_t V_{t+1}(m') \end{Bmatrix} . \tag{6}$$

Note that $\phi_t^i m u_t^i$ on the RHS of constraint (2) simply becomes $\phi_t^i m$ because the agent's own local currency is given as the CM transaction currency; that is, $u_t^i = 1$. The property that $W_t$ is linear in $m$ is crucial for the tractability of this model, as the agent's wealth is composed only of real balances and does not affect his choice of money holdings for the DM transactions.

Now, let us take into account the terms of trade $(q_t^{ei}, \Omega_t^i d_t^{ei})$ in the DM with the Nash bargaining protocol, where $q_t^{ei} \in \Re_+$ is the quantity of the search good traded between agents from countries e and i,

$$\Omega_t^i \equiv \left(E_{t(i/e)} \;\; 1 \;\; E_{t(i/v)}\right)(u_t^e \;\; u_t^i \;\; u_t^v)^T, \tag{7}$$

and $\Omega_t^i d_t^{ei}$ is the amount of money that the seller from exporting country e receives from the buyer from importing country i in exchange for $q_t^{ei}$. In double-coincidence matching, $u'(q_t^{ei*}) = c'(q_t^{ei*})$, regardless of the money holdings of the agents, where the characteristics of the DM utility are $u(0) = c(0) = 0$, $u' > 0$, $c' > 0$, $u'' < 0$, and $c'' \geq 0$. In single-coincidence matching, the terms of trade $(q_t^{ei}, \Omega_t^i d_t^{ei})$ are determined by the generalized Nash bargaining problem expressed as

$$\max_{q_t^{ei},\, d_t^{ei}} \left[u(q_t^{ei}) - (\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v) d_t^{ei}\right]^{\theta} \left[-c(q_t^{ei}) + (\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v) d_t^{ei}\right]^{1-\theta}, \tag{8}$$

subject to $d_t^{ei} \leq m_t$ and $q_t^{ei} \geq 0$, where $\theta \in [0, 1]$ is the buyer's bargaining power and $1-\theta$ is the seller's. This problem is derived based on the linearity of the CM value function specified in Equation (6). If $d_t^{ei} \leq m_t$ does not bind, the solution to the bargaining problem is $q_t^{ei} = q_t^{ei*}$ and $d_t^{ei} = m_t^* = [(1-\theta)u(q_t^{ei*}) + \theta c(q_t^{ei*})]/(\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v)$. If, however, $d_t^{ei} = m_t$, then the solution is

$$(\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v) m_t \equiv z^a(q_t^{ei}) = \frac{\theta u'(q_t^{ei}) c(q_t^{ei}) + (1-\theta) c'(q_t^{ei}) u(q_t^{ei})}{\theta u'(q_t^{ei}) + (1-\theta) c'(q_t^{ei})}. \tag{9}$$

Thus, $z'^a(q_t^{ei}) > 0$ and $q_t'^{ei}(m_t) = (\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v) / z'^a(q_t^{ei}) > 0$ given that a monetary economy is ensured. As in the Zhang's (2014) model, the agents reciprocally interact in the sense that the seller accepts the DM transaction currency that the buyer carries into the DM. This implies that there are no problems in recognizing the DM transaction currency, which can easily arise through counterfeiting, limitations on its exchange to other currencies, and high fluctuations in its value. Note that there are no shocks on currency values in this model until the seller exchanges the DM transaction currency received from the buyer for his local currency.

The matching function is borrowed from the literature on search-theoretic models of the labor market to characterize the matching probability of the DM value function. According to Pissarides (1984) and Petrongolo and Pissarides (2001), the matching function can be extended by a weight (i.e., the intensity of the search represented by the average search cost). The basic idea is that the matching probability between two agents grows as each agent searches more or increases its search expenditure. However, as this study does not consider the endogenous search effort, the intensity of search cannot be directly applied to this model. Under given search frictions, the matching probability between two agents increases when search frictions are relatively weak.[8] Suppose that an agent in the US searches for its trading partner from Canada, Columbia, and Venezuela to import a specific type of footwear. As there are more potential exporters who can communicate in English in Canada than in Columbia

[8] This is consistent with the argument of Petropoulou (2011) that the matching probability of two agents in international transactions is negatively affected by information costs.

and Venezuela, other things being equal, the matching probability between the agents in the US and Canada goes up. Thus, this study uses the relative search costs as the weights instead of the intensity of search. Note that trade costs in the gravity model are defined as search costs represented by remoteness and information costs. According to the literature (e.g., Rauch, 1999, AvW, 2003, 2004, and ST, 2006), proximity (distance/contiguity) and information costs (language/colonial ties) play a significant role as trade costs in the gravity model. Furthermore, the gravity model is based on the assumption that countries are specialized in differentiated goods, for which search efforts are essential. Thus, it is plausible that search costs (i.e., remoteness and information costs) represent trade costs in the gravity model, as supported by Rauch (1999). The relative search costs take the forms of $(1/2)(\lambda_t^e / t_t^{ei}) \in (0, 1)$ and $(1/2)(\lambda_t^i / t_t^{ei}) \in (0, 1)$ for e's agent and i's, respectively, where $\lambda_t^e$ is the weighted average of trade costs between e's agent and all of its potential trading partners, $\lambda_t^i$ is the weighted average of trade costs between i's agent and all its potential trading partners, and $t_t^{ei}$ is the trade cost factor between e's agent and i's.[9] While the intensity of search in Petrongolo and Pissarides (2001) is only applied to workers, the weights are assigned to both agents because they search together. The reason the weighted average is adopted rather than the arithmetic average is that the effective value of trade costs for each country can differ according to a country's specific characteristics. From the above example, one thousand potential exporters in Venezuela who can communicate in English can be a significant source for a trading network with American importers,

[9] As the trade cost factor $t_t^{ei}$ has a value between 1 and 2, the value of 1/2 ensures that each of the relative search costs has a value between 0 and 1.

whereas the same number in Canada may be less important in the network. This difference derives from the gap between the economic sizes of the two countries. The weights imply that for a given bilateral search friction between e's agent and i's, stronger search frictions between each agent and its other trading partners increase the matching probability between e and i. Then, the matching function takes the form of $\Pi(\lambda_t^e A_t^e / 2t_t^{ei}, \lambda_t^i A_t^i / 2t_t^{ei})$. As the specific functional form for this model, Mortensen's (1982, 2011) quadratic function is more appropriate than his linear function because neither agent knows whether its potential trading partner is matched. Thus, $\Pi_t = \eta(\lambda_t^e A_t^e / 2t_t^{ei})(\lambda_t^i A_t^i / 2t_t^{ei})$, where $\eta$ is a constant reflecting the frequency of contacts. Consequently, the matching probability between two agents is given as the number of matchings divided by the frequency of contacts, $\Pi_t / \eta = (\lambda_t^e A_t^e / 2t_t^{ei})(\lambda_t^i A_t^i / 2t_t^{ei})$. Then, the DM value function is as follows:

$$\begin{aligned} V_t(m) = & \frac{\Pi_t}{\eta}\left\{u[q_t^{ei}(m, u_t^e, u_t^v)] + W_t[\Omega_t^i m - \Omega_t^i d_t^{ei}(m)]\right\} \\ & + \frac{\Pi_t}{\eta}\int\left\{-c[q_t^{ei}(\tilde{m}, \tilde{u}_t^e, \tilde{u}_t^v)] + W_t[\Omega_t^i m + \tilde{\Omega}_t^i d_t^{ei}(\tilde{m})]\right\} dF_t(\tilde{m}) \\ & + \frac{\Pi_t}{\eta}\left[u(q_t^{ei*}) - c(q_t^{ei*}) + W_t(\Omega_t^i m)\right] + \left(1 - 3\frac{\Pi_t}{\eta}\right) W_t(\Omega_t^i m) \end{aligned}, \quad (10)$$

where "~"refers to the trading partners of i's agent and $F_t(\tilde{m})$ denotes the distribution of money holdings across these trading partners. The four terms on the RHS imply the expected payoffs for buying, selling, bartering, and not trading, respectively. Given the bargaining solution and the CM value function, the DM value function is transformed through repeated substitution, such that

$$V_t(m_t) = \Gamma_t(m_t) + \phi_t^i \overline{\Omega}_t^i m_t + E_t \sum_{s=t}^{\infty} \beta^{s-t} \max_{m_{s+1}, u_s^e, u_s^v} \left\{ \begin{array}{l} -[\phi_s^i + (\phi_s^e - \phi_s^i) u_s^e + (\phi_s^v - \phi_s^i) u_s^v] m_{s+1} \\ + \beta[\Gamma_{s+1}(m_{s+1}) + \phi_{s+1}^i \overline{\Omega}_{s+1}^i m_{s+1}] \end{array} \right\} \quad (11)$$

subject to $0 \le u_t^e \le 1$ and $0 \le u_t^v \le 1$, where

$$\begin{aligned} \Gamma_t(m_t) = & \frac{\Pi_t}{\eta} \left\{ u[q_t^{ei}(m_t, \overline{u}_t^e, \overline{u}_t^v)] - [\phi_t^i + (\phi_t^e - \phi_t^i)\overline{u}_t^e + (\phi_t^v - \phi_t^i)\overline{u}_t^v] d_t^{ei}(m_t) \right\} \\ & + \frac{\Pi_t}{\eta} \int \left\{ -c[q_t^{ei}(\widetilde{m}_t, \widetilde{u}_t^e, \widetilde{u}_t^v)] + [\phi_t^i + (\phi_t^e - \phi_t^i)\widetilde{u}_t^e + (\phi_t^v - \phi_t^i)\widetilde{u}_t^v] d_t^{ei}(\widetilde{m}_t) \right\} dF_t(\widetilde{m}_t) \\ & + \frac{\Pi_t}{\eta} [u(q_t^{ei*}) - c(q_t^{ei*})] + U(X_t^{i*}) - X_t^{i*} \end{aligned},$$

and "–" implies the predetermined DM transaction currency. As in the LW model, the choice of money holdings for the DM transactions is restricted to $m_{t+1} < m_{t+1}^*$, which means that the buyer spends all of his money $d_{t+1}^{ei} = m_{t+1}$. Thus, the first-order condition in terms of $m_{t+1}$ is

$$\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v = \beta E_t \left\{ \frac{\Pi_{t+1}}{\eta} \left[ \begin{array}{l} u'[q_{t+1}^{ei}(m_{t+1})] q_{t+1}'^{ei}(m_{t+1}) \\ -\left( \phi_{t+1}^e \overline{u}_{t+1}^e + \phi_{t+1}^i \overline{u}_{t+1}^i + \phi_{t+1}^v \overline{u}_{t+1}^v \right) \end{array} \right] + \phi_{t+1}^i \overline{\Omega}_{t+1}^i \right\}. \quad (12)$$

Using the fact that $q_{t+1}'^{ei}(m_{t+1}) = (\phi_{t+1}^e \overline{u}_{t+1}^e + \phi_{t+1}^i \overline{u}_{t+1}^i + \phi_{t+1}^v \overline{u}_{t+1}^v) / z'^a(q_{t+1}^{ei})$, Equation (12) can be rewritten as

$$\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v = \beta E_t \left\{ \phi_{t+1}^i \overline{\Omega}_{t+1}^i + \left( \phi_{t+1}^e \overline{u}_{t+1}^e + \phi_{t+1}^i \overline{u}_{t+1}^i + \phi_{t+1}^v \overline{u}_{t+1}^v \right) \frac{\Pi_{t+1}}{\eta} \left[ \frac{u'(q_{t+1}^{ei})}{z'^a(q_{t+1}^{ei})} - 1 \right] \right\}. \quad (13)$$

The price of the DM money in period t is equal to the expected discounted price of the DM money in t+1 plus the expected discounted liquidity factor that captures the marginal benefit of holding real balances in the DM. Because $m_{t+1} < m^*_{t+1}$, the liquidity factor is positive. Note that if there is a price mismatch across two periods, then the real exchange rate can be addressed under this partial equilibrium condition.

The optimal choices regarding $u_t^e$ and $u_t^v$ are summarized as follows. First, if $\phi_t^e \geq \phi_t^i$ and $\phi_t^v = \phi_t^i$ in Equation (11), then $u_t^e = 0$ and $0 < u_t^v \leq 1$, implying that the vehicle currency is either partly or fully chosen as the DM transaction currency. Second, if $\phi_t^e = \phi_t^i$ and $\phi_t^v \geq \phi_t^i$, then $0 < u_t^e \leq 1$ and $u_t^v = 0$, meaning that currency e is either partly or fully chosen as the DM transaction currency. Third, if $\phi_t^e \geq \phi_t^i$ and $\phi_t^v \geq \phi_t^i$, then $u_t^e = 0$ and $u_t^v = 0$, in other words, $u_t^i = 1$. This is the case where the agent's own local currency is fully chosen as the DM transaction currency. Finally, if $\phi_t^e = \phi_t^v = \phi_t^i$, then $0 < u_t^e < 1$ and $0 < u_t^v < 1$, indicating that the solution can be a combination of the three currencies. Consequently, all of the conditions concerning the choice of DM transaction currency can be reduced to one condition, as follows:

$$(\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v) m_t = \phi_t^i m_t \equiv z^a(q_t^{ei}) = z^i(q_t^{ei})\,. \qquad (14)$$

If $\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v > \phi_t^i$, then the buyer is unwilling to trade because he should spend more local money to prepare for the amount of the DM transaction currency. In turn, if $\phi_t^e u_t^e + \phi_t^i u_t^i + \phi_t^v u_t^v < \phi_t^i$, then the seller prefers to trade in the CM rather than the DM because he can enjoy a better return on his production in the former market. Equation (13) then can be transformed into

$$1 = E_t\{L_{t+1} R_{t+1}\} \ , \tag{15}$$

where $L_{t+1} \equiv \beta \left\{ \left[ \frac{\Pi_{t+1}}{\eta} \left( \frac{u'}{z'^{i}} \right) \right] + \left[ 1 - \frac{\Pi_{t+1}}{\eta} \right] \right\}$ and $R_{t+1} \equiv \frac{\phi_{t+1}^{i}}{\phi_t^{i}}$ . $L_{t+1}$ implies the liquidity-based discount factor and $R_{t+1}$ represents the gross rate of return for carrying money into the DM. Because the buyer in the DM is risk-averse,

$$E_t R_{t+1} = \frac{1}{E_t L_{t+1}} - \frac{COV_t(L_{t+1}, R_{t+1})}{E_t L_{t+1}} \ , \tag{16}$$

where $COV_t(L_{t+1}, R_{t+1})$ is the covariance between $L_{t+1}$ and $R_{t+1}$ conditional on information at time t. Without loss of generality, let us consider the case of $\theta \to 1$.[10] Thus, $z'^{i}(q_{t+1}^{ei}) = c'(q_{t+1}^{ei})$. As $u'(q_{t+1}^{ei})/c'(q_{t+1}^{ei}) \propto 1/q_{t+1}^{ei}$, Equation (16) characterizes the typical consumption smoothing under uncertainty feature. In other words, the buyer should take account of both stability and the level of consumption in determining the optimal type and quantity of the DM transaction currency. For example, if the vehicle currency ensures not only a better level of consumption but also higher returns when the buyer cannot consume sufficiently, then it can be selected as the optimal DM transaction currency.

The money supply changes according to the simple rule of $M_{t+1} = (1+\varepsilon)M_t$ with $\varepsilon$ constant. From Equation (14), based on the money market clearing condition $m_t = M_t$, Equation (13) can be expressed as

[10] The DM trade does not occur in the case of $\theta \to 0$. From Equations (9) and (14), $\phi_{t+1}^{i} d_{t+1}^{ei}(m_{t+1}) = u(q_{t+1}^{ei})$ in such an instance, the buyer receives no surplus from purchasing the search good, thus he optimally chooses not to trade in the DM.

$$\frac{z^i(q_t^{ei})}{M_t} = \beta E_t \left\{ \frac{z^i(q_{t+1}^{ei})}{(1+\varepsilon)M_t} \left[ \frac{\Pi_{t+1}}{\eta} \left( \frac{u'}{z'^i} \right) + 1 - \frac{\Pi_{t+1}}{\eta} \right] \right\}. \tag{17}$$

The remainder of this section focuses only on steady-state equilibria. With the typical definition of $R = (1+r)(1+\varepsilon)$, where $R$ is the gross nominal interest rate and $r = (1-\beta)/\beta$ is the real interest rate, the optimal condition is given by

$$\frac{u'(q^{ei})}{z'^i(q^{ei})} = 1 + \left( \frac{4(t^{ei})^2}{A^e A^i} \right) \left( \frac{1}{\lambda^e \lambda^i} \right) (R^i - 1). \tag{18}$$

Under the environment where trade frictions exist, ($t^{ei} \neq 0$), the buyer's consumption can approach the optimal level as he holds more money ($R^i \to 1$) given that $\theta \to 1$.

## *2.2. The Nexus between Monetary Equilibrium and the Gravity Model*

The specific forms of the terms $u(q^{ei})$ and $z^i(q^{ei})$ in Equation (18) are needed to link the derived monetary equilibrium condition to the AvW gravity model. With respect to the former, the linear plus power utility function of Dyer and Jia (1997) is chosen to ensure compatibility with the AvW gravity model, as follows:

$$u(q^{ei}) = q^{ei} - \sigma (q^{ei})^{1-\psi}, \tag{19}$$

where $\sigma > 0$ and $\psi = -\frac{u'''(1)}{u''(1)} - 1 > 1$. This utility function can be applied to both expected and non-expected utility preferences. For the latter, if $z^i(q^{ei}) = q^{ei}$, then Equations (9) and (14) yield the following DM cost function:

$$c(q^{ei}) = \frac{q^{ei}[\theta u'(q^{ei}) + (1-\theta)c'(q^{ei})] - (1-\theta)c'(q^{ei})u(q^{ei})}{\theta u'(q^{ei})}. \tag{20}$$

The cost function should meet the fundamental conditions (i.e., $c(0)=0$, $c'>0$, and $c''\geq 0$). The condition $c(0)=0$ is directly identified from Equation (20), given that $u(0)=0$. For the property of $c'>0$, the marginal cost function is

$$c'(q^{ei})=\frac{\{[\theta u'+(1-\theta)c']+q^{ei}[\theta u''+(1-\theta)c'']-(1-\theta)[c''u+c'u']\}\theta u'}{(\theta u')^2} - \frac{\{q^{ei}[\theta u'+(1-\theta)c']-(1-\theta)c'u\}\theta u''}{(\theta u')^2}. \tag{21}$$

The restriction $\theta \to 1$ makes the value of the marginal cost function so explicit that $c'(q^{ei})=1$. It is easy to show that $c''(q^{ei})=0$. Thus, $z^i(q^{ei})=q^{ei}$ is valid. With the specific forms of the marginal functions $u'$ and $z'$, Equation (18) can be expressed as

$$q^{ei}=\left\{\left[\frac{1}{\sigma(\psi-1)}\right]\left(\frac{4(t^{ei})^2}{A^eA^i}\right)\left(\frac{1}{\lambda^e\lambda^i}\right)\left(R^i-1\right)\right\}^{-\frac{1}{\psi}}. \tag{22}$$

Note that the total expenditure of the buyer from country i and the value of the gross product of the seller from country e, respectively, is $M$ in equilibrium. Thus, given that all agents from the same country are symmetric, $A^eA^i=\left(\frac{A^eM}{M}\right)\left(\frac{A^iM}{M}\right)=\left(\frac{y^e}{y^w}\right)\left(\frac{y^i}{y^w}\right)$, where $y^e$ ($y^i$) is the gross domestic product (GDP) of country e (i) and $y^w$ is the world GDP. From the money market clearing condition, $m=\frac{z^i(q^{ei})}{\phi^i}=M$. Thus, $\frac{q^{ei}}{\phi^i}=p^iq^{ei}=M$, where $p^i$ is the price level of country i. If $q^{ei}=M$, then $p^i=1$. The necessary condition for $q^{ei}=M$ is satisfied because $q'^{ei}(M)>0$. Let $\kappa$ denote the number of matched pairs where e's agent becomes a seller and i's agent becomes a buyer. Consequently, the following new gravity model is set up:

$$\ln X^{ei} = c + \frac{1}{\psi}\ln y^e + \frac{1}{\psi}\ln y^i - \frac{2}{\psi}\ln t^{ei} + \frac{1}{\psi}\ln \lambda^e + \frac{1}{\psi}\ln \lambda^i - \frac{1}{\psi}\ln(R^i - 1), \qquad (23)$$

where $X^{ei} \equiv \kappa p^i q^{ei}$ is the value of exports from country e to country i and $c \equiv \ln\kappa - \frac{1}{\psi}\ln\left[4(y^w)^2 / \sigma(\psi - 1)\right]$. The average trade resistances $\lambda^e$ and $\lambda^i$ are conceptually the same as the multilateral resistance factors of the AvW gravity model. If the multilateral resistance is stronger, which means that bilateral resistance is relatively smaller, then exports from country e to country i grow. Both the new gravity model and the AvW gravity model are the same, except for the monetary term $R^i - 1$.[11] Thus, it is informative that the new gravity model is extended from the AvW gravity model with the monetary factor.

## 3. Empirical Analysis

In this section, the empirical specification and econometric methods for estimating the model established in Section 2 are outlined, the data used are then described, and the empirical results and robustness tests are presented.

### *3.1. Empirical Specification*

To account for the unobserved multilateral resistance terms, let $\ln \lambda_e \equiv \sum_{i=1}^{N} w_i \ln t_{ei}$ and $\ln \lambda_i \equiv \sum_{e=1}^{N} w_e \ln t_{ei}$ in accordance with their definitions, where $w_i$ and $w_e$ denote the weights. Then, substituting these multilateral resistance terms

[11] Strictly speaking, there is another minor difference between Equation (23) and the AvW gravity model in the sense that it does not have the unit income elasticity. However, given that the coefficients regarding incomes usually approach the unit value in the empirical literature, this difference does not matter.

into Equation (23) yields

$$\begin{aligned}\ln X_{ei} = c + \frac{1}{\psi}\ln y_e + \frac{1}{\psi}\ln y_i - \frac{2}{\psi}\ln t_{ei} + \frac{1}{\psi}\sum_{i=1}^{N} w_i \ln t_{ei} + \frac{1}{\psi}\sum_{e=1}^{N} w_e \ln t_{ei} \\ - \frac{1}{\psi}\ln(R_i - 1)\end{aligned} \quad . \tag{24}$$

Following the literature (e.g., AvW, 2003, Hallak, 2006, Silva and Tenreyro, 2006, and Baier and Bergstrand, 2009), the unobservable trade cost is modeled as the following function of the observable variables:

$$t_{ei} = (DIS_{ei})^{\alpha_1} e^{-(\alpha_2 CON_{ei} + \alpha_3 COM_{ei} + \alpha_4 COL_{ei})}, \tag{25}$$

where $DIS_{ei}$ is the bilateral distance between countries e and i; $CON_{ei}$, $COM_{ei}$, and $COL_{ei}$ are dummy variables indicating whether the countries are contiguous, share a common official language, and have ever had a colonial link, respectively. As explained above, the bilateral distance and dummy variables on adjacency and language/colonial ties are closely associated with search costs. Inserting the trade costs in Equation (25) into Equation (24) generates

$$\begin{aligned}\ln X_{ei} = c + \frac{1}{\psi}\ln y_e + \frac{1}{\psi}\ln y_i - \frac{2}{\psi}\alpha_1 \ln DIS_{ei} + \frac{2}{\psi}\alpha_2 CON_{ei} + \frac{2}{\psi}\alpha_3 COM_{ei} \\ + \frac{2}{\psi}\alpha_4 COL_{ei} + \frac{1}{\psi}\alpha_1 MRDIS_{ei} - \frac{1}{\psi}\alpha_2 MRCON_{ei} - \frac{1}{\psi}\alpha_3 MRCOM_{ei} \\ - \frac{1}{\psi}\alpha_4 MRCOL_{ei} - \frac{1}{\psi}\ln(R_i - 1)\end{aligned} \quad , \tag{26}$$

where $MRDIS_{ei} = \sum_{i=1}^{N} w_i \ln DIS_{ei} + \sum_{e=1}^{N} w_e \ln DIS_{ei}$ .[12] If $w_i = y_i / y_w$ and $w_e = y_e / y_w$,

[12] The rest of the multilateral resistance terms follow the patterns of $MRDIS_{ei}$ .

then the multilateral resistance terms follow those of Baier and Bergstrand (2009) because an additional component of each term for the latter is constant across country pairs.[13] In line with the approach of Baier and Bergstrand (2009), this study uses the GDP shares as the weights. To estimate Equation (26), a cross-sectional analysis is used for comparison with the study of Silva and Tenreyro (2006; henceforth, ST), which uses the same econometric method. The panel analysis, however, is also offered as one of the robustness tests. The specification for estimating Equation (26) using cross-sectional data is as follows:

$$\begin{aligned}\ln X_{ei} &= c + v_1 \ln NIR_i + v_2 \ln y_e + v_3 \ln y_i + v_4 \ln DIS_{ei} + v_5 CON_{ei} + v_6 COM_{ei} \\ &\quad + v_7 COL_{ei} + MRDIS_{ei} + MRCON_{ei} + MRCOM_{ei} + MRCOL_{ei} + \ln \mu_{ei}\end{aligned} \tag{27}$$

or equivalently in its nonlinear form

$$X_{ei} = \exp\begin{bmatrix} c + v_1 \ln NIR_i + v_2 \ln y_e + v_3 \ln y_i + v_4 \ln DIS_{ei} + v_5 CON_{ei} + v_6 COM_{ei} \\ + v_7 COL_{ei} + MRDIS_{ei} + MRCON_{ei} + MRCOM_{ei} + MRCOL_{ei} + \ln \mu_{ei} \end{bmatrix}, \tag{28}$$

where $NIR_i$ is the nominal interest rate of country i, $\mu_{ei} = 1 + e_{ei} / \varphi_{ei}$ is a log-normally distributed variable with a conditional mean of one, $e_{ei}$ is a mean zero disturbance that is independent of the regressors, and $\varphi_{ei}$ is the conditional expectation of $X_{ei}$. Note that the effect of the nominal interest rate of the importing country on its imports can be identified indirectly through a domestic channel. The higher nominal interest rate causes its economic contraction, which leads to a reduction in its income. The country thereby imports less. Because the compounding effect is controlled by the GDP of the importing country in the specification, there is no need for additional treatments. The sign of the estimated parameter $v_1$ of interest in this study

[13] Baier and Bergstrand (2009) approximate multilateral resistance terms using the simple first-order log-linear Taylor-series expansion method, based on the AvW model.

is expected to be negative because the higher opportunity costs of money holdings cause the consumption demand of importers to fall. As noted in the literature, the estimated parameters $\nu_2$, $\nu_3$, and $\nu_5$ - $\nu_7$ are expected to have a positive sign, such that the relevant variables act as trade-stimulating factors, whereas the parameter $\nu_4$ is likely to have a negative sign, as distance is a trade-impeding factor.

### *3.2. Econometric Method and Data Description*

The ordinary least-squares (OLS) analysis can be used to estimate Equation (27). This method, however, cannot handle two critical problems. First, it cannot control for observations with zero trade because Equation (27) is a log-linear specification. Although either simply discarding the zeros from the sample or adding the value of one to each observation on the dependent variable is a common approach to handle the presence of zero trade, these ad hoc solutions also suffer from substantial bias. Second, the OLS estimator based on the log-linear model may be both biased and inefficient in the presence of heteroskedasticity. There is a high probability that the error term $\ln \mu_{ei}$ in the log-linear specification of the gravity model is heteroskedastic because the expected value of the logarithm of the random variable $\mu_{ei}$ is different from the logarithm of its expected value according to Jensen's inequality. Thus, the Poisson pseudo-maximum-likelihood (PPML) estimator proposed by ST is appropriate for estimating the effect of the monetary factor on bilateral trade flows. The alternative approach is to estimate the gravity model directly from its nonlinear form, Equation (28). Note that $\varphi_{ei}$ ensures nonnegative trade. Besides solving the problems of zero trade, this econometric method is also consistent in the presence of heteroskedasticity.

The data used for the benchmark regression analysis cover 168 exporting countries and 102 importing countries.[14] An average value for the 2008-2010 period is used for each variable to avoid the bias from business-cycle fluctuations. Those countries that are not consistent across the variables are excluded from further analysis. In addition, import rather than export data are used because the former are more reliable than the latter. Import data are obtained from the UNCOMTRADE dataset, and they are deflated to 2005 constant US dollar values using the US export price indices obtained from the US Bureau of Labor Statistics. The data on real GDP and per capita GDP (measured on a purchasing power parity basis), population, and nominal interest rate (i.e., the lending interest rate denominated in percent), are obtained from the World Development Indicator dataset. The distance data, which are calculated using the latitudes and longitudes of the most important cities or agglomerations of population, and the dummy data regarding the contiguity, common official language, and colonial link, are sourced from the gravity dataset of the CEPII, a French research center. The information on free trade agreements (FTA) is from Bergstrand's database on economic integration agreements, complemented with data from the World Trade Organization.[15] Appendix B summarizes the variables used in the benchmark regression analysis.

### *3.3. Regression Results*

Before investigating the benchmark regression result, let us compare the new gravity model with that of ST using the latter's dataset. Because the AvW gravity model is focused on US–Canada trade, the ST gravity model extended by many countries is appropriate for the comparison. In Table 1, the first column presents the estimation result of the ST gravity model using the PPML estimator. The ST gravity model uses

[14] See Appendix A for the list of countries used in the benchmark regression analysis.
[15] The FTA encompasses the levels of economic integration above and equal to free trade areas.

fixed effects to account for the unobserved multilateral resistance terms. This method leads to excessive control of the variable of interest in the new gravity model (i.e., the nominal interest rate of an importing country). Thus, the approach of Baier and Bergstrand (2009) is a good alternative. The second column reports the regression result of the ST gravity model estimated using the multilateral resistance terms of Baier and Bergstrand (2009) instead of the fixed effects. In this case, the effects of the GDP variables on bilateral trade flows can be also identified. This indicates that there is no significant difference between the two results. Thus, it is possible to determine whether the new gravity model is consistent with the theoretical model established in Section 2 under the empirical framework of the ST gravity model. The third column shows that the estimates of the control variables are consistent with those of the ST gravity model, and the money holding costs of interest negatively affect bilateral trade flows in line with the theoretical prediction. The fourth column demonstrates whether the multilateral resistance terms play a significant role in the new gravity model, as they do in the AvW gravity model. The results confirm that the FTA variable, which replaces the border effect between the US and Canada in the AvW gravity model, is considerably underestimated by their omission.

<Insert Table 1>

Table 2 shows the benchmark regression result obtained using the PPML estimator, along with the OLS estimation results. There are marked differences between the results of the OLS estimator in the first and second columns and that of the PPML estimator in the fifth column. First, the Poisson estimation reveals the smaller estimate regarding the distance variable, which means that the magnitude of its coefficient is overestimated in the OLS estimation. Second, while the coefficient sign of the contiguity in the case where the value of one is added to each observation on the dependent variable is negative and insignificant, it is positive and statistically significant in the PPML estimation. Third, colonial ties have strong effects under the OLS, whereas the Poisson presents no significant effect. As ST shows, these differences come from a malfunction of the OLS estimator in the presence of zero trade and heteroskedasticity. The third and fourth columns demonstrate the differences between the estimates with and without the multilateral resistance terms. Although sharing a border does not influence trade flows without them, it generates a significant effect with them. Furthermore, the role of a common official language becomes smaller with the inclusion of the multilateral resistance factors. These features are also in line with ST. The fourth and fifth columns show the change in the Poisson estimates according to whether the NIR of interest in this study is included. There is no significant change in the control variables, which means that the AvW gravity model is robust to the different specification. It confirms, however, that the NIR plays an important role in determining bilateral trade flows. As money holding costs go up, bilateral trade flows shrink because international buyers are less willing to carry money into international markets where money is essential for bilateral transactions. The results of the GDP variables follow the literature across all cases, as expected.

<Insert Table 2>

### *3.4. Robustness Tests*

Table 3 reports the estimation results using the PPML estimator, based on lagged

GDP and NIR variables.[16] The first column shows the Poisson estimates using the lagged GDP variables of exporting and importing countries to control for the simultaneous bias that can arise from their incomes, as pointed out by Anderson (1979). In this case, the estimate of the NIR is similar to that of the benchmark specification. The second column takes into account the lagged NIR variable to check whether there is reverse causality between bilateral trade flows and the opportunity costs of money holdings. The sign of its coefficient does not change but the magnitude decreases to some degree. The results in the third column using both the lagged GDP and NIR variables are similar to those in the second column. These results imply that the benchmark estimation does not suffer from significant simultaneous bias.

<Insert Table 3>

Table 4 presents the empirical results obtained by the PPML estimator across various models. The first column shows the estimates of the model incorporating the per capita GDP variables of exporting and importing countries to account for the factor endowment characteristics in line with the Hecksher-Ohlin and the non-homothetic taste factors addressed by Bergstrand (1989). The signs of the coefficients of these additional variables are consistent with those in the literature. More importantly, the sign of the NIR is equal to that of the benchmark model, although its magnitude decreases by about 50%. According to the literature (Coe et al., 2007, Melitz, 2008, and Awokuse and Yin, 2010), the populations of exporting and importing countries can be treated as a component of trade costs because larger countries tend to have higher levels of internal trade. Thus, the second column presents the result with the population variable and the

[16] The average values for the 2005-2007 period are used for the one-period lagged variables.

relevant multilateral resistance term. Its coefficient sign is in the expected direction. The NIR estimate is similar to that of the second model in Table 3. The third column reports the result with the FTA dummy variable and its multilateral resistance term, because FTA can be a factor in stimulating bilateral trade flows. The magnitude of its positive coefficient is highly similar to that of ST. The NIR estimate changes only slightly relative to that of the benchmark model. The fourth column presents the results based on the balanced model of trading partners (i.e., 101-102 country pairs), to determine whether the unbalanced model (i.e., 168-102 country pairs) leads to a biased estimation. There are no significant differences between the two models.

<Insert Table 4>

Table 5 presents the PPML estimation results extended with the use of panel data, which cover 59-60 country pairs over the following three-year averages: 1990-1992, 1993-1995, 1996-1998, 1999-2001, 2002-2004, 2005-2007, and 2008-2010. It is informative that the negative effect of the NIR on the bilateral trade flows has a one-period time lag. The estimate of the one-period lagged NIR is similar to that of the first model in Table 4.

<Insert Table 5>

In summary, it can be concluded that the main regression result of this study is indeed robust across various specifications.

## 4. Conclusion

This study theoretically and empirically confirms the argument that money functions in bilateral transactions between international buyers and sellers where trade frictions exist. The new gravity model, extended by a theoretically founded monetary model, will be useful in shedding light on various issues related to international trade. In particular, it will help to justify studies that introduce monetary factors in investigating the determinants of bilateral trade flows. The study also contributes by linking a micro-founded monetary model to open macroeconomic models by dealing with the endogenous choice of international transaction currency.

The study could be extended in a number of directions. For example, the model could be considered with an endogenous UoA of money under the economic environment, where nominal price rigidities are embedded in the modern monetary models with both competitive and decentralized markets (e.g., Aruoba and Schorfheide, 2011). In addition, although the analysis is based on the aggregate product category, something more could be learned from further disaggregation, such as the differentiated, reference-priced, and homogenous goods classification of Rauch (1999) and the consumption, intermediate, and capital goods classification of Eichengreen et al. (2007).

# APPENDIX

## *A. List of Countries used in the Benchmark Estimation*

Afghanistan, Albania, Algeria, Angola, Antigua and Barbuda, Armenia, Australia, Austria, Azerbaijan, Bahamas, Bahrain, Bangladesh, Barbados, Belarus, Belgium, Belize, Benin, Bhutan, Bolivia, Bosnia Herzegovina, Botswana, Brazil, Brunei Darussalam, Bulgaria, Burkina Faso, Burundi, Cambodia, Cameroon, Canada, Central African Rep., Chad, Chile, China, Hong Kong, Colombia, Comoros, Congo, Costa Rica, Croatia, Cyprus, Czech Rep., Côte d'Ivoire, Denmark, Dominica, Dominican Rep., Ecuador, Egypt, El Salvador, Equatorial Guinea, Eritrea, Estonia, Ethiopia, Fiji, Finland, France, Gabon, Gambia, Georgia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hungary, Iceland, India, Indonesia, Iran, Iraq, Ireland, Israel, Italy, Japan, Jordan, Kazakhstan, Kenya, Kiribati, Kuwait, Kyrgyzstan, Lao People's Dem. Rep., Latvia, Lebanon, Lesotho, Liberia, Lithuania, Luxembourg, Madagascar, Malawi, Malaysia, Maldives, Mali, Malta, Mauritania, Mauritius, Mexico, Mongolia, Morocco, Mozambique, Namibia, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Palau, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Qatar, Rep. of Korea, Romania, Russian Federation, Rwanda, Saint Kitts and Nevis, Saint Lucia, Saint Vincent and the Grenadines, Sao Tome and Principe, Saudi Arabia, Senegal, Seychelles, Sierra Leone, Singapore, Slovakia, Slovenia, Solomon Islands, South Africa, Spain, Sri Lanka, Suriname, Swaziland, Sweden, Switzerland, Syria, Tajikistan, TFYR of Macedonia, Thailand, Timor-Leste, Togo, Tonga, Trinidad and Tobago, Tunisia, Turkey, Turkmenistan, Uganda, Ukraine, United Arab Emirates, United Kingdom, United Rep. of Tanzania, Uruguay, USA, Uzbekistan, Vanuatu, Venezuela, Viet Nam, Yemen, Zambia.

*B. Statistical Summary of the Variables used in the Benchmark Estimation*

| Variable | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|
| $X_{ei}$ (unit: $ million) | 482 | 5,023 | 0 | 307,866 |
| $\ln NIR_i$ | 2.369 | 0.580 | 0.556 | 3.836 |
| $\ln GDP_e$ | 24.519 | 2.216 | 19.209 | 30.234 |
| $\ln GDP_i$ | 24.582 | 2.371 | 19.391 | 30.234 |
| $\ln DIS_{ei}$ | 8.779 | 0.780 | 4.394 | 9.894 |

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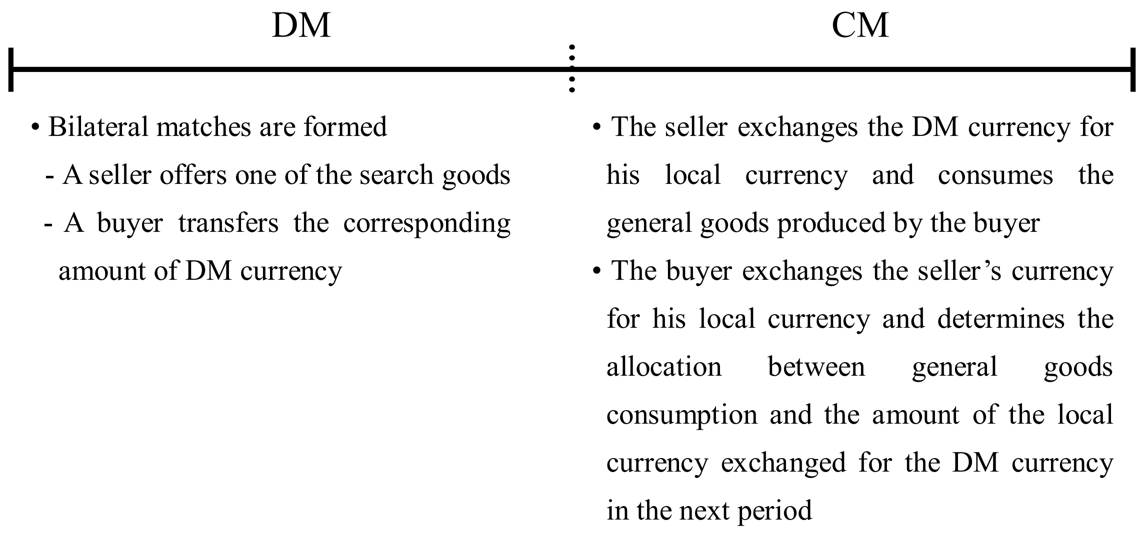


**Fig. 1.** Timing of events within a representative period.

**Table 1**

Comparison with the Silva-Tenreyro gravity model.

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $lnNIR_i$ | - | - | -0.216*** | -0.262** |
| | | | (-3.98) | (-4.42) |
| $lnGDP_e$ | - | 0.815*** | 0.823*** | 0.806*** |
| | | (40.46) | (36.74) | (36.69) |
| $lnGDP_i$ | - | 0.811*** | 0.786*** | 0.762*** |
| | | (43.79) | (40.18) | (29.42) |
| $lnDIS_{ei}$ | -0.750*** | -0.734*** | -0.718*** | -0.631*** |
| | (-18.47) | (-18.16) | (-15.34) | (-13.26) |
| $CON_{ei}$ | 0.370*** | 0.397*** | 0.368*** | 0.261** |
| | (4.08) | (3.45) | (2.86) | (2.16) |
| $COM_{ei}$ | 0.383*** | 0.355** | 0.328* | 0.486*** |
| | (4.12) | (2.03) | (1.74) | (3.29) |
| $COL_{ei}$ | 0.079 | 0.127 | 0.158 | 0.437** |
| | (0.59) | (0.57) | (0.64) | (2.44) |
| $FTA_{ei}$ | 0.376*** | 0.527*** | 0.602*** | 0.157 |
| | (4.90) | (5.56) | (5.65) | (1.38) |
| Multilateral Resistance Terms | No | Yes | Yes | No |
| Fixed effects | Yes | No | No | No |
| No. of observations | 18,360 | 18,360 | 13,500 | 13,500 |
| R-squared | 0.928 | 0.854 | 0.855 | 0.805 |

Notes: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level. The figures in parentheses are z-values. The values for the $MRDIS_{ei}$, $MRCON_{ei}$, $MRCOM_{ei}$, $MRCOL_{ei}$, $MRFTA_{ei}$, and constant do not appear in the table although they are included in the analyses.

**Table 2**

Benchmark regression result.

| Estimator: | OLS | OLS | PPML | PPML | PPML |
|---|---|---|---|---|---|
| Dependent variable: | $lnX_{ei}$ | $ln(X_{ei} +1)$ | $X_{ei}$ | $X_{ei}$ | $X_{ei}$ |
| $lnNIR_i$ | -0.591*** | -0.484*** | - | - | -0.395*** |
| | (-14.76) | (-8.50) | | | (-6.54) |
| $lnGDP_e$ | 1.333*** | 1.740*** | 0.885*** | 0.955*** | 0.953*** |
| | (123.08) | (124.37) | (31.55) | (40.69) | (38.09) |
| $lnGDP_i$ | 1.029*** | 1.476*** | 0.874*** | 0.959*** | 0.897*** |
| | (91.21) | (97.37) | (28.73) | (37.00) | (30.82) |
| $lnDIS_{ei}$ | -1.429*** | -1.975*** | -0.721*** | -0.765*** | -0.697*** |
| | (-48.87) | (-46.86) | (-16.37) | (-15.31) | (-12.72) |
| $CON_{ei}$ | 0.525*** | -0.173 | 0.262 | 0.473** | 0.512** |
| | (3.23) | (-0.75) | (1.32) | (2.23) | (2.36) |
| $COM_{ei}$ | 0.806*** | 1.324*** | 0.287** | 0.184 | 0.176 |
| | (11.34) | (13.12) | (1.99) | (1.09) | (1.08) |
| $COL_{ei}$ | 1.041*** | 1.023*** | -0.073 | 0.055 | 0.082 |
| | (8.11) | (5.53) | (-0.57) | (0.43) | (0.52) |
| Multilateral Resistance Terms | Yes | Yes | No | Yes | Yes |
| No. of observations | 14,303 | 17,136 | 17,136 | 17,136 | 17,136 |
| R-squared | 0.678 | 0.638 | 0.658 | 0.654 | 0.635 |

Notes: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level. The figures in parentheses are t-values or z-values. The values for the $MRDIS_{ei}$, $MRCON_{ei}$, $MRCOM_{ei}$, $MRCOL_{ei}$, and constant do not appear in the table although they are included in the analyses.

**Table 3**

Regression results based on lagged variables.

| | (1) | (2) | (3) |
|---|---|---|---|
| $lnNIR_i$ | -0.346*** | - | - |
| | (-5.85) | | |
| $laglnNIR_i$ | - | -0.299*** | -0.259*** |
| | | (-4.85) | (-4.33) |
| $lnGDP_e$ | - | 0.943*** | - |
| | | (39.53) | |
| $lnGDP_i$ | - | 0.925*** | - |
| | | (34.44) | |
| $laglnGDP_e$ | 0.945*** | - | 0.937*** |
| | (39.39) | | (41.09) |
| $laglnGDP_i$ | 0.901*** | - | 0.930*** |
| | (29.62) | | (33.28) |
| $lnDIS_{ei}$ | -0.683*** | -0.718*** | -0.702*** |
| | (-12.30) | (-13.76) | (-13.24) |
| $CON_{ei}$ | 0.499** | 0.518** | 0.505** |
| | (2.43) | (2.37) | (2.45) |
| $COM_{ei}$ | 0.136 | 0.182 | 0.138 |
| | (0.84) | (1.09) | (0.84) |
| $COL_{ei}$ | 0.086 | 0.040 | 0.048 |
| | (0.56) | (0.26) | (0.32) |
| Multilateral Resistance Terms | Yes | Yes | Yes |
| No. of observations | 17,136 | 17,136 | 17,136 |
| R-squared | 0.626 | 0.633 | 0.627 |

Notes: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level. The figures in parentheses are z-values. The values for the $MRDIS_{ei}$, $MRCON_{ei}$, $MRCOM_{ei}$, $MRCOL_{ei}$, and constant do not appear in the table although they are included in the analyses.

**Table 4**

Regression results across models.

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $\ln NIR_i$ | -0.188*** | -0.292*** | -0.371*** | -0.354*** |
| | (-2.69) | (-4.58) | (-5.47) | (-3.96) |
| $\ln GDP_e$ | 0.952*** | 1.134*** | 0.958*** | 0.815*** |
| | (37.20) | (27.81) | (42.73) | (27.43) |
| $\ln GDP_i$ | 0.918*** | 1.109*** | 0.914*** | 0.757*** |
| | (32.22) | (22.02) | (25.07) | (17.75) |
| $\ln percapitaGDP_e$ | 0.166** | - | - | - |
| | (2.37) | | | |
| $\ln percapitaGDP_i$ | 0.287*** | - | - | - |
| | (4.36) | | | |
| $\ln(POP_e POP_i)$ | - | -0.113** | - | - |
| | | (-2.35) | | |
| $\ln DIS_{ei}$ | -0.670*** | -0.773*** | -0.622*** | -0.663*** |
| | (-13.37) | (-15.90) | (-10.19) | (-10.55) |
| $CON_{ei}$ | 0.475** | 0.344** | 0.416** | 0.427* |
| | (2.42) | (2.03) | (2.10) | (1.66) |
| $COM_{ei}$ | 0.079 | 0.028 | 0.106 | 0.318 |
| | (0.51) | (0.19) | (0.63) | (1.35) |
| $COL_{ei}$ | 0.102 | 0.116 | 0.235 | -0.038 |
| | (0.69) | (0.69) | (1.46) | (-0.16) |
| $FTA_{ei}$ | - | - | 0.385** | - |
| | | | (2.51) | |
| Multilateral Resistance Terms | Yes | Yes | Yes | Yes |
| No. of observations | 17,136 | 17,136 | 17,136 | 10,302 |
| R-squared | 0.649 | 0.654 | 0.652 | 0.667 |

Notes: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level. The figures in parentheses are z-values. The values for the $MR(POP_e POP_i)$, $MRDIS_{ei}$, $MRCON_{ei}$, $MRCOM_{ei}$, $MRCOL_{ei}$, $MRFTA_{ei}$, and constant do not appear in the table although they are included in the analyses.

**Table 5**

Regression results based on panel data.

| | (1) | (2) | (3) |
|---|---|---|---|
| $lnNIR_{it}$ | 0.003<br>(0.03) | - | - |
| $lnNIR_{it-1}$ | - | -0.185**<br>(-2.19) | - |
| $lnNIR_{it-2}$ | - | - | -0.039<br>(-0.42) |
| $lnGDP_{et}$ | 1.527***<br>(7.46) | 1.623***<br>(6.82) | 1.522***<br>(5.44) |
| $lnGDP_{it}$ | 1.538***<br>(7.04) | 1.878***<br>(7.92) | 1.834***<br>(6.58) |
| $lnDIS_{ei}$ | -0.576***<br>(-29.08) | -0.587***<br>(-31.26) | -0.599***<br>(-30.74) |
| $CON_{ei}$ | 0.501***<br>(5.54) | 0.472***<br>(5.62) | 0.430***<br>(4.96) |
| $COM_{ei}$ | 0.098*<br>(1.71) | 0.094<br>(1.60) | 0.090<br>(1.46) |
| $COL_{ei}$ | -0.081<br>(-1.32) | -0.060<br>(-0.97) | -0.042<br>(-0.64) |
| Multilateral Resistance Terms | Yes | Yes | Yes |
| No. of observations | 24,780 | 21,240 | 17,700 |
| R-squared | 0.943 | 0.946 | 0.948 |

Notes: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level. The figures in parentheses are z-values. The values for the $MRDIS_{ei}$, $MRCON_{ei}$, $MRCOM_{ei}$, $MRCOL_{ei}$, exporter dummies, importer dummies, year dummies, and constant do not appear in the table although they are included in the analyses.