The Role of Investment-Specific Technological Change and Cointegrated Sectoral Productivities in the Business Cycles

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Abstract

This paper documents cointegration of the sectoral productivities of consumption-goods and equipment by applying the Johansen cointegration test to U.S. annual data constructed from the EU KLEMS database. I theoretically show that TFP and IST are cointegrated if and only if sectoral productivities are cointegrated. By applying the cointegration to a neoclassical two-sector framework with a non-linear vector error correction model, I investigate the role and effects of technology shocks in the U.S. business cycle. All structural parameters of the model economy are estimated via the maximum likelihood method. Unlike Ireland and Schuh (2008), all estimated innovations are statistically significant. Furthermore, the subsequent simulation analysis finds that the shocks to the common stochastic trend in sectoral productivities not only give persistent effects to consumption, investment, and hours worked but also account for over 47 percent of forecast error for consumption and over 76 percent of forecast error for investment after two years.

Keywords: cointegration; sectoral productivities; two-sector model; business cycle; investment-specific technology; non-linear error correction

JEL Classification codes: E32

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1 Introduction

This paper was motivated by Fisher (2006). It investigates the role of technology shocks in explaining the U.S. business cycles. Recent studies of Galí (1999), Francis and Ramey (2005), and Basu et al. (2006) find significant negative correlations of hours worked with neutral technology shock and argue that the technology shock is not a major cause of economic fluctuation. However, neutral technology shock is not the only technology shock. According to the seminal works of Greenwood et al. (1997, 2000), investment-specific technology (IST) is a major driver of economic growth and fluctuation rather than neutral technology. In addition, Fisher (2006) shows that technology shocks matter a great deal when investment-specific technology is considered.

Since Greenwood et al. (1997), most of the literature identifies IST as a relative price of investment in terms of consumption. It is difficult, however, to interpret the decrease in the relative price of investment as an indication of a technological progress in the equipment sector. Oulton (2007) comments that the relative price may change without a relative change in sectoral productivities between the consumption-goods and equipment sectors.[1] Furthermore, recent empirical studies show that the relative price of investment does not correctly measure the relative changes in sectoral productivities. Basu et al. (2010) estimate technological changes at a disaggregated industry level and aggregate them by the U.S. input-output tables. Their finding suggests that the relative price does not properly measure the relative technological change. Adopting the two-sector model calibrated on the U.S. input-output tables, Guerrieri et al. (2010) conclude that the effect of a productivity shock in the equipment sector is qualitatively different from that of an IST shock in a one-sector model. They argue that the productivity shock in producing equipment boosts consumption in all succeeding periods while an IST shock reduces consumption on the impact.

The objective of this paper is twofold: the first goal is to establish a model economy incorporating IST that addresses the recent critiques on the measurement of IST. I consider IST in a two-sector framework as in Whelan (2003)[2]. In the two-sector model, IST is defined as the ratio of sectoral marginal products. We, therefore, do not have to explicitly measure IST by the relative price of investment. Instead, IST is determined endogenously within the model economy. In a

[1] In the case of different factor intensity in the two sectors, Oulton (2007) points out that the relative price may change without a change in sectoral productivities.

[2] Whelan (2003) insists that a two-sector model consisting of non-durable consumption and durable equipment sectors, gives a better picture of the long-run behavior of the U.S. economy.

similar attempt, Ireland and Schuh (2008) establish a two-sector economy model incorporating the shocks of sectoral productivities to study the U.S. business cycle. Their study does not, however, reflect the fact that the sectoral productivities are cointegrated, which is implied by the empirical findings of Schmitt-Grohé and Uribe (2011)[3]. This paper examines the cointegration of sectoral productivities in the U.S. economy in order to fill the gap between the empirical findings and the available literature in a two-sector model. The second goal is to investigate the effect of external shocks including the shocks of preference and sectoral productivities on the U.S. business cycle. This paper particularly illuminates the role of cointegrated sectoral productivities in the business cycle.

What is the implication of cointegrated sectoral productivities in the business cycle? Cointegration of sectoral productivities indicates that sectoral productivities share a common stochastic trend. In turn, a shock to the common trend induces fluctuations of sectoral productivities homogeneously. As the determinants of the common trend, we may consider some elements of the nationwide environment such as the socio-economic infrastructure, education, politics, and culture. For example, the advancement of information and communication technology (ICT), by enhancing the sectoral productivities, not only causes the price of consumption-goods such as personal computers to decrease but also increases the usefulness of new equipment with enforced networkability and computerization. In a modern economy, a great deal of technological progress in fields such as ICT is sectorally non-exclusive nationwide phenomena. Therefore, it is much more appropriate to consider the cointegration of sectoral productivities when we investigate the business cycle by a multi-sector model.

Section 2 examines the cointegration in U.S. sectoral productivities. To shed light on the cointegration, two independent analyses are performed. First, I conduct the Johansen cointegration test on the sectoral productivities of the consumption-goods and equipment sectors, which are constructed from the EU KLEMS database[4]. The test statistics confirm the cointegration between sectoral productivities[5]. Second, I establish theoretical propositions based on the findings

[3] Schmitt-Grohé and Uribe (2011) introduce the cointegration between TFP and IST, which implies the existence of a common stochastic trend in TFP and IST. They further insist that the innovations in the common stochastic trend explain a sizeable percentage of the volatilities of output, consumption, investment, and hours worked.

[4] For more details about the EU KLEMS database, refer to O'Mahony and Timmer (2009). The data is available at www.euklems.net.

[5] Marquis and Trehan (2008) capture the idea that the productivities of consumption-goods and equipment may share a common shock. They fail to find, however, cointegration between the two sectoral productivities, and just

of Schmitt-Grohé and Uribe (2011) indicating that aggregate neutral productivity and investment-specific technology are cointegrated. The propositions imply that the sectoral productivities are cointegrated if and only if TFP and IST are cointegrated. Thereby, the sectoral cointegration is supported by the empirical findings of Schmitt-Grohé and Uribe (2011).

In Section 3, I apply the cointegration of sectoral productivities to establish a two-sector dynamic stochastic general equilibrium (DSGE) model to investigate the U.S. business cycle. For external shocks, as in Ireland and Schuh (2008), I consider a transitory and a permanent shock of preference and sectoral productivities. To incorporate the cointegration of sectoral productivities into the DSGE model, I employ the vector error correction model (VECM), and to ensure a stationary error correction dynamics, I introduce a smooth transition non-linear error correction (STR NEC) featured by exponential function into the vector error correction system of sectoral productivities.

In Section 4, I estimate the model parameters via the maximum likelihood method and discuss the estimates. During the estimation, I ease the symmetric assumption on the sectoral production function in order to allow that each sector may have a different factor intensity. This relaxation of the assumption is reasonable because the assumption is not based on empirical evidence, but is made in a ad-hoc fashion for computation. The estimated sectoral capital shares confirm the conventional wisdom that the consumption-goods sector is relatively labor-intensive, whereas the equipment sector is capital-intensive. More importantly, unlike Ireland and Schuh (2008), who estimate an insignificant permanent (or growth-rate) shock of the equipment (or investment-goods) sector, all estimated external shocks in this paper are statistically significant.

Section 5 carries out the impulse response analysis and forecast error variance decomposition in order to examine the contributions of external shocks to the economic fluctuations. Lastly, Section 6 offers some concluding remarks. This paper contributes to the recent literature on multi-sector business cycles by providing empirical evidence on cointegration between sectoral productivities and introducing cointegration into a typical two-sector model. Furthermore, the simulation results support the argument of Fisher (2006) by showing persistent and sizeable effects of common trend shocks in sectoral productivities to the U.S. business cycle. The shocks to the common stochastic trend in the sectoral productivities of consumption-goods and equipment sectors nearly permanently

incorporate the correlation between the productivity growth rates of consumption-goods and equipment.

increase consumption, investment, and hours worked, and they account for a large percentage of the variability in consumption and investment. Similarly as Ireland and Schuh (2008), the innovation of preference yields highly persistent and sizeable effects on hours worked. Moreover, the preference shocks account for half of the consumption variability and most of the hours-worked variability.

2 Cointegrated productivities

In this section, I ascertain whether the sectoral productivities are cointegrated in two ways. First, the cointegration is tested empirically. I construct the annual sectoral productivities, aggregate TFP, and relative price of equipment from the EU KLEMS database and use them for unit-root tests and Johansen cointegration tests. Schmitt-Grohé and Uribe (2011) have found that the aggregate TFP and IST are cointegrated using the U.S. quarterly data. I consider a neoclassical two-sector framework to derive a proposition which shows that the empirical finding of Schmitt-Grohé and Uribe (2011) implies the cointegration of sectoral productivities in the U.S. economy.

2.1 Empirical evidence

To conduct the empirical cointegration test, we need to construct the sectoral productivities, aggregate TFP, and relative price of equipment. I use the annual U.S. data of the EU KLEMS Growth and Productivity database for the period of 1970-2005. This data selection is unlike that of Schmitt-Grohé and Uribe (2011), who examine the cointegration of TFP and relative price of investment rather than sectoral cointegration. EU KLEMS is a highly disaggregated industrial productivity database that allows for direct testing of the cointegration of sectoral productivities.

Data

EU KLEMS includes the 72 sectoral definitions. To be used in the empirical tests, 72 industrial levels have to be aggregated into two sectors; consumption-goods and equipment. I define the equipment sector as the aggregation of Electrical and optical equipment (30 to 33), Machinery (29), and Transport equipment (34 to 35)[6], and the rest are aggregated for the consumption goods sector. I apply the Törnqvist index (or Divisia index) for the aggregation. For example, the log-

[6] The figure in parentheses indicates the industry code in the EU KLEMS database.

differenced capital service input of a higher sector i is the weighted average of the log-differenced capital service of its sub-sectors; the applied formula is

$$\Delta \ln K_t^i = \sum_j \bar{\omega}_{K,j,t}^i \Delta \ln K_{j,t}^i,$$

where K_t^i is the capital service of sector i, $K_j^{i,t}$ exhibits the capital demand in sub-sector j of sector i, $j \in i$, and $\bar{\omega}_{K,j,t}^i$ is the two-period moving average of the capital input share demanded by sub-sector j out of the total demand of sector i, which satisfies $\sum_j \bar{\omega}_{K,j,t}^i = 1$, $\forall t$. The aggregations for sectoral output, intermediate input, and labor services adopt the same method as capital service.

Under the growth accounting framework suggested by Jorgenson and Griliches (1967), I construct the log-differenced productivity measures by the aggregated input and output series through the following formula:

$$\Delta \ln A_t^i = \Delta \ln Y_t^i - \bar{v}_{X,t}^i \Delta \ln X_t^i - \bar{v}_{K,t}^i \Delta \ln K_t^i - \bar{v}_{L,t}^i \Delta \ln L_t^i,$$

where A_t^i represents the Solow residual (or TFP) of sector i for $i \in \{tot, cons, equip\}$.[7] Y_t^i, X_t^i, K_t^i and L_t^i respectively denote the output, intermediate input, capital service, and labor service of sector i. $\bar{v}_{l,t}^i$ indicates the two-period moving average of the share of input factor l, which satisfies $\sum_l \bar{v}_{l,t}^i = 1$, $\forall i, t$.

Price movements can be captured by the implicit GDP deflators. The log-differenced GDP deflator of sector i is formulated as

$$\Delta \ln P_t^i = \Delta \ln N.VA_t^i - \Delta \ln R.VA_t^i,$$

where $N.VA_t^i$ and $R.VA_t^i$ represent the nominal value added and real value added in sector i, $i \in \{cons, equip\}$, respectively. Then, we can construct the log-differenced relative price of equipment in terms of consumption-goods ($\Delta \ln RP$) from the following:

$$\Delta \ln RP = \Delta \ln P_t^{equip} - \Delta \ln P_t^{cons}.$$

[7] *tot*, *cons*, and *equip* stand for the aggregate economy, consumption goods sector, and equipment sector, respectively.

Using the log-differenced variables constructed above, I derive an index series of those variables with base year 1995. I set the year before the starting year of each series to 100, and then apply the following formula forwardly:

$$x_{t+1} = x_t \times \exp\left(\Delta \ln x_{t+1}\right),$$

where x_t is a time-series variable, which starts with 100 and has a known $\Delta \ln x_{t+1}$, $\forall t$. Finally, I normalize the indices by setting the base year as 1995.

Empirical findings

I conduct unit-root and cointegration tests for the logarithms of the aggregated TFP, sectoral productivities, and relative price of equipment using the data constructed above. I first perform augmented Dickey-Fuller (ADF) and Dickey-Fuller GLS (DF-GLS) tests to test the unit root. Table 1 presents the results. The ADF test fails to reject the unit-root hypothesis except for the relative price of equipment without trend. Because DF-GLS, known to have increased power, cannot reject the null hypothesis of unit-root in any of the tested variables (with or without trend), however, we can say that the variables are non-stationary. To check the order of integration of the non-stationary variables, I conduct the unit-root tests for the first-differenced logged variables, which are not reported here, and find that all test statistics reject the null hypothesis of unit-root. Based on the results so far, I can conclude that all logged variables of aggregate TFP, TFP in consumption-goods, TFP in equipment, and relative price of equipment are integrated by order one.

Schmitt-Grohé and Uribe (2011) find the cointegration of TFP and relative price of equipment with the U.S. quarterly data. To confirm the consistency of their result, I conduct Johansen cointegration tests with various sets of variables including the dataset of TFP and the relative price of equipment. The results of the Johansen trace and maximum eigenvalue tests are exhibited in Table 2 and 3, respectively.

Both Johansen tests confirm that the system of logged aggregate TFP and sectoral productivities (db1) have one cointegrating vector, which implies that logged TFP can be expressed as a linear combination of two logged sectoral productivities and a stationary series. The system of

Table 1: Unit-root tests for the logarithms of productivities and relative price of equipment

Data	Test	Trend	Lags (AIC)	Test-stats.	Critical values (5%)	Null hypothesis
TFP.cons	ADF	No	1	1.15	-1.95	Accept
	ADF	Yes	1	-2.12	-3.5	Accept
	DF-GLS	No	1	-0.319	-1.95	Accept
	DF-GLS	Yes	1	-2.38	-3.19	Accept
TFP.equip	ADF	No	1	2.72	-1.95	Accept
	ADF	Yes	1	-0.46	-3.5	Accept
	DF-GLS	No	1	1.48	-1.95	Accept
	DF-GLS	Yes	1	-0.976	-3.19	Accept
TFP.tot	ADF	No	1	1.83	-1.95	Accept
	ADF	Yes	1	-1.44	-3.5	Accept
	DF-GLS	No	1	0.901	-1.95	Accept
	DF-GLS	Yes	1	-1.93	-3.19	Accept
RP	ADF	No	1	-3.07	-1.95	Reject
	ADF	Yes	1	-0.357	-3.5	Accept
	DF-GLS	No	1	1.48	-1.95	Accept
	DF-GLS	Yes	1	-0.772	-3.19	Accept

Notes: All unit-root tests fail to reject the null hypothesis of unit-root except for the ADF test of RP without trend. Tests are conducted using the R program with the "urca" package. ADF stands for Augmented Dickey-Fuller, and DF-GLS stands for Dickey-Fuller Generalized Least Squares. TFP.cons, TFP.equip, TFP.tot, and RP denote the productivity of the consumption-goods sector, the productivity of the equipment sector, the productivity of the aggregate economy, and the relative price of equipment, respectively.

the logged relative price of equipment and sectoral productivities (db2) are cointegrated with one cointegrating vector.[8] The cointegration of logged TFP and relative price of equipment (db5) is tested, and we can confirm the result of Schmitt-Grohé and Uribe (2011). Adding each sectoral productivity on "db5", two three-variable systems (db3 and db4) are also examined for cointegration. Interestingly, both systems accept cointegration with one cointegrating vector. The simultaneous cointegrations of the two systems of variables (the system of TFP and IST (db5), and that of TFP, IST, and an augmented sectoral productivity (db3 or db4)) let us infer that sectoral productivities are cointegrated.[9] The cointegration test for sectoral productivities (db6) confirms that the inference is correct.

The cointegration among sectoral productivities indicates the possibility that the comovements

[8]According to Greenwood et al. (1997), the logged relative price of equipment equals the difference of logged productivity of equipment and that of consumption-goods; and the implied cointegrating vector is $(1, 1, -1)$ for the system of $(\ln \mathrm{RP}, \ln \mathrm{TFP.equip}, \ln \mathrm{TFP.cons})$. The estimated cointegrating vector from the Johansen test, however, fails to reproduce the implied sign of the cointegrating vector.

[9]Suppose sectoral productivities are not cointegrated. Then, to make the variable system of db3 and db4 stationary, sectoral productivities should follow a stationary stochastic process. This, however, contradicts the non-stationary assumption of sectoral productivities.

Table 2: The Johansen trace test for cointegration

Dataset	Cointegration rank	Lags (AIC)	Test stats.	Critical values (5%)	Null hypothesis
db1	r <= 2	3	0.103	8.18	-
	r <= 1		13.524	17.95	Accept
	r = 0		40.328	31.52	Reject
db2	r <= 2	3	0.35	8.18	-
	r <= 1		7.31	17.95	Accept
	r = 0		37.00	31.52	Reject
db3	r <= 2	3	0.0765	8.18	-
	r <= 1		7.4565	17.95	Accept
	r = 0		37.0785	31.52	Reject
db4	r <= 2	3	0.433	8.18	-
	r <= 1		7.375	17.95	Accept
	r = 0		36.863	31.52	Reject
db5	r <= 1	3	1.62	8.18	Accept
	r = 0		21.13	17.95	Reject
db6	r <= 1	3	0.324	8.18	Accept
	r = 0		20.898	17.95	Reject

Notes: The Johansen trace tests confirm the cointegration of all specified datasets with one cointegrating vector. Tests are conducted using the R program with the "urca" package. Test models don't include both constant and trend. The datasets used for the Johansen cointegration test are defined as follows:
db1: TFP.tot, TFP.cons, TFP.equip
db2: RP, TFP.cons, TFP.equip
db3: TFP.tot, RP, TFP.cons
db4: TFP.tot, RP, TFP.equip
db5: TFP.tot, RP
db6: TFP.cons, TFP.equip

of aggregate variables and sectoral comovements may arise not only from sectoral linkages but also from a common stochastic trend shared by sectors. Most of the literature on multi-sector business cycles has investigated the sectoral comovements with sectoral structural linkages: Hornstein and Praschnik (1997) and Horvath (2000) incorporate intermediate inputs into their model economy to develop sectoral linkages, and they find positive sectoral comovement in output and employment. However, the empirical findings in Tables 2 and 3, which exhibit the existence of a common stochastic trend in sectoral productivities, suggest that the common stochastic trend of sectoral productivities is another key source of sectoral comovement.

2.2 Theoretical approach

Schmitt-Grohé and Uribe (2011) show that TFP and IST are cointegrated, and the previous sub-

Table 3: The Johansen maximum eigenvalue test for cointegration

Dataset	Cointegration rank	Lags (AIC)	Test stats.	Critical values (5%)	Null hypothesis
db1	r = 2	3	0.103	8.18	-
	r = 1		13.421	14.9	Accept
	r = 0		26.804	21.07	Reject
db2	r = 2	3	0.35	8.18	-
	r = 1		6.96	14.9	Accept
	r = 0		29.68	21.07	Reject
db3	r = 2	3	0.0765	8.18	-
	r = 1		7.3799	14.9	Accept
	r = 0		29.6221	21.07	Reject
db4	r = 2	3	0.433	8.18	-
	r = 1		6.941	14.9	Accept
	r = 0		29.489	21.07	Reject
db5	r = 1	3	1.62	8.18	Accept
	r = 0		19.50	14.9	Reject
db6	r = 1	3	0.324	8.18	Accept
	r = 0		20.574	14.9	Reject

Notes: The Johansen maximum eigenvalue tests confirm the cointegration of all specified datasets with one conintegrating vector. Tests are conducted using the R program with the "urca" package. Test models don't include both constant and trend. The datasets used for the Johansen cointegration test are defined as follows:
db1: TFP.tot, TFP.cons, TFP.equip
db2: RP, TFP.cons, TFP.equip
db3: TFP.tot, RP, TFP.cons
db4: TFP.tot, RP, TFP.equip
db5: TFP.tot, RP
db6: TFP.cons, TFP.equip

section shows that sectoral productivities are cointegrated. Then, is there a connection between the two cointegrations? To address this question, I disentangle the linkage of the two cointegrations.

Since Greenwood et al. (1997), IST is identified as the ratio of the productivity of equipment to that of consumption-goods in much of the literature with a two-sector framework. The behavior of IST, therefore, reflects the relative change in sectoral productivities. To examine the relation formally, let us consider a simplified neoclassical two-sector model as in Oulton (2007); one sector is for producing consumption-goods and the other produces equipment. A benevolent social planner would maximize aggregate social utility, $U(C_t, N_t)$, in an infinite time horizon with the given resource constraint,

$$C_t + \tilde{I}_t = \tilde{Y}_t, \tag{1}$$

where C_t is aggregate consumption, $\tilde{I}_t$ is forgone consumption or savings for investment spending, and $\tilde{Y}_t$ is household income in terms of consumption goods. The investment spending is used for purchasing equipment and eventually contributes to capital accumulation as follows:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{2}$$

where K_t is the capital stock at the beginning of period t, δ denotes the depreciation rate of capital stock, and I_t is the amount of newly produced equipment used for gross investment during period t. Note that the gross investment, I_t, is measured in the unit of equipment, whereas the investment spending, $\tilde{I}_t$, takes the unit of consumption. In capital accumulation, the investment spending must therefore be transformed into the unit of equipment. Suppose that Q_t governs the linear transformation of the forgone consumption, then we can rewrite Eq.(2) as [10]

$$K_{t+1} = (1-\delta)K_t + \tilde{I}_t Q_t. \tag{3}$$

Since the nominal investment spending, $P_{c,t}\tilde{I}_t$, should equal the market value of investment, $P_{e,t}I_t$, Eq.(2) and Eq.(3) imply

$$Q_t \equiv \frac{P_{c,t}}{P_{e,t}}, \tag{4}$$

where $P_{c,t}$ is the market price of consumption goods, $P_{e,t}$ is the price for newly produced equipment and Q_t is known as IST by Greenwood et al. (1997).

Each representative producer in both sectors uses capital and labor in its constant return to scale production function with its own neutral technological progress as follows:

$$Y_{c,t} = Z_{c,t}F^c\left(K_{c,t}, N_{c,t}\right), \tag{5}$$

$$Y_{e,t} = Z_{e,t}F^e\left(K_{e,t}, N_{e,t}\right), \tag{6}$$

where $Y_{c,t}$ and $Y_{e,t}$ are the outputs of consumption goods and equipment, respectively. $K_{j,t}$ and $N_{j,t}$ stand for capital and labor inputs, respectively, of sector $j \in \{c, e\}$. The sum of each input across sectors satisfies the feasibility conditions: $N_t \geq N_{c,t} + N_{e,t}$ and $K_t \geq K_{c,t} + K_{e,t}$. Suppose

[10] Schmitt-Grohé and Uribe (2011) estimate the power of transformation as unity, which implies a linear transformation from consumption to investment.

that $Z_{j,t}$ represents the neutral productivity of sector j and has a random walk process as follows:

$$\ln Z_{c,t} = \ln Z_{c,t-1} + \epsilon_{c,t}, \tag{7}$$

$$\ln Z_{e,t} = \ln Z_{e,t-1} + \epsilon_{e,t}, \tag{8}$$

where both $\epsilon_{c,t}$ and $\epsilon_{e,t}$ are independent white noises. Note that both sectoral productivities follow uncorrelated random walk processes due to the independently distributed disturbances, $\epsilon_{c,t}$ and $\epsilon_{e,t}$. In addition to this, the two random walk processes are not cointegrated because there is no common trend between them by construction. Therefore, the sectoral productivities are uncorrelated and uncointegrated.

Suppose both sectors are in perfect competition. Then, the representative firms would set their prices at marginal cost, which implies

$$\frac{P_{c,t}}{P_{e,t}} = \frac{Z_{e,t}F_1^e\left(K_{e,t}, N_{e,t}\right)}{Z_{c,t}F_1^c\left(K_{c,t}, N_{c,t}\right)}, \tag{9}$$

where $F^j(\cdot,\cdot)$ is a constant-returns production function of sector j and $F_1^j(\cdot,\cdot)$ is the partial derivative with respect to the first argument. By considering the equivalence of IST and inverse relative price of equipment given by Eq.(4) with the properties of constant returns of production function, we can rewrite Eq.(9) as

$$Q_t = \frac{Z_{e,t}f^{e'}\left(k_{e,t}\right)}{Z_{c,t}f^{c'}\left(k_{c,t}\right)}, \tag{10}$$

where $k_{j,t}$ exhibits a capital per worker in sector j and $f^j(k_{j,t}) = F^j\left(K_{j,t}/N_{j,t}, 1\right)$. Suppose further that the production function is Cobb-Douglas as $f^j(k_{j,t}) = k_{j,t}^{\alpha_j}$. Then, Eq.(10) is extended by logged variables as follows:

$$\ln Q_t = \ln Z_{e,t} - \ln Z_{c,t} + S_{q,t}, \tag{11}$$

where $S_{q,t} = \ln\alpha_e - \ln\alpha_c - (1-\alpha_e)\ln k_{e,t} + (1-\alpha_c)\ln k_{c,t}$, and α_j indicates the capital share of sector j. Without loss of generality, we can assume that the capital-worker ratio of both sectors follows a stationary process, at most, with a deterministic trend; that is, the capital per worker has a trend-stationary stochastic process. Thus, $S_{q,t}$ is stationary. Since logged Q_t is composed of two uncointegrated random walk processes and a stationary process, the investment-specific technology,

Q_t, also has a random walk process.

The composite output Y_t consists of $Y_{c,t}$ and $Y_{e,t}$ with an aggregator $\Phi(\cdot)$. To make things more precise, suppose that the aggregator is Cobb-Douglas as

$$Y_t = \Phi\left(Y_{c,t}, Y_{e,t}\right) = {Y_{c,t}}^{\phi}{Y_{e,t}}^{1-\phi}, \tag{12}$$

where $\phi \in [0,1]$ indicates the share of output for consumption goods to the total output. Using the production functions given in Eq.(5) and Eq.(6), the composite output can be extended by logged variables as

$$\begin{aligned} \ln Y_t \quad = \quad & \phi \ln Z_{c,t} + (1-\phi)\ln Z_{e,t} \\ & + \alpha_c \phi \ln K_{c,t} + \alpha_e(1-\phi)\ln K_{e,t} \\ & + (1-\alpha_c)\phi \ln N_{c,t} + (1-\alpha_e)(1-\phi)\ln N_{e,t}, \end{aligned}$$

which implies that the Solow residuals of the aggregate output from a typical growth accounting method is a linear combination of $\ln Z_{c,t}$ and $\ln Z_{e,t}$:

$$\ln A_t \equiv \phi \ln Z_{c,t} + (1-\phi)\ln Z_{e,t}, \tag{13}$$

where A_t represents Solow residuals or the aggregate TFP.

Then, logged A_t has to be a random walk because logged $Z_{c,t}$ and $Z_{e,t}$ are uncointegrated $I(1)$ processes by construction. Normalizing Eq.(13) with respect to $\ln Z_{e,t}$ and substituting for Eq.(11) yields

$$\ln Q_t - (1-\phi)^{-1}\ln A_t + (1-\phi)^{-1}\ln Z_{c,t} = S_{q,t}. \tag{14}$$

According to Eq.(14), a linear combination of three $I(1)$ processes gives a stationary process, which means the cointegration system of $\ln Q_t$, $\ln A_t$, and $\ln Z_{c,t}$ with the cointegrating vector of $(1, -(1-\phi)^{-1}, (1-\phi)^{-1})$. Another cointegration is derived by substituting Eq.(13) for Eq.(11) with respect to $\ln Z_{c,t}$:

$$\ln Q_t + \phi^{-1}\ln A_t - \phi^{-1}\ln Z_{e,t} = S_{q,t}. \tag{15}$$

Eq.(15) implies that $\ln Q_t$, $\ln A_t$, and $\ln Z_{e,t}$ are cointegrated with the cointegrating vector of $(1, \phi^{-1}, -\phi^{-1})$. These results can be summarized in the following PROPOSITION 1:

Proposition 1. *Suppose that sectoral productivities, $\ln Z_{c,t}$ and $\ln Z_{e,t}$, follow uncointegrated $I(1)$ processes. Then, there exists a cointegrating vector that makes the system of three $I(1)$ processes $(\ln Q_t, \ln A_t, \ln Z_{c,t})$ (or $(\ln Q_t, \ln A_t, \ln Z_{e,t})$) stationary.*

Independent sectoral shocks are broadly assumed in most of the literature on multi-sector business cycles, including two-sector specification.[11] According to PROPOSITION 2, however, PROPOSITION 1 contradicts the empirical findings of Schmitt-Grohé and Uribe (2011), which indicate cointegration between TFP and IST.

Proposition 2. *Under the assumption of uncointegrated sectoral productivities, $\ln Z_{c,t}$ and $\ln Z_{e,t}$, which follow $I(1)$ processes, if TFP $(\ln A_t)$ and IST $(\ln Q_t)$ are cointegrated, there is no such cointegrating vector that makes the three-variable system of $(\ln A_t, \ln Q_t, \ln Z_{c,t})$ (or $(\ln A_t, \ln Q_t, \ln Z_{e,t})$) stationary.*

PROOF: refer to APPENDIX A

To reconcile PROPOSITION 1 with the empirical findings, I reconsider the underlying assumptions on PROPOSITION 1. First, I consider relaxing the random walk assumption from both sectoral productivities to either one of the two. This modification does not hurt the non-stationarity of the aggregate neutral and investment-specific technology, while ensuring cointegration between them; at least one non-stationary process is enough to make any linear combination of productivities non-stationary. However, this has not been supported by data. According to TABLE 1, U.S. sectoral productivities constructed from the EU KLEMS database reveal that the sectoral productivities have $I(1)$ processes in both sectors.

Another possible modification is to introduce a cointegration of both sectoral productivities, which is also supported by the empirical results for "db6" in TABLES 2 and 3. To derive a formal theoretical result, we first have to check if this additional assumption grants the property of $I(1)$ process to TFP and IST. To be valid, the cointegrating vector must satisfy a specific condition. It

[11] Consistent with PROPOSITION 1, Ireland and Schuh (2008) introduce growth-stationary sectoral productivities (or log-difference stationary productivities which imply $I(1)$ processes) in their two-sector model but they assume independent sectoral productivities.

is helpful to refer to IST given in Eq.(11) and aggregate TFP in Eq.(13). Both logged TFP and IST are a special linear combination of logged sectoral productivities, $\ln Z_{c,t}$ and $\ln Z_{e,t}$, with different scale vectors; respectively, $(\phi, 1-\phi)$ and $(-1, 1)$. Now, suppose that the uncovered cointegrating vector of $(\ln Z_{c,t}, \ln Z_{e,t})$ is $(1, \kappa)$. To ensure the non-stationarity of TFP and IST, κ should not be equal to $(1-\phi)/\phi$ or -1. Accordingly, as long as the cointegrating vector of sectoral productivities satisfies the conditions, the non-stationarity of TFP and IST are preserved and PROPOSITION 3 follows:

Proposition 3. *Suppose* $\ln A_t$*,* $\ln Q_t$*,* $\ln Z_{c,t}$*, and* $\ln Z_{e,t}$ *follow* $I(1)$ *processes. Then,* $\ln A_t$ *and* $\ln Q_t$ *are cointegrated if and only if* $\ln Z_{c,t}$ *and* $\ln Z_{e,t}$ *are cointegrated.*

PROOF: refer to APPENDIX A

As we have already seen in TABLES 2 and 3, PROPOSITION 3 stands on the support of empirical findings. Consequently, an appropriate model for a two-sector economy is better to introduce the cointegration of sectoral productivities. In the following section, the cointegrated sectoral productivities are incorporated into a two-sector DSGE model and are used to estimate deep parameters and analyze the role of the common stochastic trend of sectoral productivities.

3 Model

Throughout SECTION 2, I have explained why we have to consider the cointegration of sectoral productivities in a two-sector framework. Considering PROPOSITION 3, this section develops a two-sector business cycle model extended from Ireland and Schuh (2008); their model is established for a two-sector economy of consumption goods and equipment with both transitory and permanent shocks of preference and sectoral productivities. The main difference of this model is the cointegration of sectoral productivities. To ensure full mobility of capital across sector, capital accumulation is allowed only at the aggregate level. Also, capital adjustment cost and habit persistence in consumption are employed as real rigidities. Solving the competitive equilibrium, I introduce IST explicitly into the model; Ireland and Schuh (2008) regard IST as a shadow price.

3.1 The Household

Consider that the infinitely living representative household has the preference described over the habit persistent consumption, C_t, and hours worked, H_t, which is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left(C_t - \xi C_{t-1}\right) - H_t/X_t \right\}, \tag{16}$$

where β and $\xi \in [0,1)$, respectively, denote the subjective discount factor and the degree of habit persistence. X_t stands for the preference shock. The preference shock consists of two stochastic components: one is a level-stationary cyclical component, $X_{l,t}$, which indicates a transitory shock and the other is a growth-stationary trend component, $X_{g,t}$, which indicates a permanent shock. The functional form of preference shocks are given by

$$X_t = X_{l,t} X_{g,t}, \tag{17}$$

$$\ln X_{l,t} = \rho_{xl} \ln X_{l,t-1} + \epsilon_{xl,t}, \tag{18}$$

$$\ln\left(\frac{X_{g,t}/X_{g,t-1}}{\eta^{xg}}\right) = \rho_{xg} \ln\left(\frac{X_{g,t-1}/X_{g,t-2}}{\eta^{xg}}\right) + \epsilon_{xg,t}, \tag{19}$$

where $\rho_j \in [0,1)$ and ϵ_j, respectively, indicate the autoregressive coefficient and disturbance of stochastic process which is *iid* normal with mean zero and variance σ_j^2 for $j \in \{xl, xg\}$. η^{xg} stands for the steady state growth rate of preference shock.

In this model economy, the household earns income by supplying labor and renting capital to the firms, and uses the earned income for consumption and investment purposes. Hence, the household faces the budget constraint of

$$C_t + I_t/Q_t \leq \tilde{W}_t H_t + \tilde{R}_t K_t, \tag{20}$$

where $\tilde{W}_t$ and $\tilde{R}_t$ stand for the wage and rent rate in terms of the unit of consumption goods. As we have seen from Eqs.(1)-(4), investment expenditure, $\tilde{I}_t$, is equal to the gross investment in terms of consumption goods, I_t/Q_t. Capital, K_{t+1}, accumulates through investment, I_t, with capital adjustment cost and constantly depreciated previous capital stock, K_t, as follows:

$$K_{t+1} \leq (1-\delta) K_t + I_t \left[1 - \frac{\psi}{2} \left(\frac{I_t}{I_{t-1}} - \tau^I \right)^2 \right], \tag{21}$$

where $\psi > 0$ is the parameter for capital adjustment cost, and τ^I denotes the steady state level of investment growth.

The representative household maximizes its life-time utility, Eq.(16), subject to the budget constraint, Eq.(20), including the capital accumulation process, Eq.(21). The first-order conditions of solving the household's problem are derived as follows:

$$\Lambda_{1,t} = \frac{1}{C_t - \xi C_{t-1}} - \beta\xi\mathbb{E}_t \frac{1}{C_{t+1} - \xi C_t}, \tag{22}$$

$$\frac{1}{X_t} = \Lambda_{1,t}\tilde{W}_t, \tag{23}$$

$$\frac{\Lambda_{1,t}}{Q_t} = \Lambda_{2,t} \left[1 - \frac{\psi}{2} \left(\frac{I_t}{I_{t-1}} - \tau^I \right)^2 - \psi \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - \tau^I \right) \right] + \beta\mathbb{E}_t\Lambda_{2,t+1}\psi \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - \tau^I \right), \tag{24}$$

$$\Lambda_{2,t} = \beta\mathbb{E}_t \left[\Lambda_{1,t+1}\tilde{R}_{t+1} + \Lambda_{2,t+1} (1-\delta) \right], \tag{25}$$

$$C_t + I_t/Q_t = \tilde{W}_t H_t + \tilde{R}_t K_t, \tag{26}$$

$$K_{t+1} = (1-\delta) K_t + I_t \left[1 - \frac{\psi}{2} \left(\frac{I_t}{I_{t-1}} - \tau^I \right)^2 \right], \tag{27}$$

in which $\Lambda_{1,t}$ and $\Lambda_{2,t}$ stand for the Lagrange multipliers on the budget constraint, Eq.(26), and capital accumulation process, Eq.(27), respectively.

3.2 Firms

Two producing firms represent this model economy; one produces consumption-goods and the other produces equipment. For the sake of clarity, I assume that all consumption-goods are non-durables and all equipment are durables excluding structures. This assumption is consistent with the definition that I used to construct the data of two-sector productivity in Section 2.1. Equipment is usually demanded for the two purposes: durable consumption and investment. By assuming all consumption goods are non-durable, however, I justify all products of the equipment sector being used for investment without being spent for consumption. This assumption is by no means at odds; if we consider household production, the durable consumption can be regarded as an investment for the household's production. This assumption is also applied to the construction of observed data for consumption and investment.

Each firm $i \in \{c, e\}$ uses physical capital, $K_{i,t}$, and hours worked, $H_{i,t}$, as inputs to produce its output, $Y_{i,t}$, through a Cobb-Douglas type production function of homogeneous degree 1 as follows:

$$Y_{c,t} = A_{c,t} {K_{c,t}}^{\alpha_c} (Z_{c,t} H_{c,t})^{1-\alpha_c}, \tag{28}$$

$$Y_{e,t} = A_{e,t} {K_{e,t}}^{\alpha_e} (Z_{e,t} H_{e,t})^{1-\alpha_e}, \tag{29}$$

where α_i denotes the capital share of the production in sector i. The production technologies are affected by both transitory (or level) shocks, which denote $A_{i,t}$, and permanent (or growth-rate) shock, which denote $Z_{i,t}$, for $i \in \{c, e\}$. The transitory shocks of $A_{c,t}$ and $A_{e,t}$ are given as a Hicks-neutral form and are assumed to be independent of each other; the transitory productivity shocks are supposed to have mutually uncorrelated $AR(1)$ processes as follows:

$$\ln A_{c,t} = \rho_{ac} \ln A_{c,t-1} + \epsilon_{ac,t}, \tag{30}$$

$$\ln A_{e,t} = \rho_{ae} \ln A_{e,t-1} + \epsilon_{ac,t}, \tag{31}$$

where $\rho_j \in [0, 1)$ and $\epsilon_{j,t}$ denotes the autoregressive coefficient and disturbance term which is *iid* normal with mean zero and variance σ_j^2, for $j \in \{ac, ae\}$, respectively.

The permanent productivity shocks of $Z_{c,t}$ and $Z_{e,t}$ are introduced as a labor-augmented type. Following Proposition 3, I assume that $Z_{c,t}$ and $Z_{e,t}$ are cointegrated and incorporated into the system through the vector error correction model (VECM) including the smooth transition non-linear error correction (STR NEC) as

$$\begin{bmatrix} \ln \left(\dfrac{Z_{c,t}/Z_{c,t-1}}{\bar{\eta}^{zc}} \right) \\ \ln \left(\dfrac{Z_{e,t}/Z_{e,t-1}}{\bar{\eta}^{ze}} \right) \end{bmatrix} = \begin{bmatrix} \rho_{cc} & \rho_{ce} \\ \rho_{ec} & \rho_{ee} \end{bmatrix} \begin{bmatrix} \ln \left(\dfrac{Z_{c,t-1}/Z_{c,t-2}}{\bar{\eta}^{zc}} \right) \\ \ln \left(\dfrac{Z_{e,t-1}/Z_{e,t-2}}{\bar{\eta}^{ze}} \right) \end{bmatrix} + \begin{bmatrix} f_c \left(ect_{t-1} \right) \\ f_e \left(ect_{t-1} \right) \end{bmatrix} + \begin{bmatrix} D_{cc} & D_{ce} \\ D_{ec} & D_{ee} \end{bmatrix} \begin{bmatrix} \epsilon_{zc,t} \\ \epsilon_{ze,t} \end{bmatrix}, \tag{32}$$

where $\epsilon_{zc,t}$ and $\epsilon_{ze,t}$ are *iid* normal with mean zero and variance σ_{zc}^2 and σ_{ze}^2, respectively, and *ect* indicates the error correction term defined as

$$ect_t = \ln Z_{c,t} - \kappa \ln Z_{e,t}, \tag{33}$$

which implies $Z_{c,t}$ and $Z_{e,t}$ are cointegrated with cointegrating vector $(1, -\kappa)$. The functional forms of $f_i(\cdot)$ are both linear and non-linear for $i \in \{c, e\}$; if linear, it is a typical VECM. Here, I assume $f_i(\cdot)$ follows the exponential smooth transition (ESTR) functional form as

$$f_i(ect_{t-1}) = \gamma_i ect_{t-1} \left(1 - e^{-\theta(ect_{t-1}-\nu)^2}\right), \tag{34}$$

for $i \in \{c, e\}$, where $\theta \geq 0$ and ν is a transition parameter. According to Kapetanios et al. (2003), ect_t is geometrically ergodic or globally stationary as long as $\theta > 0$, $0 < \gamma_e < 2$ and $-2 < \gamma_c < 0$. In turn, the ESTR error correction function has its own benefit by ensuring that the transition dynamics are stationary. I will discuss this issue in more detail in the subsection 3.5.

Since firms would maximize profits in competitive markets subject to their production technology given in Eq.(28) and (29), their profit maximization should satisfy the following conditions:

$$\tilde{R}_t = \alpha_c Y_{c,t}/K_{c,t}, \tag{35}$$

$$\tilde{W} = (1 - \alpha_c) Y_{c,t}/H_{c,t}, \tag{36}$$

$$Q_t \tilde{R}_t = \alpha_e Y_{e,t}/K_{e,t}, \tag{37}$$

$$Q_t \tilde{W} = (1 - \alpha_e) Y_{e,t}/H_{e,t}, \tag{38}$$

Eq.(28), and Eq.(29). Accordingly, these firms' profit-maximizing conditions imply that IST is the ratio of the marginal product of capital in equipment to the marginal product of capital in the consumption-goods sector, which is given as follows:

$$Q_t = \frac{\alpha_e Y_{e,t}/K_{e,t}}{\alpha_c Y_{c,t}/K_{c,t}}. \tag{39}$$

3.3 Market Clearing

In equilibrium, the four markets of consumption goods, equipment, capital, and labor must be cleared. Hence, the following market clearing conditions should be satisfied:

$$C_t = Y_{c,t}, \tag{40}$$

$$I_t = Y_{e,t}, \tag{41}$$

$$K_t = K_{c,t} + K_{e,t}, \tag{42}$$

$$H_t = H_{c,t} + H_{c,t}. \tag{43}$$

The aggregate output measured by unit of consumption goods is defined as

$$\tilde{Y}_t = Y_{c,t} + Y_{e,t}/Q_t. \tag{44}$$

3.4 Solution

The variables of this model economy possess non-stationary properties granted by Z_c, Z_e, and X_g of $I(1)$ stochastic processes. Consequently, we need to transform each non-stationary variable into a stationary one on the balanced growth path. Since each variable changes at different growth rates along the balanced growth path, the functional form of the transformation depends on each of them. Through the following transformation equations, each non-stationary variable, denoted in upper-case, is replaced by its stationary form, denoted in lower-case, : $\tilde{Y}_t = \tilde{y}_t \mathrm{T}^c_{t-1}$; $C_t = c_t \mathrm{T}^c_{t-1}$; $H_t = h_t \mathrm{T}^h_{t-1}$; $\Lambda_{1,t} = \lambda_{1,t}/\mathrm{T}^c_{t-1}$; $\Lambda_{2,t} = \lambda_{2,t}/\mathrm{T}^i_{t-1}$; $\tilde{R}_t = \tilde{r}_t \mathrm{T}^c_{t-1}/\mathrm{T}^i_{t-1}$; $\tilde{W}_t = \tilde{w}_t \mathrm{T}^c_{t-1}/\mathrm{T}^h_{t-1}$; $Q_t = q_t \mathrm{T}^i_{t-1}/\mathrm{T}^c_{t-1}$; $K_t = k_t \mathrm{T}^i_{t-1}$; $I_t = i_t \mathrm{T}^i_{t-1}$; $Y_{c,t} = y_{c,t} \mathrm{T}^c_{t-1}$; $Y_{e,t} = y_{e,t} \mathrm{T}^i_{t-1}$; $K_{c,t} = k_{c,t} \mathrm{T}^i_{t-1}$; $K_{e,t} = k_{e,t} \mathrm{T}^i_{t-1}$; $H_{c,t} = h_{c,t} \mathrm{T}^h_{t-1}$; $H_{e,t} = h_{e,t} \mathrm{T}^h_{t-1}$; $X_{l,t} = x_{l,t}$; $A_{c,t} = a_{c,t}$; $A_{e,t} = a_{e,t}$, where $\mathrm{T}^c_t = {Z_{c,t}}^{1-\alpha_c} {Z_{e,t}}^{\alpha_c} X_{g,t}$, $\mathrm{T}^i_t = Z_{e,t} X_{g,t}$ and $\mathrm{T}^h_t = X_{g,t}$.

Applying the above transformation to the non-stationary system of equations, Eqs.(17)-(44) except for the redundant Eqs.(37) and (38), we obtain the stationary system of equations: the equations are presented in Appendix B. In the substitution process, I define the exogenous fundamental growth rates, denoted ηs, and the growth rates of endogenous variables, denoted τs, as follows: $\eta^{zc}_t = Z_{c,t}/Z_{c,t-1}$, $\eta^{ze}_t = Z_{e,t}/Z_{e,t-1}$ and $\eta^{xg}_t = X_{g,t}/X_{g,t-1}$; $\tau^c_t = \mathrm{T}^c_t/\mathrm{T}^c_{t-1}$, $\tau^i_t = \mathrm{T}^i_t/\mathrm{T}^i_{t-1}$

and $\tau_t^h = \mathrm{T}_t^h / \mathrm{T}_{t-1}^h$.

To solve the stationary non-linear system, I employ the method of Klein (2000). Since this solution method requires a linearized system, I log-linearize the stationary non-linear system on the steady-state values.[12]

3.5 Non-linear Error Correction

Before moving to the next section, we need to address one question: Why is a non-linear error correction considered in this model economy? A linear error correction is predominantly applied in cointegration models; Schmitt-Grohé and Uribe (2011) incorporate VECM into their model with a linear error correction. The estimated adjustment-speed coefficient with linear assumption, such as Johansen test statistics, however, does not guarantee the dynamic global stationary process of the cointegration system. We, therefore, need to ensure the dynamic stability of the system for the structural model.

Table 4: Cointegrated relation of sectoral productivities

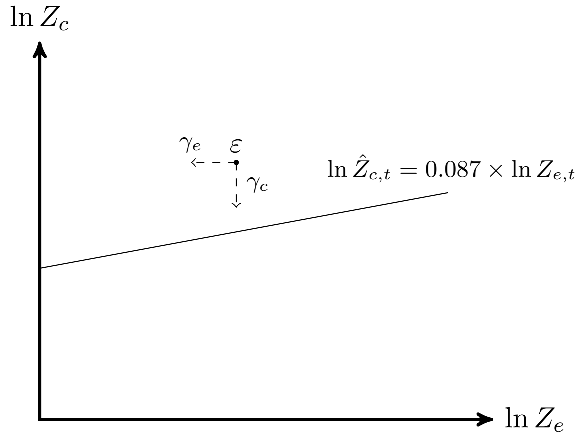
	TFP.cons	TFP.equip
Cointegration Vector	1	-0.087
Adjustment parameter	-0.653	-0.613

Notes: The estimated cointegrating vector and adjustment parameters are obtained by the Johansen test for the dataset named "db6" represented in Table 2 and 3. The cointegrating vector is normalized by TFP.cons. TFP.cons and TFP.equip stand for the productivity of consumption goods and equipment, respectively.

Table 4 exhibits the estimated cointegration parameters from the Johansen test for the dataset "db6" represented in Tables 2 and 3. From Table 4, we can see that the estimated cointegrating vector, $(1, \kappa)$, is $(1, -0.087)$ and the adjustment-speed, $(\gamma_{zc}, \gamma_{ze})$, is revealed $(-0.653, -0.613)$. The adequate adjustment-speed vector, which induces stationary adjustment dynamics, is necessarily near orthogonal to the cointegrating vector. The estimated adjustment-speed vector, however, is far from the orthogonal cointegrating vector of sectoral productivities. The estimated cointegrating vector and adjustment-speed vector in Table 4 indicate that the sign of the estimated adjustment-speed vector is unlike that of the orthogonal vector to the long-run equilibrium represented by the

[12] The steady-state values are explicitly derived and presented in Appendix ??. Also, the log-linearization method applied is explained in Appendix ??.

Figure 1: Linear adjustment of the cointegrated sectoral productivities



cointegrating vector. Furthermore, FIGURE 1 illustrates that if the deviation point, ε, is far enough from the long-run equilibrium path, the linear adjustment from the deviation may not lead it back on the long-run equilibrium; this long travel of adjustment may cause dynamic instability in the vector error correction system.

Table 5: Cointegration test under non-linear error correction assumptions

	Case	Lags(AIC)	Test statistic	Critical value(95%)	Null hypothesis
F_{nec}	Constant	3	0.908	13.73	Accept
	Trend	3	1.112	16.13	Accept
F^*_{nec}	Constant	3	1.459	12.17	Accept
	Trend	3	1.873	15.07	Accept
t_{nec}	Constant	3	-3.224	-3.22	Reject
	Trend	3	-4.477	-3.59	Reject

Notes: The statistics of F_{nec} tests the null hypothesis of no cointegration with no under-lying assumptions. The statistics of F^*_{nec} tests the null hypothesis of no cointegration with the assumption that the switching point is zero. The statistic of t_{nec} tests the null hypothesis of no cointegration with the assumption that the switching point is zero and the error correction term follows the unit roots process in the middle regime.

How can we then ensure the global stability in the cointegration system? One possible answer is by introducing non-linear error correction dynamics into the cointegration; more specifically, the exponential smooth transition (ESTR) in the error correction term. This is motivated by Kapetanios et al. (2006), who develop test statistics for cointegration under the non-linear error correction assumption. To check the applicability of their model (ESTR), I test the non-linear cointegration of the annual sectoral productivities constructed from the EU KLEMS database by using the test statistics of Kapetanios et al. (2006).[13] TABLE 5 shows the results of the non-linear cointegration test for sectoral productivities. The test statistics without underlying assumption (F_{nec}) and with the assumption of zero switching point (F^*_{nec}) fail to reject the null hypothesis of no cointegration. The test statistics with the assumption of zero switching point and the unit roots process in the middle regime (t_{nec}), however, significantly reject the null hypothesis of no cointegration.

The non-linear error correction dynamics between sectoral productivities is confirmed by the cointegration tests with non-linear error correction. Accordingly, if we push the assumption of linear error correction, the dynamic instability is likely to hinder the estimation of structural parameters discussed in the next section. To ensure the dynamic stationary process on the DSGE model with the VECM of the sectoral productivities, I assume non-linear error correction featuring exponential adjustment function.

4 Estimation

One goal of this paper is to investigate the role of the common stochastic trend of sectoral productivities in the U.S. business cycle. This requires that we estimate the structural parameters in the model economy, especially those in the external stochastic processes, such as the autoregressive coefficients and the standard errors of disturbances. As in Ireland and Schuh (2008) and Schmitt-Grohé and Uribe (2011), I adopt the maximum likelihood estimation to estimate the deep parameters that lie on the structural model economy. The linear solution method of Klein (2000) provides the approximated solution of the non-linear system, which is defined on a state-space. We can, accordingly, employ the Kalman filter with given observable variables and construct a likelihood function.

For estimation, the growth rate of consumption, investment, and hours worked are adopted as observable variables. I construct the series of consumption and investment from the U.S. quarterly data of national income and product accounts (NIPAs) available on the BEA website.[14] To be

[13]This paper considers three test statistics of Kapetanios et al. (2006); F_{nec}, F^*_{nec}, and t_{nec}. The statistic of F_{nec} tests the null hypothesis of no cointegration with no underlying assumptions. The statistic of F^*_{nec} tests the null hypothesis of no cointegration with the assumption that the switching point is zero. The statistic of t_{nec} tests the null hypothesis of no cointegration with the assumption that the switching point is zero and the error correction term follows the unit roots process in the middle regime.

[14]Table 1.1.4 (Price index for GDP) and Table 1.1.5 (Nominal GDP) of NIPAs are used to construct real consump-

consistent with the model economy, consumption data is constructed by aggregating non-durables and service consumption. Also, investment is constructed by aggregating "durable consumption," and "equipment and software" in NIPAs. For aggregation, as in SECTION 2.1, the Törnqvist index is applied. Hours worked are obtained from the Federal Reserve Bank of St. Louis' FRED website, under "hours of all persons for nonfarm business sector." All data, ranging from 1948:Q2 to 2011:Q4, are seasonally adjusted and reconstructed in per-capita terms by applying "the civilian non-institutional population age 16 and over," which is available on the BLS website.

A subset of the structural parameters is calibrated. It is quite well known that the maximum likelihood estimates of the discount factor, β, and the capital depreciation rate, δ, are extremely difficult to obtain. Hence, as in Ireland and Schuh (2008), I impose $\beta = 0.99$ and $\delta = 0.025$. The diagonal elements of innovation coefficients (D_{cc} and D_{ee}) of VECM, without loss of generality, are normalized to unity. The steady state quarterly growth rates of consumption, investment, and hours worked ($\bar{\tau}^c$, $\bar{\tau}^i$, and $\bar{\tau}^h$) are calibrated as 1.0042, 1.0092, and 0.9995, respectively, from the average growth rate of the corresponding observables constructed above. The cointegrating vector, $(1, \kappa)$, and the steady-state growth rate of sectoral productivities, and preference ($\bar{\eta}^{zc}$, $\bar{\eta}^{ze}$, and $\bar{\eta}^{xg}$) are calculated from the steady state conditions of the model economy and the steady state growth rate of consumption, investment, and hours worked.

The rest of the structural parameters are estimated via maximum likelihood. TABLE 6 presents the estimated 27 parameters with standard errors, which are computed by a parametric bootstrapping procedure as in Ireland and Schuh (2008). I generate 1000 sets of artificial data, which contain the same number of observations as the original sample, from the estimated model by assigning random disturbances for each period. The artificially generated 1000 sets of data are used to estimate 1000 sample parameters. The reported standard errors in TABLE 6 are the standard deviations of the samples. Additionally, during estimation, I allow the existence of measurement errors in the observables of the growth rates for consumption, investment, and hours worked series, which are denoted by μ_c, μ_i, and μ_h, respectively. The estimates of these measurement errors are curbed to not exceed 25 percent of the standard error of each series.

The model estimates a significant habit-persistence parameter, ξ, of 0.2028; it is much higher

tion for non-durables and services, and real investment, which is redefined as the aggregate of "durable consumption," and "equipment and software" in NIPAs.

Table 6: The maximum likelihood estimates and standard errors of the structural parameters

Parameter	Description	Estimate	Standard error
ξ	habit persistence	0.2028	0.0324
ψ	parameter for capital adjustment	0.3148	0.0392
θ	identification parameter for cointegration	0.9384	0.0267
ν	transition parameter	0.0550	0.0752
α_c	capital share of consumption-goods production	0.3307	0.0325
α_e	capital share of equipment production	0.4009	0.0747
ρ_{cc}	autoregressive parameter in VECM	0.2986	0.1479
ρ_{ce}	autoregressive parameter in VECM	0.0000	0.0525
ρ_{ec}	autoregressive parameter in VECM	0.0000	0.0757
ρ_{ee}	autoregressive parameter in VECM	0.0000	0.0402
γ_c	adjustment-speed of error correction	-0.1822	0.5258
γ_e	adjustment-speed of error correction	1.7947	0.0601
D_{ce}	correlation of innovations in VECM	0.3000	0.0762
D_{ec}	correlation of innovations in VECM	0.0225	0.1135
ρ_{xl}	autoregressive parameter of x_l	0.8910	0.1060
ρ_{xg}	autoregressive parameter of η^{xg}	0.5493	0.1200
ρ_{ac}	autoregressive parameter of a_c	0.0000	0.0829
ρ_{ae}	autoregressive parameter of a_e	0.0000	0.0476
σ_{xl}	standard error of x_l	0.0033	0.0014
σ_{xg}	standard error of η^{xg}	0.0046	0.0010
σ_{ac}	standard error of a_c	0.0029	0.0005
σ_{ae}	standard error of a_e	0.0086	0.0020
σ_{zc}	standard error of η^{zc}	0.0042	0.0010
σ_{ze}	standard error of η^{ze}	0.0200	0.0055
μ_c	measurement error of c	0.0004	0.0003
μ_i	measurement error of i	0.0078	0.0000
μ_h	measurement error of h	0.0023	0.0002

Notes: Sample period is 1948:Q2 to 2011:Q4. The observables are the growth rates of consumption, investment, and hours worked. Each of the observables is assumed to possess measurement error. During estimation $\beta = 0.99$ and $\delta = 0.025$ are imposed. The diagonal elements of VECM innovations (D_{cc} and D_{ee}) are normalized to unity.

than 0.08 of Ireland and Schuh (2008) but a little bit lower than 0.31 of Schmitt-Grohé and Uribe (2011). The capital adjustment-cost parameter is estimated as 0.3148, which is much lower those in other literature; however, the estimate is significant. Most of the two-sector models, including Ireland and Schuh (2008), assume a symmetric sectoral production technology. The symmetry assumption, however, does not reflect the actual situation but is done for convenience. This paper discards the symmetry assumption across sectoral production in estimating the capital share of each sector. The maximum likelihood method estimates the capital share of consumption-goods production, α_c, as 0.3307 and that of equipment, α_e, as 0.4009; both estimated capital shares are statistically significant. The estimated sectoral capital shares are worthy of comparison in the literature: Ireland and Schuh (2008) estimate the capital share of 0.39 with a standard error of 0.06, and Schmitt-Grohé and Uribe (2011) estimate 0.37 with a standard error of 0.03. Therefore, we cannot say that the estimates of sectoral capital shares are statistically different from other literature; that is, the estimates are consistent in the literature. Moreover, the estimates correspond to the conventional wisdom that consumption-goods production is relatively labor-intensive while equipment production is capital-intensive.

The most interesting features of the estimation are the parameters for cointegration, volatility, and persistence of external innovations. The existence of cointegration can be tested by evaluating the estimate of θ.[15] If $\theta = 0$, the error-correction term of non-linear VECM will vanish; it implies a regular VAR model. Applying the standard deviation of estimated θ, we can easily test the null hypothesis of $\theta = 0$; we can reject the null because the estimated θ of 0.9349 lies far outside two standard deviations from the null. Accordingly, we can confirm the cointegration of sectoral productivities with the maximum likelihood estimates. The persistence parameters of the innovations in common trend (ρ_{cc}, ρ_{ce}, ρ_{ec}, and ρ_{ee}) are estimated as 0.2986, zero, zero, and zero, respectively, which means that the persistence of common trend shocks is delivered to the next period only through the consumption goods channel. The correlation parameters of innovations in VECM (D_{ce} and D_{ec}) indicate that the innovations are significantly correlated: about 30 percent of the growth-rate innovation of equipment, $\epsilon_{ze,t}$, is correlated to that of consumption goods, $\epsilon_{zc,t}$. Plus, the estimated adjustment-speed parameters (γ_c and γ_e), respectively -0.1825 and 1.7946, indicate that most of the error-correction adjustment occurs in the equipment sector; that is, the productivity of the consumption-goods sector is weakly exogenous.

One of the most important features of the estimates is that all estimated external innovations are statistically significant. In the case of Ireland and Schuh (2008), the growth component of equipment productivity turned out to be statistically insignificant. Based on the estimation, they conclude that no equipment-sector-specific technology has had persistent effects on the postwar U.S. economy. The maximum likelihood estimates of this paper suggest, however, that the insignificant external innovation of equipment productivity is due to misspecification of their structural model, which

[15] The maximum likelihood estimates have asymptotically normal distributions. Therefore, for hypothesis tests, we can apply a t-test. See Canova (2007), pp. 225-228, for details.

assumes independent sectoral productivities rather than sectoral cointegration. Furthermore, it turned out that the largest disturbance among the external innovations is generated by a stochastic trend of sectoral productivities in the postwar U.S. data. The volatility of innovations in the common trend (σ_{zc} and σ_{ze}) are estimated as 0.0042 and 0.0200, respectively. The estimated volatilities of the other innovations (σ_{xl}, σ_{xg}, σ_{ac}, and σ_{ae}) are 0.0033, 0.0046, 0.0029, and 0.0086, respectively. The transitory level shock and the permanent growth-rate shock of preference are estimated with high persistence: the estimated autoregressive coefficients of the transitory and permanent shocks (ρ_{xl} and ρ_{xg}) are 0.8911 and 0.5493, respectively. However, the persistence of the transitory shocks of sectoral productivities (ρ_{ac} and ρ_{ae}) are estimated as zero; that is, there is no persistence in the transitory innovations of sectoral productivities.

Table 7: Empirical and simulated moments

	Relative volatility			Correlation with output growth		
	Data	Model	SU2011	Data	Model	SU2011
τ^Y	1.00	1.11	0.98			
τ^C	0.58	0.64	0.56	0.76	0.69	0.41
τ^I	3.33	3.18	2.45	0.88	0.92	0.67
τ^H	0.99	0.90	-	0.59	0.65	-

Notes: τ^i, for $i \in \{Y, C, I, H\}$, denotes the growth rate of output, consumption, investment, and hours worked, respectively. Relative volatility is computed as the standard deviation of a variable divided by the standard deviation of observed output. The column of "SU2011" is the second moments reported in Schmitt-Grohé and Uribe (2011). It is noteworthy that the consumption data of this paper only includes non-durable goods and service consumption and the investment data is constructed by aggregating durable consumption and equipment. The consumption in Schmitt-Grohé and Uribe (2011) includes durable goods, and the investment includes structure.

It is also worth investigating how the estimated model economy fits the real data. Table 7 presents observed and simulated second moments of the growth rate of output, consumption, investment, and hours worked. The second moments indicate that the model fits the data very well. The model replicates the volatility ranking of investment growth, output growth, hours worked growth, and consumption growth. The model also captures the procyclicality of consumption, investment, and hours worked. Because this paper considers production in a general way, which admits household production[16], and excludes structure investment and government spending from the data, a direct comparison of moments in the literature is somewhat difficult. In spite of the

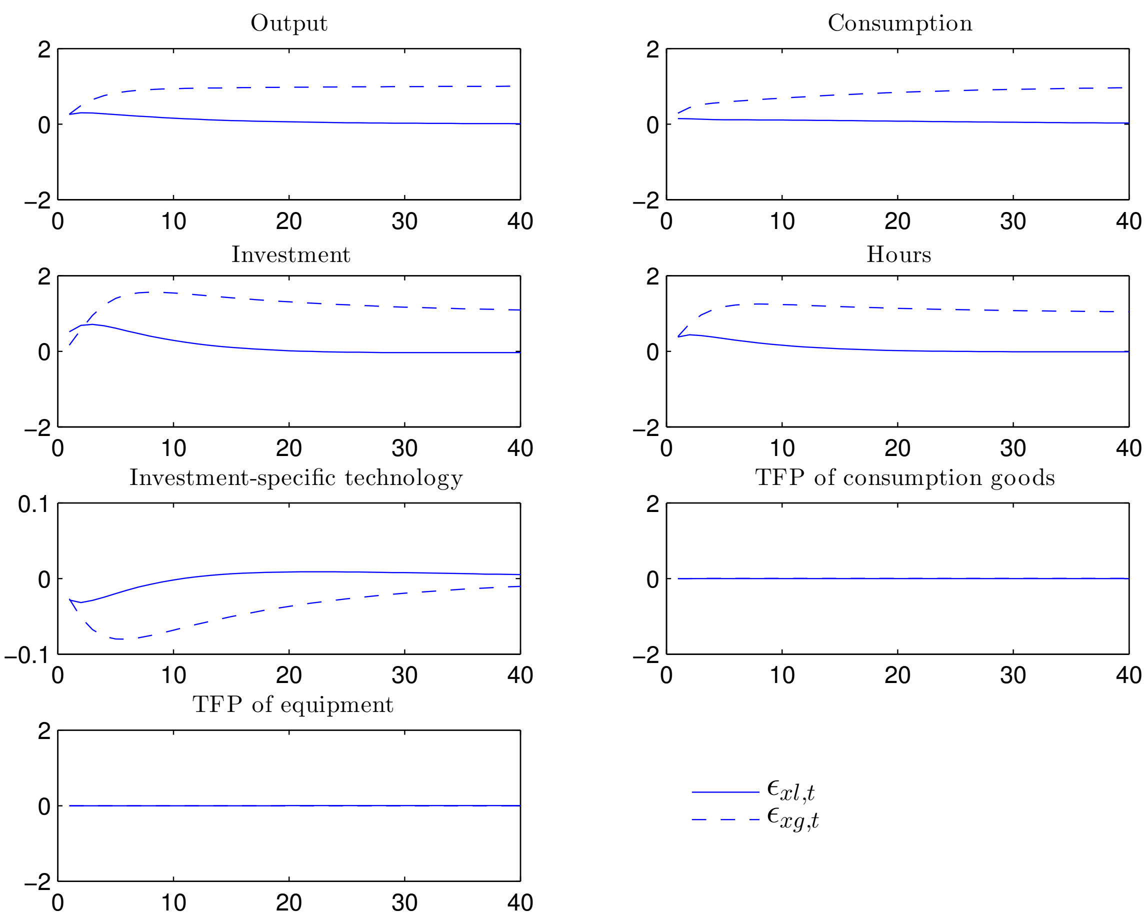
[16] By accepting household production, we can reclassify durable consumption into investment category because durable consumption is used to accumulate household capital, which is an input of household production.

difference in constructing consumption and investment, the volatility ranking and procyclicality of consumption and investment are consistent with Schmitt-Grohé and Uribe (2011).

5 Results

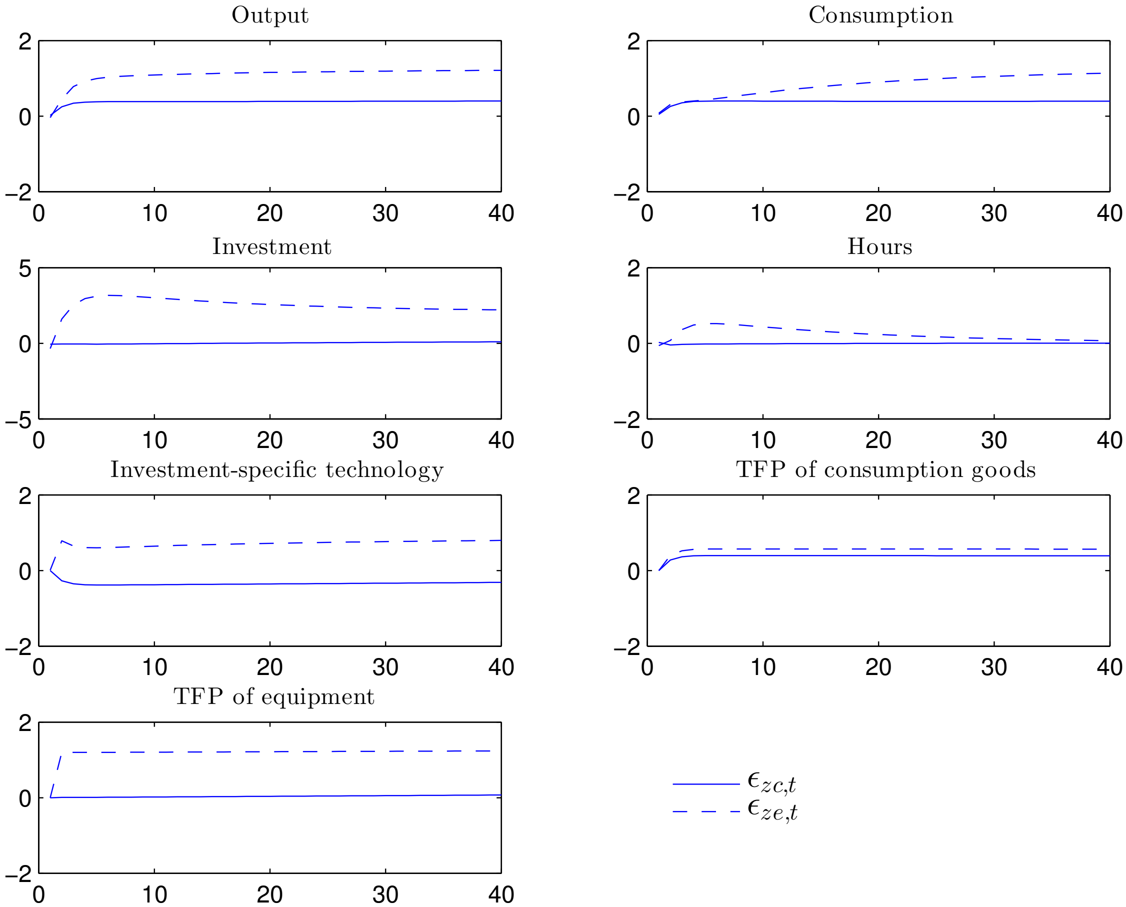
This section investigates the effects of each external shock and discusses its roles in the business cycle by analyses of impulse responses and variance decomposition. The impulse responses in FIGURES 2-4 depict the responses of output, consumption, investment, hours, IST, and sectoral productivities to a one-standard-deviation shock of each external innovation. FIGURE 2 displays the impulse responses to the transitory and permanent shocks in preference. FIGURES 3 and 4 exhibit the impulse responses to the shocks of sectoral productivities.

Figure 2: Impulse responses on preference shocks



Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, productivity of the consumption-goods sector, and productivity of the equipment sector to one-standard-deviation shocks of transitory and permanent innovations in preference.

Figure 3: Impulse responses on common trend shocks of sectoral productivities

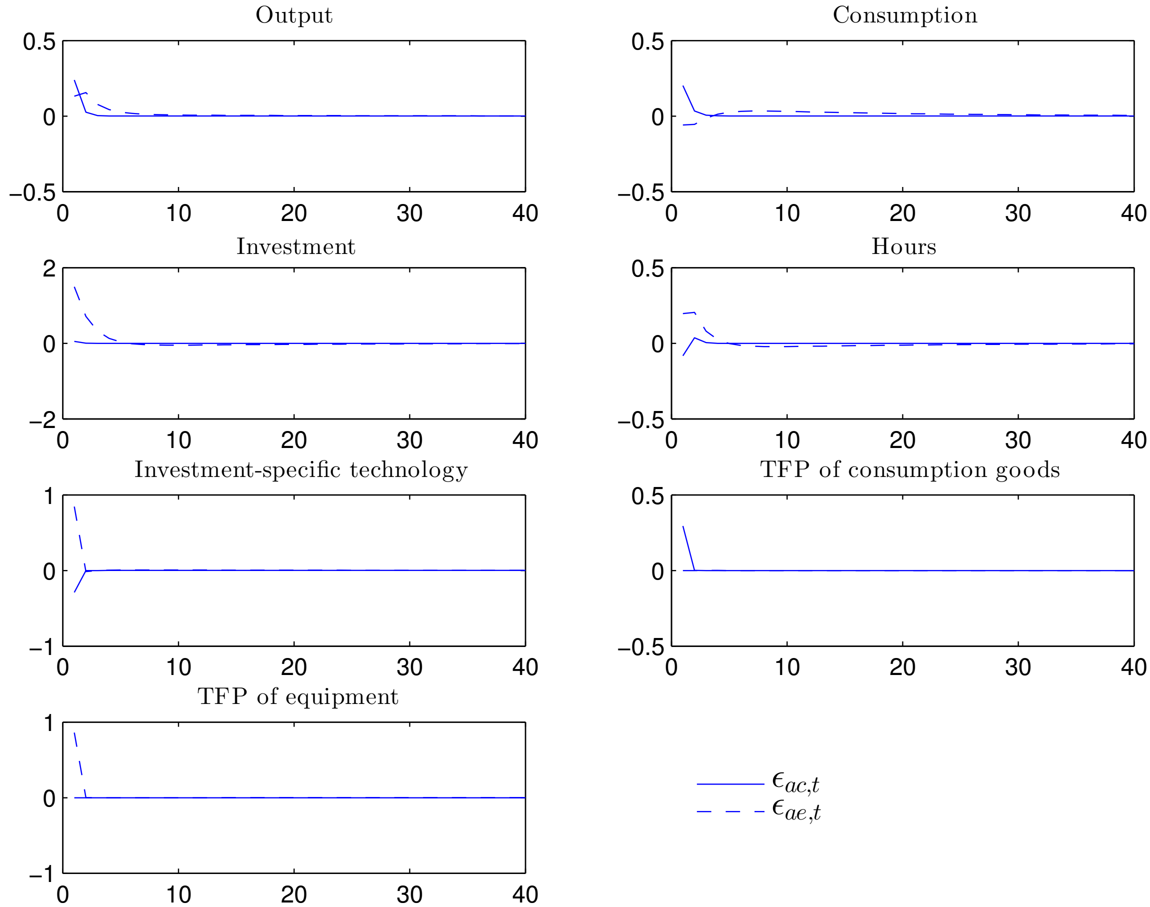


Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, productivity of the consumption-goods sector, and productivity of the equipment sector to one-standard-deviation shocks of common stochastic trend in sectoral productivities.

FIGURE 2 indicates that both transitory and permanent shocks in preference have positive effects on all four macroeconomic aggregates: output, consumption, investment, and hours worked. In particular, the permanent shock in preference has equally sizeable effects on all macroeconomic aggregates with high persistence. This result is consistent with Ireland and Schuh (2008); they find that, among other innovations, only the permanent shock of preference has a sizeable effect on hours worked over time. Since the preference shocks are not related to the changes in productivities, they have no effect on sectoral productivities.

Another notable implication of FIGURE 2 is the decrease of IST in the short run, which recovers its original level in the long run. This fact confirms Oulton (2007)'s argument: the relative price of equipment can change without the relative change of sectoral productivities. In the model economy,

Figure 4: Impulse responses on transitory productivity shocks



Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, productivity of the consumption goods sector, and productivity of the equipment sector to one-standard-deviation shocks of transitory innovations in sectoral productivities.

equipment production is capital-intensive while consumption production is labor-intensive; these are estimated rather than assumed. The positive preference shock increases the labor supply and subsequently pushes down the equilibrium wage. Accordingly, the production of consumption goods, which is labor-intensive, rises by accompanying a decrease in the price of consumption-goods. IST is therefore decreasing in the short run. As we can see in Figure 2, however, the magnitude of the effect is very limited. Consequently, we can say that Oulton's argument is correct but does not explain a significant degree of the fluctuation in IST.

As we can see in Figure 3, the shocks of common stochastic trend generally have persistent effects to the model, but the propagation paths differ for each source of shocks. The shock from $\epsilon_{ze,t}$ has a very sizeable effect on output, consumption, and investment. In particular, the effect on investment is much greater than that on consumption and remains for a long period of time. The shock on $\epsilon_{ze,t}$ increases the hours worked in the short run, and then hours decline rapidly to the original level. The shock from $\epsilon_{zc,t}$ primarily affects the productivity of consumption-goods. Whereas the shock of $\epsilon_{zc,t}$ also increases consumption persistently, the effect of $\epsilon_{zc,t}$ on the productivity of the equipment is negligible; subsequently, IST decreases almost permanently. Despite the decrease in IST, however, investment does not contract; it remains nearly unchanged.

The impulse responses to the transitory shocks of sectoral productivities have effects only for short periods of time. Since these shocks are not mutually correlated, there is no cross-over effect. As we can see in FIGURE 4, the transitory productivity shock to the consumption-goods sector, $\epsilon_{ac,t}$, has an effect only on consumption, while a positive transitory shock in equipment productivity, $\epsilon_{ae,t}$, leads to an increase of investment.

Table 8: Forecast-error variance decomposition

Quarters ahead	ϵ_{xl}	ϵ_{xg}	ϵ_{ac}	ϵ_{ae}	ϵ_{zc}	ϵ_{ze}
			Consumption			
1	14.0	52.5	26.2	2.2	1.3	3.8
4	4.2	49.5	2.5	0.4	20.0	23.4
8	2.5	48.6	0.9	0.2	20.1	27.6
12	1.9	47.3	0.5	0.2	17.7	32.5
20	1.1	44.9	0.2	0.1	13.7	40.0
40	0.5	41.7	0.1	0.0	9.4	48.4
			Investment			
1	10.1	1.0	0.1	84.0	0.2	4.6
4	6.8	10.9	0.0	11.3	0.0	70.9
8	3.7	15.8	0.0	3.8	0.0	76.7
12	2.5	17.6	0.0	2.4	0.0	77.4
20	1.6	18.9	0.0	1.5	0.0	78.0
40	0.9	19.4	0.0	0.9	0.0	78.8
			Hours worked			
1	41.9	43.8	2.0	11.1	0.2	1.0
4	16.5	71.4	0.2	2.2	0.1	9.5
8	8.6	78.1	0.1	0.8	0.0	12.5
12	5.8	81.9	0.0	0.5	0.0	11.7
20	3.6	86.5	0.0	0.3	0.0	9.5
40	2.0	91.8	0.0	0.2	0.0	6.0

Notes: The decomposed forecast error variances in consumption, investment, and hours worked are exhibited. The decomposition consists of the contribution of all 6 shocks to each forecast error variance.

TABLE 8 exhibits the decomposed forecast error variances of consumption, investment, and hours worked; the decomposition indicates the contribution of all six external shocks to the forecast error variances. About half of the error in the consumption forecast after two years depends on the permanent preference shocks. The transitory preference shock accounts for a small fraction of consumption variability. The other half of consumption variability is explained by the technology shocks: Among them, over 47 percent of consumption variability comes from the common trend shocks of sectoral productivities, whereas only a small fraction depends on the transitory shocks of sectoral productivities. The forecast error for investment is mostly explained by the common trend shocks of sectoral productivities; it is responsible for over 76 percent of the forecast error after two years. The transitory productivity shock in equipment accounts for most of the one-period-ahead forecast error for investment; however, its explanatory power declines rapidly with the increase in the forecast period. The remainder, around 19 percent, of the investment forecast error is due to preference shocks. Most of the forecast error for hours worked, over 86 percent, is associated with preference shocks after two years: the rest of the hours forecast error is explained by common trend shocks of sectoral productivities.

6 Conclusion

In this paper, I have investigated the role and effects of technology shocks in the U.S. business cycles by resolving the recently raised problems such as measuring IST and the existence of cointegration in sectoral productivities. Since Greenwood et al. (1997), most of the literature has identified IST as the relative price of investment, but this identification is rebutted because of a large measurement error in the relative price. The problem of measurement error is resolved in this paper by employing a neoclassical two-sector model where IST is identified as a ratio of sectoral marginal products. Besides, this paper documents the existence of cointegration in sectoral productivities, which is supported by the findings of Schmitt-Grohé and Uribe (2011). Furthermore, by introducing a non-linear vector error correction model into a neoclassical two-sector framework, I establish a dynamic stochastic general equilibrium model incorporating the cointegration of sectoral productivities. The estimated model via maximum likelihood implies that all estimated external innovations are statistically significant: This estimation result is unlike Ireland and Schuh (2008) that find an insignificant role of a permanent productivity shock of the equipment sector by assuming independent sectoral productivities. The subsequent simulation analysis indicates that the shocks to common stochastic trend in sectoral productivities not only have persistent effects on consumption, investment, and hours worked but also account for a large degree of the variability of consumption and investment.

This paper contributes to the literature in two ways. First, this paper supports the results of Fisher (2006) by resolving the recent critiques on the IST measurement and sectoral cointegration problems. Second, this paper documents that the sectoral productivities are cointegrated. To my knowledge, this paper is the first to show that the cointegration of sectoral cointegration can be non-linear. Furthermore, this paper suggests that to establish a stable DSGE model with VECM it is better to assume a non-linear function, such as an exponential smooth-transition (STR) function, for the error correction term.

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Appendix

A Proofs

A.1 Proof for Proposition 2

Suppose $\ln A_t$ and $\ln Q_t$ are cointegrated. Then, there exists a cointegrating vector $(1, \psi)$ such that $\ln A_t + \psi \ln Q_t = S_t^1$, where S_t^1 is a stationary stochastic process. Suppose a negation that there exist a cointegrating vector $(1, \mu_1, \mu_2)$ in the system of $(\ln A_t, \ln Q_t, \ln Z_{c,t})$ and assume that S_t^2 is another stationary process which is independent of S_t^1. Then,

$$
\begin{aligned}
& \ln A_t + \mu_1 \ln Q_t + \mu_2 \ln Z_{c,t} = S_t^2 \\
\rightarrow \quad & S_t^1 - \psi \ln Q_t + \mu_1 \ln Q_t + \mu_2 \ln Z_{c,t} = S_t^2 \\
\rightarrow \quad & (\mu_1 - \psi) \ln Q_t + \mu_2 \ln Z_{c,t} = S_t^2 - S_t^1 \\
\rightarrow \quad & (\mu_1 - \psi)(\ln Z_{e,t} - \ln Z_{c,t}) + \mu_2 \ln Z_{c,t} = S_t^2 - S_t^1 \\
\rightarrow \quad & (\mu_1 - \psi) \ln Z_{e,t} + (\mu_2 - \mu_1 + \psi) \ln Z_{c,t} = S_t^2 - S_t^1.
\end{aligned}
$$

Since the *RHS* of the above equation is stationary, the *LHS* must also be stationary. Since $\ln Z_e$ and $\ln Z_c$ are not cointegrated, the following conditions must be satisfied to make the *LHS* stationary:

$$
\begin{aligned}
& \mu_1 - \psi = 0, \ \ and \\
& \mu_2 - \mu_1 + \psi = 0,
\end{aligned}
$$

which implies $\mu_2 = 0$. However, $\mu_2 = 0$ contradicts the assumption that $(\ln A_t, \ln Q_t, Z_{c,t})$ is a cointegrated system. Therefore, $(\ln A_t, \ln Q_t, \ln Z_{c,t})$ is not cointegrated.

The proof for $(\ln A_t, \ln Q_t, \ln Z_{e,t})$ is omitted because of its similarity to the above $\square$

A.2 Proof for Proposition 3

Case1: $\ln A_t$ and $\ln Q_t$ are cointegrated $\Longrightarrow$ $\ln Z_{c,t}$ and $\ln Z_{e,t}$ are cointegrated.

Suppose $\ln Z_{c,t}$ and $\ln Z_{e,t}$ consist of random walk components ($\mu_{c,t}$ and $\mu_{e,t}$) and stationary components ($e_{c,t}$ and $e_{e,t}$) as follows:

$$\ln Z_{c,t} = \mu_{c,t} + e_{c,t}$$
$$\ln Z_{e,t} = \mu_{e,t} + e_{e,t},$$

then $\ln A_t$ and $\ln Q_t$ are represented as follows:

$$\begin{aligned}
\ln A_t &= \phi \ln Z_{c,t} + (1-\phi)\ln Z_{e,t} \\
&= \phi\mu_{c,t} + (1-\phi)\mu_{e,t} + \phi e_{c,t} + (1-\phi)e_{e,t} \\
\ln Q_t &= \ln Z_{e,t} - \ln Z_{c,t} \\
&= \mu_{e,t} - \mu_{c,t} + e_{e,t} - e_{c,t}.
\end{aligned}$$

Since $\ln A_t$ and $\ln Q_t$ are cointegrated, there exists a cointegrating vector $(1, \psi)$ such that $\ln A_t + \psi \ln Q_t = S_t^1$ where S_t^1 is a stationary process. $\ln A_t + \psi \ln Q_t$ can be rewritten as

$$\begin{aligned}
\ln A_t + \psi \ln Q_t &= \phi\mu_{c,t} + (1-\phi)\mu_{e,t} + \psi\mu_{e,t} - \psi\mu_{c,t} + S_t^3 \\
&= (\phi-\psi)\mu_{c,t} + (1-\phi+\psi)\mu_{e,t} + S_t^3,
\end{aligned}$$

where S_t^3 is a stationary process, defined as $\phi e_{c,t} + (1-\phi)e_{e,t} + \psi e_{e,t} - \psi e_{c,t}$. Suppose further that $\mu_{c,t}$ and $\mu_{e,t}$ are not cointegrated. Then, the cointegrated $\ln A_t$ and $\ln Q_t$ requires the following conditions:

$$\phi - \psi = 0, \;\; \textit{and}$$
$$1 - \phi + \psi = 0.$$

The two equations, however, cannot be solved simultaneously. Therefore, $\mu_{c,t}$ and $\mu_{e,t}$ have to be cointegrated, which further implies the cointegration of $\ln Z_{c,t}$ and $\ln Z_{e,t}$.

Case2: $\ln Z_{c,t}$ and $\ln Z_{e,t}$ are cointegrated $\Longrightarrow$ $\ln A_t$ and $\ln Q_t$ are cointegrated.

Since $\ln Z_{c,t}$ and $\ln Z_{e,t}$ are cointegrated, there exists a cointegrating vector $(1, \kappa)$ such that

$\ln Z_{c,t} + \kappa \ln Z_{e,t} = S_t$ with a stationary S_t. $\ln A_t$ and $\ln Q_t$ can be rewritten as

$$\ln A_t = (1 - \phi - \kappa\phi) \ln Z_{e,t} + \phi S_t$$
$$\ln Q_t = (1 + \kappa) \ln Z_{e,t} - S_t.$$

Then, there exists a linear combination for $\ln A_t$ and $\ln Q_t$ such that

$$\begin{aligned} \ln A_t - \frac{1 - \phi - \phi\kappa}{1+\kappa} \ln Q_t &= (1 - \phi - \kappa\phi) \ln Z_{e,t} + \phi S_t - (1 - \phi - \kappa\phi) \ln Z_{e,t} + \frac{1 - \phi - \kappa\phi}{1+\kappa} S_t \\ &= \frac{1}{1+\kappa} S_t. \end{aligned}$$

Therefore, $\ln A_t$ and $\ln Q_t$ are cointegrated with the cointegrating vector $\left(1, -\frac{1-\phi-\phi\kappa}{1+\kappa}\right)$ $\square$

B Stationary system of equations

The Household's Conditions

$$\lambda_{1,t} = \frac{1}{c_t - \xi c_{t-1}/\tau^c_{t-1}} - \beta\xi\mathbb{E}_t \frac{1}{c_{t+1}\tau^c_t - \xi c_t} \tag{B.0.1}$$

$$\frac{1}{x_{l,t}\eta^{xg}_t} = \lambda_{1,t}\tilde{w}_t \tag{B.0.2}$$

$$\begin{aligned} \lambda_{1,t}/q_t = \lambda_{2,t} &\left[1 - \frac{\psi}{2}\left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \tau^I\right)^2 - \psi\frac{i_t}{i_{t-1}}\tau^i_{t-1}\left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \tau^I\right)\right] \\ &+\beta\psi\mathbb{E}_t\lambda_{2,t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 \tau^i_t \left(\frac{i_{t+1}}{i_t}\tau^i_t - \tau^I\right) \end{aligned} \tag{B.0.3}$$

$$\lambda_{2,t}\tau^i_t = \beta\mathbb{E}_t\left\{\lambda_{1,t+1}\tilde{r}_{t+1} + \lambda_{2,t+1}(1-\delta)\right\} \tag{B.0.4}$$

$$c_t + i_t/q_t = \tilde{w}_t h_t + \tilde{r}_t k_t \tag{B.0.5}$$

$$k_{t+1}\tau^i_t = (1-\delta)k_t + i_t\left[1 - \frac{\psi}{2}\left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \tau^I\right)^2\right] \tag{B.0.6}$$

The Firms' Conditions

$$y_{c,t} = a_{c,t}(k_{c,t})^{\alpha_c}(\eta_t^{zc}h_{c,t})^{1-\alpha_c} \tag{B.0.7}$$

$$y_{e,t} = a_{e,t}(k_{e,t})^{\alpha_e}(\eta_t^{ze}h_{e,t})^{1-\alpha_e} \tag{B.0.8}$$

$$\tilde{r}_t = \alpha_c y_{c,t}/k_{c,t} \tag{B.0.9}$$

$$\tilde{w}_t = (1-\alpha_c)y_{c,t}/h_{c,t} \tag{B.0.10}$$

$$q_t = \frac{\alpha_e y_{e,t}/k_{e,t}}{\alpha_c y_{c,t}/k_{c,t}} \tag{B.0.11}$$

Market Clearing Conditions

$$k_t = k_{c,t} + k_{e,t} \tag{B.0.12}$$

$$h_t = h_{c,t} + h_{e,t} \tag{B.0.13}$$

$$c_t = y_{c,t} \tag{B.0.14}$$

$$i_t = y_{e,t} \tag{B.0.15}$$

$$\tilde{y}_t = y_{c,t} + y_{e,t}/q_t \tag{B.0.16}$$

Growth Rates

$$\tau_t^c = (\eta_t^{zc})^{1-\alpha_c}(\eta_t^{ze})^{\alpha_c}\eta_t^{xg} \tag{B.0.17}$$

$$\tau_t^i = \eta_t^{ze}\eta_t^{xg} \tag{B.0.18}$$

$$\tau_t^h = \eta_t^{xg} \tag{B.0.19}$$

Observable Variables

$$\tau_t^C = \tau_{t-1}^c \frac{c_t}{c_{t-1}} \tag{B.0.20}$$

$$\tau_t^I = \tau_{t-1}^i \frac{i_t}{i_{t-1}} \tag{B.0.21}$$

$$\tau_t^H = \tau_{t-1}^h \frac{h_t}{h_{t-1}} \tag{B.0.22}$$

Exogenous Stochastic Processes

$$ect_t - ect_{t-1} = \ln \eta_t^{zc} - \kappa \ln \eta_t^{ze} \tag{B.0.23}$$

$$\begin{bmatrix} \ln\left(\eta_t^{zc}/\eta^{zc}\right) \\ \ln\left(\eta_t^{ze}/\eta^{ze}\right) \end{bmatrix} = \begin{bmatrix} \rho_{cc} & \rho_{ce} \\ \rho_{ec} & \rho_{ee} \end{bmatrix} \begin{bmatrix} \ln\left(\eta_{t-1}^{zc}/\eta^{zc}\right) \\ \ln\left(\eta_{t-1}^{ze}/\eta^{ze}\right) \end{bmatrix} + \begin{bmatrix} f_c(ect_{t-1}) \\ f_e(ect_{t-1}) \end{bmatrix} + \begin{bmatrix} D_{cc} & D_{ce} \\ D_{ec} & D_{ee} \end{bmatrix} \begin{bmatrix} \epsilon_{zc,t} \\ \epsilon_{ze,t} \end{bmatrix} \tag{B.0.24}$$

$$\ln x_{l,t} = \rho_{x,l} \ln x_{l,t-1} + \epsilon_{xl,t} \tag{B.0.25}$$

$$\ln(\eta_t^{xg}/\eta^{xg}) = \rho_{xg} \ln(\eta_{t-1}^{xg}/\eta^{xg}) + \epsilon_{xg,t} \tag{B.0.26}$$

$$\ln a_{c,t} = \rho_{ac} \ln a_{c,t-1} + \epsilon_{ac,t} \tag{B.0.27}$$

$$\ln a_{e,t} = \rho_{ae} \ln a_{e,t-1} + \epsilon_{ae,t} \tag{B.0.28}$$