Liquidity Risk and Volatility under Two-Tiered Asymmetric Information

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Abstract

The paper investigates how the 'degree of ambiguity' affects liquidity risk, expected price sensitivity, and price volatility in asset markets. We analyze asset market equilibrium under two-tiered asymmetric information by introducing uninformed traders with ambiguity into the model of Grossman and Stiglitz (1980) without endogenous information acquisition.

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1. Introduction

Asset prices under asymmetric information have been extensively studied for several decades. Among others, Grossman and Stiglitz (1980) examine rational expectations equilibrium under asymmetric information assuming that uninformed traders know the distribution of a risky asset's true value. Recently, financial crises have motivated economists to pay more attention to ambiguity, which represents uncertainty about the distribution of the risky asset's true value. Along the line of Grossman and Stiglitz (1980), Mele and Sangiorgi (2011) and Ozsoylev and Werner (2011) study asset markets with ambiguity, assuming that all the uninformed traders face ambiguity.

In reality, however, uninformed traders may or may not face ambiguity according to the levels of their knowledge, education, and experiences. In other words, uninformed traders with and without ambiguity may coexist in asset markets. Unlike the aforementioned literature, the paper reflects this point by dividing the uninformed traders into traders with and without ambiguity.[1] This scheme leads to two-tiered asymmetric information: the first tier lies between informed and uninformed traders, while the second tier between uninformed traders with and without ambiguity.

The paper attempts to characterize equilibrium asset price under two-tiered asymmetric information. In particular, we investigate how the 'degree of ambiguity' of asset markets affects liquidity risk, expected price sensitivity, and price volatility in asset market equilibrium. To do this, the uninformed traders with ambiguity are introduced into the model of Grossman and Stiglitz (1980). The two-tiered asymmetric information allows us to conduct a refined analysis of ambiguity effects on asset market equilibrium. Asset market equilibrium is analyzed by taking into account both the individual degree of ambiguity and the fraction of traders with ambiguity, which constitute the degree of ambiguity at the market level.

2. The Model

In a two-period economy, there are two assets: a risk-free bond and a risky asset. The economy is populated by a continuum of traders, indexed in the interval $[0, 1]$. Taking the bond as the numeraire, let p be the price of the risky asset in the first period, when trader t invests his initial wealth w_t between b_t shares of the bond and x_t shares of the risky asset with the budget constraint $b_t + px_t = w_t$. In the second period, the bond and the risky asset yield 1 and $\tilde{v}$, respectively, and thus his portfolio (b_t, x_t) yields wealth $w'_t = w_t + (\tilde{v} - p)x_t$. The payoff $\tilde{v}$ of the risky asset is the sum of true value $\tilde{\theta}$ and noise $\tilde{\varepsilon}$: $\tilde{v} = \tilde{\theta} + \tilde{\varepsilon}$, where $\tilde{\theta}$ and

[1] Easley and O'Hara (2010) consider this case without informed traders in a different context.

$\tilde{\varepsilon}$ are normal random variables with means μ and 0 and variances σ_θ^2 and σ_ε^2, respectively. Random supply $\tilde{z}$ of the risky asset is also assumed to be normally distributed with mean 0 and variance σ_z^2. All random variables are independent. All the traders have rational expectations so that they understand the functional relationship $\tilde{p}$ between p and (θ, z) with $\tilde{p}(\theta, z) = p$. They have the same CARA utility function with the coefficient of constant absolute risk aversion $\alpha > 0$: $u(c) = -\exp(-\alpha c)$.

As mentioned in the introduction, all the traders are divided into three groups: informed traders, uninformed traders without ambiguity (we refer to as *non-ambiguous traders*), and uninformed traders with ambiguity (we refer to as *ambiguous traders*). Informed traders observe realization θ of $\tilde{\theta}$ with p, while uninformed traders only observe p. Non-ambiguous traders know the distribution of $\tilde{\theta}$, while ambiguous traders only know that $\mu \in [\underline{\mu}, \bar{\mu}]$ with the exact information about σ_θ^2.[2] Length $\Delta\mu = \bar{\mu} - \underline{\mu}$ of the interval is called *individual degree of ambiguity*. Thus, the first tier of asymmetric information about θ exists between informed traders and uninformed traders, while the second tier of asymmetric information about the distribution of θ exists between non-ambiguous traders and ambiguous traders.

All traders in each group are identical. Let $\lambda_1 \in (0, 1)$ denote the fraction of informed traders and $\lambda_2 \in [0, 1]$ that of non-ambiguous traders among uninformed traders. Note that we exclude the case where all traders are either informed or uninformed. It is assumed that λ_1 and λ_2 are exogenously given so that there is no endogenous information acquisition. Our model reduces to that of Grossman and Stiglitz (1980) when $\lambda_2 = 1$ or $\Delta\mu = 0$ and to that of Mele and Sangiorgi (2009) when $\lambda_2 = 0$.[3]

For the optimal portfolio choice, informed trader i with initial wealth w_i solves

$$\max_{x_i} \mathbb{E}[-\exp\left(-\alpha[w_i + (\tilde{v} - p)x_i]\right) |(\tilde{p}, \tilde{\theta}) = (p, \theta)]$$

and his demand for the risky asset is given by

$$x_i(p, \theta) = \frac{\theta - p}{\alpha\sigma_\varepsilon^2}. \tag{2.1}$$

Non-ambiguous trader n with initial wealth w_n solves

$$\max_{x_n} \mathbb{E}\left[-\exp\left(-\alpha[w_n + (\tilde{v} - p)x_i]\right) |\tilde{p} = p\right]$$

and his demand for the risky asset is given by

$$x_n(p, \tilde{p}) = \frac{\mathbb{E}_\mu[\tilde{v}|\tilde{p} = p] - p}{\alpha \text{Var}[\tilde{v}|\tilde{p} = p]}. \tag{2.2}$$

[2]In general, ambiguous traders may not know either the distribution of μ or σ_θ^2. For simplicity, however, the paper considers only ambiguous information about μ.

[3]Here we consider the case where there is no information acquisition in their models.

Ambiguous trader a chooses the optimal portfolio according to the maxmin expected utility of Gilboa and Schmeidler (1989). Thus he solves

$$\max_{x_a} \min_{\mu_a \in [\underline{\mu}, \bar{\mu}]} \mathbb{E}_{\mu_a} \left[-\exp\left(-\alpha[w_a + (\tilde{v} - p)x_a]\right) \middle| \, \tilde{p} = p \right],$$

where w_a is his initial wealth. Then his demand for the risky asset is given by

$$x_a(p, \tilde{p}) = \begin{cases} \dfrac{\mathbb{E}_{\underline{\mu}}[\tilde{v}|\tilde{p} = p] - p}{\alpha \mathrm{Var}[\tilde{v}|\tilde{p} = p]}, & \text{if} \quad p < \mathbb{E}_{\underline{\mu}}[\tilde{v}|\tilde{p} = p], \\ 0, & \text{if} \quad \mathbb{E}_{\underline{\mu}}[\tilde{v}|\tilde{p} = p] \leq p \leq \mathbb{E}_{\bar{\mu}}[\tilde{v}|\tilde{p} = p], \\ \dfrac{\mathbb{E}_{\bar{\mu}}[\tilde{v}|\tilde{p} = p] - p}{\alpha \mathrm{Var}[\tilde{v}|\tilde{p} = p]}, & \text{if} \quad p > \mathbb{E}_{\bar{\mu}}[\tilde{v}|\tilde{p} = p]. \end{cases} \tag{2.3}$$

It is noted that ambiguous traders participate in trading of the risky asset when its price is sufficiently low or sufficiently high for them. This means that ambiguous traders are cautious to take positions in the risky asset.

3. Asset Market Equilibrium

We adopt the notion of rational expectations equilibrium in Grossman and Stigltiz (1980). By (2.1)–(2.3), the (risky) asset market is cleared at p if

$$\lambda_1 x_i(p, \theta) + (1 - \lambda_1)\lambda_2 x_n(p, \tilde{p}) + (1 - \lambda_1)(1 - \lambda_2)x_a(p, \tilde{p}) = z. \tag{3.1}$$

Following Grossman and Stiglitz (1980), we define a function $\tilde{s}$ as

$$\tilde{s}(\theta, z) = \theta - \frac{\alpha \sigma_\varepsilon^2}{\lambda_1} z,$$

whose realization is denoted by s. Note that $\tilde{s}$ is a normal random variable with mean μ and variance $\sigma_s^2 = \sigma_\theta^2 + \alpha^2 \sigma_\varepsilon^4 \sigma_z^2 / \lambda_1^2$. Given θ, function $\tilde{s}$ provides a partial information about θ, in which sense it is sometimes called a signal function. Let us conjecture that equilibrium asset price is represented by a function P of s such that $P(s) = P(\tilde{s}(\theta, z)) = \tilde{p}(\theta, z)$ with $\tilde{s}(\theta, z) = s$, which is verified by Theorem 3.1 below. For simplicity, henceforth we set $\mu = 0$ and $\underline{\mu} = -\bar{\mu}$, so that $\Delta\mu = 2\bar{\mu}$.

Theorem 3.1. *There exists a unique rational expectations equilibrium asset price function given by*

$$P(s) = (\underline{\kappa} + \zeta s)\mathbf{1}_{(-\infty, \underline{s})}(s) + \zeta_1 s \mathbf{1}_{[\underline{s}, \bar{s}]}(s) + (\bar{\kappa} + \zeta s)\mathbf{1}_{(\bar{s}, \infty)}(s), \tag{3.2}$$

where $\mathbf{1}_A(\cdot)$ *is an indicator function for a set* A *in* $\mathbb{R}$ *and*

$$\begin{aligned}
\zeta &= \frac{\lambda_1(\lambda_1\sigma_\theta^2+\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2+\alpha^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda_1^2\sigma_\theta^2+\lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2+\alpha^2\sigma_\varepsilon^4\sigma_z^2},\\
\zeta_1 &= \frac{\lambda_1[(1-\lambda_1)\lambda_1\lambda_2\sigma_\theta^2+\lambda_1^2\sigma_\theta^2+\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2+\alpha^2\sigma_\varepsilon^4\sigma_z^2]}{\{(1-\lambda_1)\lambda_2+\lambda_1\}(\lambda_1^2\sigma_\theta^2+\alpha^2\sigma_\varepsilon^4\sigma_z^2)+\lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2},\\
\bar{\kappa} = -\underline{\kappa} &= \frac{(1-\lambda_1)(1-\lambda_2)\alpha^2\sigma_\varepsilon^4\sigma_z^2\Delta\mu}{2(\lambda_1^2\sigma_\theta^2+\lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2+\alpha^2\sigma_\varepsilon^4\sigma_z^2)},\\
\bar{s} = -\underline{s} &= \frac{[\{(1-\lambda_1)\lambda_2+\lambda_1\}(\lambda_1^2\sigma_\theta^2+\alpha^2\sigma_\varepsilon^4\sigma_z^2)+\lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2]\Delta\mu}{2\lambda_1(\lambda_1^2\sigma_\theta^2+\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2+\alpha^2\sigma_\varepsilon^4\sigma_z^2)}.
\end{aligned}$$

PROOF : See the appendix. ■

The equilibrium asset price function P is piecewise linear in s. Since P strictly increases in s by (3.2), the information from observed asset price p is equivalent to that from s and therefore it gives only partial information about θ to the uniformed traders. The price function has kinks at $\underline{s}$ and $\bar{s}$. This is because ambiguous traders do not trade when $s \in [\underline{s}, \bar{s}]$. Thus we call the interval $[\underline{s}, \bar{s}]$ *non-participation region* of ambiguous traders. The size of non-participation region is given by $\Delta s = 2\bar{s}$, which increases in individual degree of ambiguity $\Delta\mu$ and decreases in the fraction $(1-\lambda_2)$ of ambiguous traders among uninformed traders.

Let $\rho \equiv (1-\lambda_2)\Delta\mu \geq 0$. We say that ambiguity is absent if there are no ambiguous traders or the individual degree of ambiguity is zero, i.e., $\rho = 0$ and that ambiguity is present otherwise.

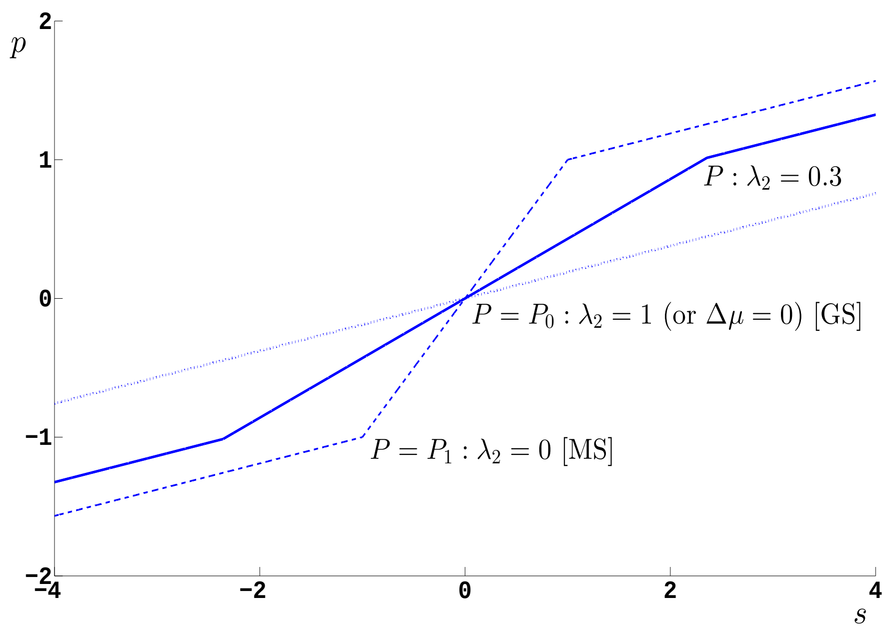

Fig. 1. Equilibrium price function when $(\lambda_1, \Delta\mu, \alpha, \sigma_\theta^2, \sigma_\varepsilon^2, \sigma_z^2) = (0.1, 2, 1, 1, 1, 1)$

Corollary 3.1. *The following hold.*

(1) If ambiguity is absent, i.e., $\rho = 0$, then the equilibrium asset price P becomes

$$P_0(s) := \zeta s, \ \forall\, s \in \mathbb{R}.$$

(2) If all the uniformed traders are ambiguous, i.e., $\lambda_2 = 0$, then P becomes

$$P_1(s) := (\underline{\kappa}' + \zeta s)\mathbf{1}_{(-\infty,\underline{s})}(s) + s\mathbf{1}_{[\underline{s},\bar{s}]}(s) + (\bar{\kappa}' + \zeta s)\mathbf{1}_{(\bar{s},\infty)}(s), \ \forall\, s \in \mathbb{R}$$

where

$$\bar{\kappa}' = -\underline{\kappa}' = -\frac{(1-\lambda_1)\alpha^2\sigma_\varepsilon^4\sigma_z^2\Delta\mu}{2(\lambda_1^2\sigma_\theta^2 + \lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)}.$$

Grossman and Stiglitz (GS, 1980) and Mele and Sangiorgi (MS, 2011) correspond to (1) and (2) of Corollary 3.1, respectively, which are illustrated in Figure 1.

4. Effects of Ambiguity on Asset Market Equilibrium

Using a slightly different model where there are a single risk-averse informed trader, a single risk-neutral uninformed trader with ambiguity, and noisy traders, Ozsoylev and Werner (2011) analyze effects of ambiguity on liquidity risk, price sensitivity, and excess volatility.[4] Under two-tiered asymmetric information, we examine similar issues in view of both the individual degree of ambiguity and the fraction of ambiguity traders.

Now let us define

$$\eta = \frac{\bar{s}}{\sqrt{2\sigma_s^2}}, \quad \mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}}\int_0^\eta \exp\left(-t^2\right) dt, \ \text{ and } \ f_s(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}}\exp\left(-\frac{s^2}{2\sigma_s^2}\right).$$

Note that $\mathrm{erf}(\eta) = 2\int_0^{\bar{s}} f_s(s)ds$, which is the probability that an ambiguous trader does not trade.

4.1. Liquidity Risk

In Ozsoylev and Werner (2011), liquidity risk is defined as the probability that asset price lies in the non-participation region of ambiguous traders. In our model, the liquidity risk is defined by the probability that a trader does not trade the risky asset, i.e.,

$$L = (1-\lambda_1)(1-\lambda_2)\int_{\underline{s}}^{\bar{s}} f_s(s)ds = (1-\lambda_1)(1-\lambda_2)\mathrm{erf}(\eta).$$

This consists of two parts: the population of ambiguous traders and the probability that an ambiguous trader does not trade. In particular, $L = 0$ if $\rho = 0$. When $\rho > 0$, the next

[4] In their model, ambiguity is the uncertainty about both mean and variance of the true value.

proposition shows how ambiguity affects liquidity risk. Let us consider a strictly increasing function of $\Delta\mu > 0$, which is given by

$$h(\Delta\mu) \equiv \sqrt{\pi}\mathrm{erf}\left(\frac{\Delta\mu}{2\sqrt{2\sigma_s^2}}\right)\left[\Delta\mu \exp\left(-\frac{(\Delta\mu)^2}{8\sigma_s^2}\right)\right]^{-1}.$$

Proposition 4.1. *If $\rho > 0$, then the following hold.*

(1) Liquidity risk L increases in the individual degree of ambiguity $\Delta\mu$.

(2) L increases in the fraction $(1-\lambda_2)$ of ambiguous traders among uninformed traders if $\Delta\mu$ is sufficiently large such that $h(\Delta\mu) \geq 1/\lambda_1 - 1$.

PROOF : (1) It is straightforward since $\mathrm{erf}(\eta)$ increases in $\Delta\mu$. □

(2) Since $\partial\eta/\partial\lambda_2 < (1/\lambda_1 - 1)\Delta\mu/2$ and $h(\Delta\mu) \geq 1/\lambda_1 - 1$, we have

$$\begin{aligned}
\left.\frac{\partial L}{\partial \lambda_2}\right|_{\lambda_2=0} &= -(1-\lambda_1)\left[\mathrm{erf}\left(\frac{\Delta\mu}{2\sqrt{2\sigma_s^2}}\right) - \frac{2}{\sqrt{\pi}}\exp\left(-\frac{(\Delta\mu)^2}{8\sigma_s^2}\right)\frac{\partial\eta}{\partial\lambda_2}\right] \\
&< -\frac{(1-\lambda_1)\Delta\mu}{\sqrt{\pi}}\exp\left(-\frac{(\Delta\mu)^2}{8\sigma_s^2}\right)\left[h(\Delta\mu) - \left(\frac{1}{\lambda_1} - 1\right)\right] \\
&\leq 0.
\end{aligned}$$

Moreover, we also have $\partial^2 L/\partial\lambda_2^2 < 0$ for all $\lambda_2 \in [0,1]$. Consequently, it follows that $\partial L/\partial\lambda_2 < 0$ for all $\lambda_2 \in [0,1]$, which implies the claim. ■

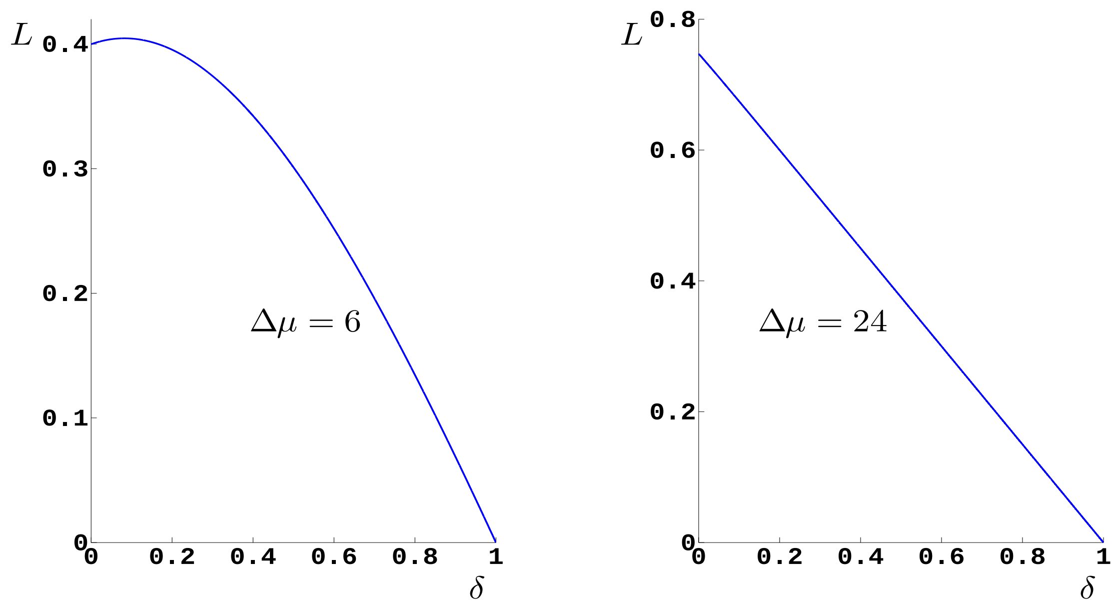

Fig. 2. Liquidity risk when $(\lambda_1, \alpha, \sigma_\theta^2, \sigma_\varepsilon^2, \sigma_z^2) = (0.25, 1, 1, 1, 1)$

If the individual degree of ambiguity $\Delta\mu$ increases, so does the probability that an ambiguous trader does not trade, which implies that liquidity risk increases. When $(1-\lambda_2)$ increases, it increases the population of ambiguous traders but decreases the probability that an ambiguous trader does not trade. However, if $\Delta\mu$ is sufficiently large, it turns out that the first effect dominates the second, so as to increase liquidity risk. Otherwise, liquidity risk may increase, achieve the maximum, and then decrease in $(1-\lambda_2)$, as illustrated by the left graph of Figure 2. The right graph of Figure 2 illustrates the case where liquidity risk always increases in $(1-\lambda_2)$.

4.2. Price Sensitivity

One unit change of asset supply z moves the asset price by $\alpha\sigma_\varepsilon^2\zeta/\lambda_1$ on $[\underline{s},\bar{s}]^c$ and by $\alpha\sigma_\varepsilon^2\zeta_1/\lambda_1$ on $[\underline{s},\bar{s}]$. Thus we can define price sensitivity to asset supply z as

$$\psi(s)=\frac{\alpha\sigma_\varepsilon^2}{\lambda_1}\left[\zeta\mathbf{1}_{[\underline{s},\bar{s}]^c}(s)+\zeta_1\mathbf{1}_{[\underline{s},\bar{s}]}(s)\right].$$

In particular, the price sensitivity in Grossman and Stiglitz (1980) is given by $\alpha\sigma_\varepsilon^2\zeta/\lambda_1$. The presence of ambiguity leads to the relation $\zeta_1>\zeta$, implying that asset price is more sensitive on $[\underline{s},\bar{s}]$ than on $[\underline{s},\bar{s}]^c$. Since the price sensitivity depends on s, it seems to be appropriate to use *expected price sensitivity* $\mathbb{E}[\psi]$ in analyzing the overall effect of ambiguity on asset market equilibrium.

Proposition 4.2. *If $\rho>0$, then the following hold.*

(1) Expected price sensitivity $\mathbb{E}[\psi]$ is greater under ambiguity than under no ambiguity by $\alpha\sigma_\varepsilon^2(\zeta_1-\zeta)\mathrm{erf}(\eta)/\lambda_1>0$.
(2) $\mathbb{E}[\psi]$ increases in the individual degree of ambiguity $\Delta\mu$.
(3) $\mathbb{E}[\psi]$ increases in the fraction $(1-\lambda_2)$ of ambiguous traders among uninformed traders.

PROOF : (1) It is obvious that $\mathbb{E}[\psi]=\alpha\sigma_\varepsilon^2\zeta/\lambda_1$ when $\rho=0$. If $\rho>0$, we have

$$\mathbb{E}\left[\psi\right]=\frac{\alpha\sigma_\varepsilon^2}{\lambda_1}\left[\zeta+(\zeta_1-\zeta)\int_{\underline{s}}^{\bar{s}}f_s(s)ds\right]=\frac{\alpha\sigma_\varepsilon^2}{\lambda_1}\zeta+\frac{\alpha\sigma_\varepsilon^2}{\lambda_1}(\zeta_1-\zeta)\mathrm{erf}(\eta).$$

Since $\zeta_1>\zeta$ and $\eta>0$, it holds that $(\zeta_1-\zeta)\mathrm{erf}(\eta)>0$, which implies the claim. ☐

(2) It holds since $\mathrm{erf}(\eta)$ increases in $\Delta\mu$. ☐

(3) The partial derivative of $\mathbb{E}[\psi]$ with respect to λ_2 is given by

$$\frac{\partial\mathbb{E}\left[\psi\right]}{\partial\lambda_2}=\frac{\alpha\sigma_\varepsilon^2}{\lambda_1}\left[\frac{\partial\zeta_1}{\partial\lambda_2}\mathrm{erf}(\eta)+\frac{2(\zeta_1-\zeta)}{\sqrt{\pi}}\exp\left(-\eta^2\right)\frac{\partial\eta}{\partial\lambda_2}\right].$$

Let $g(\Delta\mu) \equiv \partial\mathbb{E}[\psi]/\partial\lambda_2$. Since $\text{erf}(\eta) = \partial\eta/\partial\lambda_2 = 0$ if $\Delta\mu = 0$, we have $g(0) = 0$ and, for all $\Delta\mu > 0$,

$$g'(\Delta\mu) = -\frac{2\alpha\sigma_\varepsilon^2}{\sqrt{\pi}\lambda_1}\exp(-\eta^2)\left[\frac{(1-\lambda)\alpha^2\sigma_\varepsilon^4\sigma_z^2}{\sqrt{2\sigma_s^2(\lambda_1^2\sigma_\theta^2 + \lambda\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)}} + 2(\zeta_1 - \zeta)\eta\frac{\partial\eta}{\partial(\Delta\mu)}\right] < 0.$$

Thus $g(\Delta\mu) < 0$ for all $\Delta\mu > 0$, so that $\partial\mathbb{E}[\psi]/\partial\lambda_2 < 0$. Hence the claim follows. ■

Intuitively, (2) of Proposition 4.2 follows since an increase of $\Delta\mu$ widens the non-participation region holding the corresponding slope constant $\alpha\sigma_\varepsilon^2\zeta_1/\lambda_1$. If $(1-\lambda_2)$ increases, the size Δs of non-participation region decreases, while $\alpha\sigma_\varepsilon^2\zeta_1/\lambda_1$ on $[\underline{s}, \bar{s}]$ increases since

$$\frac{\partial\zeta_1}{\partial\lambda_2} = \frac{-(1-\lambda_1)\lambda_1\alpha^2\sigma_\varepsilon^4\sigma_z^2(\lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)}{[(1-\lambda_1)\lambda_2(\lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2) + \lambda_1(\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)]^2} < 0.$$

However, from (3) of Proposition 4.2, we see that the latter effect dominates the former one.

A closely related notion with price sensitivity is market depth. The asset market is said to be deep when the price can absorb random supply without much variation. Kyle (1985) measures market depth by the inverse of price sensitivity. In our model, we can define *expected market depth* by the reciprocal of the expected price sensitivity. Hence, Proposition 4.2 implies that the asset market is deeper under no ambiguity than under ambiguity. Furthermore, the market depth decreases in $\Delta\mu$ and $(1-\lambda_2)$.

4.3. *Price Volatility*

From the results for expected price sensitivity of Proposition 4.2, one can expect that price volatility σ_P^2 is greater under ambiguity than under no ambiguity and moreover increases as the 'degree of ambiguity' increases. The following proposition verifies that this is true.

Proposition 4.3. *If $\rho > 0$, then the following hold.*

(1) Price volatility σ_P^2 is greater under ambiguity than under no ambiguity by 2ξ where

$$\xi = \bar{\kappa}^2\int_{\bar{s}}^{\infty} f_s(s)ds + 2\bar{\kappa}\zeta\int_{\bar{s}}^{\infty} sf_s(s)ds + \left(\zeta_1^2 - \zeta^2\right)\int_0^{\bar{s}} s^2 f_s(s)ds > 0.$$

(2) σ_P^2 increases in the individual degree of ambiguity $\Delta\mu$.

(3) σ_P^2 increases in the fraction $(1-\lambda_2)$ of ambiguous traders among uninformed traders.

Proof : (1) Noting $\mathbb{E}[P(s)] = 0$, we see

$$\sigma_P^2 = \zeta^2\sigma_s^2 + 2\left[\bar{\kappa}^2\int_{\bar{s}}^{\infty} f_s(s)ds + 2\bar{\kappa}\zeta\int_{\bar{s}}^{\infty} sf_s(s)ds + \left(\zeta_1^2 - \zeta^2\right)\int_0^{\bar{s}} s^2 f_s(s)ds\right] = \sigma_{P_0}^2 + 2\xi$$

where $\sigma^2_{P_0}$ is price volatility when $\rho = 0$. Since $\xi > 0$ if and only if $\rho > 0$, we have $\sigma^2_P > \sigma^2_{P_0}$. $\square$

(2) Noting that $\zeta_1 \bar{s} = \bar{\kappa} + \zeta \bar{s}$, we have

$$\frac{\partial \sigma^2_P}{\partial(\Delta\mu)} = 2\left(2\bar{\kappa}\int_{\bar{s}}^{\infty} f_s(s)ds + 2\zeta\int_{\bar{s}}^{\infty} s f_s(s)ds\right)\frac{\partial\bar{\kappa}}{\partial(\Delta\mu)} > 0,$$

which implies the claim. $\square$

(3) Similarly, the claim holds since

$$\frac{\partial \sigma^2_P}{\partial \lambda_2} = 4\left(\bar{\kappa}\frac{\partial\bar{\kappa}}{\partial\lambda_2}\int_{\bar{s}}^{\infty} f_s(s)ds + \zeta\frac{\partial\bar{\kappa}}{\partial\lambda_2}\int_{\bar{s}}^{\infty} s f_s(s)ds + \frac{\partial\zeta_1}{\partial\lambda_2}\int_0^{\bar{s}} s^2 f_s(s)ds\right) < 0. \qquad \blacksquare$$

5. Conclusion

Introducing ambiguous traders into Grossman and Stiglitz's (1980) model, the paper examines rational expectations equilibrium under the two-tiered asymmetric information. If the individual degree of ambiguity or the fraction of ambiguous traders among uninformed traders increases, then liquidity risk, expected price sensitivity, and price volatility would increase. Obviously, an important direction for future research is to incorporate endogenous information acquisition into our model.

Appendix: Proof of Theorem 3.1

Suppose $p < \mathbb{E}_{\underline{\mu}}[\tilde{v}|P = p]$. We assume that P is a linear function of s such that $P(s) = \underline{\kappa} + \zeta s$. Then the information from p becomes equivalent to that from s and hence we have

$$\mathbb{E}_{\underline{\mu}}[\tilde{v}|P = p] = \mathbb{E}_{\underline{\mu}}[\tilde{v}|\tilde{s} = s] = \frac{\alpha^2\sigma_\varepsilon^4\sigma_z^2\underline{\mu} + \lambda_1^2\sigma_\theta^2 s}{\lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2},$$

$$\text{Var}[\tilde{v}|P = p] = \text{Var}[\tilde{v}|\tilde{s} = s] = \frac{\sigma_\varepsilon^2(\lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2}.$$

From (2.1)–(2.3) and (3.1), we obtain

$$\begin{aligned}
P(s) &= \frac{(1-\lambda_1)(1-\lambda_2)\sigma_\varepsilon^2\mathbb{E}_{\underline{\mu}}[\tilde{v}|\tilde{s} = s] + (1-\lambda_1)\lambda_2\sigma_\varepsilon^2\mathbb{E}[\tilde{v}|\tilde{s} = s] + \lambda_1 s\text{Var}[\tilde{v}|\tilde{s} = s]}{(1-\lambda_1)(1-\lambda_2)\sigma_\varepsilon^2 + (1-\lambda_1)\lambda_2\sigma_\varepsilon^2 + \lambda_1\text{Var}[\tilde{v}|\tilde{s} = s]} \\
&= -\frac{(1-\lambda_1)(1-\lambda_2)\alpha^2\sigma_\varepsilon^4\sigma_z^2\Delta\mu}{2(\lambda_1^2\sigma_\theta^2 + \lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)} + \frac{\lambda_1(\lambda_1\sigma_\theta^2 + \alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda_1^2\sigma_\theta^2 + \lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2}s.
\end{aligned}$$

Similarly, equilibrium asset price function for $p > \mathbb{E}_{\underline{\mu}}[\tilde{v}|P = p]$ is given by

$$P(s) = \frac{(1-\lambda_1)(1-\lambda_2)\alpha^2\sigma_\varepsilon^4\sigma_z^2\Delta\mu}{2(\lambda_1^2\sigma_\theta^2 + \lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)} + \frac{\lambda_1(\lambda_1\sigma_\theta^2 + \alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda_1^2\sigma_\theta^2 + \lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2}s$$

and, when $\mathbb{E}_{\underline{\mu}}[\tilde{v}|P=p] \leq p \leq \mathbb{E}_{\underline{\mu}}[\tilde{v}|P=p]$, it is given by

$$P(s) = \frac{\lambda_1[(1-\lambda_1)\lambda_1\lambda_2\sigma_\theta^2 + \lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2]}{\{(1-\lambda_1)\lambda_2+\lambda_1\}(\lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2) + \lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2} s.$$

Breaking points $\underline{s}$ and $\bar{s}$ are obtained by solving $\mathbb{E}_{\underline{\mu}}[\tilde{v}|\tilde{s}=\underline{s}] = P(\underline{s})$ and $\mathbb{E}_{\bar{\mu}}[\tilde{v}|\tilde{s}=\bar{s}] = P(\bar{s})$. Then we have

$$\bar{s} = -\underline{s} = \frac{[\{(1-\lambda_1)\lambda_2+\lambda_1\}(\lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2) + \lambda_1\alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2]\Delta\mu}{2\lambda_1(\lambda_1^2\sigma_\theta^2 + \alpha^2\sigma_\theta^2\sigma_\varepsilon^2\sigma_z^2 + \alpha^2\sigma_\varepsilon^4\sigma_z^2)}.$$

Since $p < \mathbb{E}_{\underline{\mu}}[\tilde{v}|P=p]$ if and only if $s < \underline{s}$ and $p > \mathbb{E}_{\bar{\mu}}[\tilde{v}|P=p]$ if and only if $s > \bar{s}$, we obtain P as in (3.2). ■

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