# Explaining Equity Home Bias using Labor Income and Inflation Risk

Kyounghun Kim* Sunghyun Kim**

Abstract

This paper derives an optimal portfolio equilibrium using a two-country dynamic stochastic general equilibrium (DSGE) model with two assets, home and foreign equities. We solve the model using a second-order approximation method proposed by Devereux and Sutherland (2011) and Till and Van Wincoop (2010). Unlike in previous studies, this paper derives analytically tractable closed form solutions for the covariance term between wages and equity returns in the one-good model, and that between home goods price and equity returns in the two-good model, respectively. The simulation results show that the smaller the covariance between equity returns and labor income, or the higher the covariance between home goods price and equity returns, the greater the share of home equity in optimal portfolio. While the one-good model cannot generate the Equity Home Bias through endogenously generated covariance between equity returns and wage, the two-good model can produce the Equity Home Bias when the covariance between home goods prices and home equity returns is large and the risk aversion of the consumer is sufficiently high. In addition, empirical tests for 23 countries during 2006-2015 verify that the correlation between home goods price and equity returns has a positive relationship with home equity share, indicating that the motive to hedging a inflation risk can explain the observed Equity Home Bias.

Keyword: Equity Home Bias, Labor Income Risk, Inflation Risk, Second-Order Approximation, Non-Separable Utility

JEL Codes: F30, F41, G11, G15

---

* First Author, Ph.D. Candidate, Department of Economics, Sungkyunkwan University, Seoul, Republic of Korea. E-mail: kim.kh.8312@gmail.com

** Corresponding Author, Professor, Department of Economics, Sungkyunkwan University, Seoul, Republic of Korea. E-mail: shenrykim@skku.edu

## Ⅰ. Introduction

The Equity Home Bias Puzzle has been widely studied in international finance-macro literature in both theoretical and empirical frameworks. In general, Equity Home Bias refers to when a country holds a relatively larger share of home equities than the optimal level predicted by model. Theoretically, one country's optimal share of domestic equities should be equal to the market share of domestic equities in the global equity market. However, according to empirical analysis, it has been observed that the proportion of domestic equities owned by each country is much higher than expected, named as the Equity Home Bias Puzzle by French and Porterba (1991).[1]

This so-called puzzle is not yet completely solved even though the degree of home bias has been significantly reduced over time. Some researchers focus on the possibility that the home bias can be derived from optimal hedging behavior of consumers facing shocks. For example, Coeurdacier and Rey (2013) use the hedging behavior against labor income and inflation risks (real exchange rate risk) to try to explain the Equity Home Bias.[2] This paper derives an optimal portfolio equilibrium in an international DSGE model with a more general setup than that in Coeurdacier and Rey (2013) and examines whether hedging behavior against labor income and inflation risks is able to generate the Equity Home Bias.

We set up a DSGE model with two equity assets (home and foreign), and derive a portfolio equilibrium (steady state of home equity) by using a second-order approximation method proposed by Devereux and Sutherland (2011) and Tille and Van Wincoop (2010). This method enables us to obtain analytically tractable closed form solutions for the steady state asset holdings, despite the presence of multiple assets.[3] Unlike most papers using a second-order approximation method who have made a strong assumption of an endowment economy and a separable utility, we use labor production economy with one good or two goods and the utility function being

1) French and Porterba (1991) shows that the share of home equities in the U.S., U.K., France, and Germany was 92%, 96%, 92%, and 89%, respectively, in 1989. And according to Coeurdacier and Rey (2013), U.S. investors hold above 80% domestic equities in 2007.

2) Theoretically, the real exchange rate is the relative inflation between countries.

3) A typical DSGE model solution is based on perturbation (mostly linearization) around a deterministic steady state. Thus, to explain Equity Home Bias with a standard DSGE model, it is necessary to derive a steady state of equity share. In the standard linearized solution for the DSGE model, the first-order approximation should be conducted near the steady state value. However, at the first-order approximation, the returns of the two assets are equalized and are perfectly substitutable; thus, the steady state value of assets cannot be determined. Accordingly, the financial market has traditionally been incorporated into the macro model in its simplest form by assuming that the financial asset comprises Arrow–Debreu securities, or single tradable bonds.

modified to encompass a non-separable utility.[4] Under this general assumption, we derive closed form solutions for the covariance term between wages and equity returns in the one-good model, and that between home goods prices and equity returns in the two-good model. We also derived endogenously a function between optimal share of home equity and the covariance terms in the model.[5] Using these equilibrium solutions, we can confirm whether the covariance term influence the optimal share of home equity, and whether the Equity Home Bias would be an optimal portfolio equilibrium under certain conditions.

Using a one-good model, we demonstrate that the smaller the covariance between equity returns and labor income, the greater the optimal share of home equity. This means that household hedge labor income risks by holding home equities. When households have both labor income and capital income, they tend to hedge labor income risks by holding assets that have low or negative correlation with wage. Furthermore, if home equity returns have a negative relationship with wage, or the covariance between home equity returns and wage is small, then households prefer to hold domestic equity to foreign equity. However, the one-good model cannot generate the Equity Home Bias through endogenously generated covariance between equity returns and wage under reasonable parameter values.

Using a two-good model, we show that the greater the covariance between home goods price and equity returns, the greater the optimal share of home equity. This indicates that households tend to hold home equity to hedge inflation risks. Households tend to hold equity that generates a higher yield when the purchasing power declines due to inflation (appreciation of the real exchange rate). Particularly, in the two-good model, when the covariance between home goods prices and equity returns is high and the consumers' risk-aversion is sufficiently high, the optimal portfolio can generate the Equity Home Bias.

For a robustness check, this paper conducts simple empirical tests for 23 countries during 2006-2015, and the results show that the correlation between home goods price and equity returns has a positive relationship with home equity share. This result explains that the motive to hedge inflation risk can generate the Equity Home Bias. On the other hand, empirical test results show that the correlation between wage and equity returns is positively related to home equity share, which is inconsistent with the theoretical predictions from the model. Thus, the Equity Home Bias is better explained by the motive for hedging inflation risk than labor income risk.

---

4) By using a non-separable utility, we can explicitly derive the covariance term without assumption of perfect correlation between consumption and real exchange rate.

5) Existing papers measure the covariance term by using data, or derive the steady state of home equity, and the covariance terms, by assuming perfect risk sharing.

The paper is organized as follows. Section 2 reviews literature related to the paper. Section 3 lays out a one-good labor production model and a two-good labor production model. Section 4 derives each portfolio equilibrium from the models shown in section 3. Section 5 provides simulation results of portfolio equilibrium solutions. Section 6 conducts simple empirical test for a robustness check. Section 7 concludes the paper.

## Ⅱ. Literature Review

There are mixed results regarding whether theoretical models can generate Equity Home Bias. Baxter and Jermann's (1997) argue that a change in output is distributed at a constant ratio towards labor and capital in the Cobb-Douglas production function, which leads to a very high correlation between labor income and capital income. Thus, households should have a greater share of foreign equity relative to home equity in order to hedge wage fluctuation risk, strengthening the puzzle. However, Heathcote and Perri (2013) contend that the Equity Home Bias is an optimal portfolio equilibrium, when considering a two-good model and capital accumulation. They show that positive production shocks increase wages, whereas home country firms' dividends decline as investment increases, illustrating the negative co-movement between wages and dividends, explaining the Home Bias puzzle. In the case of Heathcote and Perri (2013), the portfolio equilibrium can be derived under a strong assumption for utility and technology functions, such as log-separable utility. Engel and Matsumoto (2009) also show that home equity is useful for hedging labor income risk in the labor production economy model with money and sticky prices.

Meanwhile, Kollmann (2006) explains the Equity Home Bias by a feature of consumption home bias and low elasticity of substitution between home and foreign goods in a two-good endowment economy. Coeurdacier, Kollmann and Martin (2010) extend Kollmann's (2006) model to a two-good production economy with multiple assets (bonds and equities) and argue that the Equity Home Bias occurs due to an investment bias. They claim that capital income and labor income are negatively correlated because home investment boom decreases home dividends and increases output and wages.

Coeurdacier and Rey (2013) further added shocks to the disutility of leisure in Coeurdacier, Kollmann, and Martin's (2010) model, and consider two kinds of assets – equities and bonds. They show that, when considering only equity as an asset, there is a relatively larger share of foreign equity than home equity for hedging the fluctuation of domestic labor income; this is consistent with the results from Baxter and Jermann (1997). However, when considering bonds and equity together, they confirm that the labor income risk is hedged by home equity and the real exchange rate risk is hedged by home bonds. Thus, they conclude that the correlation between labor income and capital income is conditionally negative.

Previous papers related to real exchange rate risk are predominantly based on the theoretical aspect of the general equilibrium model. For instance, Obstfeld and Rogoff (2000) argue that under certain parameter values (i.e. risk aversion is lower than unity and is equal to the inverse of the elasticity of substitution between two goods), an agent prefers local equity when local consumption is expensive. Coeurdacier (2009) demonstrates that the Equity Home Bias can occur when investors hedge real exchange rate risk. They argue that if the correlation between real exchange rate and equity returns is highly positive, the incentive to hold domestic equity increases.[6)]

There are a few empirical studies delving into the relationship between real exchange rate risk and the Equity Home Bias. According to Fidora (2006), only Cooper and Kaplains (1994) have conducted a systematic analysis by developing a test that indirectly determines the impact of domestic inflation risk. Van Wincoop and Warnock (2006) empirically show that the correlation between excess equity returns and real exchange rate is very low; they also note that the motivation to hedge real exchange rate risk offers a limited explanation of the Equity Home Bias. Utilizing the international Capital Asset Pricing Model (CAPM), Fidora (2006) reports that exchange rate volatility increases domestic equity holdings, as holding foreign securities can generate additional risk for home investors.

In general, previous studies regarding the Equity Home Bias in the DSGE model have derived the steady state for home equity share by using a first-order approximation with a perfect risk sharing condition. These studies derive the consumption allocation under the condition of a complete market, and then replicate it forward to the financial market to determine the portfolio equilibrium. However, this approach which exogenously derives the portfolio equilibrium has limitations, because the portfolio equilibrium must be derived simultaneously (and endogenously) with macro variables. Additionally, since it is necessary to consider risks such as variance of the portfolio equilibrium across countries, the use of the first-order approximation method does have its limitations. In order to obtain an optimal portfolio equilibrium, it is essential to derive not only the average returns of the asset, but also risk information such as variance and covariance. According to Kim (1997), if first-order approximation is only used, any relevant information contained in the second or higher-order approximation will be lost.

Devereux and Sutherland (2011) and Tille and Van Wincoop (2010), show that a

6) According to Coeurdacier and Rey (2013), Solink (1974), Adler and Dumas (1983), Krugman (1981), Stulz (1981), and Cooper and Kaplains (1994) argue that real exchange rate risk is the cause of the Equity Home Bias.

portfolio equilibrium can be derived in a more general environment without specific constraints by using a second-order approximation. According to Devereux and Sutherland (2011), the second-order approximation is used to obtain the steady state of the optimal holdings for assets, a method for which information related to risk—such as variance—is preserved. Various studies following the methods of Devereux and Sutherland (2011) and Tille and Van Wincoop (2010) have been conducted. However, many of these studies assume an endowment economy and a separable utility. For example, Amdur (2010) assumes an endowment economy and derives a portfolio equilibrium for bonds in a two-good DSGE model. Ke Pang (2013) derives a portfolio equilibrium in a two-good labor production economy model with nominal rigidities and money using a second-order approximation. She shows that Equity Home Bias can be generated by nominal price rigidities. However, her conclusion is based on the assumption of separable utility, which may not go through under the non-separable utility assumption.

## Ⅲ. Model

This section constructs and solves portfolio equilibrium in two models: a one-good labor production economy and a two-good labor production economy. In the one-good labor production economy, we examine how the optimal share of home equity changes by labor income risk. In the two-good labor production economy, we investigate whether Equity Home Bias occurs in the process of hedging inflation risk. Both models assume that there are two symmetric countries, Home and Foreign, which have the same preference and production technology. Each country consists of a representative household, and a representative firm.

### 1. One-Good Labor Production Economy

We consider the following non-separable utility function. Household maximizes the expected lifetime utility, given by equation (1), where $C_t$ and $L_t$ denote consumption and labor, respectively. Households in both countries have the same discount factor $\beta$.

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \text{ where } U(C_t, L_t) = \frac{[C_t^{\theta} (1-L_t)^{1-\theta}]^{1-\sigma}}{1-\sigma} \tag{1}$$

Budget constraint of the household is given by equation (2). $w_t$ represents the wage at home country. $Z_t$ and $D_t$ denote home equity price and dividend, respectively. $S_{H,t}$ is the share of home equity held by the home country, $S_{F,t}$ is the share of foreign equity held by the home country: $S_{H,t} + S^*_{H,t} = 1$. Foreign variables are denoted by asterisks.

$$C_t + Z_t S_{H,t+1} + Z^*_t S_{F,t+1} = w_t L_t + (Z_t + D_t) S_{H,t} + (Z^*_t + D^*_t) S_{F,t} \tag{2}$$

To obtain portfolio equilibrium using a second-order approximation method proposed by Devereux and Sutherland (2011), the budget constraint should be transformed into equation (3). $NFA_t$ represents net foreign assets of the home country, which is expressed as in equation (4). Sum of the net foreign assets of home and foreign countries becomes zero as in equation (5). $R_t$ and $R^*_t$ are equity returns of home and foreign countries, respectively, as in equation (6).

$$C_t + NFA_t = w_t L_t + \alpha_{H,t-1}(R_t - R^*_t) + R^*_t NFA_{t-1} + D_t \tag{3}$$

$$NFA_t = \alpha_{H,t} + \alpha_{F,t}, \quad NFA^*_t = \alpha^*_{H,t} + \alpha^*_{F,t} \tag{4}$$

where $\alpha_{H,t} = Z_t (S_{H,t+1} - 1)$ and $\alpha_{F,t} = Z^*_t S_{F,t+1}$

$$NFA_t + NFA^*_t = 0 \tag{5}$$

$$R_t = \frac{Z_t + D_t}{Z_{t-1}}, \ R^*_t = \frac{Z^*_t + D^*_t}{Z^*_{t-1}} \tag{6}$$

The first order conditions for $C_t, L_t, \alpha_{H,t}, \alpha_{F,t}$ are in equations (7) - (9)

$$\frac{\theta}{1-\theta} = \frac{C_t}{w_t (1 - L_t)} \tag{7}$$

$$E_t \left[ C_{t+1}^{\theta(1-\rho)-1} (1 - L_{t+1})^{(1-\theta)(1-\sigma)} R_{t+1} \right] = E_t \left[ C_{t+1}^{\theta(1-\rho)-1} (1 - L_{t+1})^{(1-\theta)(1-\sigma)} R^*_{t+1} \right] \tag{8}$$

$$C_t^{\theta(1-\sigma)-1} (1 - L_t)^{(1-\theta)(1-\rho)} = \beta E_t \left[ C_{t+1}^{\theta(1-\sigma)-1} (1 - L_{t+1})^{(1-\theta)(1-\sigma)} R_{t+1} \right] \tag{9}$$

Dividend is defined in equation (10). Firm has an objective to maximize profit and the firm's profit is distributed to the household in the form of dividend. This paper assumes that productivity shocks follow an i.i.d process, where $E_t(\epsilon_{a,t}) = E_t(\epsilon^*_{a,t}) = 0$, $E_t({\epsilon_{a,t}}^2) = E_t({\epsilon^*_{a,t}}^2) = \sigma^2_a$, $\rho(\epsilon_a \epsilon^*_a) = 0$ for all *t*. Equation (11) is derived from the firm's maximization problem.

$$D_t = Y_t - w_t L_t \tag{10}$$

where $Y_t = A_t L_t^{\alpha}$ and $\log A_t = \epsilon_{a,t}$

$$w_t L_t = \alpha Y_t \tag{11}$$

**2. Two-Good Labor Production Economy**

The two-good model in this section is based on Coeurdacier and Rey (2013). Main difference is the utility function where we use a non-separable utility function unlike Coeurdacier and Rey(2013). We assume that each country produces its own good: home good and foreign good. Utility function of households are same as in equations (1), where aggregate consumption follows equation (12). $C_t$ is the aggregate consumption of the home country, where $c_{H,t}$ is home country consumption of the good produced by the home country's firm, and $c_{F,t}$ is home country consumption of the good produced by the foreign country's firm. $\phi$ is the elasticity of substitution between home and foreign goods. $\lambda$ is the share of consumption spending on the home good. We assume that there is consumption home bias: $1/2<\lambda<1$.

$$C_t = [\lambda^{1/\phi}(c_{H,t})^{(\phi-1)/\phi} + (1-\lambda)^{1/\phi}(c_{F,t})^{(\phi-1)/\phi}]^{\phi/(\phi-1)} \tag{12}$$

Consumption price index follows equation (13). $P_t$ and $P^*_t$ are price index of home and foreign countries, respectively. $p_{H,t}$ is the price of the home good, and $p_{F,t}$ is the price of the foreign good.

$$\begin{aligned} P_t &= [\lambda(p_{H,t})^{1-\phi} + (1-\lambda)(p_{F,t})^{1-\phi}]^{1/(1-\phi)} \\ P^*_t &= [(1-\lambda)(p_{H,t})^{1-\phi} + \lambda(p_{F,t})^{1-\phi}]^{1/(1-\phi)} \end{aligned} \tag{13}$$

$e_t$ and $TOT_t$ denote real exchange rate and terms of trade, respectively.

$$e_t = \frac{P_t}{P_t^*} \quad , \quad TOT_t = \frac{p_{H,t}}{p_{F,t}} \tag{14}$$

Household of home country's budget constraint is:

$$P_t C_t + Z_t S_{H,t+1} + Z_t^* S_{F,t+1} = w_t L_t + (Z_t + D_t) S_{H,t} + (Z_t^* + D_t^*) S_{F,t} \tag{15}$$

To obtain a portfolio equilibrium, the budget constraint should be transformed into equation (16).

$$P_t C_t + NFA_t = w_t L_t + \alpha_{H,t-1}(R_t - R_t^*) + R_t^* NFA_{t-1} + D_t \tag{16}$$

Equations (17) - (19) are the first conditions for $C_{t,}, L_t, \alpha_{H,t}, \alpha_{F,t}$ derived from the household's maximization problem.

$$P_t C_t = \frac{\theta}{1-\theta}(1-L_t) w_t \tag{17}$$

$$E_t\left[P_{t+1}^{-1} C_{t+1}^{\theta(1-\sigma)-1}(1-L_{t+1})^{(1-\theta)(1-\sigma)} R_{t+1}\right] = E_t\left[P_{t+1}^{-1} C_{t+1}^{\theta(1-\sigma)-1}(1-L_{t+1})^{(1-\theta)(1-\sigma)} R_{t+1}^*\right] \tag{18}$$

$$P_t^{-1} C_t^{\theta(1-\sigma)-1}(1-L_t)^{(1-\theta)(1-\sigma)} = \beta E_t\left[P_{t+1}^{-1} C_{t+1}^{\theta(1-\sigma)-1}(1-L_{t+1})^{(1-\theta)(1-\sigma)}\right] \tag{19}$$

Optimal allocation across two consumption goods can be derived from the household's intra-temporal maximization problem and the resulting expressions are in the following equation (20).

$$c_{H,t} = \lambda\left(\frac{p_{H,t}}{P_t}\right)^{-\phi} C_t, \quad c_{F,t} = (1-\lambda)\left(\frac{p_{F,t}}{P_t}\right)^{-\phi} C_t,$$

$$c_{H,t}^* = (1-\lambda)\left(\frac{p_{H,t}}{P_t^*}\right)^{-\phi} C_t^*, \quad c_{F,t}^* = \lambda\left(\frac{p_{F,t}}{P_t^*}\right)^{-\phi} C_t^* \tag{20}$$

Firm's profit is defined as in equation (21). Productivity shock follows an i.i.d process, where $E_t(\epsilon_{a,t}) = E_t(\epsilon_{a,t}^*) = 0$, $E_t(\epsilon_{a,t}{}^2) = E_t(\epsilon_{a,t}^*{}^2) = \sigma_a^2$, $\rho(\epsilon_a \epsilon_a^*) = 0$ for all *t*. Equation (22) is derived from the firm's maximization problem.

$$D_t = p_{H,t} Y_t - w_t L_t \tag{21}$$

where $Y_t = A_t L_t^{\alpha}$ and $\log A_t = \epsilon_{a,t}$

$$w_t L_t = \alpha p_{H,t} Y_t \tag{22}$$

## Ⅳ. Portfolio Equilibrium

### 1. One-Good Labor Production Economy

In the one-good labor economy, a second-order approximation for equation (8) yields the following equation (23). Hereinafter, the 'hat variable' is defined as deviation from a steady state and the upper bar is defined as a steady state ($\hat{x} = \log\frac{x_t}{\overline{x}} = \frac{x_t - \overline{x}}{\overline{x}}$). Exceptionally $N\hat{F}A_t$ is defined as $N\hat{F}A_t = \frac{NFA_t - \overline{NFA}}{\overline{Y}}$

$$E_t[\{(\hat{C}_{t+1} - \hat{C}^*_{t+1}) - X_2(\hat{L}_{t+1} - \hat{L}^*_{t+1})\}\hat{R}_{X,t+1}] = 0 \tag{23}$$

where $X_2 = \frac{\theta(1-\sigma)\alpha}{\theta(1-\sigma)-1}$

$\hat{R}_{X,t+1}$ denotes $\hat{R}_{t+1} - \hat{R}^*_{t+1}$. Since $E_t[\hat{R}_{X,t+1}] = 0$ by equation (8), equation (23) can be expressed as equation (24):

$$E_t[(\hat{C}_{t+1} - \hat{C}^*_{t+1})\hat{R}_{X,t+1}] = \frac{X_2 X_3}{1+X_2 X_3} E_t\left[(\hat{A}_{t+1} - \hat{A}^*_{t+1})\hat{R}_{X,t+1}\right] \tag{24}$$

where $X_3 = (1-\theta)/\{\theta\alpha - (1-\theta)(\alpha-1)\}$

$\hat{C}_{t+1} - \hat{C}^*_{t+1}$ and $\hat{R}_{X,t+1}$ can be written as in equations (25) and (26), respectively.

$$\begin{aligned}\zeta(\hat{C}_{t+1} - \hat{C}^*_{t+1}) = {} & 2\frac{\beta}{1-\beta} N\hat{F}A_t - \beta Z\{2\tilde{\alpha}_H(1-\beta)\alpha X_3 + (1+\alpha X_3)\}(\hat{A}_{t+1} - \hat{A}^*_{t+1}) \\ & + (1-\beta)(1+\alpha X_3)\{2\tilde{\alpha}_H(1-\beta)+1\}(\hat{A}_{t+1} - \hat{A}^*_{t+1}) \\ & + 2\tilde{\alpha}_H(1-\beta)\alpha X_3(\hat{C}_t - \hat{C}^*_t) + 2\tilde{\alpha}_H(1-\beta)\alpha X_3 Z(\hat{A}_t - \hat{A}^*_t)\end{aligned} \tag{25}$$

where $\tilde{\alpha}_H = \frac{\overline{\alpha}_H}{\beta\overline{Y}}$, $\zeta = 2\tilde{\alpha}_H(1-\beta)\alpha X_3 + (1+\alpha X_3)$, and $Z = -\frac{X_2 X_3}{1+X_2 X_3}$

$$\hat{R}_{X,t+1} = -\alpha X_3(\hat{C}_{t+1} - \hat{C}^*_{t+1}) + \{(1-\beta)(1+\alpha X_3) - \beta\alpha X_3 Z\}(\hat{A}_{t+1} - \hat{A}^*_{t+1}) \\ + \alpha X_3(\hat{C}_t - \hat{C}^*_t) + \alpha X_3 Z(\hat{A}_t - \hat{A}^*_t) \tag{26}$$

Finally, the portfolio equilibrium in equation (27) can be obtained by substituting equations (25) and (26) into equation (24):

$$\overline{S} = 1 - \frac{1}{2(1-\alpha)} \frac{1+\alpha X_3}{(\alpha X_3 + 1 + X_2 X_3)} \tag{27}$$

First order conditions for consumption and labor yield the following relationship:

$$(\hat{C}_{t+1} - \hat{C}^*_{t+1}) = H_2(\hat{A}_{t+1} - \hat{A}^*_{t+1}) - H_1(\hat{w}_{t+1} - \hat{w}^*_{t+1}) \tag{28}$$

where $H_1 = \frac{1}{(\alpha-1)X_3}$ and $H_2 = 1 + H_1$

Substituting equation (28) into equation (24), the covariance between wages and equity returns is endogenously derived in the model as in equation (29).

$$Cov(\hat{w}_{t+1}\hat{R}_{t+1}) = \frac{1}{H_2}(H_2 + Z)(1-\beta)\{1 + \alpha X_3(1+Z)\}\sigma_a^2 \tag{29}$$

The portfolio equilibrium can be expressed in the covariance term between wages and equity returns, as in equation (30), by using equations (25), (26), and (29). This function is equal to equation (27):

$$\overline{S} = 1 - \frac{1}{2(1-\alpha)} \frac{(1+\alpha X_3)H_1(1+Z)}{(H_2 + Z)(1-\beta)\{1 + \alpha X_3(1+Z)\}^2\sigma_a^2} Cov(\hat{w}_{t+1}\hat{R}_{t+1}) \tag{30}$$

**2. Two-Good Labor Production Economy**

To examine the inflation risk (real exchange rate risk) on portfolio equilibrium, this paper derives the covariance term between home goods price and equity returns in a two-good model. Equation (31) is derived from the second-order approximation of equation (18).

$$E_t\left[\left\{(\hat{C}_{t+1}-\hat{C}^*_{t+1})-X_2(\hat{L}_{t+1}-\hat{L}^*_{t+1})-\frac{1}{X_1-1}(\hat{P}_{t+1}-\hat{P}^*_{t+1})\right\}\hat{R}_{X,t+1}\right]=0 \quad (31)$$

where $X_2=\frac{X_1\alpha}{X_1-1}$ and $X_1=\theta(1-\sigma)$

Equation (31) can be expressed as in equation (32).

$$E_t[(\hat{C}_{t+1}-\hat{C}^*_{t+1})\hat{R}_{X,t+1}]=\frac{(X_1\alpha X_3\Lambda_0-1)j_3-\alpha X_1X_3}{(X_1\alpha X_3\Lambda_0-1)j_4-X_1(1+\alpha X_3)+1}E_t\left[(\hat{A}_{t+1}-\hat{A}^*_{t+1})\hat{R}_{X,t+1}\right] \quad (32)$$

where, $\Lambda_0=\frac{2(\lambda-1)}{2\lambda-1}$, $j_3=\frac{1+\alpha X_3}{1+\alpha X_3\Lambda_0}$, and $j_4=\frac{1+\alpha X_3\Lambda_0}{2\lambda-1+\alpha X_3}$

$\hat{C}_{t+1}-\hat{C}^*_{t+1}$ and $\hat{R}_{X,t+1}$ can be expressed as in equations (33), (34), respectively.

$$\begin{aligned}\kappa(\hat{C}_{t+1}-\hat{C}^*_{t+1})=2\frac{\beta}{1-\beta}N\hat{F}A_t+2\tilde{\alpha}_H(1-\beta)\{(1-\beta)d_2-\beta d_3b_5\}(\hat{A}_{t+1}-\hat{A}^*_{t+1})\\ -u_2\{\beta(b_5-1)+1\}(\hat{A}_{t+1}-\hat{A}^*_{t+1})\\ +2\tilde{\alpha}_H(1-\beta)d_3(\hat{C}_t-\hat{C}^*_t)+2\tilde{\alpha}_H(1-\beta)d_3b_5(\hat{A}_t-\hat{A}^*_t)\end{aligned} \quad (33)$$

where, $\tilde{\alpha}_H=\frac{\overline{\alpha}_H}{\beta\overline{Y}}$, $\kappa=2\tilde{\alpha}_H(1-\beta)d_3+u_1$, $d_3=d_1j_4+\alpha X_3$, $d_1=\frac{1-2(\lambda-1)\alpha X_3}{2\lambda-1}$, $u_1=(1+\alpha X_3)-(1-\alpha X_3)\Lambda_0j_4$, $d_2=d_1j_3+1+\alpha X_3$, $u_2=(\Lambda_0j_3-1)(1+\alpha X_3)$, $b_5=\frac{(X_1\alpha X_3\Lambda_0-1)j_3-\alpha X_1X_3}{X_1(1+\alpha X_3)-1-(X_1\alpha X_3\Lambda_0-1)j_4}$, $b_1=X_1(1+\alpha X_3)-1$, $b_2=X_1\alpha X_3\Lambda_0-1$,

$$\begin{aligned}\hat{R}_{X,t+1}=-d_3(\hat{C}_{t+1}-\hat{C}^*_{t+1})+\{(1-\beta)d_2-\beta d_3b_5\}(\hat{A}_{t+1}-\hat{A}^*_{t+1})\\ +d_3(\hat{C}_t-\hat{C}^*_t)+d_3b_5(\hat{A}_t-\hat{A}^*_t)\end{aligned} \quad (34)$$

Substituting equation (33) into equation (32) yields portfolio equilibrium as in equation (35).

$$\overline{S}=1+\frac{1}{2(1-\alpha)}\frac{u_2(\beta(b_5-1)+1)-u_1b_5}{(1-\beta)(d_2+d_3b_5)} \quad (35)$$

The covariance between home goods price and home equity returns is endogenously determined in the model as in equation (36).

$$Cov(\hat{P}_{t+1}\hat{R}_{t+1}) = \frac{(b_1 b_5 + X_1 \alpha X_3)(1-\beta)(d_3 b_5 + d_2)}{b_2}\sigma_a^2 \tag{36}$$

The portfolio equilibrium for the covariance between home goods price and home equity returns is derived as in equation (37) by using equation (36).

$$\overline{S} = 1 + \frac{1}{2(1-\alpha)}\frac{b_2\{u_2(\beta(b_5-1)+1)-u_1 b_5\}}{(b_1 b_5 + X_1\alpha X_3)(1-\beta)^2(d_2+d_3 b_5)^2\sigma_a^2} Cov(\hat{P}_{t+1}\hat{R}_{t+1}) \tag{37}$$

## Ⅴ. Simulation of Labor Production Economy

### 1. One-Good Labor Production Economy

By using the linearized first-order conditions, functions for profit and wage are expressed as $\hat{D}_t = \hat{A}_t + \alpha\hat{L}_t$ and $\hat{w}_t = \hat{A}_t + (1-\alpha)\hat{L}_t$, respectively. Both variables (wages and dividends) are linked to labor share. Equity return is also a function of dividend as shown in equation (6). We therefore examine how the covariance of wage and equity returns varies with labor share, and measure the optimal share of home equity by using the covariance term between wage and equity returns.

For calibration, we use the parameter values taken from previous studies. Risk aversion is 2, which is commonly used in macro models. The discount factor is set at 0.96, so that the steady state annual real interest rate is equal to 4%. Consumption share in utility function is 0.25, which is between 0.2 and 0.3 that are used in the values of King, Plosser, and Rebelo (1988), and Backus, Kehoe, and Kydland (1992), respectively. In this paper, the covariance between wage and equity returns is expressed as a function of productivity shocks, so we normalize the productivity shock variance to 1%. We experiment with different values for labor share ($\alpha$) in production function between 0.2 to 0.8 in order to observe how changes in labor share affects covariance between wage and equity returns.

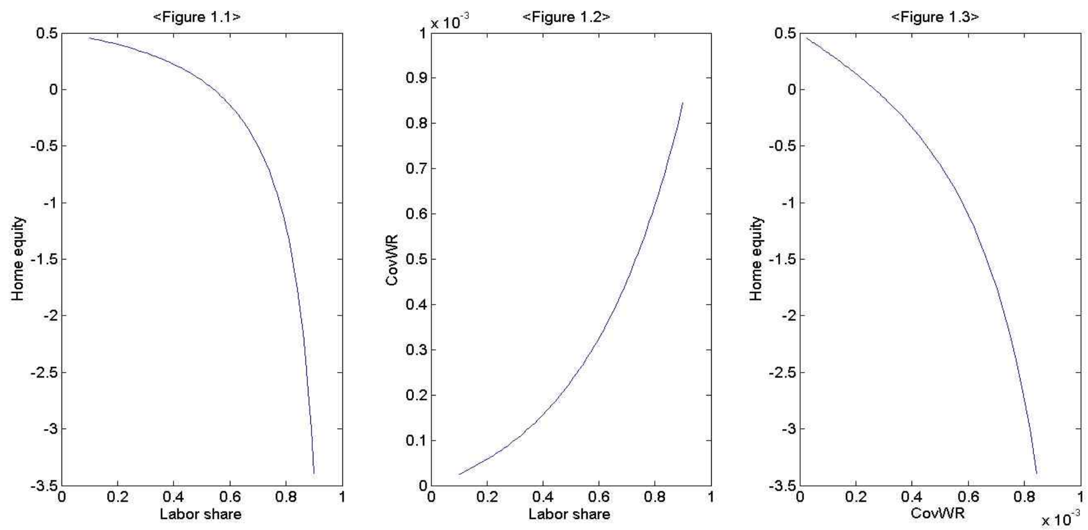
<Figure 1.1> displays the optimal share of home equity when the labor share $\alpha$ rises from 0.2 to 0.8. <Figure 1.2> confirms that as the labor share increases from 0.2 to 0.8, the covariance between wage and equity returns increases.[7] <Figure 1.3> shows

7) The absolute value of covariance is very small, as determined endogenously in the model, which does not affect the interpretation of the results.

how the optimal share of home equity varies, depending on the covariance between wage and equity returns, which is derived from <Figure 1.1>.

In <Figure 1.3>, one can see that the larger the covariance between wage and equity returns of the home country, the larger the optimal share of home equity. This means that households tend to have home equity which have low covariance with labor income, in order to hedge fluctuations of wage (non-tradable income). This result is consistent with those reported in both Baxter and Jermann (1997) and Heathcote and Perri (2013). Baxter and Jermann (1997) argue that households' holding of foreign equity should be high because the covariance between wage and equity returns is positive. In contrast, Heathcote and Perri (2013) show that the Equity Home Bias is an optimal portfolio equilibrium because the covariance between wage and equity returns is negative. Both studies imply that the smaller the covariance, the greater the optimal holding of home equity, which is shown in our analysis of the one good model.

However, the one-good model in this paper cannot generate the Equity Home Bias because the optimal share of home equity derived in the model is always less than 1/2. This result supports the conclusion of Baxter and Jermann (1997) that the Equity Home Bias could not be explained by the covariance value between wage and equity returns generated by the model.




Notes 1). **Home equity** denotes the optimal holding of home equity

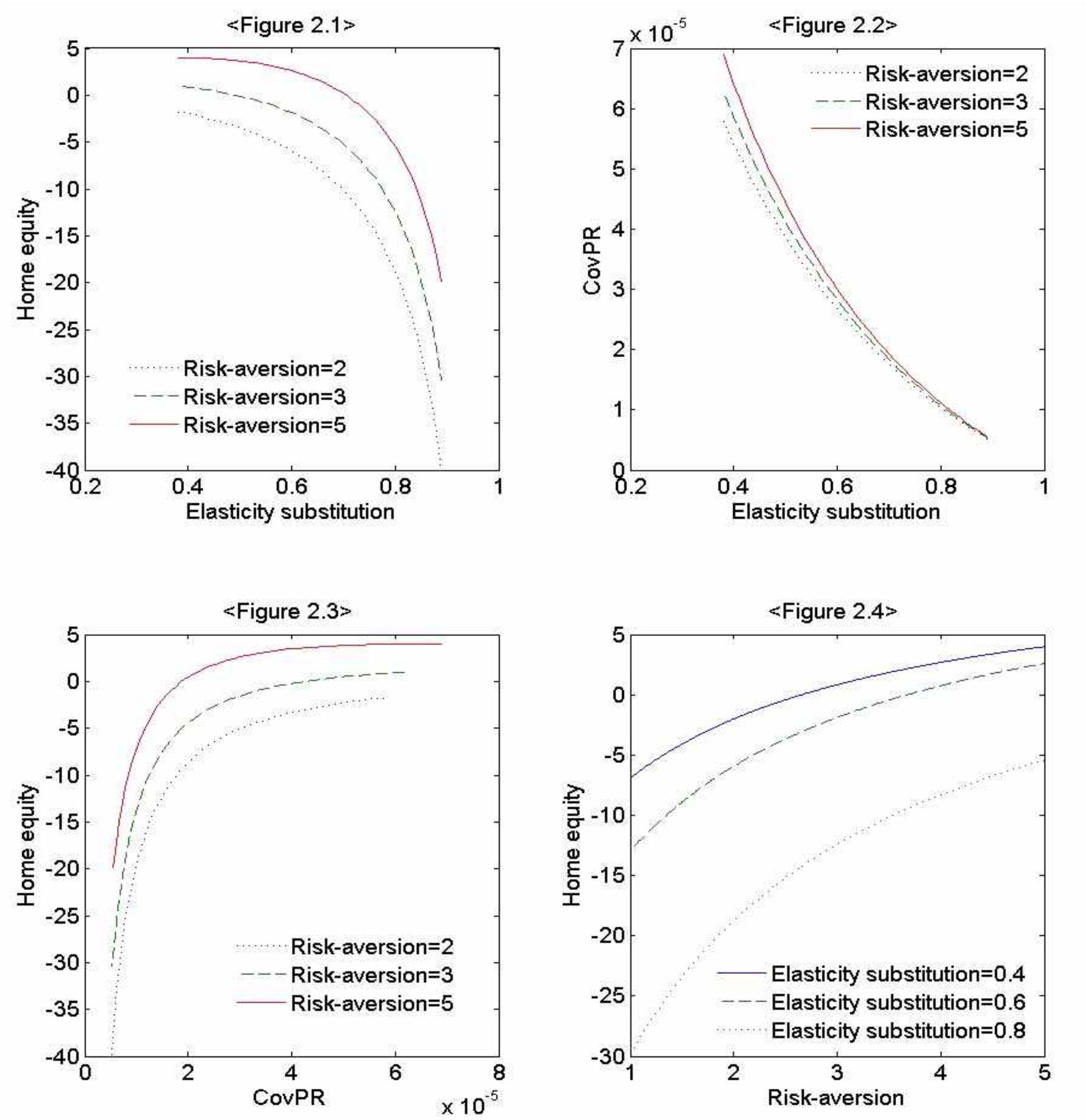
2). **CovWR** denotes the covariance between wage and equity returns

## 2. Two-Good Labor Production Economy

In the two-good model, most parameters are equal to those of the one-good model: $\beta$=0.96, $\sigma_a^2$=0.01, $\theta$=0.25. Labor share ($\alpha$) in production function is fixed at 0.6, which is a typical value used in the literature. For home good share $\lambda$ in the aggregate consumption is set at 0.55, assuming a modest degree of consumption home bias. In this paper, the elasticity of substitution between home and foreign goods is set between 0.38 and 0.89, as shown in Bayoumi (1999). Bayoumi (1999) conducts empirical analysis of 21 countries, and finds that the long-term elasticity of substitution between home and foreign goods lay between 0.38 and 0.89. In addition, in the macro literature, the low elasticity of substitution between home and foreign goods (roughly below 'unity') better fits to the international business cycle data. Finally, this paper uses three values of the risk-aversion: $\sigma$=2, $\sigma$=3, $\sigma$=5.

<Figure 2.1> displays how the optimal share of home equity changes when the elasticity of substitution between home and foreign goods changes. The graph indicates that the smaller the elasticity of substitution between home and foreign goods, the higher the share of home equity. <Figure 2.2> shows that the smaller the elasticity substitution between home and foreign goods, the higher the covariance between home goods prices and equity returns. These results can be interpreted that household holds an asset which has a positive covariance with the home goods price to prevent the decline in the purchasing power due to an increase in the price of home goods (inflation). To be specific, the low elasticity of substitution between home and foreign goods makes it difficult to substitute domestic goods for foreign goods, which leads to the greater covariance between home equity returns and home prices. In general, home equity returns and home goods price (inflation) move in the same direction. <Figure 2.3> directly depicts the channel mentioned above: plot of optimal share of home equity on the covariance between home prices and equity returns derived from the model. The graph shows that the larger the covariance between home goods price and equity returns, the higher the optimal share of home equity.

Particularly, as in <Figure 2.3>, when risk aversion is above 2 and the covariance between home and foreign goods is high, the optimal share of home equity exceeds 1/2. This means that, if home equity returns are sufficiently correlated with home goods price and households have a high tendency for hedging consumption risk, households prefer to hold home equity rather than foreign equity. In other words, the stronger the motive for hedging consumption risks, the higher the optimal home equity holding is. <Figure 2.4> supports these results by directly showing the effect of household's preference on the holdings of home equity, indicating that the optimal share of home equity is large when the risk aversion is high.




Notes 1). **Home equity** denotes the optimal holding of home euqity
2). **CovPR** denotes the covariance between home goods price and equity returns
3). **Elasticity substitution** denotes the elasticity substitution between home and foreign goods

## Ⅵ. Empirical Evidence of Equity Home Bias

We have theoretically confirmed that the Equity Home Bias can be an optimal portfolio equilibrium when the covariance between home goods prices and equity returns is large and the risk aversion of the consumer is sufficiently high. Therefore, this section conducts simple empirical analysis to show whether the correlation between home goods prices and equity returns has a positive affect on home equity holdings. In addition, empirical tests are conducted to verify whether the correlation between wage and equity returns has a negative effect on home equity holdings. These empirical evidence allow us to prove theoretical predictions in the previous section.

Previous empirical studies have mainly focused on the relationship between real exchange rate and equity returns, and between wage and equity returns. For instance, Coeurdacier and Gourinchas (2016) show that real exchange rate and financial income are positively correlated. Unlike in previous studies, we directly examine the effect of the correlation terms on home equity holdings by conducting empirical tests with balanced panel data of 23 OECD countries during 2006-2015.[8)]

### 1. Constructing Equity Home Bias Index

Equity Home Bias is measured as in equation (38) following Coeurdacier and Rey (2014), Mishra (2015), and Ahearne et al. (2004). In equation (38), $EHB_{i,t}$ denotes the level of equity home bias for country *i* in period *t*. If $EHB_{i,t}$ is above zero, the Equity Home Bias occurs. And if $EHB_{i,t}$ is one, there is perfect Equity Home Bias (100% of share on home equity). *Share of Foreign Equities in the World Market Portfolios*$_{i,t}$ is foreign countries' share in the world equity market, excluding country *i*. This is the optimal foreign weight in equity portfolio of country *i*. $FES_{i,t}$ in equation (39) is the foreign equity held by country *i*, which represents the actual share of foreign equity in equity portfolio of country i. Year-end data of IFS are used as *Foreign Equity Asset*$_{i,t}$ and F*oreign Equity Liability*$_{i,t}$, and M*arket Capitalization*$_{i,t}$ is based on national year-end data in Bloomberg.

$$EHB_{i,t} = 1 - \frac{Share\ of\ Foreign\ Equities\ in\ Country\ i\ Equity\ Holding_t\ (FES_{i,t})}{Share\ of\ Foreign\ Equities\ in\ the\ World\ Maket\ Portfolios_{i,t}} \tag{38}$$

$$FES_{i,t} = \frac{Foreign\ Equity\ Asset_{i,t}}{Foreign\ Equity\ Asset_{i,t} + Market\ Capitalization_{i,t} - Foreign\ Equity\ Liability_{i,t}} \tag{39}$$

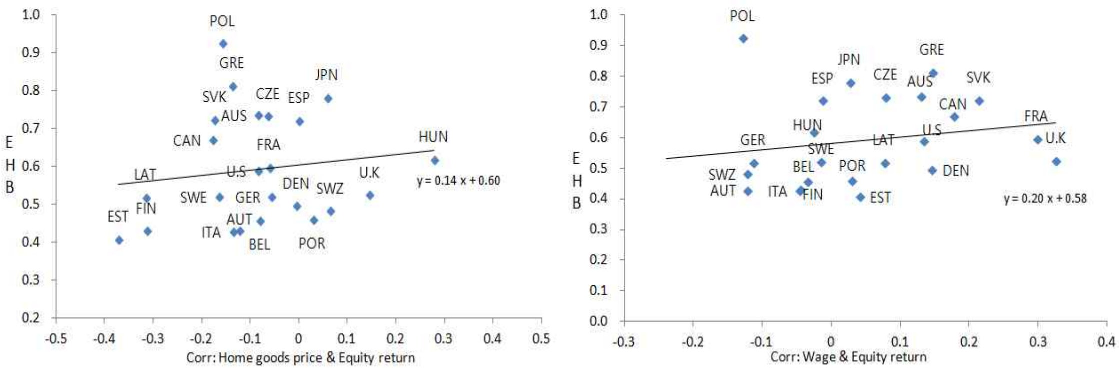
To derive the correlation between home goods price and equity returns, equity returns are measured as each countries' stock price index in Datastream and Bloomberg. Core CPI is used as home goods price, which is obtained from OECD database. Finally, compensation of employees in national income is used for wage variable. Based on these data, the correlations are calculated by quarterly log differences of each variables for the 5-year period, and the correlations at the fourth quarter in each year is used in

8) Australia, Austria, Belgium, Canada, Czech, Denmark, Estonia, Finland, France, Germany, Japan, Slovakia, Spain, Sweden, Switzerland, the UK, the US, Greece, Hungary, Poland, Italy, Latvia, and Portugal

actual regression with annual data.

First, we examine the relationship between the Equity Home Bias and the correlations among the examined variables (between home goods price and equity returns, between wage and equity returns). <Figure 3> shows the plot of EHB on these correlations in turn over the 2006-2015 period. On the left panel of <figure 3>, the Equity Home Bias is positively related with the correlation between home goods price and equity returns. As a result, it can be intuitively interpreted that the larger the correlation between home goods price and equity returns, the greater the optimal share of home equity. This result is consistent with that derived from the two-good labor production model. On the right panel of <figure 3>, as the correlation between wage and equity returns increases, the Equity Home Bias increases. This is in conflict with the theoretical prediction in the one-good model that the smaller the covariance of wage and equity returns, the greater the optimal share of home equity.

<Figure 3> Relationship between Equity Home Bias and Correlation by Country




Notes 1). EHB denotes the Equity Home Bias

2). Corr:Home goods price & Equity return denotes the correlation between goods price and equity returns

3). Corr:Wage & Equity return denotes the correlation between wage and equity returns

Source : Authors' calculations

## 2. Results of Empirical Test

In this section, we formally test the effect of the correlation terms (between home goods price and equity returns, between wage and equity returns) on the Equity Home Bias by using balanced panel regression. We assume that the correlation terms affect the Equity Home Bias with some time lags, as in the following equations (40) and (41). In these regressions, $EHB_{i,t}$ denotes the Equity Home Bias of country $i$ in period $t$. $CorrPR_{i,t}$ and $CorrWR_{i,t}$ represent the correlation between home goods price and equity returns, and between wage and equity returns, respectively.

$$EHB_{i,t+k} = \beta_0 + \beta_1 CorrPR_{i,t} + \epsilon_{i,t} \quad \text{, where } k \geq 0 \tag{40}$$

$$EHB_{i,t+k} = \beta_0 + \beta_1 CorrWR_{i,t} + \epsilon_{i,t} \quad \text{, where } k \geq 0 \tag{41}$$

<Table 1> shows the fixed-effect panel regression results of equations (40) and (41). We can confirm that the correlation between home goods price and equity returns has a significant and positive effects on the Equity Home Bias for the first three years, which is consistent with the theoretical predictions from the two-good model.

On the other hand, empirical test results show that the correlation between wages and equity returns has a significant and positive effect on the Equity Home Bias in some cases. This result is not consistent with the previous theoretical prediction: the one-good model shows that the smaller the covariance of wages and equity returns, the greater the optimal holding of home equity. These results suggest that hedging motive for inflation risk is a better explanation for the Equity Home Bias, not the motive to hedge labor income risk.

<Table 1> Panel regression results with different lags (fixed effects)

| lag($k$) | Eqs.(40) | | Eqs.(41) | |
|---|---|---|---|---|
| | $CorrPR_{i,t}$ | $Constant$ | $CorrWR_{i,t}$ | $Constant$ |
| 0 | 0.07 ** (0.03) | 0.60 *** (0.01) | 0.06 ** (0.03) | 0.59 *** (0.01) |
| 1 | 0.05 * (0.03) | 0.58 *** (0.01) | 0.04 * (0.02) | 0.57 *** (0.01) |
| 2 | 0.09 *** (0.03) | 0.57 *** (0.01) | 0.05 ** (0.02) | 0.56 *** (0.01) |
| 3 | 0.05 * (0.03) | 0.56 *** (0.00) | 0.03 (0.02) | 0.55 *** (0.00) |
| 4 | 0.04 (0.03) | 0.55 *** (0.00) | 0.04 (0.02) | 0.54 *** (0.00) |
| 5 | 0.02 (0.03) | 0.54 *** (0.00) | -0.01 (0.02) | 0.54 *** (0.01) |

Note : ***, **, * denote significance at the 1%, 5%, 10% level, respectively.

## Ⅶ. Conclusion

So far, we have examined theoretical implications for optimal portfolio holdings in labor production economy model with one and two goods by using a second-order approximation method. Contribution of this paper, compared to previous studies, is that we adopt non-separable utility function and explicitly derive the covariance terms between home goods price and equity returns, and between wage and equity returns, into the model.

Our results show that the portfolio equilibrium depends on macro parameters. In the one-good labor production model, we show that households hedge labor income risk by holding home equity, so that the optimal share of home equity is large when the covariance between labor income and equity return is low. However, the Equity Home Bias cannot be endogenously generated by this model. Rather, the Foreign Equity Bias is an optimal outcome, as Baxter and Jermann (1997) have emphasized. On the other hand, in the two-good labor production model, the Equity Home Bias is the optimal portfolio equilibrium, conditional on the covariance between goods prices and the equity returns being high and on households having risky preferences. This is because households hedge inflation risk by holding a home equity. Our empirical tests support these results, as they confirm that the correlation between home goods and equity yields positive effects on the Equity Home Bias. Thus, under certain circumstances, the Equity Home Bias can be an optimal portfolio equilibrium, not a puzzle

Recently, as financial markets have become larger and more complex, increased efforts have been made to incorporate financial markets into standard macroeconomic models. However, it is difficult to derive a portfolio equilibrium by way of the standard linearization method in a DSGE model. Ever since Devereux and Sutherland (2011) and Till and Van Wincoop (2010) derived a portfolio equilibrium for the multiple asset model by using a second-order approximation method, many studies have continued to use this method to derive a portfolio equilibrium; although, they have assumed either an endowment economy or a separable utility. Unlike previous studies, this paper extended the model to incorporate a labor production economy and non-separable utility. Additionally, the covariance between equity returns and macro variables (home goods price and labor income) have been explicitly derived in our model-one of the technical contributions of this paper to the macro economy model. We believe that this paper is relevant to the efforts being made to develop and enhance the combination of macroeconomic variables and financial variables in the DSGE model. Nonetheless, this paper still has some limitations, such as not including a government agent, capital accumulation, etc. In order to provide a more precise study, it will be necessary to extend the model by introducing more variables and agents in the model.

**References**


Adler, M. and Dumas, B., (1983). International Portfolio Choice and Corporation Finance: A Synthesis. Journal of Finance, 38(3), pages 925-84.

Ahearne, A. G., Griever, W.L. and Warnock, F.E., (2004). Information costs and home bias: an analysis of US holdings of foreign equities, Journal of International Economics, 62(2), 313-336

Amdur, D., (2010), International cross-holdings of bonds in a two-good DSGE model, Economics Letters, 108(2), 163-166.

Baxter, M. and Urban J.Jermann, (1997), The International Diversification Puzzle Is Worse Than You Think, American Economic Review, 87(1), 170-80.

Bayoumi, T., (1999). Estimating Trade Equations from Aggregate Bilateral Data, IMF Working Paper, 99/77.

Backus, D.K., Kehoe, P.J., and F.E. Kydland, (1994). International Real Business Cycles, The Journal of Political Economy, 100(4), 745-775.

Coeurdacier, N., (2009). Do Trade Costs in Goods Markets Lead to Home Bias in Equities?, Journal of International Economics, 77, 86-100.

Coeurdacier, N., and Rey, H., (2013). Home Bias in Open Economy Financial Macroeconomics, Journal of Economic Literature, 51(1), 63-115.

Coeurdacier, N., and Gourinchas, P.O., (2016). When Bonds Matter: Home Bias in Goods and Assets, Journal of Monetary Economics, 82, 116-137.

Coeurdacier, N., Kollmann, R., and Martin, P., (2010). International Portfolios, Capital Accumulation and Foreign Assets Dynamics, Journal of International Economics, 80(1), 100-112.

Cooper, I., Kaplanis, E., (1994). Home Bias in Equity Portfolios, Ination Hedging, and International Capital Market Equilibrium, The Review of Financial Studies, 7(1), 45-60.

Devereux, M.B., and Sutherland, A. (2011). Country Portfolios in Open Economy Macro Models, Journal of the European Economic Association, 9(2), 337-369.

Engel, C., and Matsumoto, A., (2009). The International Diversification Puzzle When Goods Prices Are Sticky: It's Really about Exchange-Rate Hedging, Not Equity Portfolios, American Economic Journal: Macroeconomics, 1(2), 155-88.

Fidora, M., Fratzscher, M., and Thimann, C., (2007). Home bias in global bond and equity markets: the role of real exchange rate volatility, Journal of International Money and Finance. 26(4), 222-225.

French, K.R., and Poterba, J.M., (1991). Investor Diversication and International Equity Markets, American Economic Review, 81(2), 222-26.

Heathcote, J., and Perri, F., (2013). The International Diversification Puzzle Is Not As Bad As You Think, Journal of Political Economy, 121(6), 1108-1159.

Ke Pang, (2013). Financial Integration, Nominal Rigidity, and Monetary Policy, International Review of Economics and Finance, 25, 75-90.

Kim, S.H., (1997). International Business Cycles in a Partial-Risk-Sharing Market with Capital Income Taxation, chapter 2 of Ph.D. dissertation, Yale University.

King, R.G., Plosser, C.I. and Rebelo S.T., (1998). Production, Growth and Business cycles: I. The Basic Neoclassical Model, Journal of Monetary Economics, 21(2/3), 195-232.

Mishra, A.V., (2015). Measures of equity home bias puzzle, Journal of Empirical Finance, 34, 293-312.

Obstfeld, M. and Rogoff, K., (2000). The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?, NBER Macroeconomics Annual, 339-390.

Krugman, R.P., (1981). Consumption Preferences, Asset Demands, and Distribution Eects in International Financial Markets, NBER Working Papers 0651, National Bureau of Economic Research.

Kollmann, R., (2006). International Portfolio Equilibrium and the Current Account, unpublished manuscript, University of Paris XII.

Solnik, B., (1974). An equilibrium model of the international capital market, Journal of Economic Theory, 4, 500-524.

Stulz, R.M., (1981). A Model of International Asset Pricing, Journal of Financial Economics, 9(4), 383-406.

Tille, C., and Van Wincoop, E., (2010). International Capital Flows, Journal of International Economics, 80(2), 157-175.

Van Wincoop, E., and Warnock, F., (2006). Is Home Bias in Assets Related to Home Bias in Goods?, NBER Working Papers 12728, National Bureau of Economic Research.